Holography and (Lorentzian) black holes

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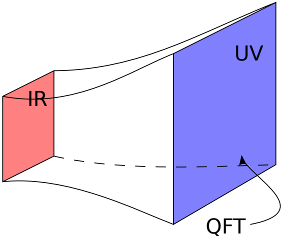
Outline

- Holography, Black holes as thermal states
- Some questions about Lorentzian black holes
- Static solution
- Time-dependent perturbations
- Fluid-gravity correspondence
- Black hole formation and thermalization
- Black holes on boundary

Holography

Maldacena

A class of QFTs are dual to gravity in at least one more dimension.



- For CFT_d, dual is $\mathrm{AdS}_{d+1} \times X$. $SO(d,2)$ isometries of AdS.
- Large class of well-understood examples - large N gauge theories
- Radial direction geometrizes RG flow: Dilatation $D : x^\mu \to \lambda x^\mu, r \to \lambda^{-1} r$.
- $\langle e^{\int \phi_0 \mathcal{O}} \rangle = Z_{string}[\phi \to \phi_0] \approx e^{-S[\phi_0]}$. Witten Gubser Klebanov Polyakov
  - Bulk fields $\leftrightarrow$ gauge-invariant operators.
  - Boundary conditions $\leftrightarrow$ sources.
  - Choice of solutions $\leftrightarrow$ choice of state.

Black holes as thermal states

Witten

▷ Think about finite temperature in Euclidean space: field theory partition function $Z = \mathrm{tr}(e^{-\beta H})$ on flat space.
Dominant bulk saddle-point is the Schwarzschild-AdS black hole

$$ds^2 = \frac{r^2}{\ell^2}[f(r)dt^2 + d\vec{x}^2] + \frac{\ell^2 dr^2}{r^2 f(r)}.$$

Horizon at r_+ where $f(r_+) = 0$, $S^1 \to 0$. "IR end" to geometry
Corresponds to a deconfined phase in CFT: $S \sim \mathcal{O}(N^2)$

▷ Can have non-trivial phase structure if you introduce a scale

Hawking Page

Recent example: hairy black holes.

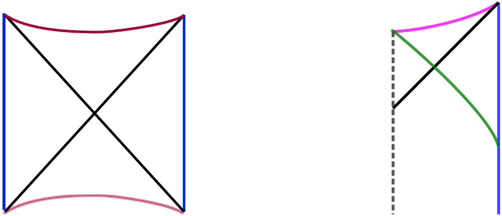
Gubser

Hartnoll Herzog Horowitz

Charged scalars can condense in bulk at non-zero chemical potential.
⇒ Superfluid/superconducting transition where global $U(1)$ in boundary theory spontaneously broken.

Lorentzian black holes

Qualitatively different picture:



New questions:

- Does the holographic description extend inside the horizon?
- How is black hole formation described?
  - Might be easier to understand the region inside the horizon in this case.
- How is an observer crossing the horizon described? What do they see?

General arguments

Classical evolution:

- Event horizon is not locally special; would expect a complete dynamical description to include the region inside the horizon.
- Causally, observations outside the horizon should be insensitive to the interior — challenge for holography.

Boundary observables: Marolf

- There is a set of "boundary observables" in spacetime which are identified with CFT observables.
- Gravitational Hamiltonian is a boundary term, so time evolution preserves the set of boundary observables.
- $\Rightarrow$ Information about anything created by acting with boundary operators is available in these observables at all times.

Eternal black hole

Maldacena

Kraus
Ooguri
Shenker

Fidkowski
Hubeny
Kleban
Shenker

Elegant picture from analytic continuation

- Analytic continuation of Euclidean field theory $\to CFT_1 \otimes CFT_2$, in an entangled state.

Schwinger

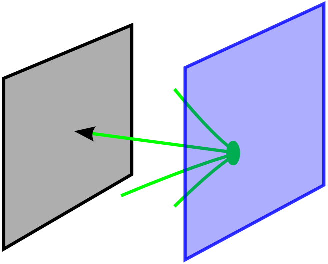
$$|\Psi\rangle = \Sigma_n e^{-\beta E_n/2} |E_n\rangle_1 \otimes |E_n\rangle_2$$

- Observations on a single boundary thermal, related to spacetime outside horizon.
- Correlation functions $\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \rangle$ approximated by geodesics which cross horizon.
- Integration over the bulk in higher point functions can be taken either outside horizon or over whole spacetime.

Relies on analyticity of correlation functions.

Time-dependent perturbations

Perturbations decay: corresponds to thermalization in field theory.



- Initial decay characterised by quasi-normal modes, $\tau_{therm} \sim T^{-1}$.
- Perturbations spread from UV to IR, interact with IR thermal modes. Holographic description complicated for scales $> T^{-1}$?
- Membrane paradigm: dynamics outside horizon can be described as interaction with a fluid membrane at horizon.

Hydrodynamics

Integrating out modes with $\lambda < T^{-1}$ gives a universal low-energy theory: Fluid dynamics, in local thermal equilibrium.

$$T_{\mu\nu} \sim ((\rho + p) u_\mu u_\nu + p\, \eta_{\mu\nu}) - 2\eta\sigma_{\mu\nu} + \dots$$

- From studying perturbations, Kubo formula relating transport coefficients to microscopic correlators:

$$\eta = - \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{xy,xy}(\omega, 0).$$

- Universal result from Einstein gravity in bulk,

Damour

Policastro
Son
Starinets

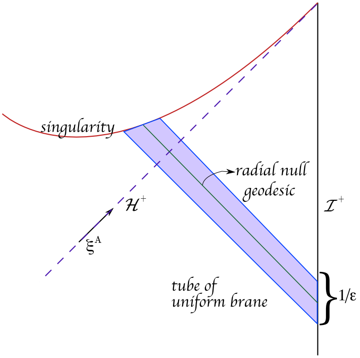
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Fluid-gravity correspondence

Bhattacharyya
Hubeny
Minwalla
Rangamani

Consider arbitrary fluid in local thermal equilibrium.
Holographic description:

- Take uniform black hole solution, make $T(t, \mathbf{x})$, $u^{\mu}(t, \mathbf{x})$ functions.
- Correct order by order in derivative expansion.
- Read off stress tensor from asymptotics of bulk solution.



Fluid-gravity

Membrane at the boundary:

- Focusing on long wavelength physics: thermal scale close to boundary.
- Fluid dynamics maps naturally onto full bulk spacetime.

Bulk dynamics gives two things:

- Conservation equations $\nabla_\mu T^{\mu\nu} = 0$: direct relation of bulk and boundary dynamics.
- Transport coefficients from regularity in interior.

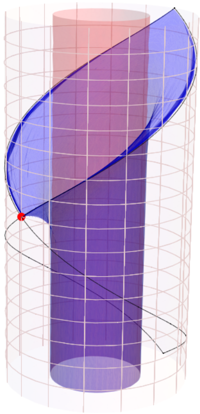
Area of event horizon defines an entropy current in field theory.

Time dependence and entropy

Gubser

Figueras
Hubeny
Rangamani
SFR

Event horizon is teleological; entropy in CFT shouldn't be.



Eg: Conformal soliton – global Schwarzschild-AdS BH in "Poincare" coordinates

- Restrict to a portion of $\mathcal{I}^+$:
  event horizon $>$ black hole horizon.
- In field theory, time-dependent lump of fluid, but conformally related to thermal eq on S^3.
  $\Rightarrow$ Entropy unchanged.
- CFT entropy not given by event horizon area
- Apparent horizon?

Black hole formation

Describes process of thermalization in field theory.

- Add energy by a change in boundary conditions: e.g., perturbation of boundary metric

  Chesler
  Yaffe

  $$ds^2 = -dt^2 + e^{B_0(t)}dx_\perp^2 + e^{-2B_0(t)}dx_{||}^2$$

  Bulk geometry constructed numerically, $\langle T_{\mu\nu}\rangle(t)$ read off from asymptotic behaviour. Anisotropic perturbation: $\tau_{iso} \approx 0.7\,T^{-1}$.

- Isotropic case simpler: model by a Vaidya metric in bulk,

  $$ds^2 = \frac{1}{z^2}[-(1-m(v)z^d)dt^2 - 2dzdv + d\vec{x}^2]$$

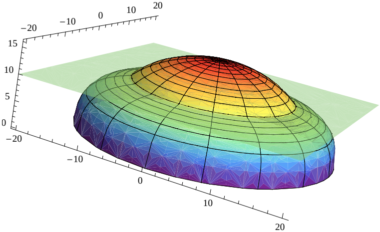
  Local vevs like $\langle T_{\mu\nu}\rangle(v)$ thermalize instantaneously.

  Bhattacharyya
  Minwalla

Black hole formation

Balasubramanian SFR Balasubramanian et al

Study non-local observables: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle$, Wilson loops, entanglement entropy. Approximated by minimal surfaces in bulk.



- For entanglement entropy, $\tau_{crit} \sim \ell/2$ - saturates causality bound.
- Observables involving lower-dim regions thermalize more quickly.
- Some of the minimal surfaces which cross shell do so inside horizon.
- Thermalization from UV: local observables thermalize first, long-distance correlations thermalize later.

Black holes on boundary

Study geometries with a black hole on the boundary of the spacetime – learn about CFT on a curved background, step towards braneworld black holes.

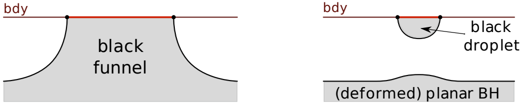
- Eg: Schwarzschild-AdS boundary:

  Gregory
  SFR Zegers

  a bulk solution is $ds^2 = \frac{\ell^2}{\cos^2\theta}[d\theta^2 + ds^2_{S-AdS}]$.
  Surprise: boundary CFT stress tensor is not thermal at leading order.

- For asymp flat boundary, funnel and droplet solutions:

  Hubeny
  Marolf
  Rangamani



(a) (b)

Describe field theory at finite temperature. In droplet, horizons can have different temperatures.

- Laboratory for exploring description of region inside horizon

Discussion

Insights into field theory:

- Calculations of transport coefficients
- "Top-down" thermalization
- Field theory on curved backgrounds.

Discussion

Progress in understanding time-dependent black holes:

- In late-time hydrodynamic regime, simple, direct map of long-wavelength excitations to bulk geometry
- Boundary theory thermalizes causally, information about initial matter available on the boundary at all times — importance of non-local observables.
- Focused on "large", planar black holes; interesting new phenomena in global case - see Gay Horowitz's talk.
- Further work needed to understand infalling observers

Horowitz
Lawrence
Silverstein

Happy Birthday Stephen!