The cosmic censorship conjectures in classical general relativity

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Gravity and black holes
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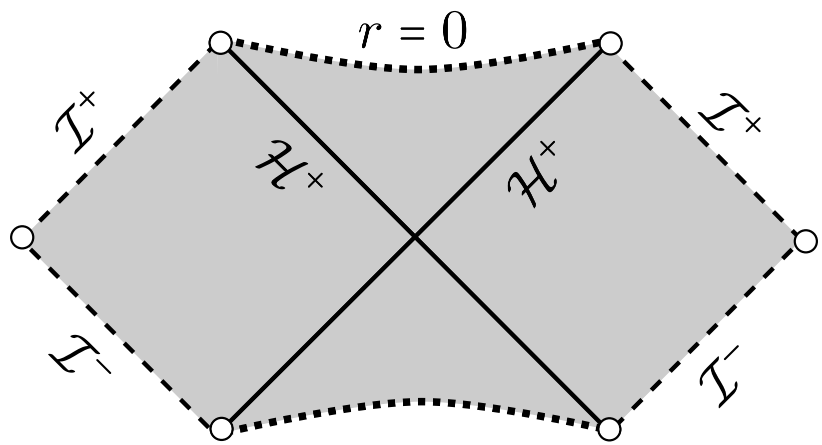
“God abhors a naked singularity”

Outline

1. Background: Schwarzschild, Reissner–Nordström and Kerr
2. The modern formulation of the cosmic censorship conjectures
3. Spherical symmetry
4. Beyond symmetry

1. Background: Schwarzschild, Reissner–Nordström and Kerr

The prototype for a singular spacetime is (maximal analytic) **Schwarzschild** (Lemaitre 1932, Synge 1949, Kruskal 1959)

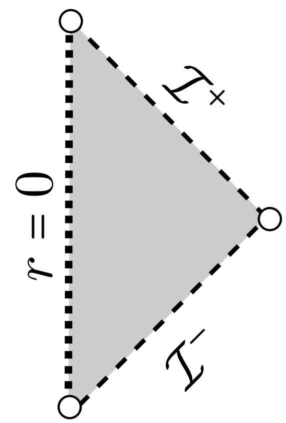


This has a **singularity** $\mathcal{S} = \{r = 0\}$ (Kretschmann scalar blows up), which is **spacelike**
and is cloaked behind an **event horizon** $\mathcal{H}^+$,

$$\mathcal{S} \cap J^-(\mathcal{I}^+) = \varnothing.$$

Singularities need not necessarily be "cloaked" behind horizons!

The quintessential example is **negative-mass Schwarzschild**



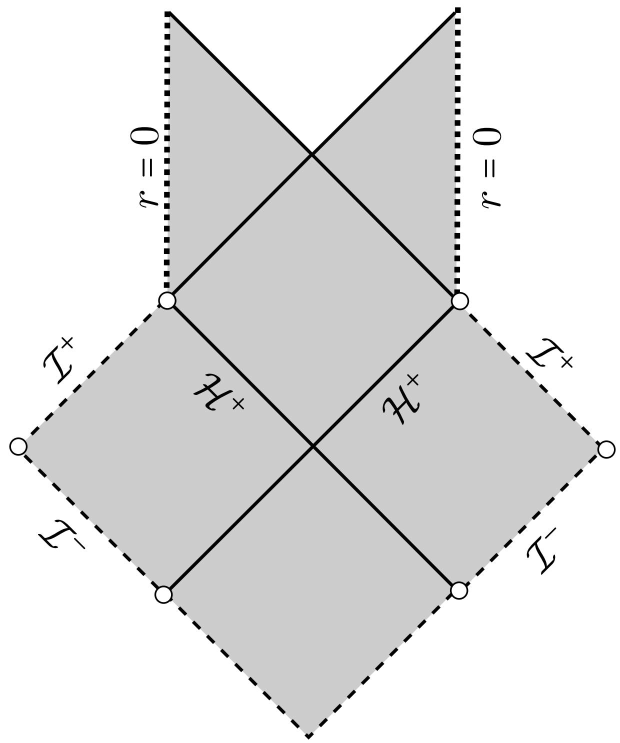
Again $\mathcal{S} = \{r = 0\}$ is a curvature singularity, which is now **timelike**.

Moreover it is "visible to infinity" in the sense that $\mathcal{S} \subset J^-(\mathcal{I}^+)$.

This is the textbook **naked singularity**.

Another example of a **“timelike singularity”**:
deep inside (maximal analytic) Reissner–Nordström and Kerr black holes.

Graves–Brill 1960, Carter 1968



$\mathcal{S} = \{r = 0\}$ timelike means that it can be thought of as a **“locally naked singularity”**

Old-style formulations of cosmic censorship I

"Weak cosmic censorship" (Penrose, c. 1969)
In gravitational collapse,
singularities are always cloaked by horizons, i.e. $\mathcal{S} \cap J^-(\mathcal{I}^+) = \emptyset$.

"Strong cosmic censorship" (Penrose, c. 1972)
In gravitational collapse,
generically,
there are no "locally naked singularities",
i.e. singularities are generically spacelike or null, *not timelike*.

Vague evidence for the latter was the "blue-shift" instability
associated with the inner horizon of Reissner–Nordström
(Penrose 1969, Simpson–Penrose 1972)

Old-style formulations of cosmic censorship II

“Very strong cosmic censorship”
In gravitational collapse, singularities are generically spacelike.
(cf. “BKL picture”)

2. The modern formulation of the cosmic censorship conjectures

The primacy of the Cauchy problem

Theorem (Choquet-Bruhat 1952, Choquet-Bruhat–Geroch 1969)**.**
Let $(\Sigma, \bar{g}, K)$ *be a smooth vacuum initial data set.*
There exists a unique smooth spacetime $(\mathcal{M}, g)$ *such that*

1. $\mathrm{Ric}(g) = 0$
2. $(\mathcal{M}, g)$ *is* **globally hyperbolic** *with Cauchy surface* Σ*, with induced first and second fundamental form* $\bar{g}$*,* K *respectively*
3. *Any other smooth spacetime with properties 1., 2., isometrically embeds into* $\mathcal{M}$*.*

We call $(\mathcal{M}, g)$ the **maximal Cauchy development**.
Similar theorems can be proven for suitable coupled Einstein–matter systems, like Einstein–Maxwell, Einstein–dust and Einstein–scalar field.

Taking the Cauchy problem to heart means that

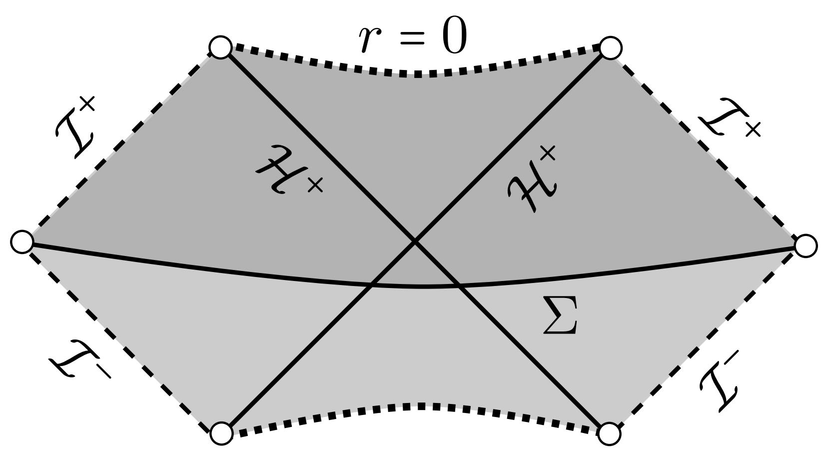
- Assumptions can only be made on *initial data* Σ
- Properties must be described in terms of maximal Cauchy developments $(\mathcal{M}, g)$

In particular, since $(\mathcal{M}, g)$ is **by definition** globally hyperbolic,
it follows that the "finite boundary of $\mathcal{M}$", call it $\mathcal{S}$,
is <u>nowhere timelike</u>.

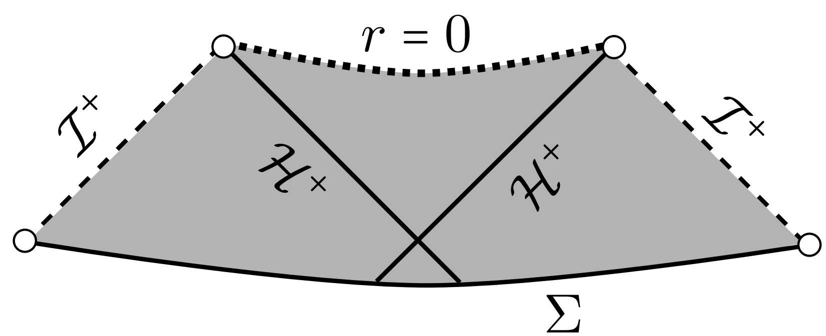
Moreover, again if $\mathcal{I}^+$ denotes the future null infinity of $\mathcal{M}$,
then necessarily $\mathcal{S} \smallsetminus J^-(\mathcal{I}^+) = \varnothing$.

Thus, there is no such thing as a "timelike singularity" and we have to learn to talk about cosmic censorship **without ever saying those words**.

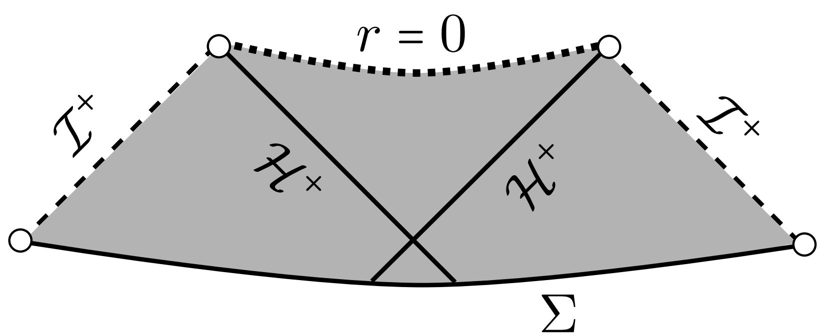
The maximal (future) Cauchy development of Schwarzschild data



Schwarzschild as a maximal (future) Cauchy development

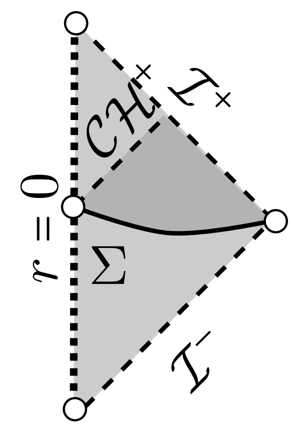


Recap properties of Schwarzschild as seen from perspective of Cauchy evolution

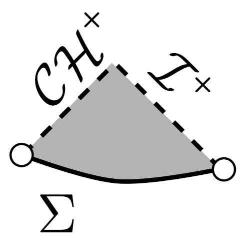


- Σ complete, asymptotically flat but $(\mathcal{M}, g)$ future geodesically **incomplete**
- geodesic incompleteness of $(\mathcal{M}, g)$ **stable** to perturbation of data on Σ by Penrose's 1965 "singularity" theorem
- $\mathcal{I}^+$ is complete (cf. Geroch–Horowitz 1978)
- future inextendible as a C^2 Lorentzian manifold, in fact (Sbierski 2015) as a continuous Lorentzian manifold ("observers torn apart")
- Can think of singularity as a spacelike boundary $\mathcal{S}$.

The maximal future Cauchy development of negative-mass Schwarzschild data

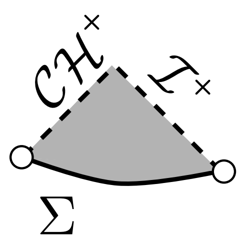


Negative-mass Schwarzschild as a maximal future Cauchy development



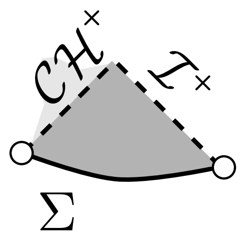
How can we identify this as possessing "a naked singularity"?

A new way to think of what it means to "possess a naked singularity":



$\mathcal{I}^{+}$ is itself **incomplete** (cf. Geroch–Horowitz 1978)

What about the "locally naked" singularity property?



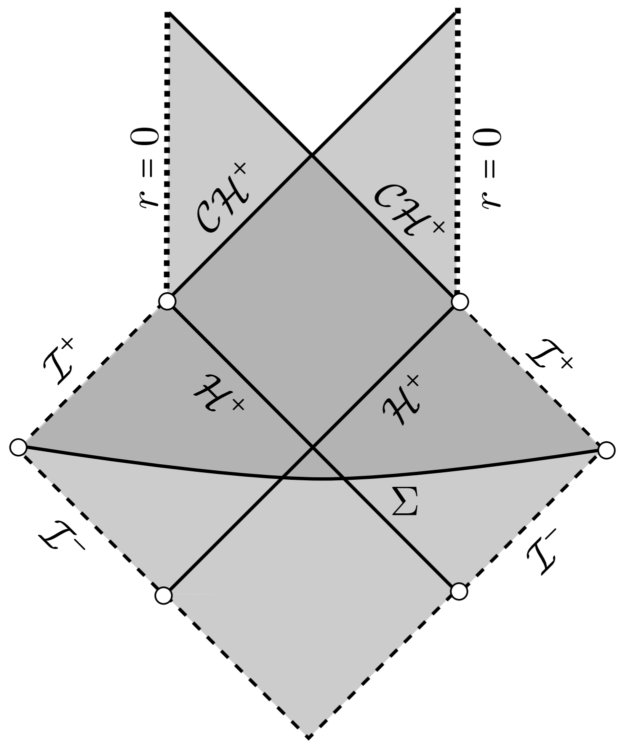
$\mathcal{CH}^+$ is a **Cauchy horizon**.

The maximal Cauchy development $(\mathcal{M}, g)$ is extendible (smoothly!) to a larger spacetime $(\widetilde{\mathcal{M}}, \tilde{g})$ across a null hypersurface $\mathcal{CH}^+$.

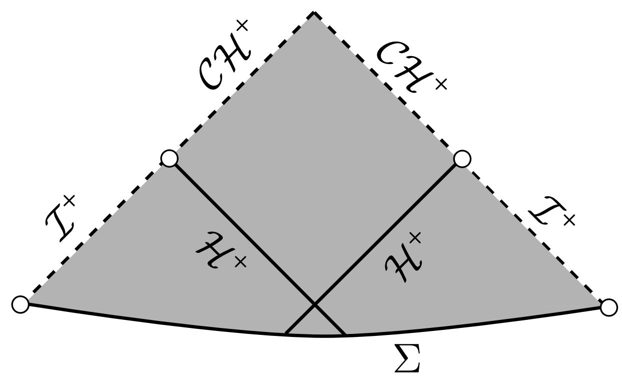
These extensions $\widetilde{\mathcal{M}}$ are severely non-unique.
Failure of determinism!

But initial **data** Σ is itself incomplete,
so we are "allowed" to rule this spacetime inadmissible.

The maximal future Cauchy development of Reissner–Nordström data

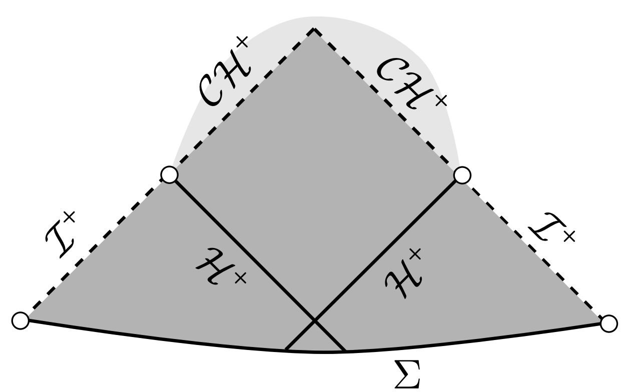


Reissner–Nordström/Kerr as a maximal future Cauchy development

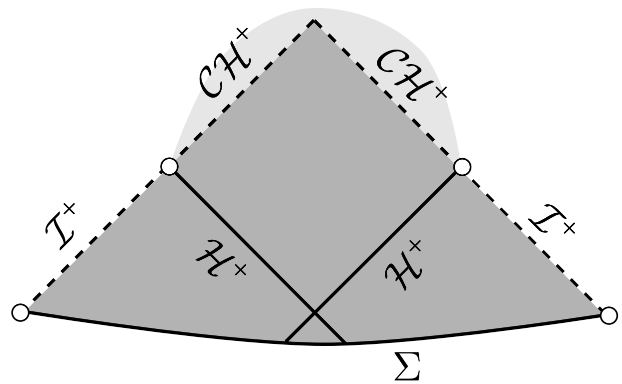


(Aside: interpretation of the Kerr Penrose diagram Pretorius–Israel 1998)

Reissner–Nordström and Kerr are future extendible,
in fact as smooth solutions,
but these extensions are non-globally hyperbolic and thus severely non-unique.



Reissner–Nordström and Kerr are not just extendible,
but can be extended such that **all** incomplete geodesics
pass safely into the extension



Thus, these spacetimes, thought of as maximal Cauchy developments,
demonstrate that PENROSE's original 1965 "singularity" theorem
may have <u>nothing</u> to do with singularities.

The occurrence of singularities in cosmology.

III. Causality and singularities†

By S. W. Hawking

Department of Applied Mathematics and Theoretical Physics, University of Cambridge

(*Communicated by H. Bondi, F.R.S.—Received* 26 *September* 1966—*Revised* 24 *February* 1967)

The physical significance of a partial Cauchy surface is that data on it determine events in some region $\mathscr{D}(\mathscr{H})$ of $\mathscr{M}$ which we shall call the Cauchy development of $\mathscr{H}$. We may define $\mathscr{D}(\mathscr{H})$ as $\mathscr{D}^+(\mathscr{H}) \cup \mathscr{D}^-(\mathscr{H})$ where $\mathscr{D}^+(\mathscr{H})$ is called the future Cauchy development of $\mathscr{H}$ and is defined as the set of all points q such that every past directed non-spacelike line through q intersects $\mathscr{H}$. Clearly points sufficiently near $\mathscr{H}$ will be in $\mathscr{D}(\mathscr{H})$. We shall call $\dot{\mathscr{D}}^+(\mathscr{H}) - \mathscr{H}$, the future Cauchy horizon, $\mathscr{L}^+(\mathscr{H})$. The set $\mathscr{N} = \langle \mathscr{H}) - \mathscr{D}^+$ satisfies the conditions of lemmas 2 and 3. Thus $\mathscr{N}$ is a future null horizon. It consists of two disconnected components, $\langle\dot{\mathscr{H}})$ and $\mathscr{L}^+(\mathscr{H})$. Thus $\mathscr{L}^+(\mathscr{H})$ is a future null horizon.

Modern formulation of weak cosmic censorship (first attempt)

Conjecture. *For complete asymptotically flat vacuum initial data, the maximal Cauchy development has a complete null infinity* $\mathcal{I}^+$.

One can think of this as a statement of **global existence**, still compatible with the singularity theorems of Penrose and Hawking.

Penrose 1969, Geroch–Horowitz 1978, Christodoulou 1999

Strong cosmic censorship

Conjecture. *For generic, complete asymptotically flat vacuum initial data, the maximal Cauchy development is future inextendible as a suitably regular Lorentzian manifold.*

One should think of this conjecture as a statement of **global uniqueness**, or *determinism*.

Penrose 1972, Geroch–Horowitz 1979, Wald 1984, Christodoulou 1999

“Very” strong cosmic censorship

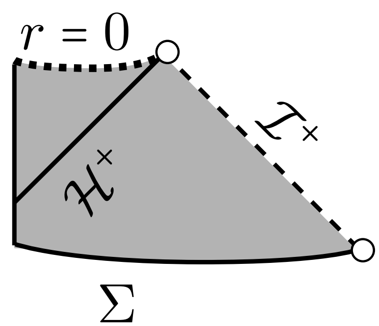
Conjecture. *For generic asymptotically flat vacuum initial data, the maximal Cauchy development is future inextendible as a Lorentzian manifold with metric assumed merely continuous.*

Moreover, the finite boundary of spacetime is spacelike.

This formulation is related to the statement that incomplete classical observers not only encounter infinite *curvature*,
but are in fact **torn apart** by infinite **tidal deformations**.

3. Spherical symmetry

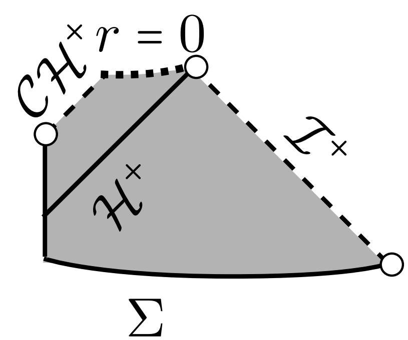
Gravitational collapse of a homogeneous dust ball



Oppenheimer–Snyder 1939

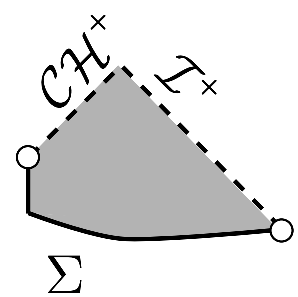
Consistent with all formulations of cosmic censorship.

Theorem (Christodoulou 1983)**.**
For the spherically symmetric Einstein–dust system,
generic, arbitrarily small perturbations of homogeneous data on Σ
give rise to a maximal Cauchy devlopment $(\mathcal{M}, g)$
smoothly extendible *across a Cauchy horizon* $\mathcal{CH}^+$*.*



Thus **strong cosmic censorship fails** for the Einstein–dust system.

Theorem (Christodoulou 1983).
Again for the spherically symmetric Einstein–dust system, there is an open set in the moduli space of data on Σ *for which the maximal Cauchy devlopment* $(\mathcal{M}, g)$ *is bounded by a Cauchy horizon* $\mathcal{CH}^+$ *intersecting an* *<u>in</u>complete* $\mathcal{I}^+$.



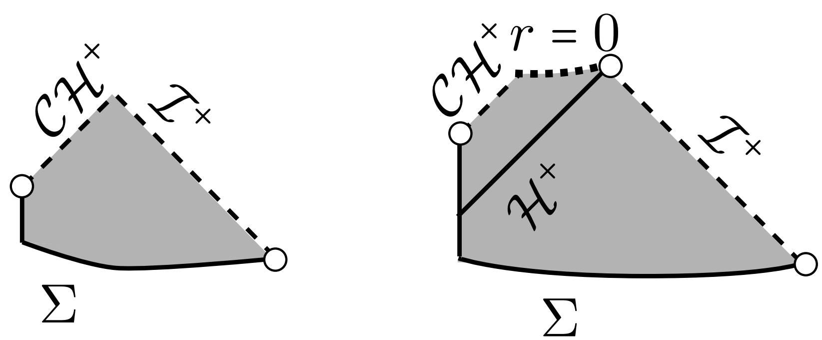
Thus **weak cosmic censorship also fails** for the Einstein–dust system.

Dust is a “bad” model for matter.

In retrospect, it is not suprising that the analogues
of the cosmic censorship conjectures do not hold for the Einstein–dust system.

What about Einstein–scalar field?

Theorem (Christodoulou 1990)**.**
For the spherically symmetric Einstein–scalar field system, there exist regular complete asymptotically flat initial data on Σ *giving rise to a maximal Cauchy devlopment* $(\mathcal{M}, g)$ *with Penrose diagrams depicted*



In the first example $\mathcal{I}^+$ *is incomplete, while in both examples* $(\mathcal{M}, g)$ *is extendible beyond* $\mathcal{CH}^+$*.*

Thus the analogue of weak cosmic censorship as we formulated before is false for Einstein–scalar field under spherical symmetry.

See also subsequent numerics by Choptuik.

Weak cosmic censorship (second attempt)

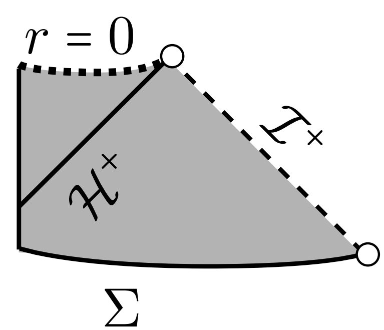
This can be fixed easily enough!

Conjecture. *For <u>generic</u>, complete asymptotically flat vacuum initial data, the maximal Cauchy development has a complete null infinity* $\mathcal{I}^+$.

Theorem (Christodoulou, 1999)**.**

Both *weak and (very) strong cosmic censorship are* **true** *for the Einstein–scalar field system* **under spherical symmetry***.*

For generic spherically symmetric initial data, the maximal future Cauchy development has Penrose diagram

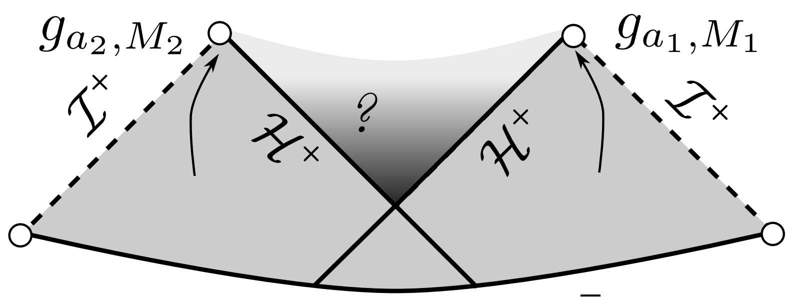


with complete $\mathcal{I}^+$ *and a spacelike singularity* $\mathcal{S} = \{r = 0\}$*.*

4. Beyond symmetry

A partial result on **weak cosmic censorship** beyond symmetry would be to prove the full nonlinear stability of Kerr

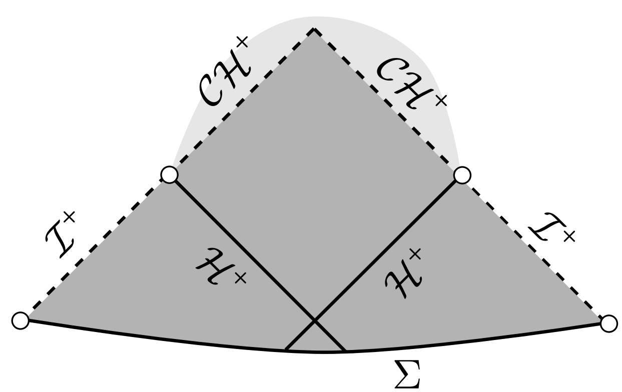
Conjecture. *The maximal Cauchy development* $(\mathcal{M}, g)$ *of small peturbations of two-ended Kerr initial data*



settles down to two nearby Kerr solutions in the exterior with a future complete bifurcate event horizon $\mathcal{H}^+$ *and a complete future null infinity* $\mathcal{I}^+$.

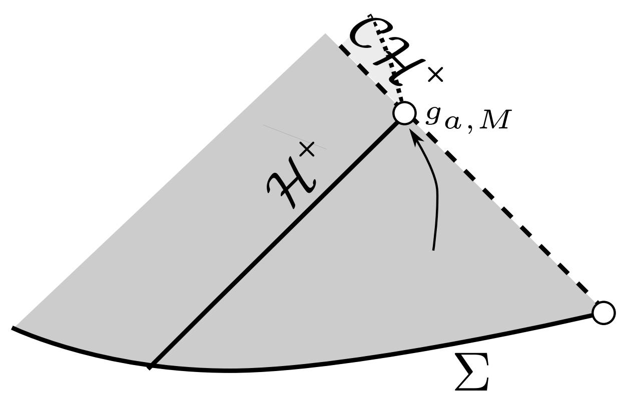
Theorem (M.D.–J. Luk, upcoming 2017)**.**
If the above nonlinear stability of Kerr conjecture is true,
then **very strong cosmic censorship** *is* *false.*

The Penrose diagram of Kerr is globally stable
and spacetime is extendible beyond a bifurcate null Cauchy horizon $\mathcal{CH}^+$
as a Lorentzian manifold with countinuous metric.



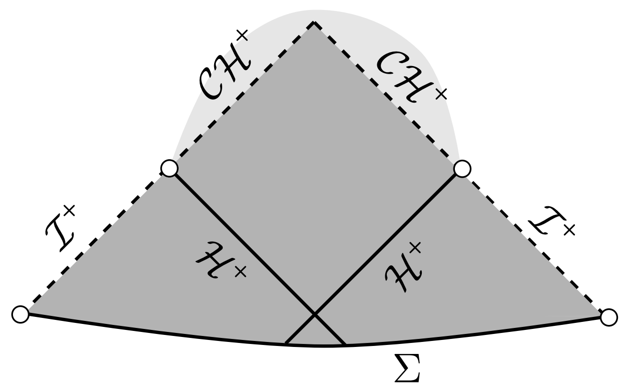
What about the physical one-ended case?

Corollary. *Any spacetime forming in gravitational collapse whose exterior settles down to a sub-extremal Kerr* $0 \neq |a| < M$ *will contain a* **piece** *of Cauchy horizon* $\mathcal{CH}^+$ *across which the metric is continuously extendible.*



Open problem I

Is the null boundary $\mathcal{CH}^+$ generically **singular** *in a* **weaker** *sense?*

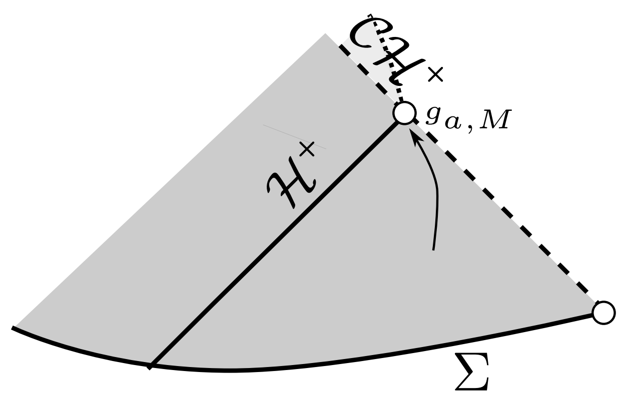


Proven for a spherically symmetric model problem (Poisson–Israel 1990, Ori 1991, M.D. 2001, Luk–Oh 2016)

$\Longrightarrow$ revised "Christodoulou formulation" of SCC may still be true.

Open problem II

In gravitational collapse
from complete initial data Σ *with* **one** *asymptotically flat end,*
is there generically
an additional non-empty spacelike piece of the spacetime boundary, or
can the Cauchy horizon close off the spacetime?



Happy birthday Stephen!