CMB spectroscopy at third order in cosmological perturbations

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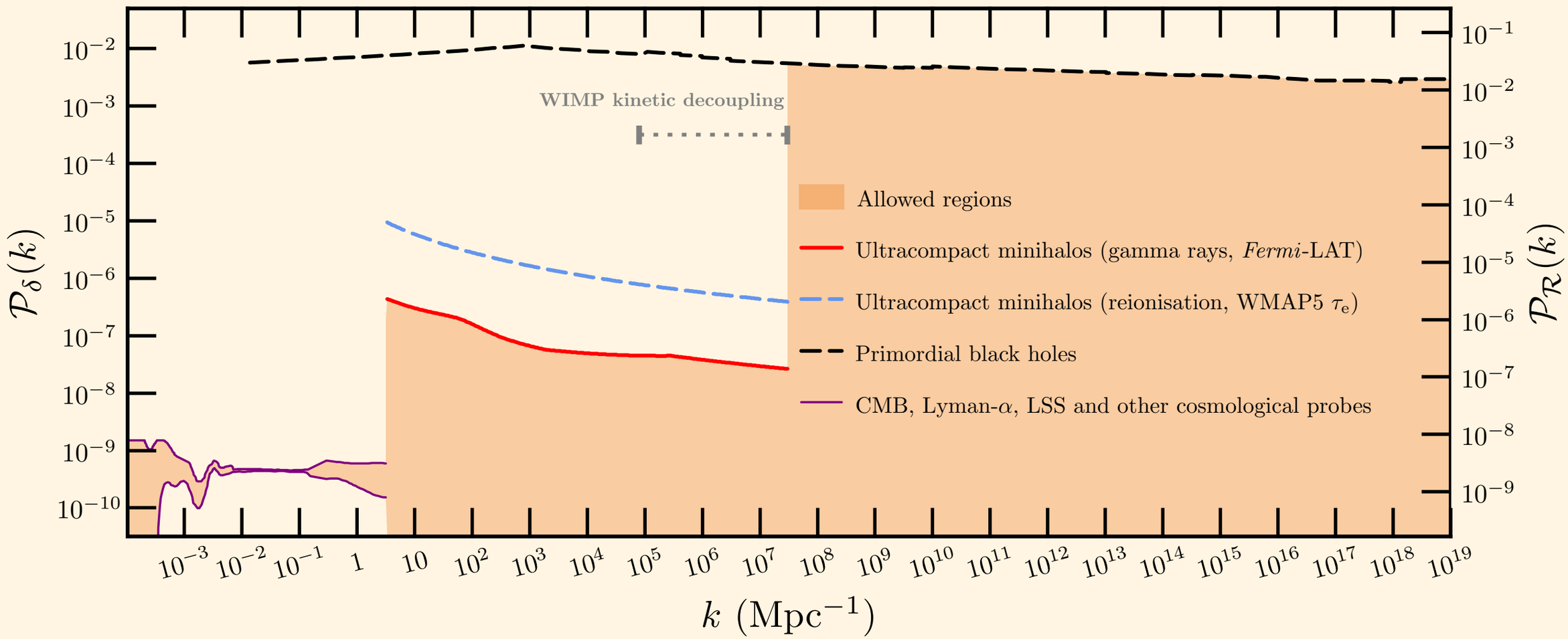
Linear CMB was successful

Density fluctuations are

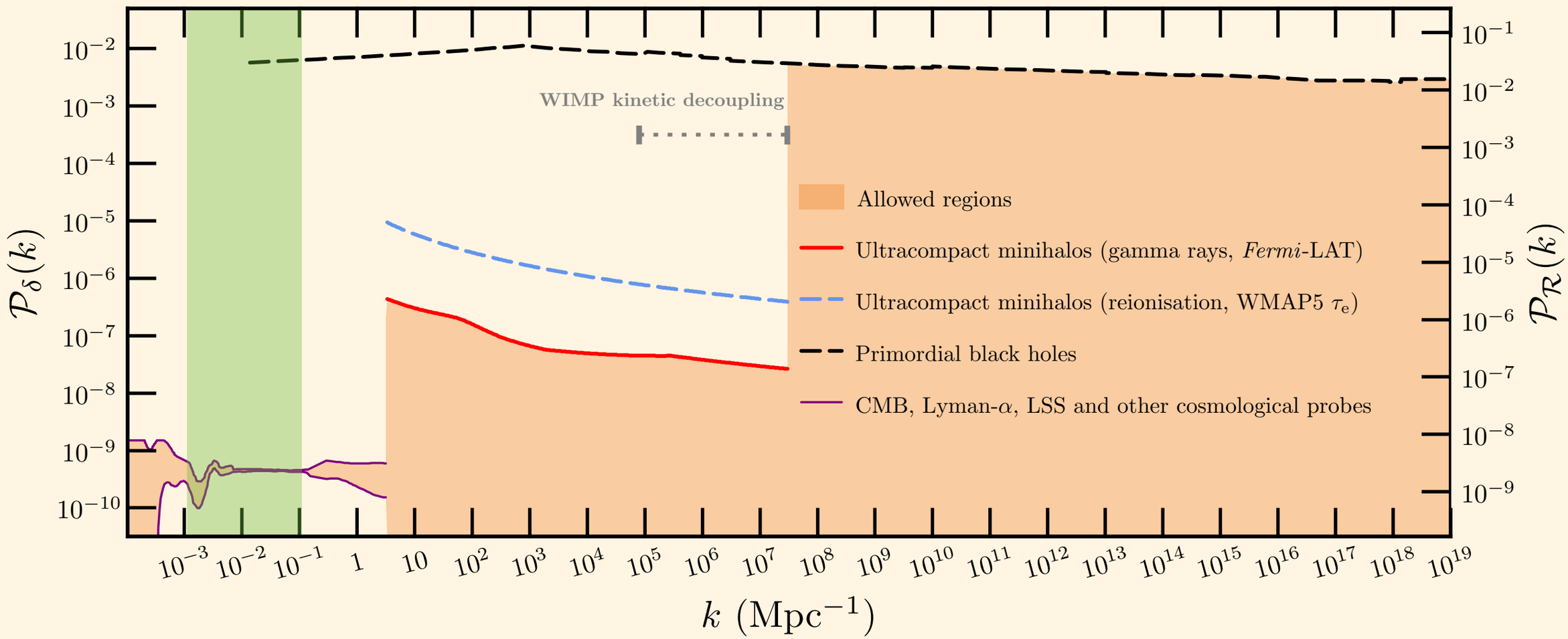
- Almost scale invariant
- Adiabatic
- Gaussian



Torsten Bringmann, Pat Scott, Yashar Akrami (2011)

Small scales are not observed!

Neither is non-Gaussianity (Bispectrum)



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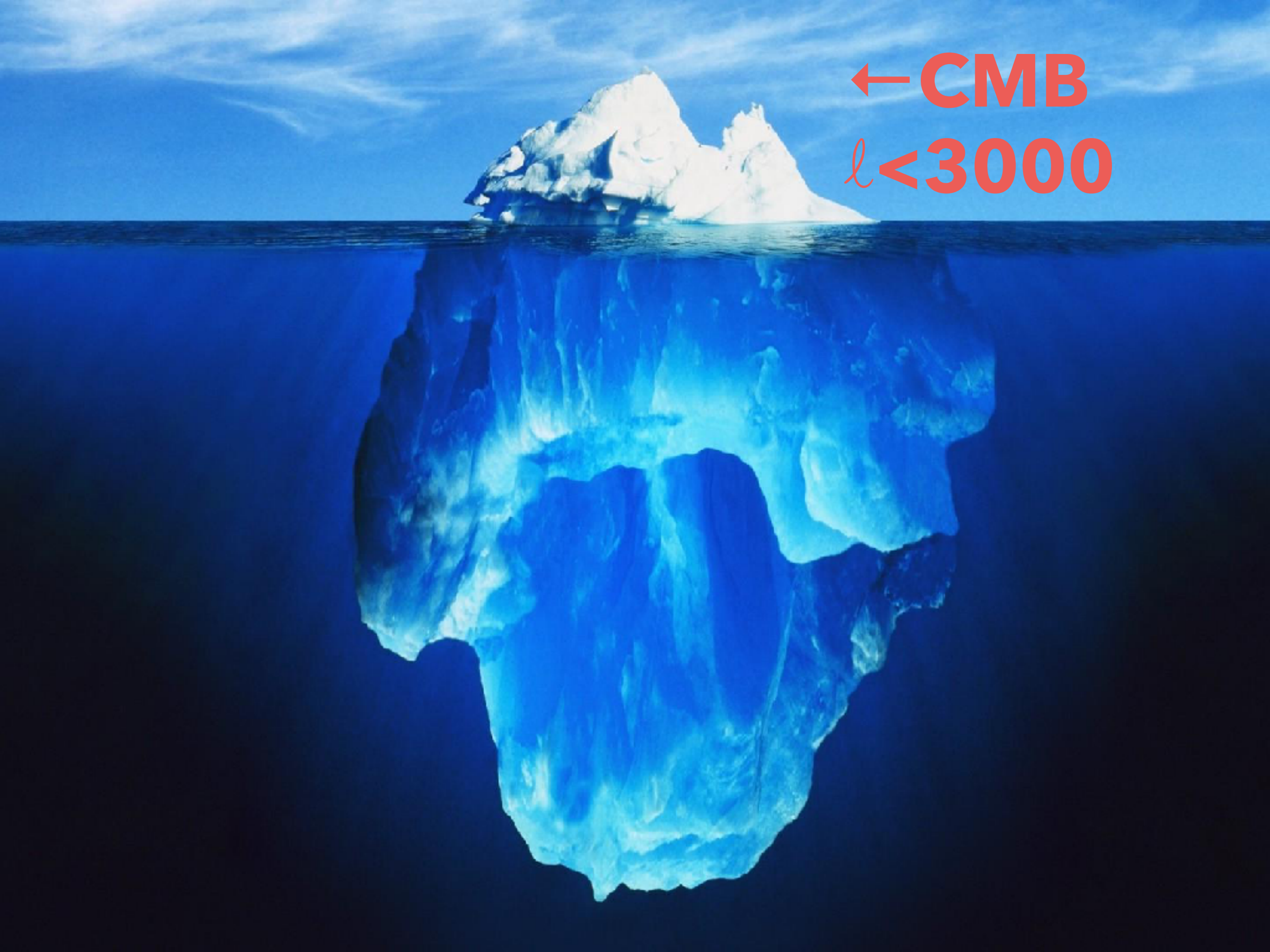

←CMB
ℓ<3000

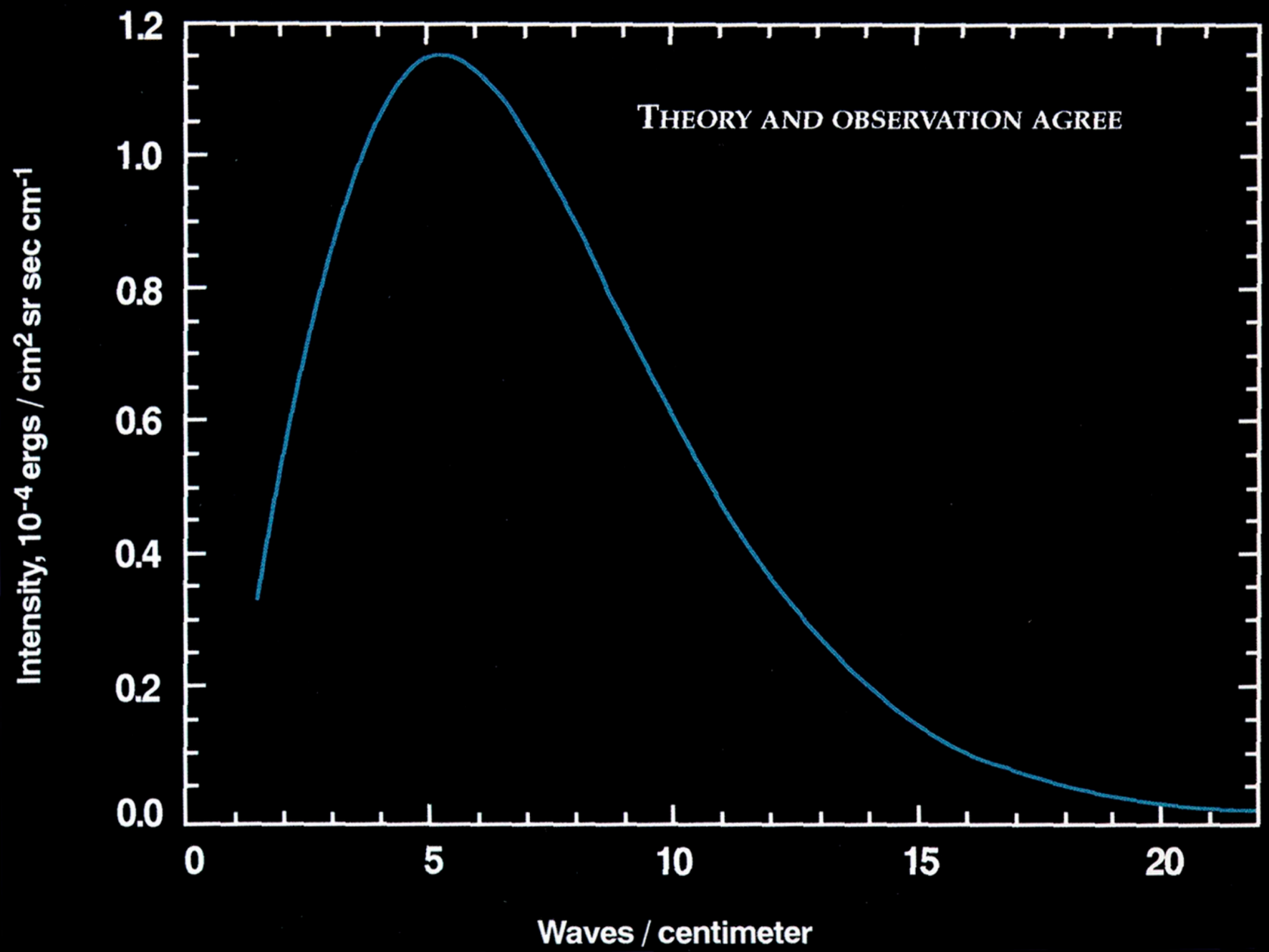
We know very little on

$k > 1\ \mathrm{Mpc}^{-1}$

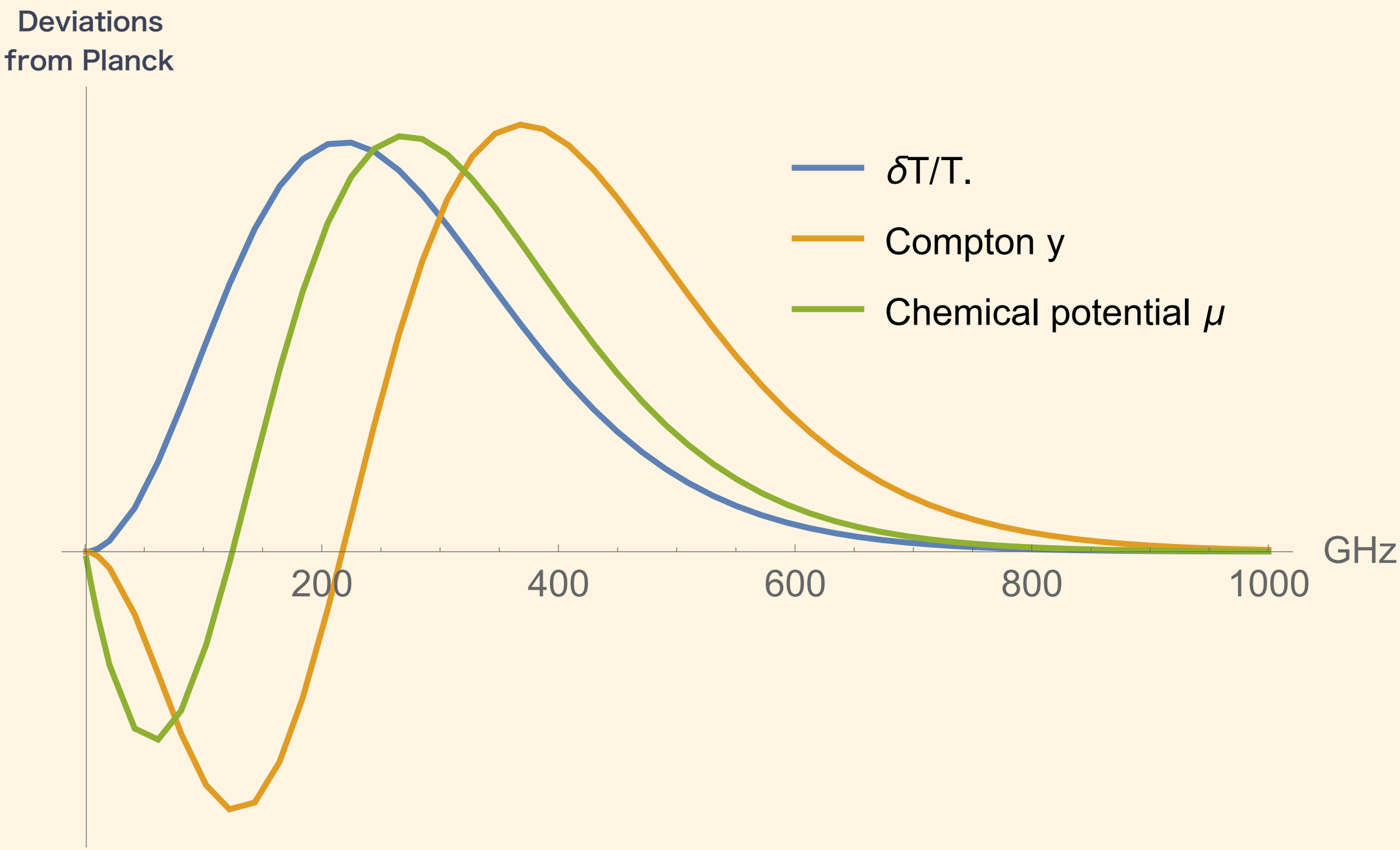
How can we observe small scales?

CMB spectral distortions

Cosmic Microwave Background Spectrum from COBE



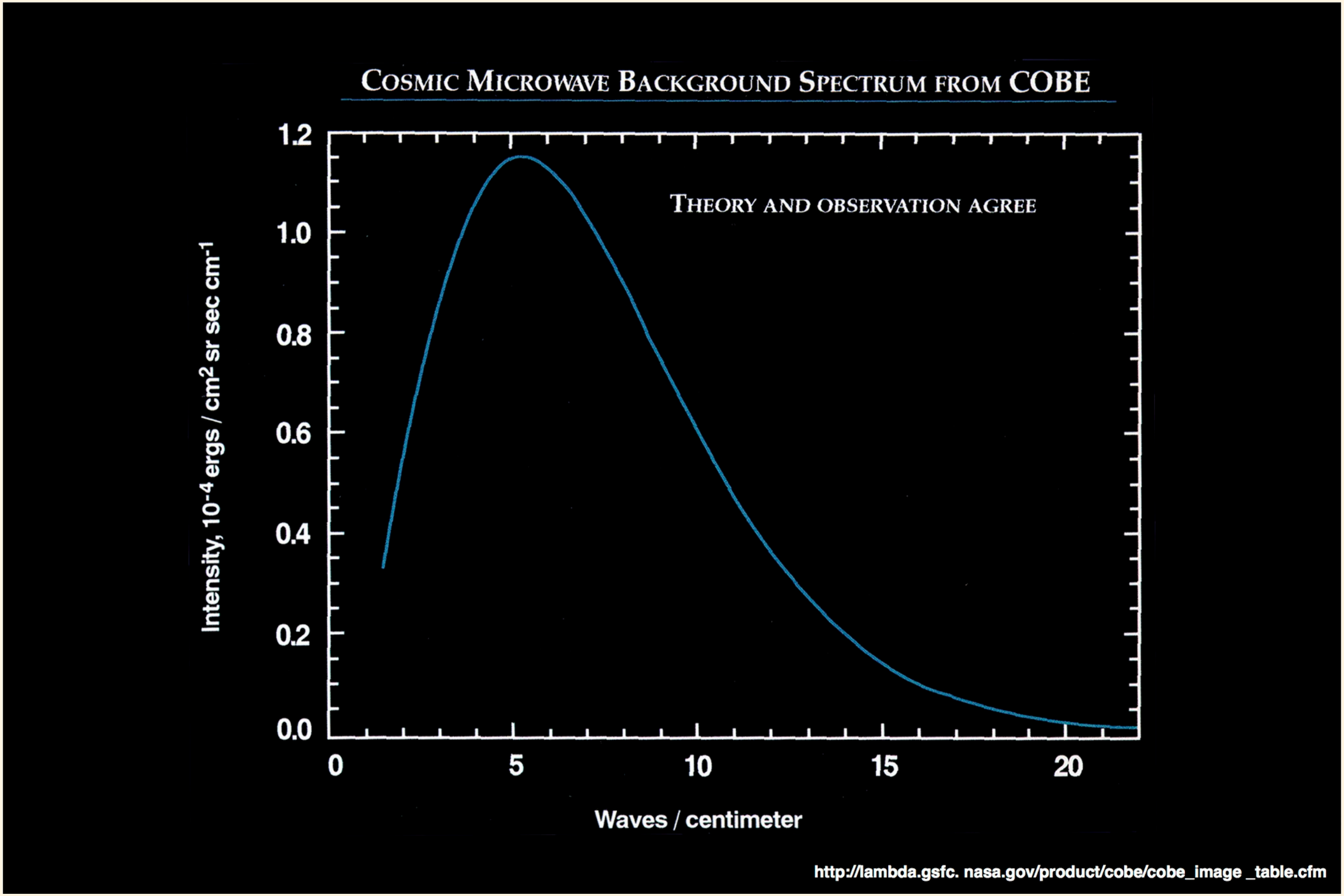
http://lambda.gsfc. nasa.gov/product/cobe/cobe_image _table.cfm

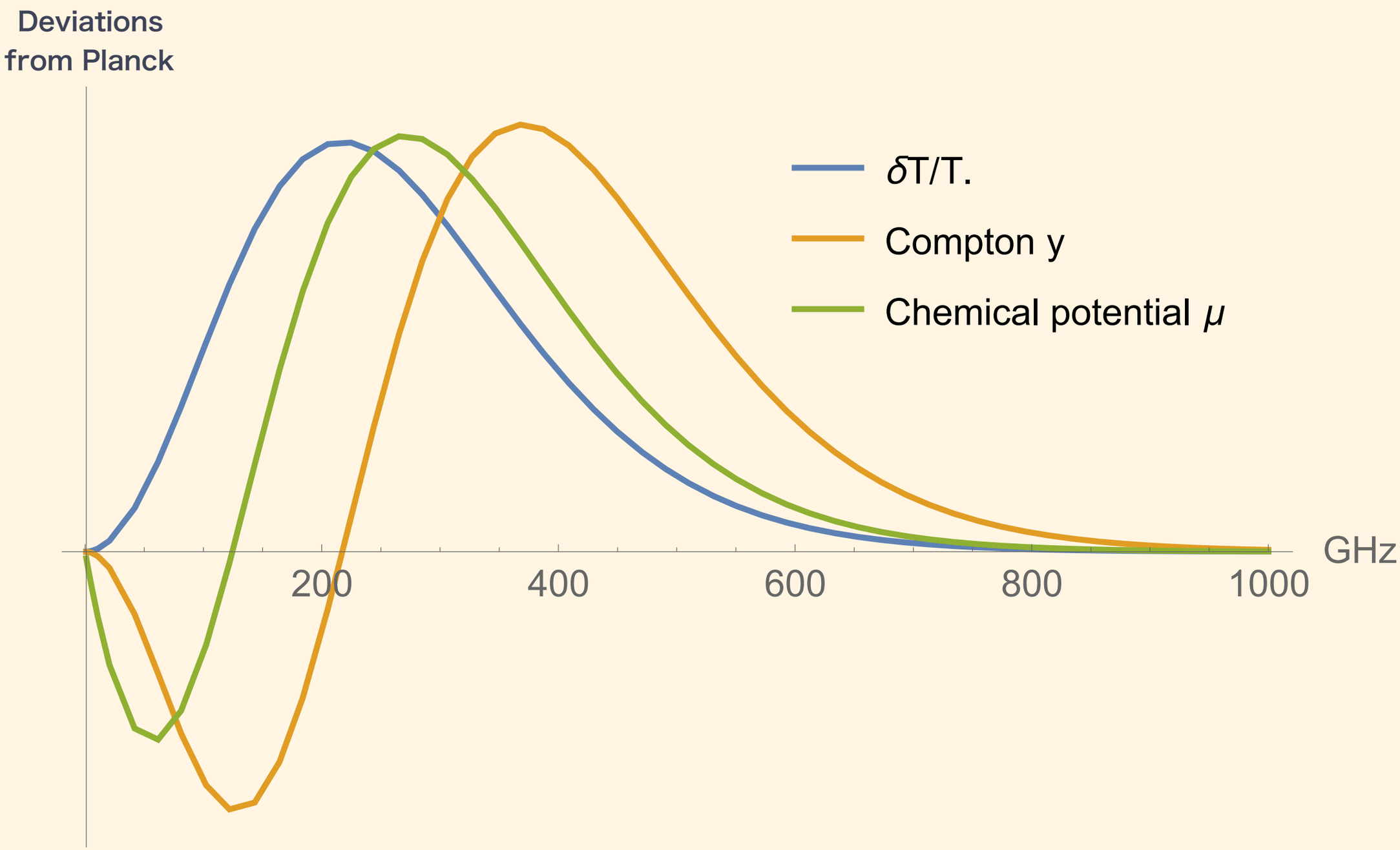


Spectral distortions

Deviations from the isotropic Planck distributions.

Deviations
from Planck





Spectral distortions

Deviations from the isotropic Planck distributions.

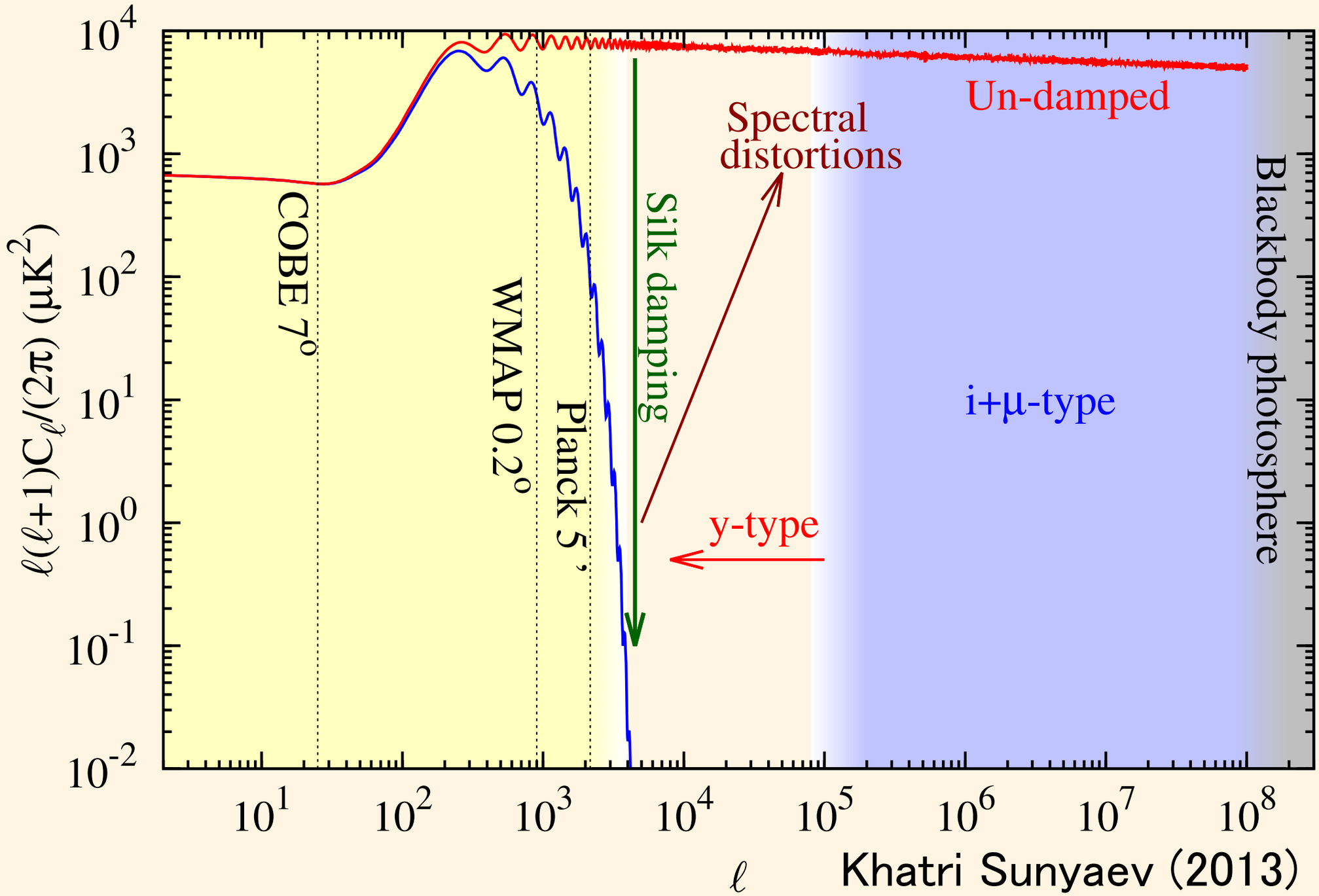
Spectral distortions and energy release in the early Universe

$$\mu = 1.4 \frac{\Delta\rho}{\rho}\Bigg|_{5\times10^4 < z < 2\times10^6}$$

$$y = \frac{1}{4}\frac{\Delta\rho}{\rho}\Bigg|_{z < 5\times10^4}$$

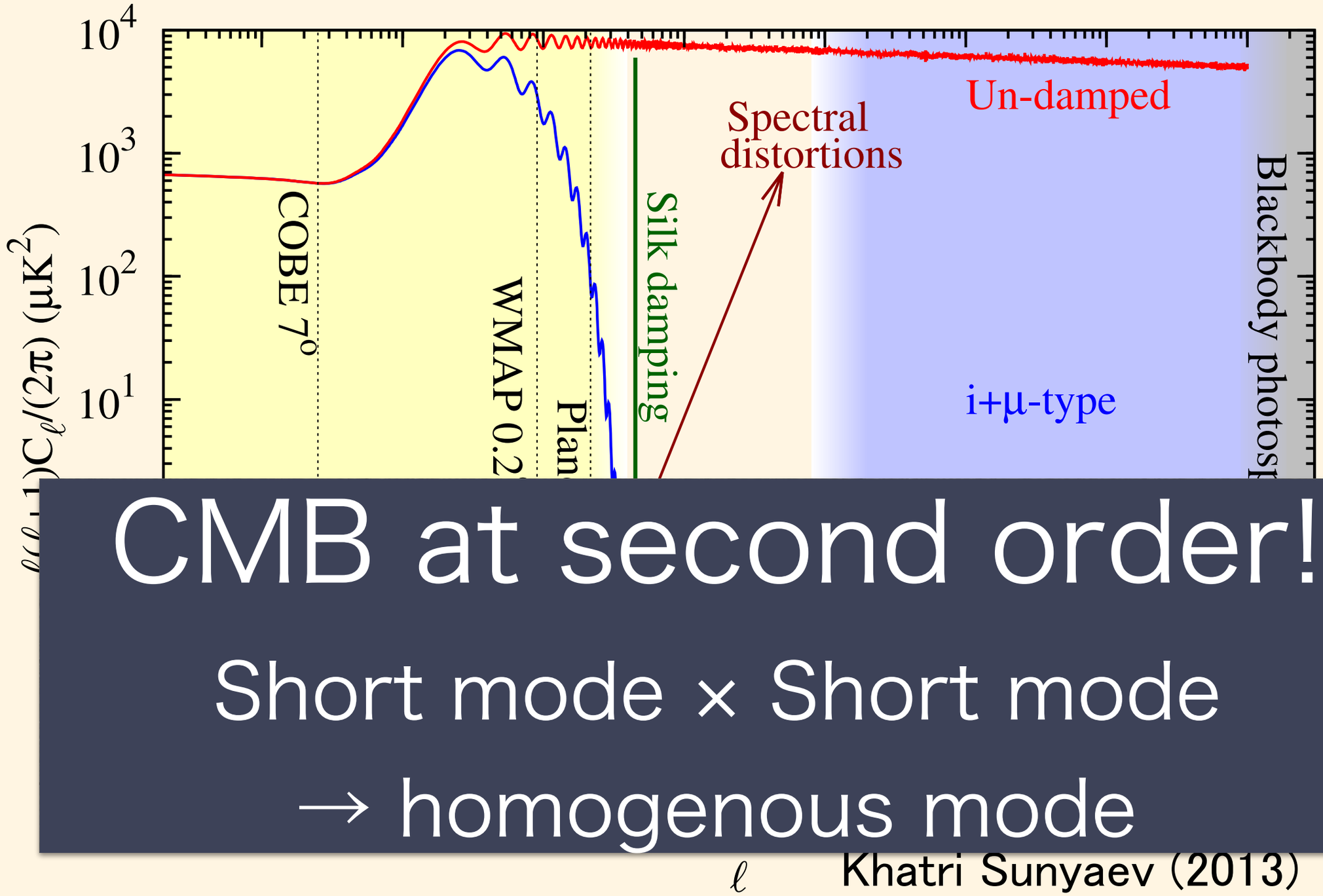
[Sunyaev+1970,Daly1991, Hu+1994]

- **Acoustic damping**
- Annihilation of darkmatter
- PBH evapoation
- etc…



Khatri Sunyaev (2013)

$$\left.\frac{\Delta\rho}{\rho}\right|_{\rm Silk} = 6\left\langle\left(\frac{\delta T}{T}\right)^2\right\rangle \sim \langle\zeta^2\rangle$$



CMB at second order!

Short mode × Short mode

→ homogenous mode

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Summary at second order

- COBE FIRAS constraints on **0.1< k Mpc < 10^4**

$$\mu, y < 10^{-5} \leftrightarrow \mathcal{P}_\zeta < 10^{-5}$$

- **Complementary constraints** on density perturbations.

- Future measurements: $\mu<10^{-8}$, $y<10^{-9}$

CMB at third order

→ Bispectrum?

Boltzmann equation

$$\frac{\partial f_\gamma}{\partial t} + \frac{d\mathbf{x}}{dt}\frac{\partial f_\gamma}{\partial \mathbf{x}} + \frac{d\mathbf{p}_\gamma}{dt}\frac{\partial f_\gamma}{\partial \mathbf{p}_\gamma} = \int_{\mathbf{p}'_\gamma, \mathbf{p}_\mathrm{e}, \mathbf{p}'_\mathrm{e}} \delta^{(3)}(\mathbf{p}_\mathrm{tot.}) \sum_{\mathrm{spins}} |\mathcal{M}|^2$$
$$\times \left[f'_\gamma f'_\mathrm{e}(1+f_\gamma)(1-f_\mathrm{e}) - f_\gamma f_\mathrm{e}(1+f'_\gamma)(1-f'_\mathrm{e}) \right]$$

Expand this up to **third order** in perturbations...

n^{th} order Boltzmann equations

If $T_{\rm e}/m_{\rm e} \ll 1$*, then derivative operators in the Compton scattering collision terms are* $(p\partial/\partial p)^n$

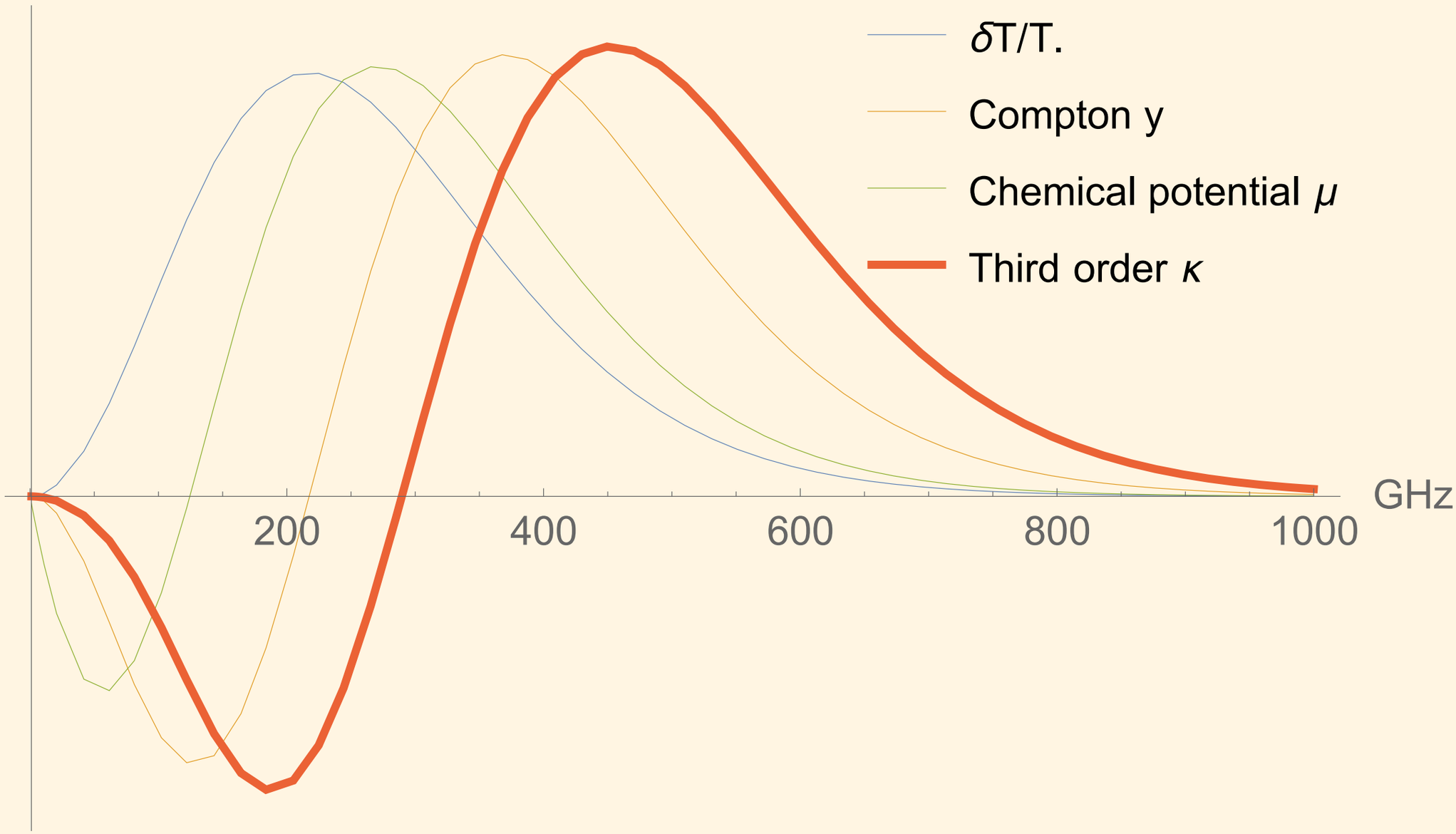
[**AO** 2016]

3rd order Boltzmann equations

Up to third order we only have

$$(p\partial/\partial p)^1,\ (p\partial/\partial p)^2,\ (p\partial/\partial p)^3$$

- Ansatz can be written as some derivatives of Planck dist.
- No need to follow full phase space evolution.



Solution at third order

Sum of δT/T, y, and κ (ignore μ for simplicity)

Equation to solve

- Simple even at third order!

$$\frac{d\langle\langle\kappa\rangle\rangle}{d\eta} = -2\langle\langle y\mathcal{A}\rangle\rangle$$

First order temperature collision term:

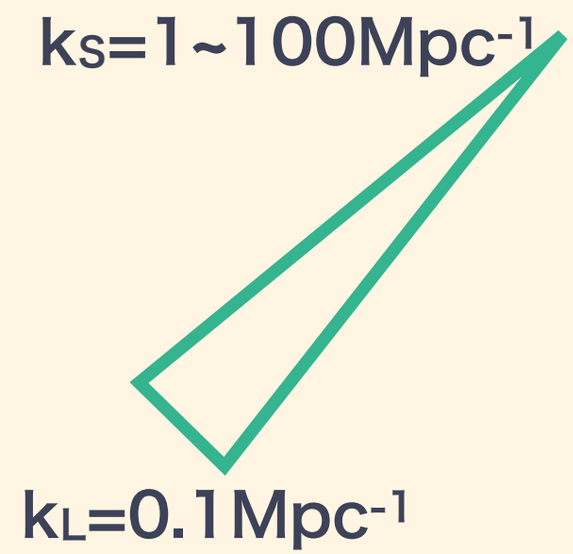
$$-\dot{\tau}^{-1}\mathcal{A} = \frac{\Theta_{00}}{\sqrt{4\pi}} - \Theta + V + \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}\Theta_{2m},$$

[See textbook by Dodelson]

Results

- For scale invariant perturbations (local):

$$\kappa \approx 10^{-18} f_{\rm NL}^{\rm loc.} \left(\frac{y}{4.0 \times 10^{-9}} \right)$$



- $\kappa < 10^{-9}$ in Future space mission.

- **Too small (?)**

Results

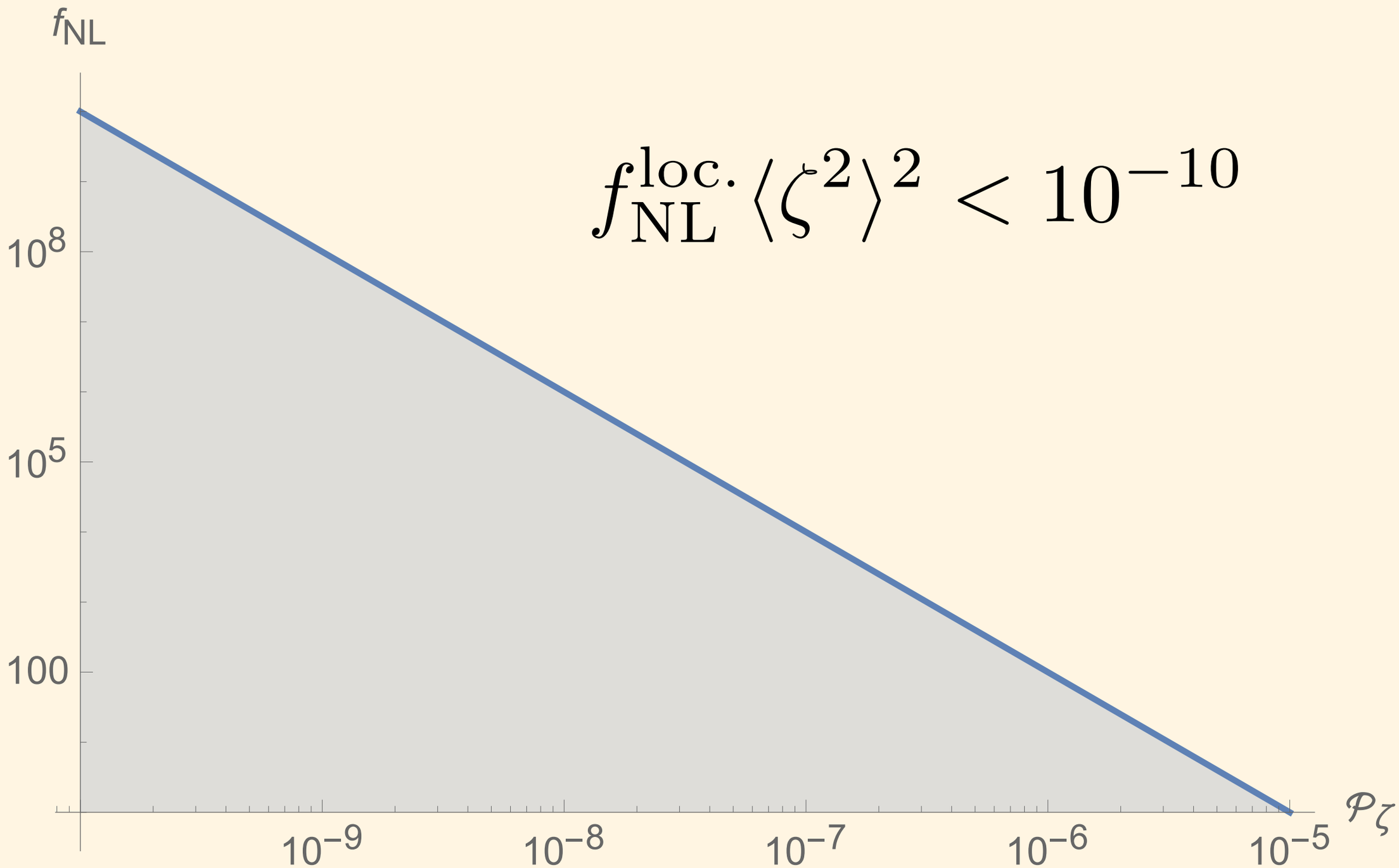
- Normalization of nonlinear parameter:

$$f_{\rm NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2}$$

- No one knows the power on small scales

$$10^{-18} f_{\rm NL}^{\rm loc.} \left(\frac{y}{4.0 \times 10^{-9}} \right) \to \langle \zeta^2 \rangle^2 f_{\rm NL}^{\rm loc.}$$

Forecast in P_ζ-f_{NL} plane



$$f_{\rm NL}^{\rm loc.}\langle\zeta^2\rangle^2 < 10^{-10}$$

Summary

- Third order distortion arises from non-Gaussianity.
- It is small for scale invariant perturbations.
- But, no one knows both bispectrum and powerspectrum on small scales.
- non-Gaussianity can be constrained for bigger powerspectrum.

Future works

- Are cluster contaminations (relativistic SZ effects) crucial?
- What about equilateral NG?

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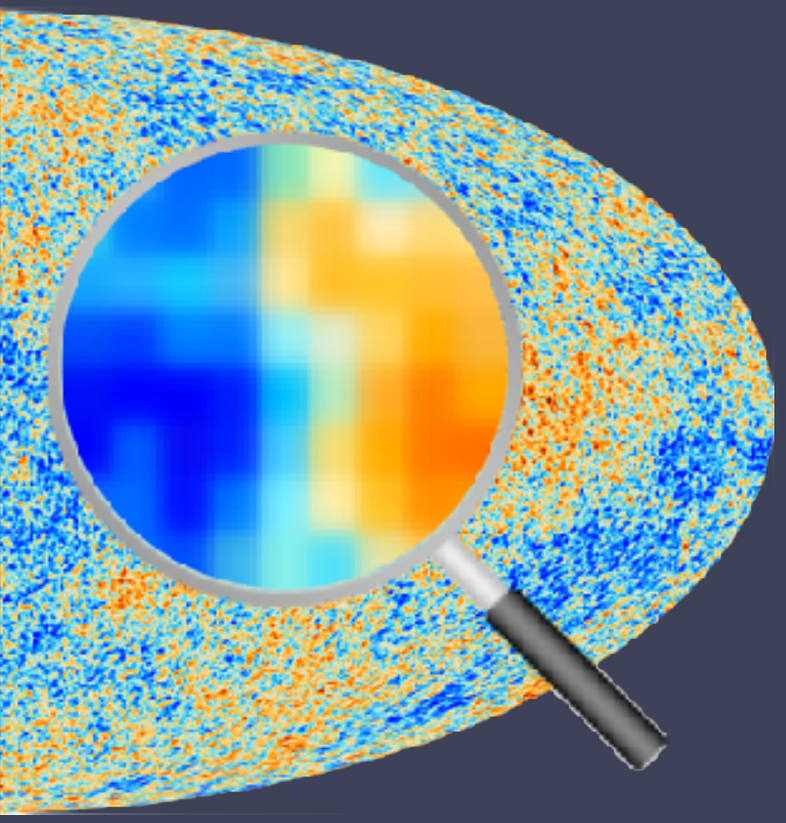

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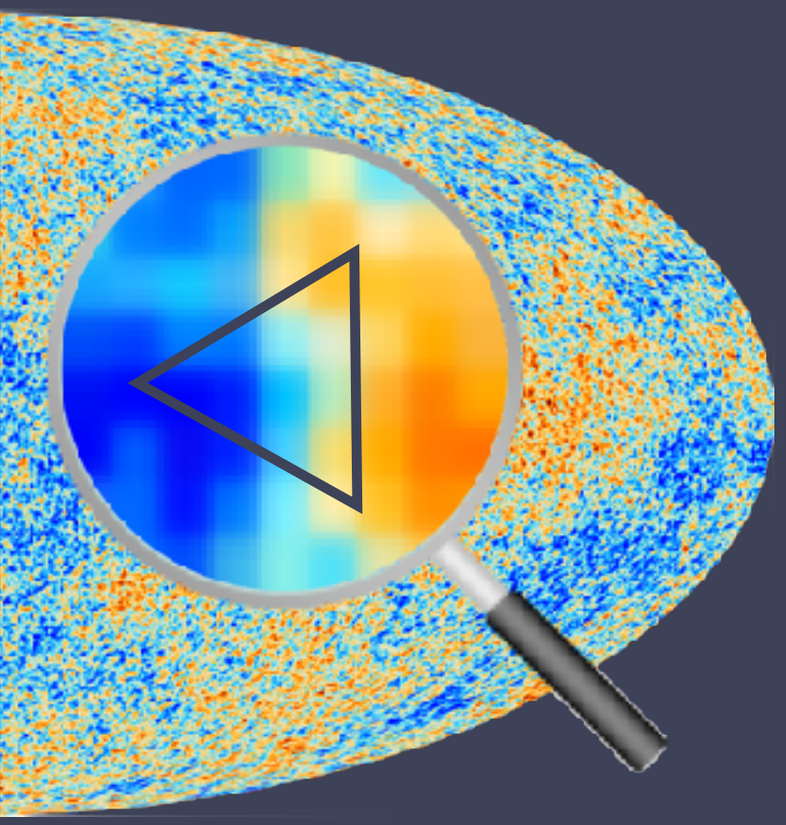

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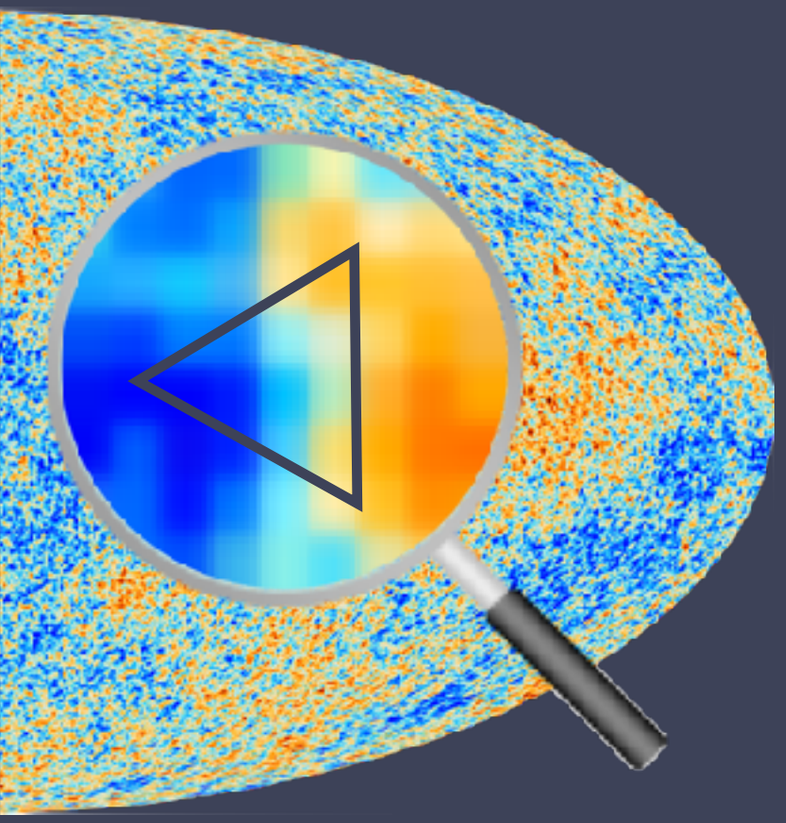

$k_1=k_2=k_3=1\sim100\mathrm{Mpc}^{-1}$