A Bit Rate Bound on Superluminal Communication

With Xi Tong (童曦) and Yuhang Zhu (祝浴航)

Based on 2012.11278

Yi Wang (王一), Hong Kong University of Science and Technology

Image: Artist impression of GJ 273b. DANIELLE FUTSELAAR/METI

How far can our civilization spread?

How far can our civilization spread?

With c constraint | Without c constraint

Possibilities to get rid of the c limitation?

Approach	Definition	Matter violates	CTC	Sample Reference
P(X)	$L = P(X), \qquad X \equiv -\frac{1}{2}(\partial\phi)^2$	positivity	N	Armendariz-Picon, Damour, Mukhanov, 1999
Alcubierre Warp-drive	$ds^2 = -dt^2 + dx^2 + dy^2 + (dz - v_s f(r_s)dt)^2$	DEC	Y	Alcubierre 1994
Krasnikov Tube	$ds^2 = -(dt - dx)(dt + k(t,x)dx)$	WEC	Y	Krasnikov 1995, Everett, Roman 1997
Wormhole	$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2 d\Omega^2$	WEC	N:static Y:move	Morris, Thorne 1988
Extra D	$ds^2 = e^{2A}(-hdt^2 + dx^2) + e^{2B}d\tilde{s}^2_{D-4}$	NEC	N	Rubakov, Shaposhnikov, 1983

See also: Einstein-Aether Waves (Jacobson, Mattingly 2004), QED with plates (Scharnhorst 1990) or gravity (Drummond, Hathrell 1980), Gödel Universe (Gödel 1949), Tipler cylinder (Tipler 1974), van Stockum dust (Lanczos 1924), …

For reviews, see Lobo 2017a, Lobo 2017b, Krasnikov 2018, Shoshany 2019.

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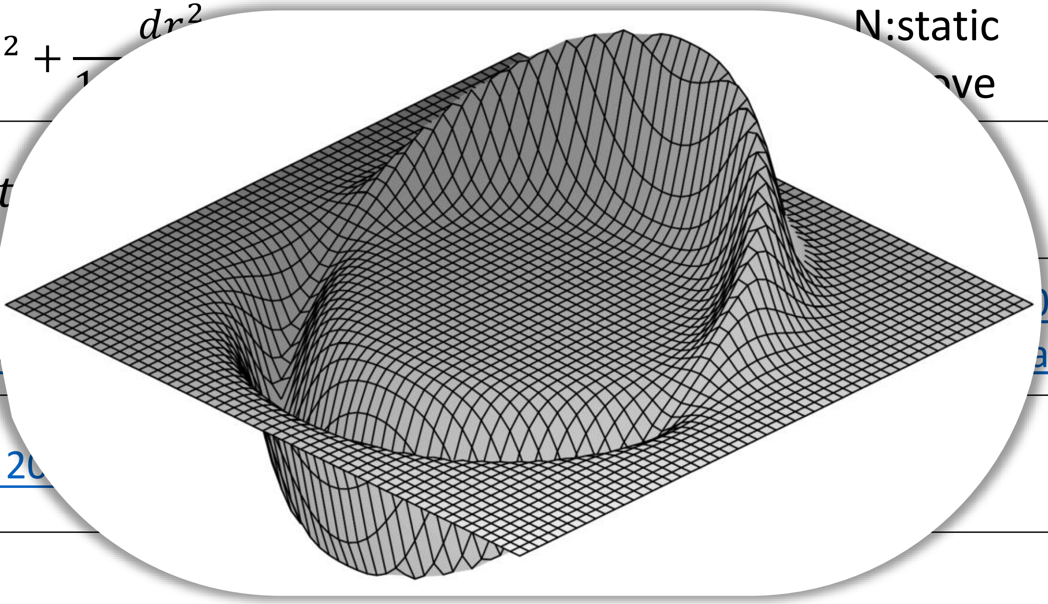

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Exotic matter needed: positivity violation, DEC, WEC, NEC violation, …

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$$\mathcal{L} = \qquad X \; - V$$

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$$\mathcal{L} = P(\phi, X)$$

“k-inflation”, Armendariz-Picon, Damour, Mukhanov (1999)

Exotic matter needed: positivity violation, DEC, WEC, NEC violation, …

$$\mathcal{L} = P(\phi, X) - G_3(\phi, X)\Box\phi$$

"G-inflation", Kobayashi, Yamaguchi, Yokoyama (2010)
(See also "Galileons", Nicolis, Rattazzi, Trincherini 2009)

Exotic matter needed: positivity violation, DEC, WEC, NEC violation, …

$$
\begin{aligned}
\mathcal{L} =& P(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
&+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]
\end{aligned}
$$

“Horndeski”: Horndeski (1974), Deffayet, Gao, Steer, Zahariade (2011), Kobayashi, Yamaguchi, Yokoyama (2011)

Exotic matter needed: positivity violation, DEC, WEC, NEC violation, …

$$\mathcal{L} = P(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+ G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$L_4^{\rm bH} \equiv F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_\sigma\,\epsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\ ,$$
$$L_5^{\rm bH} \equiv F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

“Beyond Horndeski”, or “GLPV”, Gleyzes, Langlois, Piazza, Vernizzi, 2015

$$S[g,\phi] = \int d^4x \sqrt{-g} \left[f_0(X,\phi) + f_1(X,\phi)\Box\phi + f_2(X,\phi)R + C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} + f_3(X,\phi)G_{\mu\nu}\phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\phi_{\mu\nu}\phi_{\rho\sigma}\phi_{\alpha\beta} \right],$$

where X is the kinetic energy of the scalar field, $\phi_{\mu\nu} = \nabla_\mu \nabla_\nu \phi$, and the quadratic terms in $\phi_{\mu\nu}$ are given by

$$C_{(2)}^{\mu\nu\rho\sigma}\phi_{\mu\nu}\phi_{\rho\sigma} = \sum_{A=1}^{5} a_A(X,\phi) L_A^{(2)},$$

where

$$L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}, \quad L_2^{(2)} = (\Box\phi)^2, \quad L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu, \quad L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu, \quad L_5^{(2)} = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2,$$

and the cubic terms are given by

$$C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\phi_{\mu\nu}\phi_{\rho\sigma}\phi_{\alpha\beta} = \sum_{A=1}^{10} b_A(X,\phi) L_A^{(3)},$$

where

$$L_1^{(3)} = (\Box\phi)^3, \quad L_2^{(3)} = (\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu}, \quad L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi^\mu_\rho, \quad L_4^{(3)} = (\Box\phi)^2\phi_\mu\phi^{\mu\nu}\phi_\nu,$$
$$L_5^{(3)} = \Box\phi\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho, \quad L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma, \quad L_7^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma,$$
$$L_8^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda, \quad L_9^{(3)} = \Box\phi(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2, \quad L_{10}^{(3)} = (\phi_\mu\phi^{\mu\nu}\phi_\nu)^3.$$

The a_A and b_A are arbitrary functions of ϕ and X.

“DHOST”, Langlois, Noui (2016)

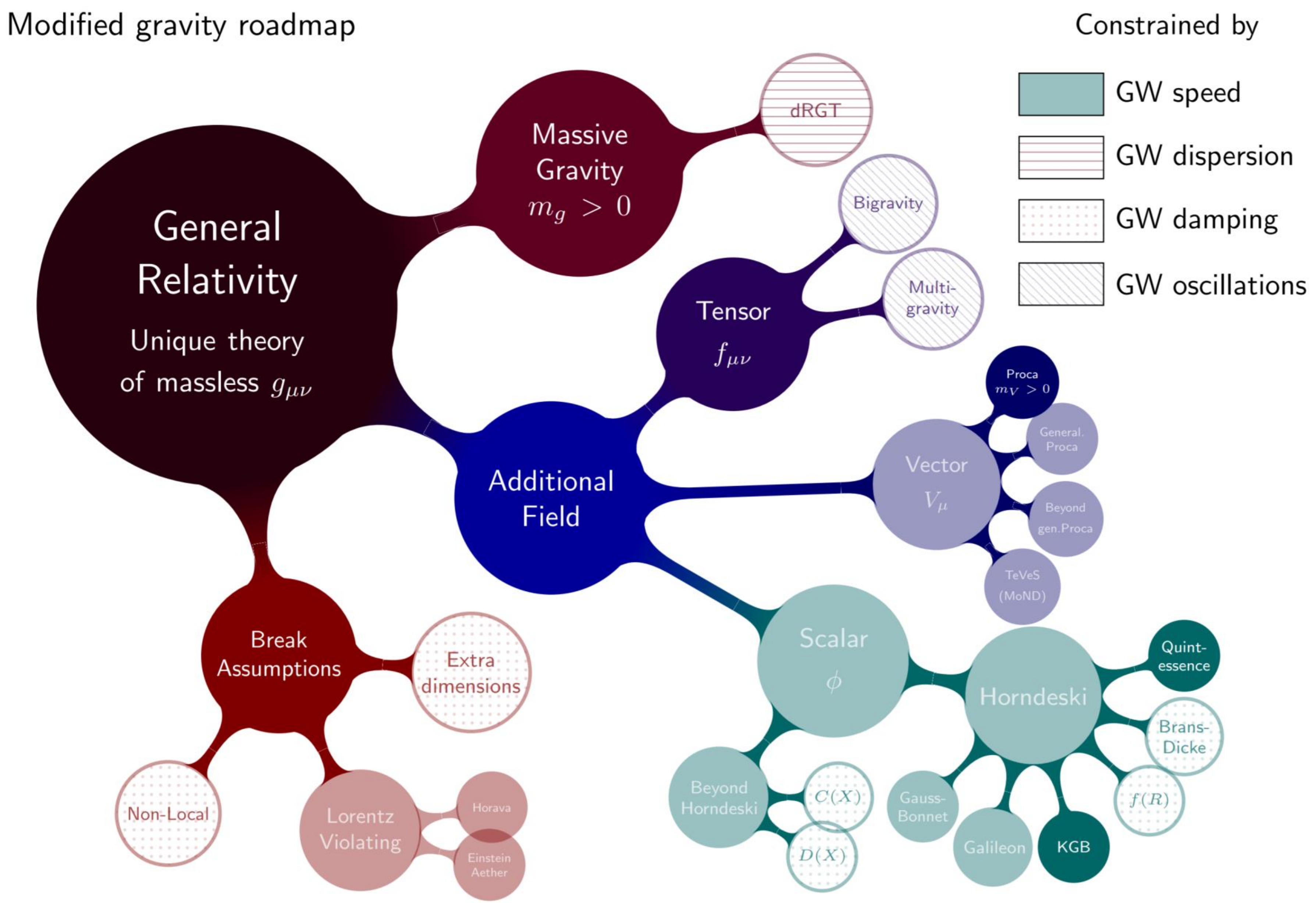

Figure from Ezquiaga, Miguel Zumalacarregui (2018)

See also Shinji's & Lavinia's & Filippo's talks on Monday

Exotic matter needed: positivity violation, DEC, WEC, NEC violation, …

Considering a huge literature of exotic matter discussed in

inflationary cosmology, dark energy, etc,

wouldn’t it make more sense to study exotic matter for superluminal travel?

“The Rise of Field Theory”

Outline:

- Motivation (✓)
- Theory and Intuition
- Derivation of Bit Rate Bound
- Discussions

Theory and Intuition: The K-Essence Theory

K-essence: $\mathcal{L} = \mathcal{L}(X), X = -\frac{1}{2}(\partial\phi)^2$

Sound speed: c_s: $c_s^{-2} = 1 + 2X\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}$

Thus, if $\mathcal{L}_{,XX} < 0$, $c_s > 1$.

Armendzriz-Picon, Damour, Mukhanov 1999,

Garriga, Mukhanov 1999,

Babichev, Mukhanov, Vikman 2007

Theory and Intuition: The K-Essence Theory

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Comment on the $\mathcal{L}_{,XX} < 0$ branch:

- No CTC
- Positivity violation:

  UV competition cannot be

  local & analytical & unitary & Lorentz inv.

  Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 2006,

  Shore 2007, c.f. Jackson 3rd Edition, Sec 7.10

  See also Claudia's & Brando's talks yesterday.

Theory and Intuition: Superluminality and Non-Linearity

K-essence: $\mathcal{L} = \mathcal{L}(X), X = -\frac{1}{2}(\partial\phi)^2$

Sound speed: c_s: $c_s^{-2} = 1 + 2X\frac{\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}$

Thus, if $\mathcal{L}_{,XX} < 0$, $c_s > 1$.

Non-linear for $\mathcal{L}_{,XX} \neq 0$

Consider $\phi = \phi_0(t) + \varphi(\mathbf{x}, t)$

2nd order $\Rightarrow$ $\dot{\phi}_0^2\,\dot{\varphi}^2 \Rightarrow c_s$

3rd order $\Rightarrow$ $\dot{\phi}_0\,\dot{\varphi}^3 \Rightarrow \varphi$ cannot be large

c.f. EFT of Inflation
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

Theory and Intuition: Superluminality and Non-Linearity

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This talk

For $c_s > 1$,

(bit rate) = (decreasing function of c_s)

Thus:

For low latency: $c_s > 1$ preferred

For high bit rate: $c_s \leq 1$ preferred

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- Theory and Intuition (✓)

- Derivation of Bit Rate Bound

- Discussions

$$\mathcal{L}(X) = \sum_{n=0}^{\infty} \frac{1}{n!} \partial_X^n \mathcal{L}(X_0)(X - X_0)^n$$

$$\mathcal{L}(X) = (\text{total derivatives}) + \mathcal{L}^{(2)} + \mathcal{L}_{\text{int}} \ ,$$

with

$$\mathcal{L}^{(2)} \equiv \frac{1}{2}\left[\left(c_1 + \frac{c_2\dot{\phi}_0^2}{2\Lambda^4}\right)\dot{\varphi}^2 - c_1(\nabla\varphi)^2\right]$$

$$\mathcal{L}_{\text{int}} \equiv \left(\frac{c_2\dot{\phi}_0}{2\Lambda^4} + \frac{c_3\dot{\phi}_0^3}{6\Lambda^8}\right)\dot{\varphi}^3 - \frac{c_2\dot{\phi}_0}{2\Lambda^4}\dot{\varphi}(\nabla\varphi)^2 + \cdots \ .$$

Requiring $\mathcal{L}_{\text{int}} < \mathcal{L}^{(2)}$

$\Rightarrow \; |\dot{\varphi}| < \frac{\dot{\phi}_0}{2\, c_s^2}$ (note: naively it would be $|\dot{\varphi}| < \dot{\phi}_0$)

$\Rightarrow$ Constraint on stress tensor: $\left|T_{\mu\nu}^{(2)}\right| \sim c_s^{-1}\dot{\varphi}^2 < \frac{\dot{\phi}_0^2}{c_s^5}$

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How should this constraint be imposed?

- Locally (LC) on $\left|T_{\mu\nu}^{(2)}(\mathbf{x})\right|$: Pointwise in spacetime
- Globally (GC) on $E(\mathrm{mode})$: Each mode is linear
- Averaged & Globally (AGC) on $\int d(\mathrm{mode})P(\mathrm{mode})E(\mathrm{mode})$:

  An average mode is linear

Theoretically: LC and GC too strong, AGC more reasonable

Operationally: AGC is simple to use (more later)

This is the classical constraint on information

Quantum mechanical: quanta

- Short wavelength: large energy per quanta
- Long wavelength: slowly varying, thus low bit rate

Model of semi-classical signal: coherent state

$$|z\rangle = \exp\left(-\frac{1}{2}\int_{\vec{k}} |z_{\vec{k}}|^2\right) \exp\left(\int_{\vec{k}} z_{\vec{k}} a^\dagger_{\vec{k}}\right) |0\rangle$$

Shannon entropy:

$$S[P] = -\int \mathcal{D}^2\hat{z} P[\hat{z}] \ln P[\hat{z}] \quad \text{with} \quad \int \mathcal{D}^2\hat{z} P[\hat{z}] = 1 \ .$$

Area-density of energy:

$$E[\hat{z}] \equiv \frac{\langle z|H^{(2)}|z\rangle}{L_y L_z} = \int \frac{dk}{2\pi} \omega_k |\hat{z}_k|^2$$

AGC:

$$\int \mathcal{D}^2\hat{z} P[\hat{z}] E[\hat{z}] \lesssim \varepsilon_{\max} L_x \qquad \varepsilon_{\max} \sim \frac{\dot{\phi}_0^2}{c_s^6}$$ (maximal energy density)

Model of semi-classical signal: coherent state

$$|z\rangle = \exp\left(-\frac{1}{2}\int_{\vec{k}} |z_{\vec{k}}|^2\right) \exp\left(\int_{\vec{k}} z_{\vec{k}} a^\dagger_{\vec{k}}\right)|0\rangle$$

The quantity to maximize

$$S[P] = -\int \mathcal{D}^2\hat{z}\, P[\hat{z}] \ln P[\hat{z}] \quad \text{with} \quad \int \mathcal{D}^2\hat{z}\, P[\hat{z}] = 1\ .$$

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AGC:

$$\int \mathcal{D}^2\hat{z}\, P[\hat{z}] E[\hat{z}] \lesssim \varepsilon_{\max} L_x$$

constraints

Using methods in statistical physics:

$$0 = \delta \left(S[P] - (\alpha - 1) \int \mathcal{D}^2 \hat{z} P - \beta \int \mathcal{D}^2 \hat{z} P E \right)$$

$$P_* = e^{-\alpha - \beta E}$$

$$S_* = \frac{L_x \Lambda}{2\pi c_s} \ln \left(1 + \frac{1}{\gamma} e^{-\gamma} \right) + \frac{2\dot{\phi}_0^2 L_x L_y L_z}{c_s^6 \Lambda} \gamma$$

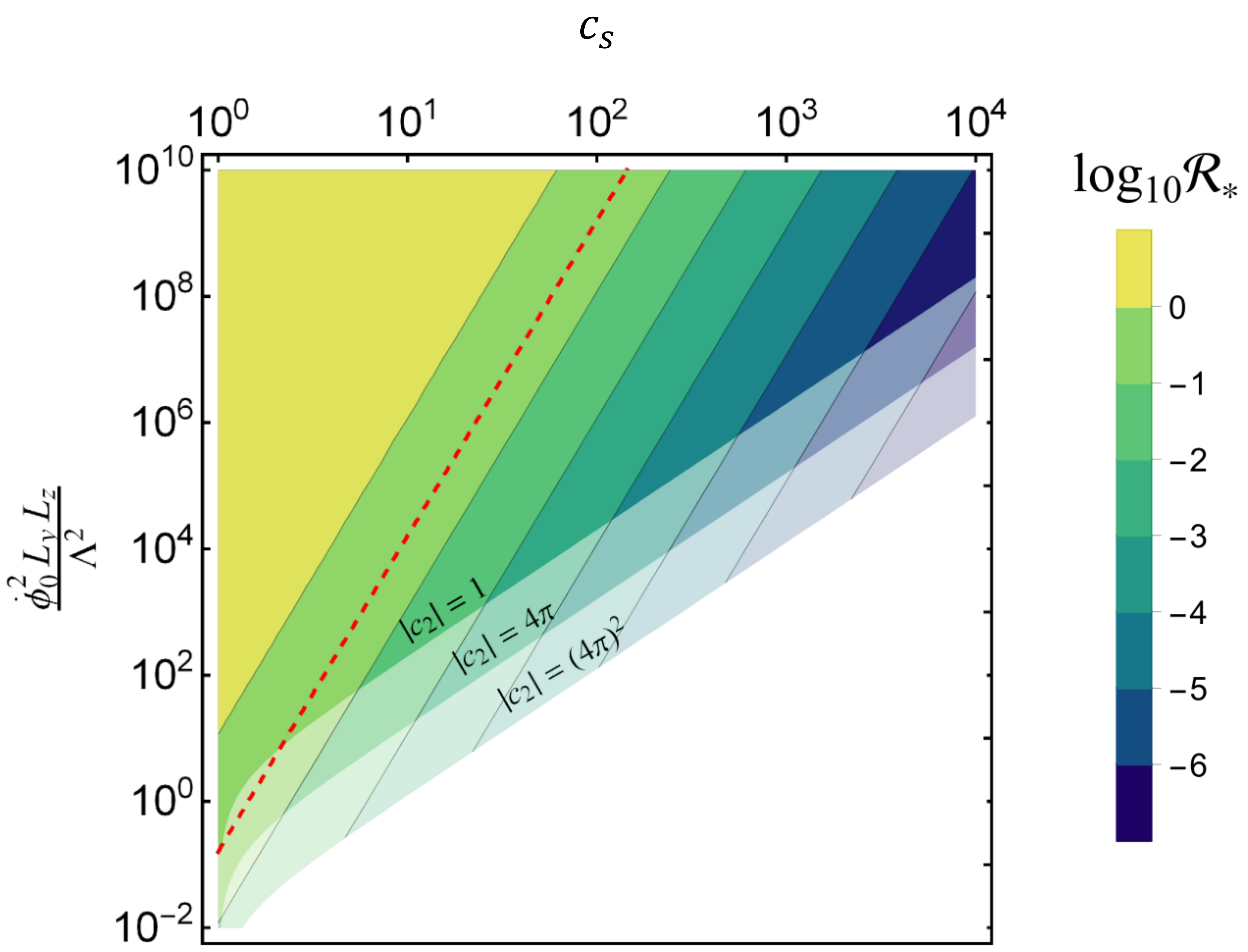
⇨

AGC bit rate bound:

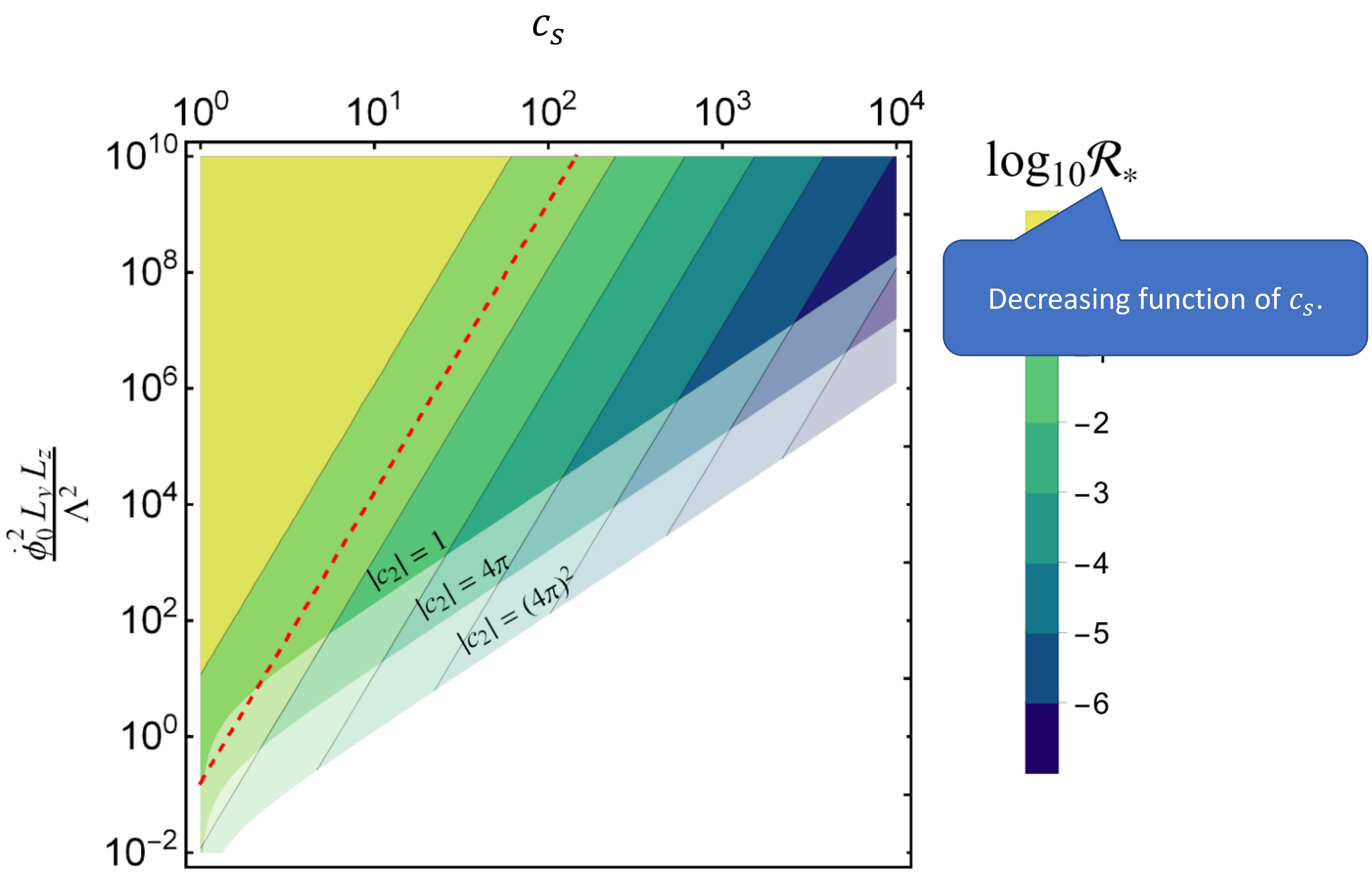
$$R_* = \frac{c_s S_*}{L_x} \equiv \Lambda \mathcal{R}_*$$

where γ is the solution to

$$\frac{1}{\gamma^2} \int_0^\gamma dx \frac{1+x}{1+x e^x} = \frac{2\pi \dot{\phi}_0^2 L_y L_z}{c_s^5 \Lambda^2} \; .$$



c_s
10^0
10^1
10^2
10^3
10^4
10^{10}
10^8
10^6
10^4
10^2
10^0
10^{-2}
$\frac{\dot{\phi}_0^2 L_y L_z}{\Lambda^2}$
$|c_2| = 1$
$|c_2| = 4\pi$
$|c_2| = (4\pi)^2$
$\log_{10}\mathcal{R}_*$
0
−1
−2
−3
−4
−5
−6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10^0
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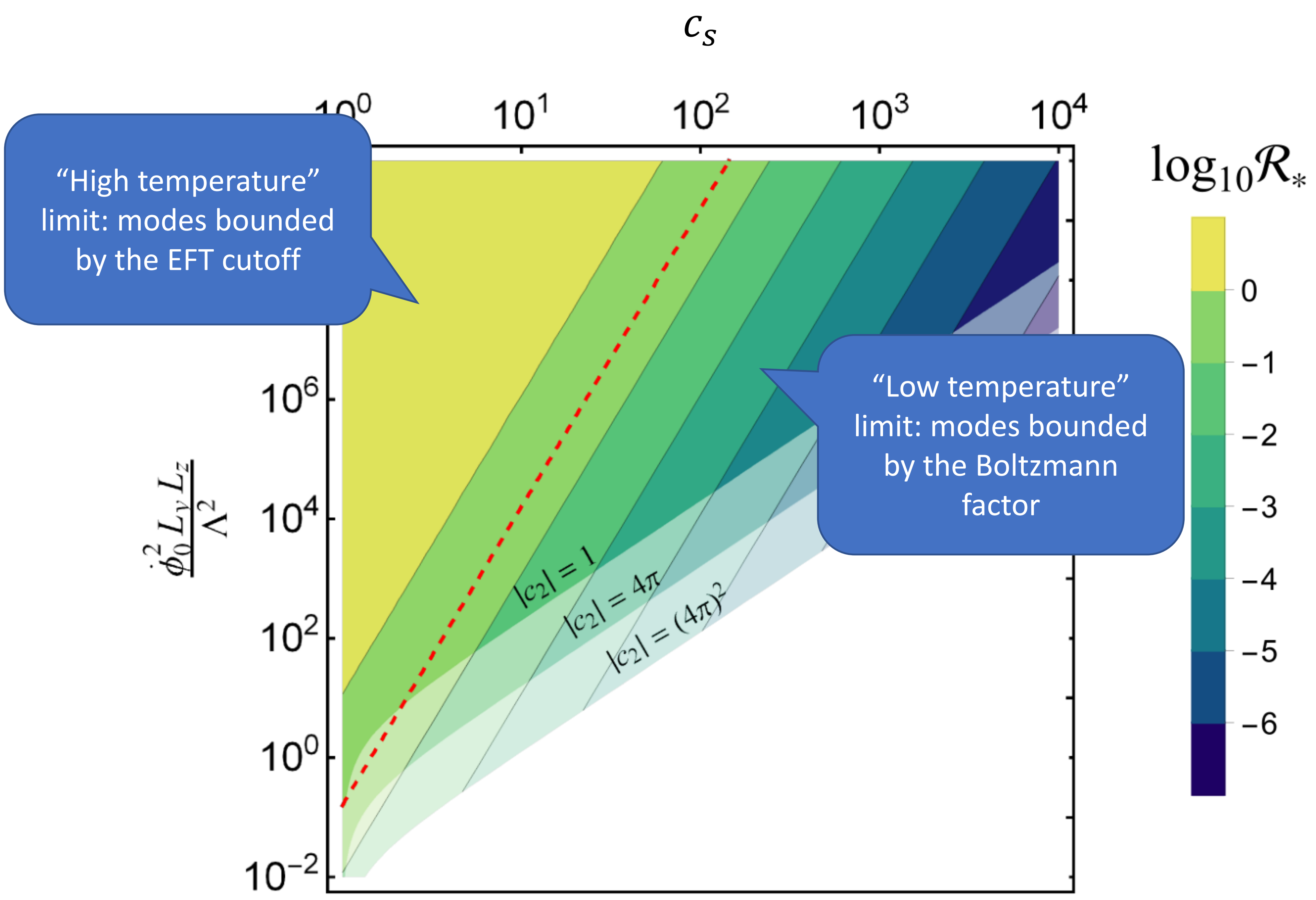
−2
−3
−4
−5
−6
Decreasing function of c_s.



c_s
10^0 10^1 10^2 10^3 10^4
$\frac{\dot{\phi}_0^2 L_y L_z}{\Lambda^2}$
10^{-2} 10^0 10^2 10^4 10^6
$|c_2| = 1$
$|c_2| = 4\pi$
$|c_2| = (4\pi)^2$
$\log_{10}\mathcal{R}_*$
0 −1 −2 −3 −4 −5 −6
“High temperature” limit: modes bounded by the EFT cutoff
“Low temperature” limit: modes bounded by the Boltzmann factor

"Low temperature" limit: $\frac{\dot{\phi}_0^2 L_y L_z}{\Lambda^2} \ll \frac{c_s^5}{2\pi}$

$$R_* \approx \left(\frac{2.64}{\pi} \dot{\phi}_0^2 L_y L_z \right)^{1/2} c_s^{-5/2}$$

"High temperature" limit: $\frac{\dot{\phi}_0^2 L_y L_z}{\Lambda^2} \gg \frac{c_s^5}{2\pi}$

$$R_* \approx \frac{\Lambda}{2\pi} \ln \frac{2\pi \dot{\phi}_0^2 L_y L_z}{c_s^5 \Lambda^2}$$

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Conclusion:

- Superluminal ⇒ nonlinearity
- Nonlinearity ⇒ energy density bound, AGC
- + quantization ⇒ bit rate bound

Discussions:

- SM constraints on Lorentz-violating sectors?
- More general scalar-tensor theories or higher-dim EFT operators?
- More information theory justifications of AGC?
- Other superluminal mechanisms, say, extra-dim?
- Semi-classical. Go fully quantum?

Thank you!

Appendix: Energy Conditions

Energy Condition	Definition	Eigenvalues $\hat{T}_{\mu\nu} = \mathbf{diag}\,\{\rho, p, p, p\}$ [3]	Implies
WEC (weak)	$T_{\mu\nu}V^\mu V^\nu \geq 0$ [1]	$\rho \geq 0, \quad \rho + p_i \geq 0$	NEC
DEC (dominant)	WEC & $T_{\mu\nu}V^\nu$ not spacelike [2]	$\rho \geq 0, \quad -\rho \leq p_i \leq \rho$	WEC, NEC
SEC (strong)	$\left(T_{\mu\nu} - ½\, g_{\mu\nu}T\right)V^\mu V^\nu \geq 0$	$\rho + p_i \geq 0, \quad \rho + \sum_i p_i \geq 0$	NEC
NEC (null)	$T_{\mu\nu}L^\mu L^\nu \geq 0$	$\rho + p_i \geq 0$	

Remarks:

[1] By continuity, this also implies $T_{\mu\nu}L^\mu L^\nu \geq 0$.

[2] Equivalent definition: For orthonormal basis, $T^{00} \geq |T^{\mu\nu}|$.

[3] See Hawking & Ellis for other classes of $T_{\mu\nu}$.

Notations: V^μ, W^μ are general time-like vectors; L^μ is a general null vector; $\mu = 0, 1, 2, 3$ and $i = 1, 2, 3$.