A new twist on the No-Boundary State

Stephen Hawking's 70th Birthday Conference

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Thomas Hertog

Institute for Theoretical Physics
University of Leuven, Belgium

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Stephen's "recent" work...

No-boundary cosmology:

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$\rightarrow$ local observations: *conditional, coarse-grained*

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$\rightarrow$ eternal inflation

2010: local observations in eternal inflation;

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Higher order corrections?

$\rightarrow$ 2011: dual formulation no-boundary state?

Outline

- No-Boundary state
- Its AdS representation $\rightarrow$ dual formulation
- Implications for eternal inflation

No-Boundary Wave Function

$$\Psi[b,h,\chi] = \int_C \delta g \delta\phi \exp(-I[g,\phi])$$

"The amplitude of configurations (b,h,χ) *on a three-surface* Σ *is given by the integral over all regular metrics* g *and matter fields* ϕ *that match* (b,h,χ) *on their only boundary."* [Hartle & Hawking '83]

Motivation: analogy w/ ground state wave function

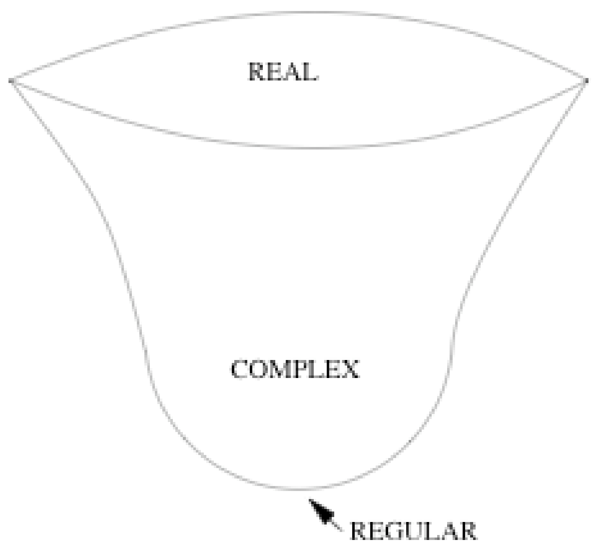
Semiclassical Limit

$$\Psi(b,h,\chi) \approx \sum_e \exp\{[-A_e(b,h,\chi)]/\hbar\}$$

with

$$A_e = I(b,h,\chi) + \hbar I^{(1)}(b,h,\chi) + \cdots$$

Extremal geometries generally complex:



$$I(b,h,\chi) = -I_R(b,h,\chi) + iS(b,h,\chi)$$

WKB Interpretation

$$\Psi(b,h,\chi) \approx \exp\{[-I_R(b,h,\chi) + iS(b,h,\chi)]/\hbar\}$$

The semiclassical wave function predicts Lorentzian, classical evolution in regions of superspace where [Hawking '84, Grischuk & Rozhansky '90]

$$|\nabla_A I_R| \ll |\nabla_A S|$$

The resulting coarse-grained classical histories of the universe are given by the integral curves of S_L:

$$p_A = \nabla_A S$$

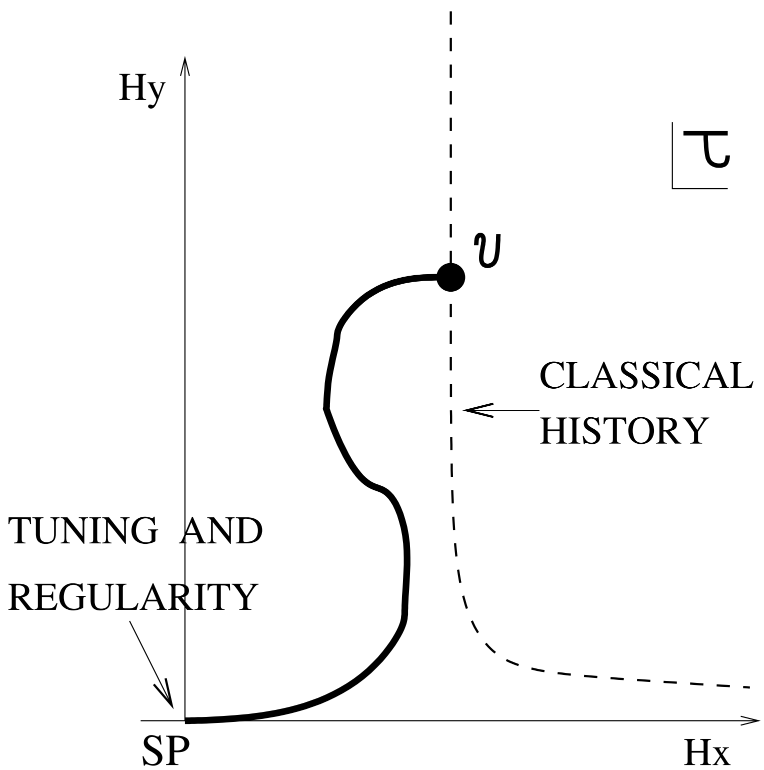
and have conserved probabilities

$$P_{history} \propto \exp[-2I_R/\hbar]$$

$\rightarrow$ no-boundary measure: prior on multiverse.

Complex Saddle points

$$ds^2 = d\tau^2 + g_{ij}(\tau, x)dx^i dx^j, \quad \phi(\tau, x)$$



Regularity at SP: $g_{ij}(0) \to 0, \quad \dot{\phi}(0) \to 0$

At final boundary: $g_{ij}(\upsilon) = b^2 h_{ij}, \quad \phi(\upsilon) = \chi$

Tuning at SP: $\phi(0) = \phi_0 e^{i\gamma}, \ldots$

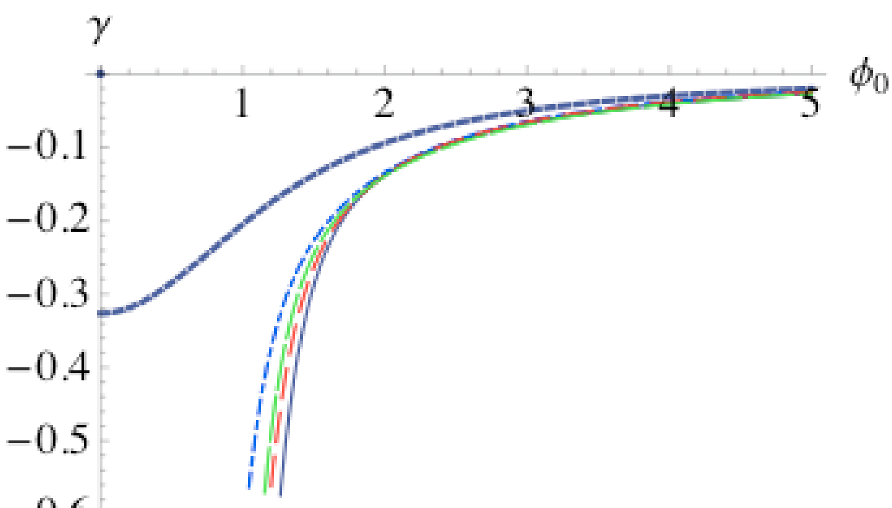
Example

Consider homogeneous/isotropic multiverse:

$$ds^2 = d\tau^2 + a^2(\tau) d\Omega_3, \quad \phi(\tau)$$

$$V(\phi) = \Lambda + \frac{1}{2} m^2 \phi^2$$

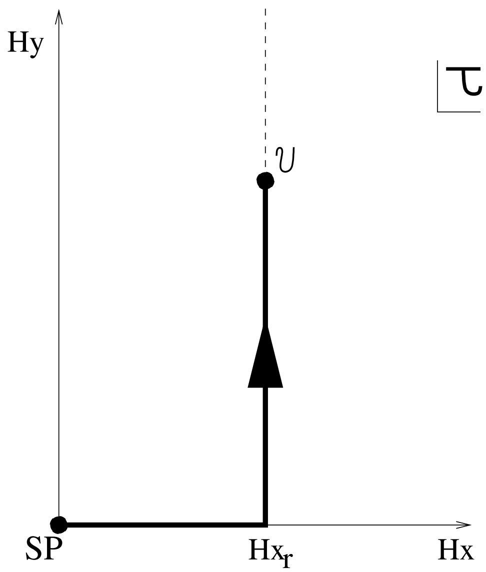
Classical evolution requires tuning of $\phi(0) = \phi_0 e^{i\gamma}$ at the South Pole of the saddle point: ($m_c^2 = 9/4H^2$)



→ $|\nabla_A I_R| \ll |\nabla_A S|$ at large scale factor

→ 1-parameter set of FLRW backgrounds

Saddle point Action



with $Hx_r \to \pi/2$ for $\phi_0 \to 0$

$$I(\upsilon) = \frac{3\pi}{2} \int_{C(0,\upsilon)} d\tau a[a^2(H^2 + 2V(\phi)) - 1]$$

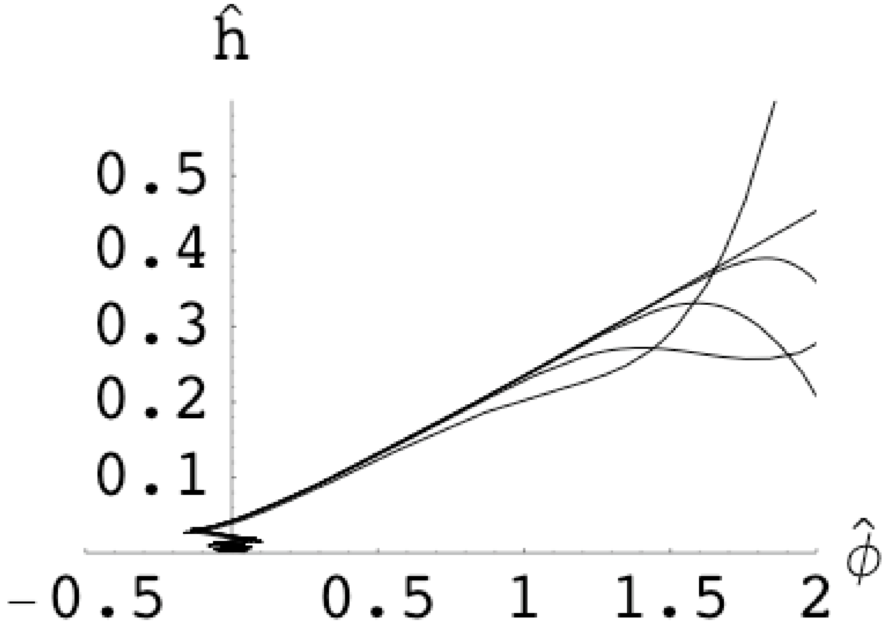
I_R tends to a constant along vertical part

$\to$ probability measure on *classical histories.*

Inflation

Background histories:

$$p_A = \nabla_A S$$

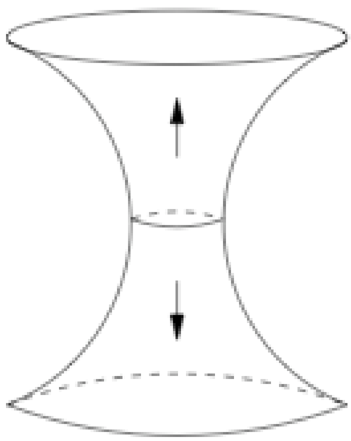


$\rightarrow$ no-boundary state predicts inflation

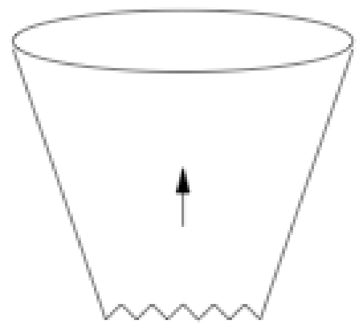
Origin

Background histories:

$$p_A = \nabla_A S$$



large ϕ_0

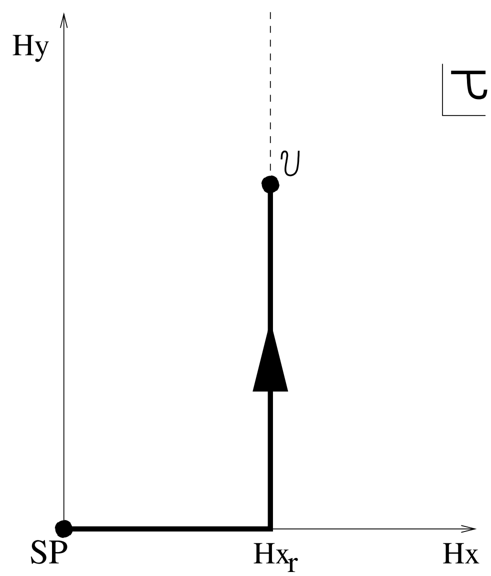


small ϕ_0

Part II: Towards a dual description of the no-boundary state

Complex Saddle points

Lorentzian histories always lie on asymptotically vertical curves in complex τ-plane:



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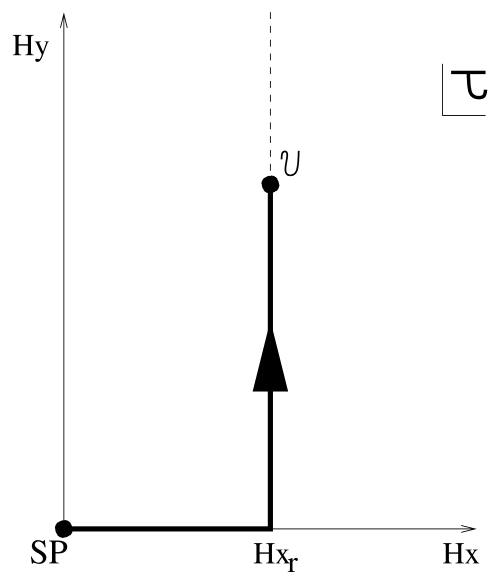

e.g. pure deSitter: $a(\tau) = \frac{1}{H}\sin(H\tau)$

horizontal part: $ds^2 = d\tau^2 + \frac{1}{H^2}\sin^2(H\tau)d\Omega_3^2$

vertical part: $ds^2 = -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

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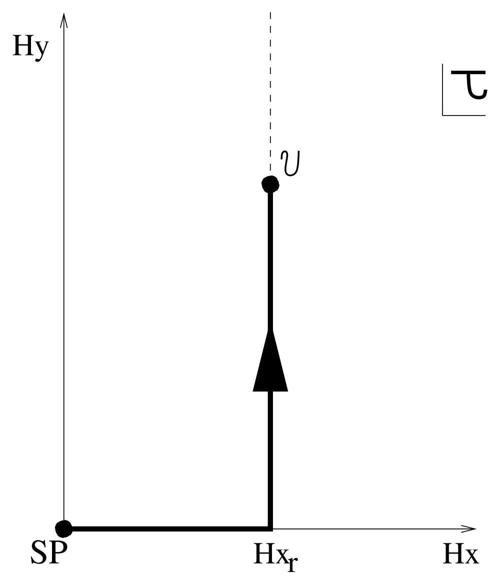
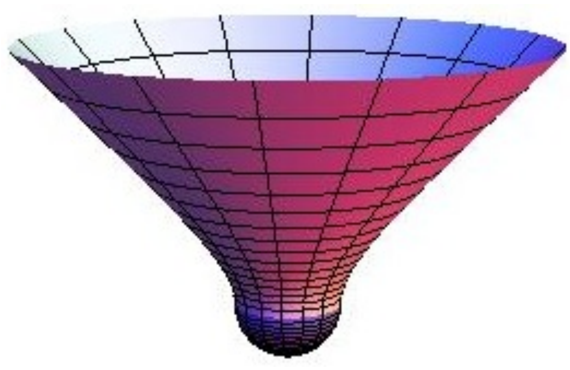
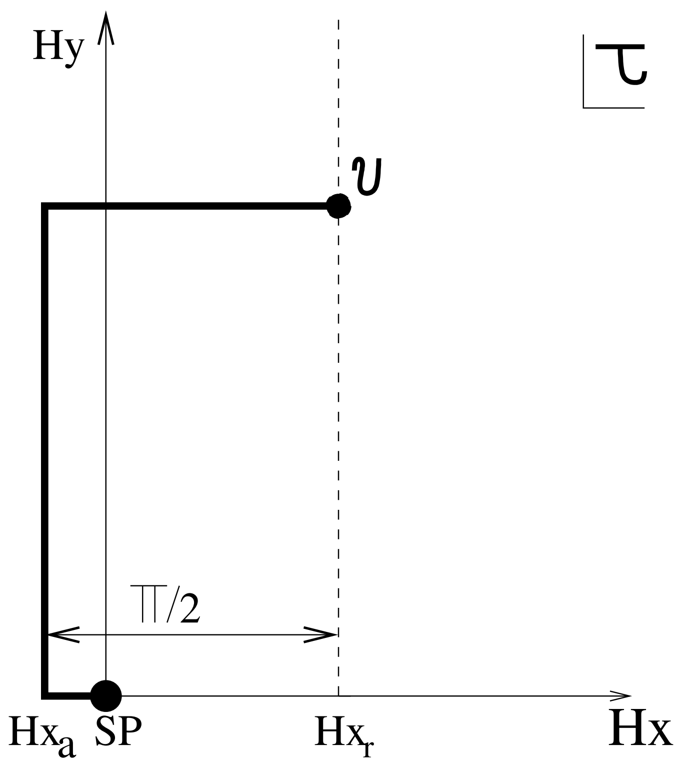




Representations of Saddle points



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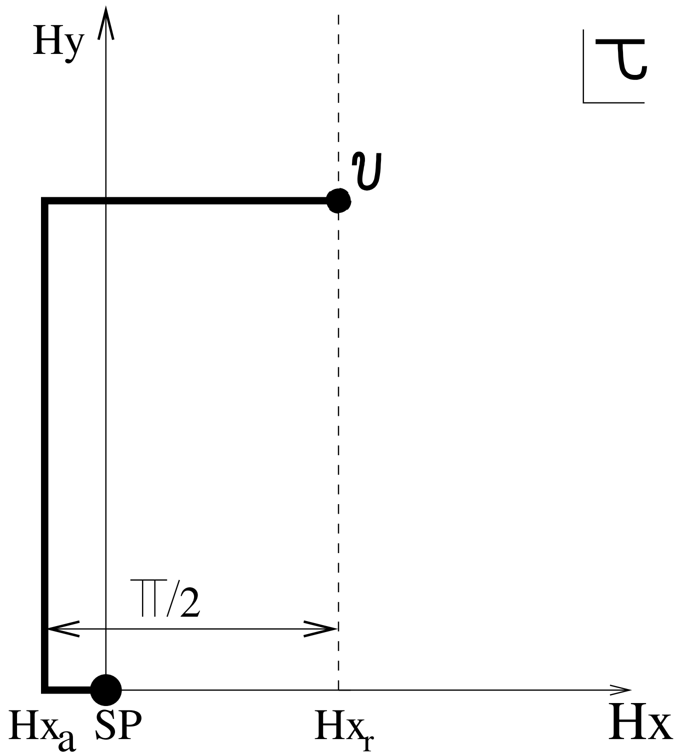

vertical part: Euclidean ADS

$$ds^2 = -dy^2 - \frac{1}{H^2}\sinh^2(Hy)d\Omega_3^2$$

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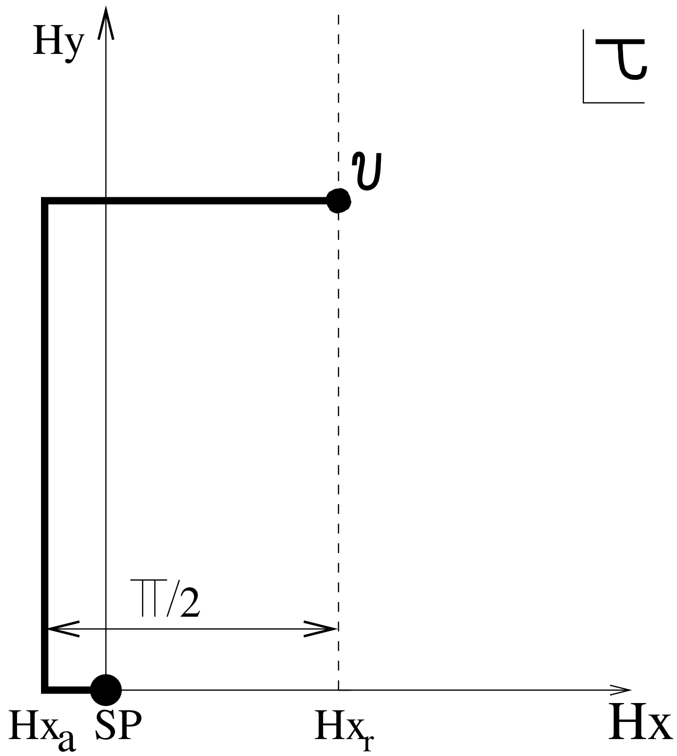

With matter: Euclidean ADS domain wall

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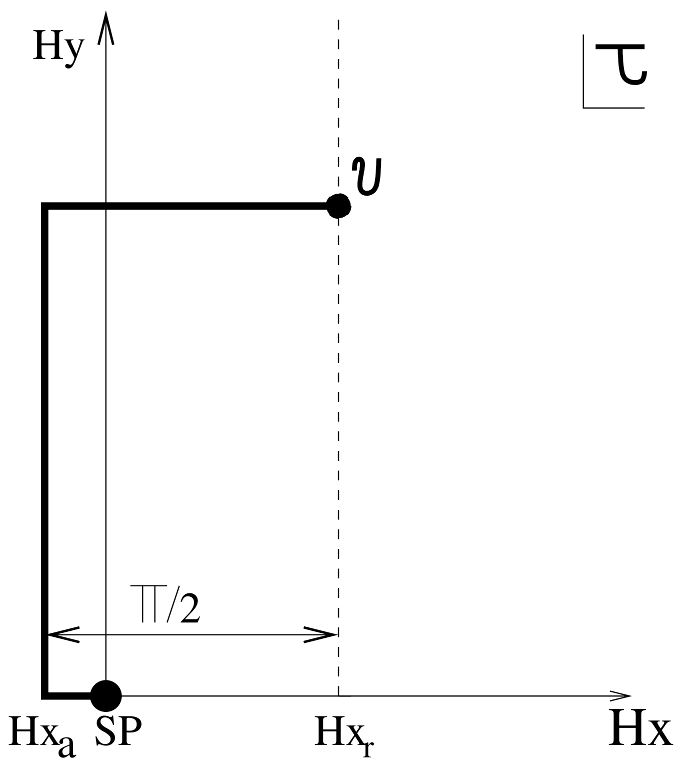

With matter: Euclidean ADS domain wall

- signature complex saddle pt metric varies in τ-plane

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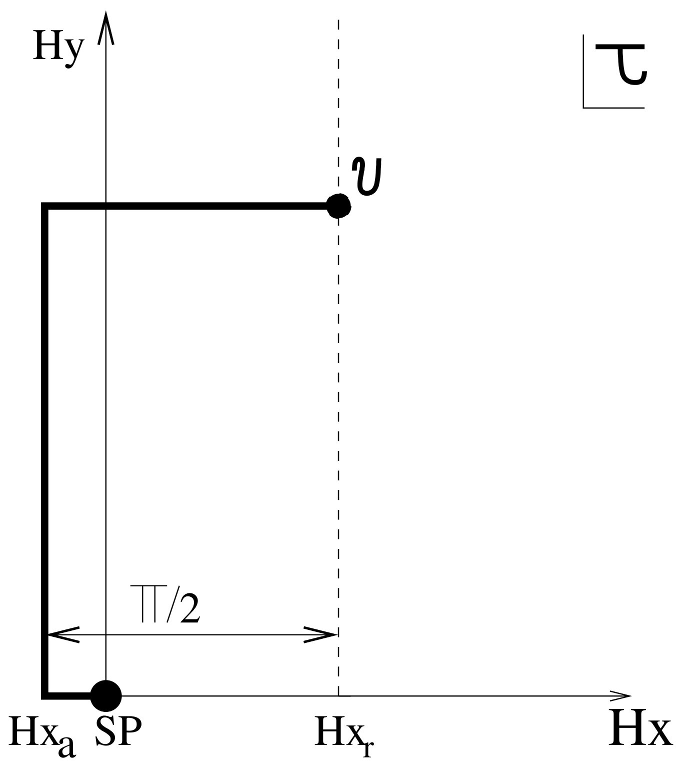

With matter: Euclidean ADS domain wall

- Domain Wall/Cosmology correspondence in SUGRA [Cvetic '96; Skenderis, Townsend, Van Proeyen '07]

→ realized here through universe's quantum state

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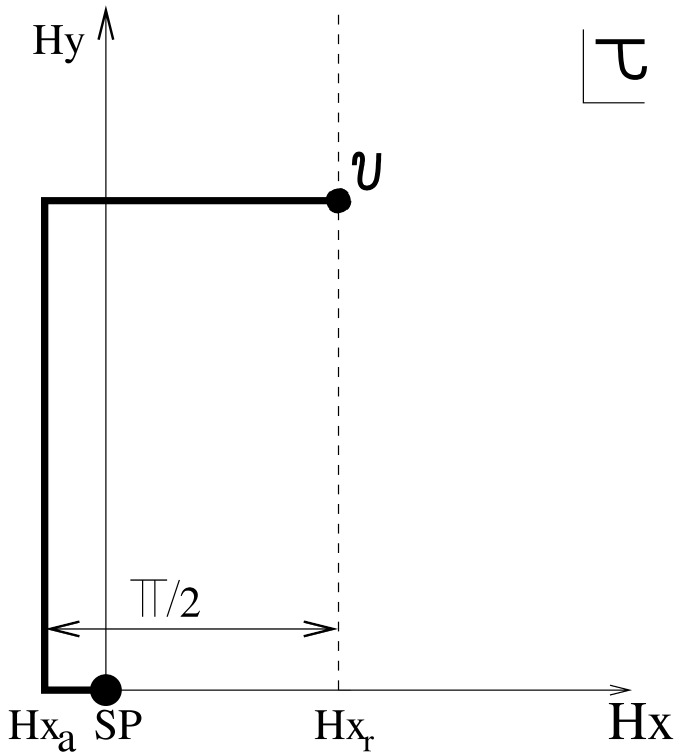

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$$V^{eff}_{AdS} = -\Lambda - V$$

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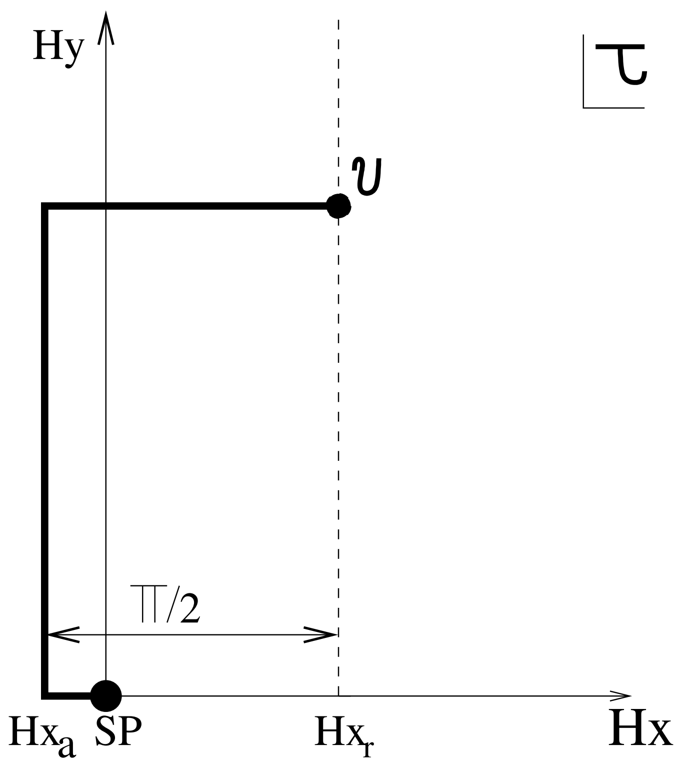

Vice versa: starting from Euclidean AdS SUGRA we predict Lorentzian asymptotically deSitter universes.

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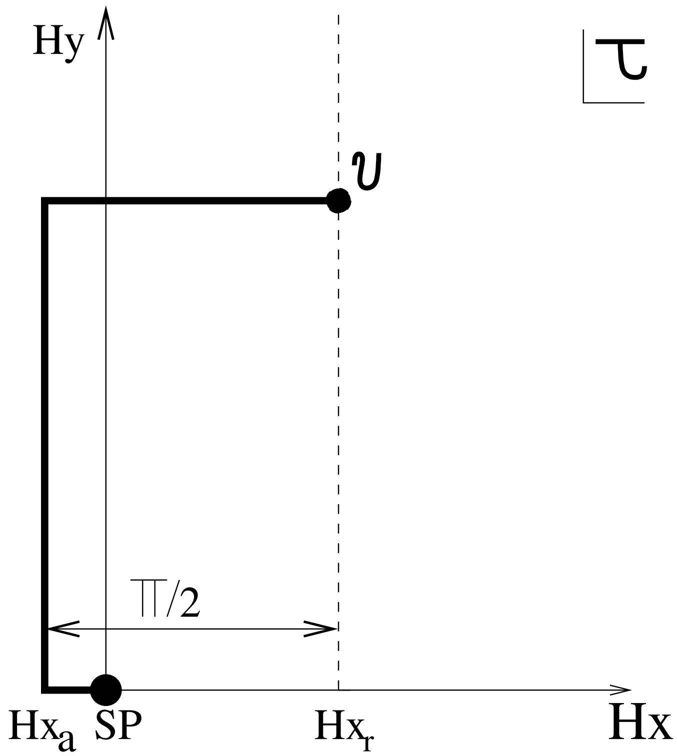

- Does the AdS/dS connection hold independently of the saddle pt approximation?

- What happens along horizontal branch?

Asymptotic Analysis

Asymptotic expansion of metric and fields in small $u \equiv e^{i\tau} = e^{-y+ix}$ [Skenderis,...]:

$$g_{ij}(u,\Omega) = \tfrac{-1}{4u^2}[h_{ij}(\Omega) + h^{(2)}_{ij}(\Omega)u^2 + h^{(-)}_{ij}(\Omega)u^{\lambda_-} + h^{(3)}_{ij}(\Omega)u^3 + \cdots]$$

$$\phi(u,\Omega) = u^{\lambda_-}(\alpha(\Omega) + \alpha_1(\Omega)u + \cdots) + u^{\lambda_+}(\beta(\Omega) + \beta_1(\Omega)u + \cdots)$$

with $\lambda_\pm \equiv \frac{3}{2}[1 \pm \sqrt{1-(2m/3)^2}]$

and (arbitrary) 'boundary values' (h_{ij}, α).

$\rightarrow$ AdS/dS connection

Action integral

- Action integral along horizontal part:

$$I_h = \int_h I[g,\phi] = +S_{ct}(b,h,\chi) - iS_{ct}(b,h,\chi)$$

and no finite contribution.

$$S_{ct} = a_0 \int \sqrt{h} + a_1 \int \sqrt{h} R^{(3)} + a_2 \int \sqrt{h}\phi^2$$

- Action integral along vertical part:

$$I_v = \int_v I[g,\phi] = -I^R_{AdS}(h,\chi) - S_{ct}(b,h,\chi)$$

where I^R_{AdS} is finite when $a \to \infty$.

$\to$ starting from AdS SUGRA, horizontal branch amounts to adding the usual counterterms.

$$I(b,h,\chi) = -I^R_{AdS}(h,\chi) - iS_{ct}(b,h,\chi)$$

Towards a holographic dual

$$\Psi(b,h,\chi) \approx \exp\{[+I^R_{AdS}(h,\chi) + iS_{ct}(b,h,\chi)]/\hbar\}$$

AdS/CFT [Maldacena,Witten,...]:

$$\exp(-I^R_{ADS}[h,\chi]/\hbar) = Z_{QFT}[h,\tilde{\chi}] = \langle \exp \int d^3x \sqrt{h}\tilde{\chi}\mathcal{O}\rangle$$

$\rightarrow$ 'dS/CFT dual' formulation of NBWF:

$$\Psi(b,h,\chi) \approx \frac{1}{Z_{QFT}[h,\tilde{\chi},\epsilon]} \exp\{[iS_{ct}(b,h,\chi)]/\hbar\}$$

where UV cutoff $\epsilon \sim 1/b$

Remarks

$$\Psi(b,h,\chi) \approx \frac{1}{Z_{QFT}[h,\tilde{\chi},\epsilon]} \exp\{[iS_{ct}(b,h,\chi)]/\hbar\}$$

- Z provides measure on configurations (b, h, χ).
- Scale factor evolution as inverse RG flow [Strominger '02]
- Physical interpretation of counterterms in AdS
- Coarse-graining over UV modes at finite scale factor [see also Vilenkin '11]
- no-boundary condition of regularity implemented

Part III: Eternal Inflation

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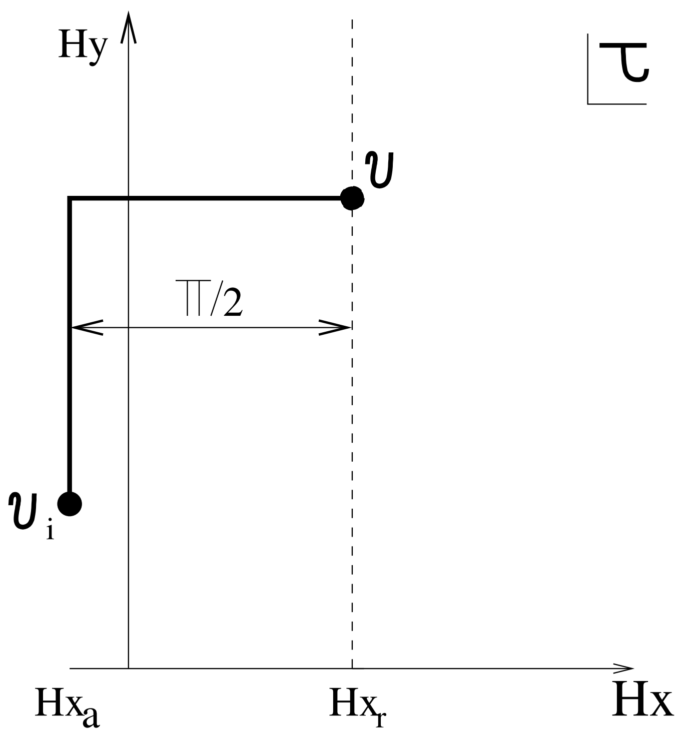
Can the dual formulation of no-boundary state be applied to regime of eternal inflation only?

Euclidean Eternal Inflation

[Hartle, Hawking & TH, in progress]

Replace inner region of eternal inflation by dual CFT on its boundary:



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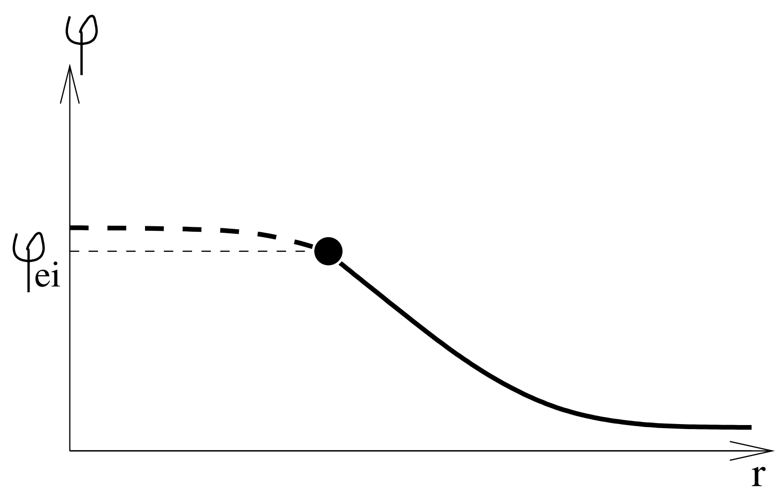

- IR CFT with deformation given by $\phi = \phi_{EI}$. (similar to [Maldacena '10])
- $< \mathcal{O} >$ on inner boundary replaces regularity at SP.

$\rightarrow$ remaining saddle point with inner boundary

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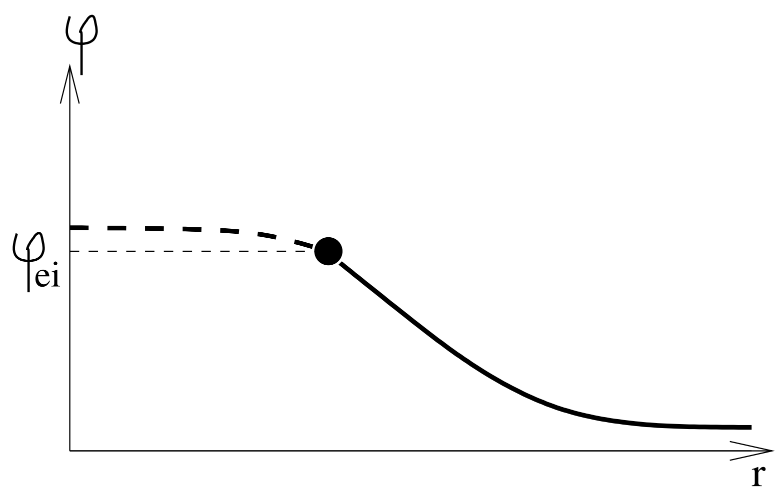

$$|\Psi(b,h,\chi)|^2 \approx \frac{1}{|Z_{QFT}[\phi_{EI},h_e,\epsilon]|^2}\exp\{[+2\tilde{I}^R_{AdS}/\hbar\}$$

Conclusion

- EAdS/dS connection in no-boundary cosmology
- dual description in terms of relevant deformations of the CFTs that occur in AdS/CFT.
- Reinterpretation of eternal inflation

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Happy Birthday Stephen!