Bounds on non-geodesics from the Swampland Distance Conjecture

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2012.00034 with Calderon-Infante, Uranga

2006.15154 with Lanza,Marchesano,Martucci

+ previous work with Gendler, Grimm, Li, Palti

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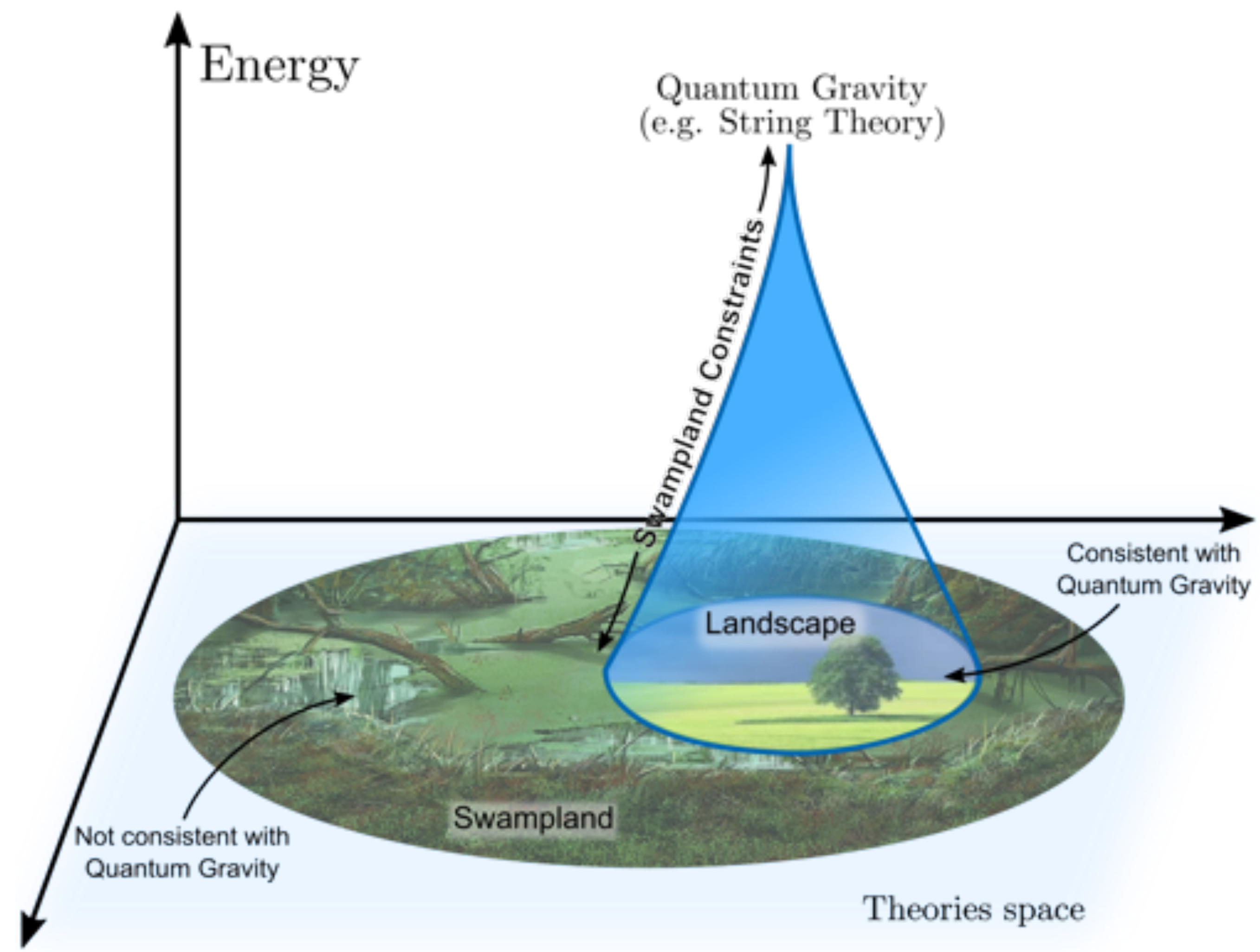
Swampland:

Apparently consistent (anomaly-free) quantum effective field theories that cannot be UV embedded in quantum gravity (they cannot arise from string theory)



Not everything is possible in string theory/quantum gravity!!!

Goal of the Swampland program:

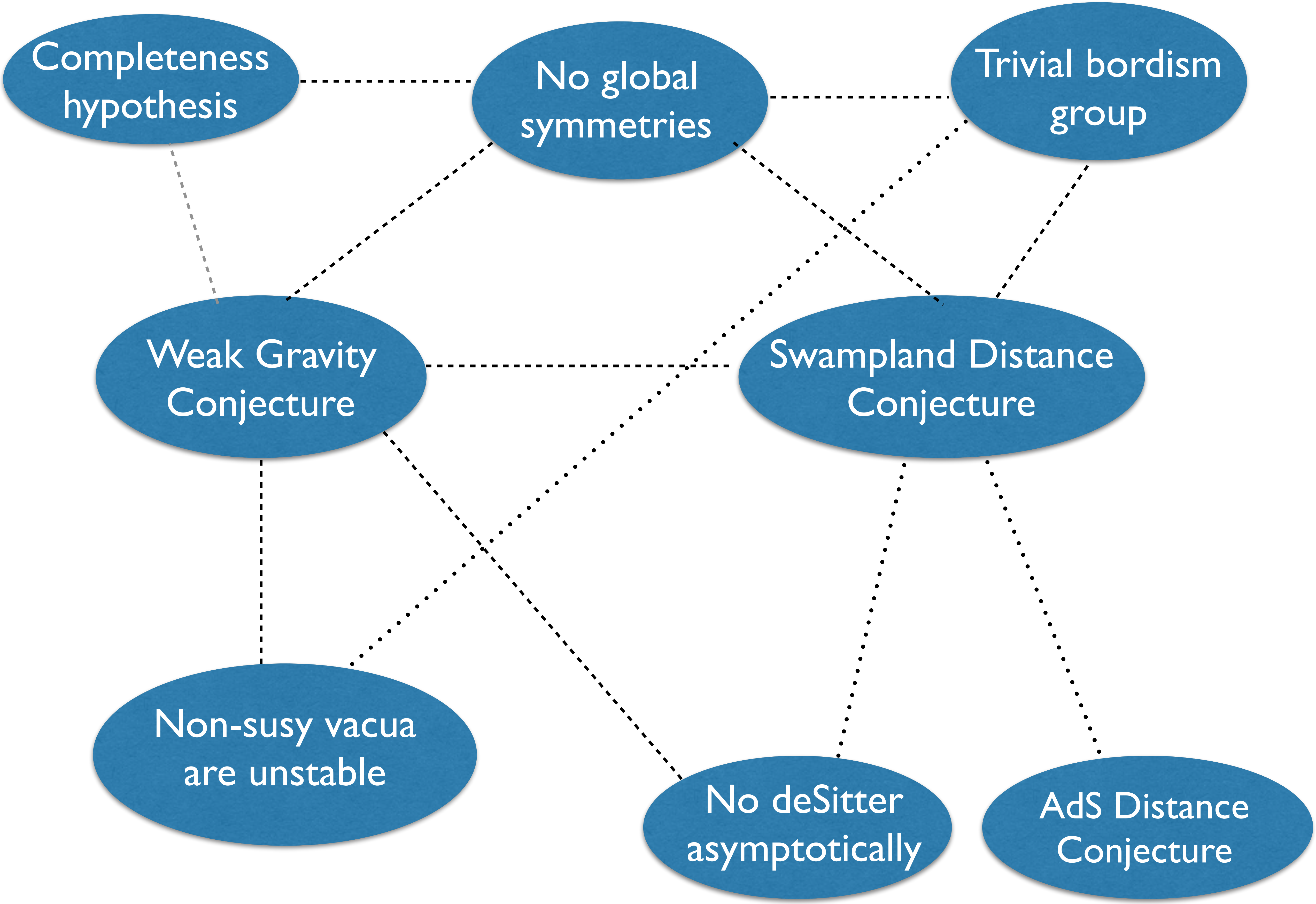
> What are the constraints that an effective theory must satisfy to be consistent with quantum gravity?

What distinguishes the landscape from the swampland?

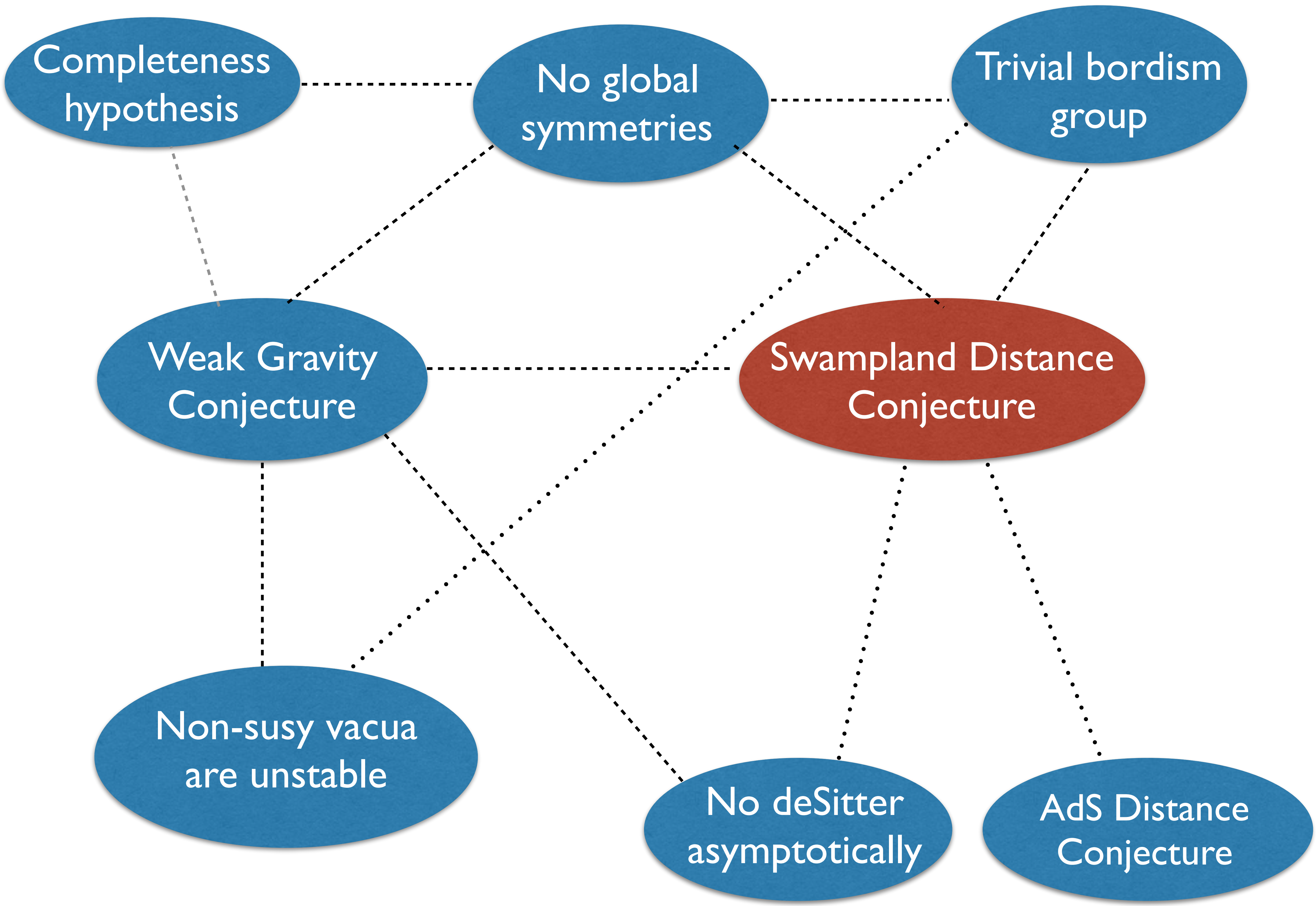
→ Potential phenomenological implications

- *Guiding principles to construct BSM models*
- *New insights to solve naturalness issues in our universe*

Swampland Conjectures



Swampland Conjectures



Outline:

(1) Review of Distance Conjecture

(2) Bounds on non-geodesics in the presence of a potential

→ Convex Hull SDC (in analogy to scalar WGC)

(3) Sharpening the value of order one factors

(1) Distance conjecture

Swampland Distance Conjecture (SDC)

An effective theory is valid only for a finite scalar field variation $\Delta\phi$

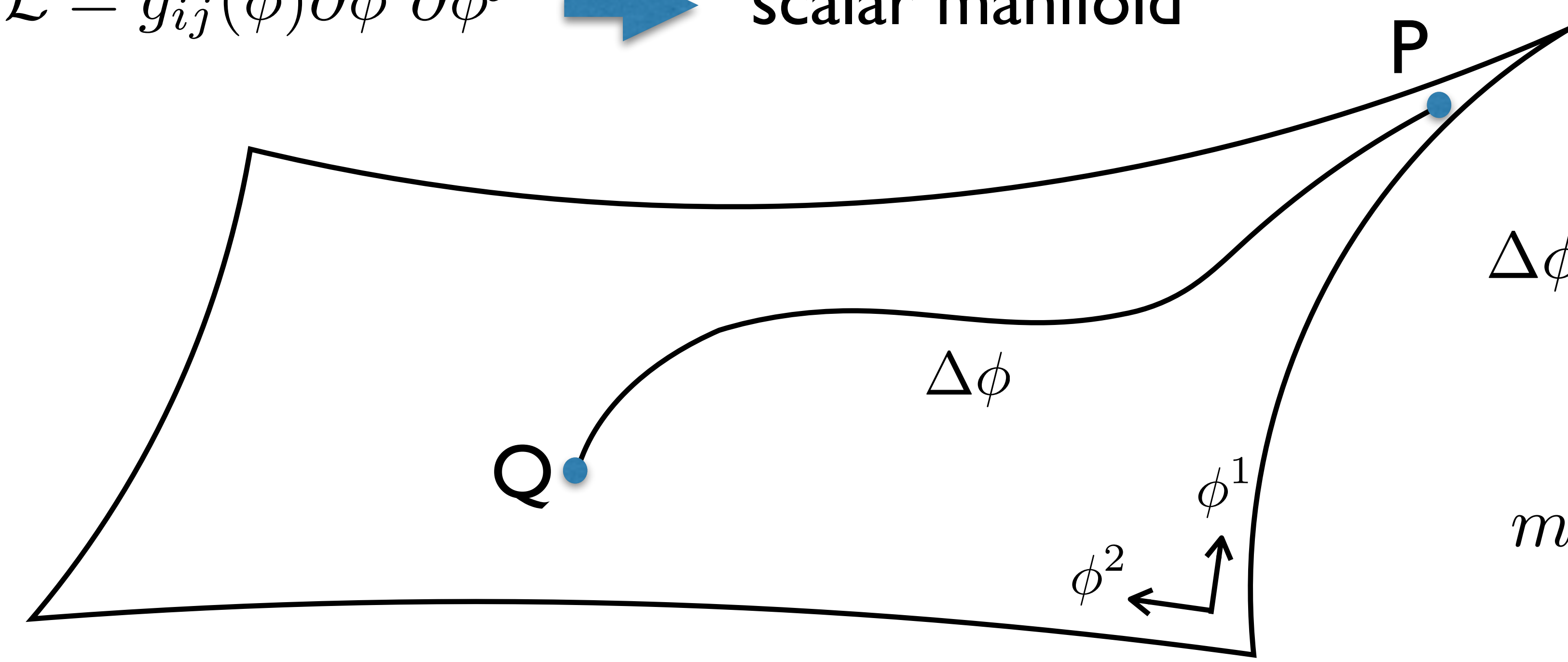
because an infinite tower of states become exponentially light

$$m \sim m_0 e^{-\alpha\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty$$

[Ooguri-Vafa'06]

Consider the moduli space of an effective theory:

$$\mathcal{L} = g_{ij}(\phi)\partial\phi^i\partial\phi^j$$ → scalar manifold



$\Delta\phi =$ geodesic distance between P and Q

$$m(P) \sim m(Q)e^{-\alpha\Delta\phi}$$

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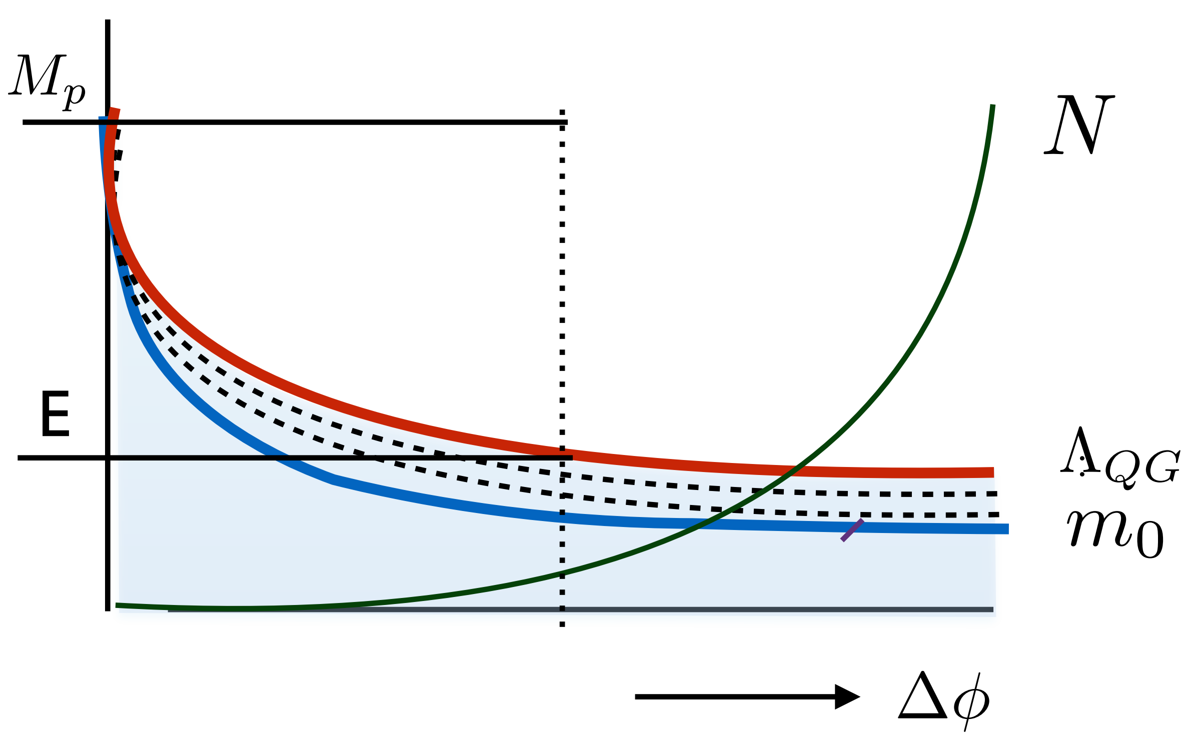
$$m \sim m_0 e^{-\alpha\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty$$

This signals the breakdown of the effective theory:



$$\Lambda_{QG} \sim M_p \exp(-\alpha\Delta\phi)$$

Species scale: $\Lambda_{QG} = \frac{M_p}{\sqrt{N}}$

[Dvali'07]

(scale at which QG effects become important)

String theory evidence

Recent works:

[Grimm, Palti, IV'18] [Grimm,Palti,Li'18] [Gendler,IV'20]

- 4d N=2 theories: Calabi-Yau compactifications of Type II

  Proven for towers of BPS states → $\alpha \geq \frac{1}{\sqrt{6}}$

- 5d/6d N=1 theories: F-theory CY compactifications [Lee,Lerche,Weigand'18-19] [Corvilain, Grimm, IV'18]

  Tensionless strings (wrapping M2-branes)

- 4d N=1 theories: Orientifold flux compactifications

  BPS strings [Lanza,Marchesano,Martucci, IV'20]

→ *uncover interesting relations between swampland conjectures and theorems of algebraic geometry (limiting mixed hodge structures), BPS counting and modular forms*

see also [Cecotti'20][Marchesano,Wiesner'19] [Grimm,van de Heisteeg'19][Baume,Marchesano,Wiesner'19] [Font,Herraez,Ibanez'19] [Grimm et al'20] [Lee et al'20]

Phenomenological implications

It gives an upper bound on the scalar field range that can be described by an effective field theory with finite cut-off

$$\Delta\phi \leq \frac{1}{\alpha}\log\frac{M_p}{\Lambda}$$

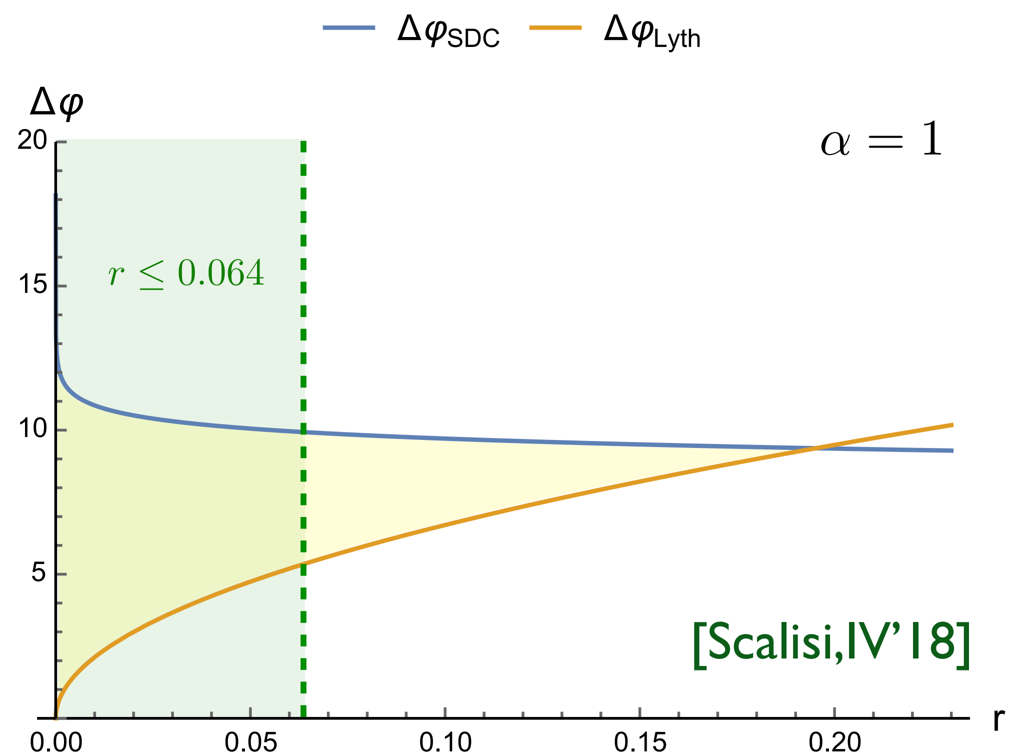
Refined SDC: $\alpha \sim \mathcal{O}(1)$

- *Large field inflation*
- *Cosmological relaxation of the EW scale*

For $H \leq \Lambda$:

$$\Delta\phi \leq \frac{1}{\alpha}\log\frac{M_p}{H} = \frac{1}{\alpha}\log\sqrt{\frac{2}{\pi^2 A_s r}}$$

Opposite scaling than Lyth bound!



Large field inflation is not ruled out but constrained

Cosmological signatures of the tower?

Relevant questions for phenomenology

- A lot of evidence for the SDC in theories with extended supersymmetry

  What happens in the presence of a potential and for non-geodesic trajectories?

- Undetermined O(1) factor in the conjecture

$$m \sim m_0 e^{-\alpha \Delta\phi} \quad , \quad \alpha \sim \mathcal{O}(1)$$

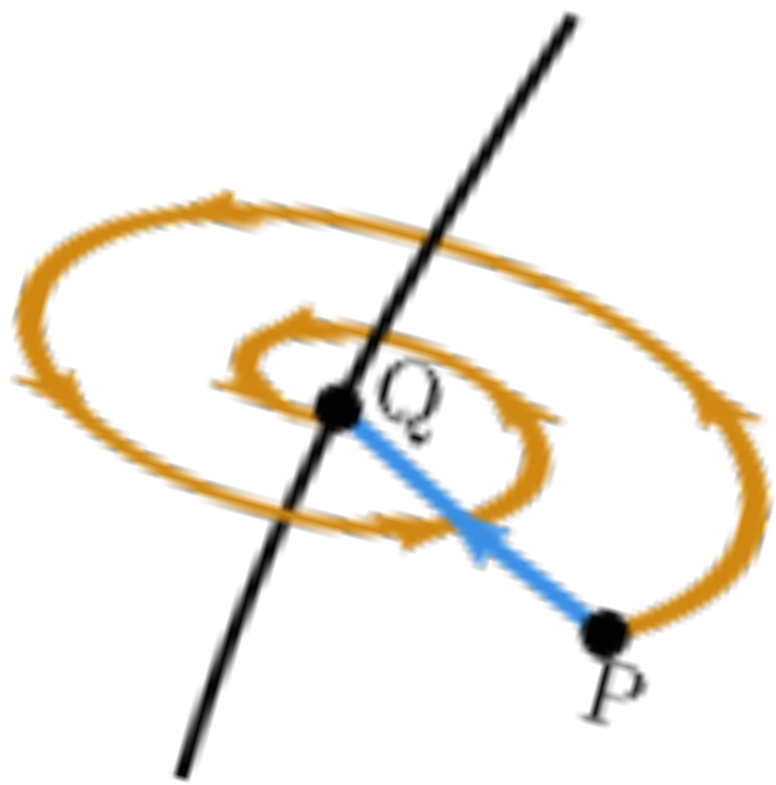
  Can we be precise about its value?

(2) Bounds on non-geodesics

[Calderon-Infante, Uranga, IV ’20]

Caveat…

What about non-geodesic trajectories?



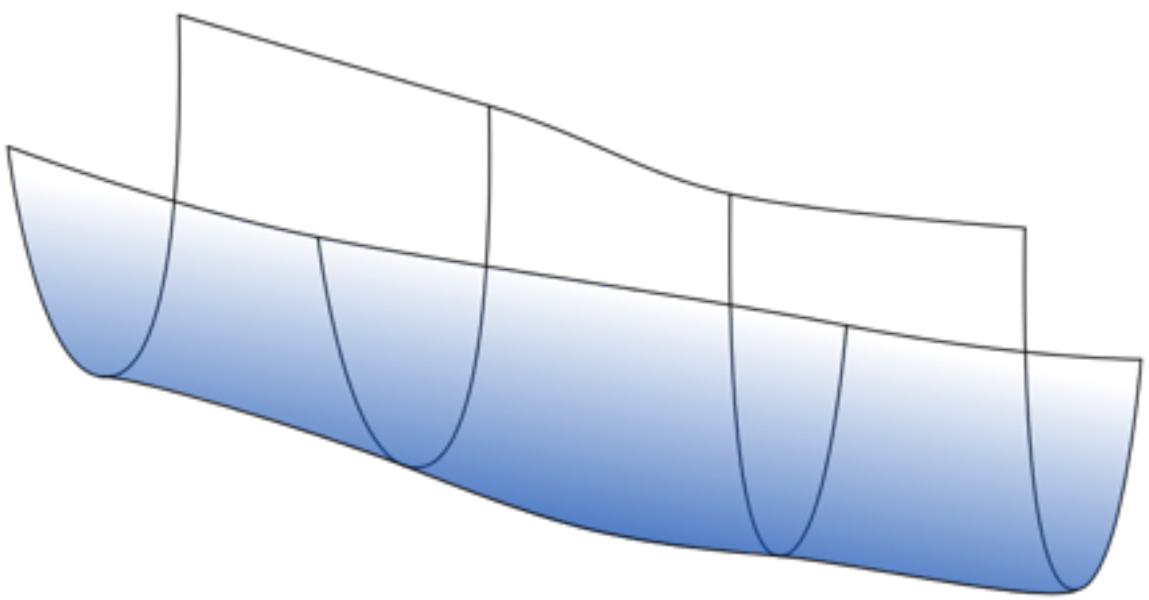
Very important for inflation!

Caveat…

…but notice

The SDC constraints geodesics in pseudo-moduli spaces

(as long as masses are small compared to cut-off)



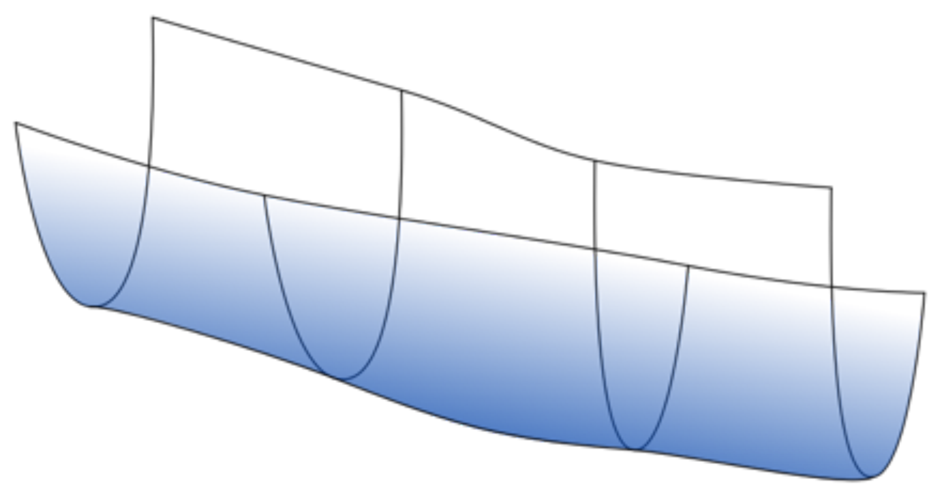
Valleys of the potential should also obey the SDC

Puzzle

But… Notion of geodesic changes at different energy scales in the presence of a potential

$\mathcal{M}$: *moduli space before adding the potential*

$\overline{\mathcal{M}}$: *moduli space after adding the potential*



SDC might be satisfied in $\mathcal{M}$ but not in $\overline{\mathcal{M}}$

At which energy regime should we apply the SDC?

Proposal

SDC should apply at any energy scale, i.e. in any EFT valid at any intermediate energy scale

Implication:

Consistency of SDC at any energy scale → Constraints on potentials consistent with quantum gravity

(it restricts the amount of non-geodesicity allowed for trajectories along valleys of the potential)

Example

Hyperbolic moduli space: $\mathcal{L} \supset \frac{n^2}{s^2} \left(\partial_\mu s \, \partial^\mu s + \partial_\mu \phi \, \partial^\mu \phi \right)$

Tower: $M \sim s^{-a} \sim \exp\left(-\alpha_{\text{geod}} \Delta\right) \;, \quad \alpha_{\text{geod}} = \frac{a}{n}$

SDC satisfied along (geodesic) saxionic trajectories

Consider trajectory $\phi = f(s)$ $\qquad d\Delta = \frac{n}{s} \sqrt{1 + f'(s)^2} \, ds$

- $f'(s) \to 0$ *Asymptotically geodesic trajectory* $\alpha = \alpha_{\text{geod}}$ ✓
- $f'(s) \to \beta \equiv \text{const.}$ *Critical trajectory* $\alpha = \frac{\alpha_{\text{geod}}}{\sqrt{1+\beta^2}}$ ✓
- $f'(s) \to \infty$ *Swampy trajectory* *(No exponential decay)* ✗

Geometric formulation

Exponential rate of the tower: $\alpha(\Delta) = -\frac{d \log M}{d\Delta} = -T^i \, \partial_i \log M$

tangent vector

$\mathbb{G}$: *subspace spanned by tangent vectors of asymptotically geodesic trajectories*

$\mathbb{M}$: *subspace spanned by gradient vectors of logM for all towers of states*

For any vector in $\mathbb{G}$, there must be at least one non-orthogonal vector in $\mathbb{M}$ (i.e. a suitable tower of states becoming massless)

→ similar to Completeness Hypothesis

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Exponential rate of the tower: $\alpha(\Delta) = -\dfrac{d\log M}{d\Delta} = -T^i\,\partial_i \log M$

Stronger version: Lower bound on $\alpha(\Delta)$

Proposals: $\alpha \geq \dfrac{1}{\sqrt{2n}}$ for CY_n ; $\alpha \geq \dfrac{1}{\sqrt{(d-2)(d-3)}}$; $\alpha \geq \dfrac{1}{2}\left(\dfrac{Q}{m}\right)_{\mathrm{extr.}}$

[Grimm, Palti, IV'18] [Gendler,IV'20] [Bdroya,Vafa'19] [Andriot et al'20] [Gendler,IV'20] [Lanza et al'20]

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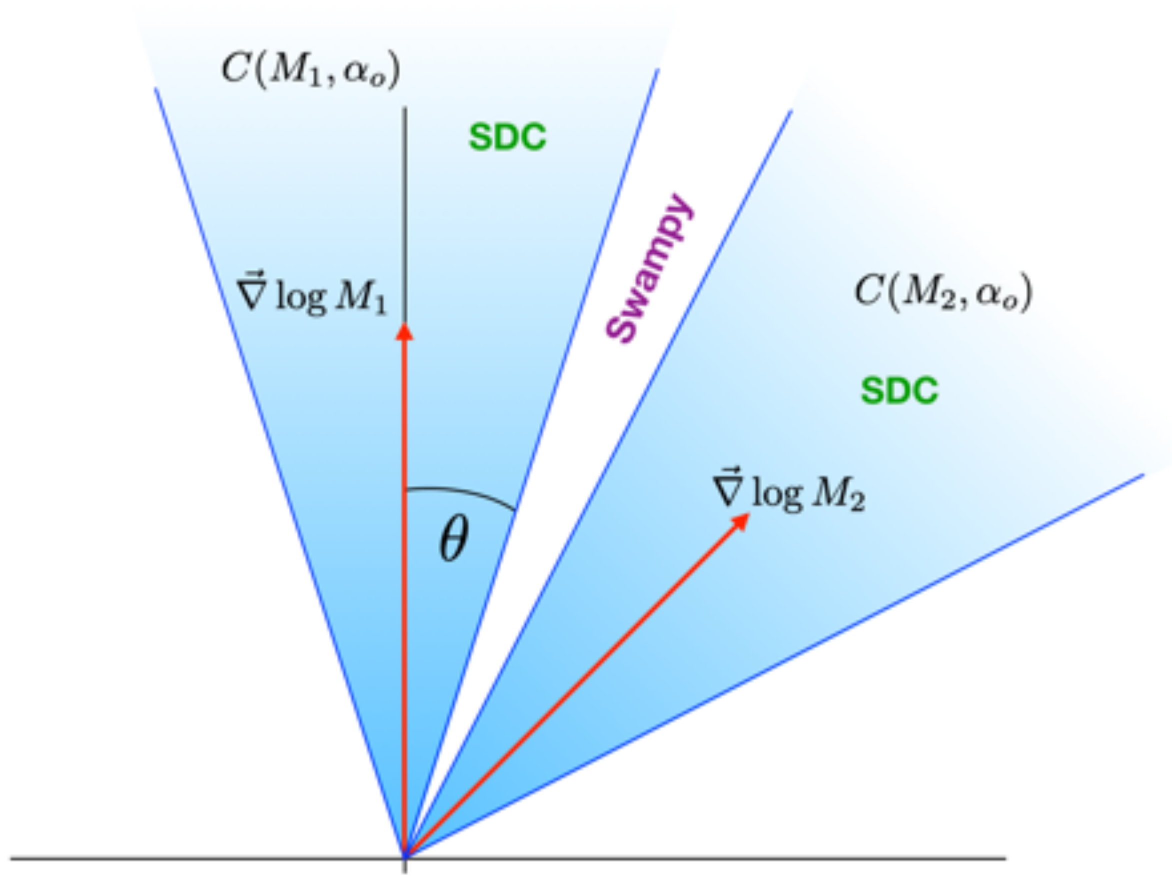
For any vector in $\mathbb{G}$, there must be at least one non-orthogonal vector in $\mathbb{M}$ (i.e. a suitable tower of states becoming massless) with $\alpha(\Delta) \geq \alpha_0$

→ similar to WGC $\frac{Q}{M} \geq \left(\frac{Q}{M}\right)_{\text{extr.}}$

Trajectories allowed by SDC

A tower allows the SDC to be satisfied along trajectories with a certain level of non-geodesicity

$$\mathcal{T}_{SDC} = \bigcup \mathcal{C}_{M_i}(\alpha_0)$$



→ convex hull condition

SDC satisfied if $\mathbb{G} \subset \mathcal{T}_{SDC}$

Convex Hull SDC

Define scalar charge to mass ratio: $\vec{z} = -g^{-\frac{1}{2}} \vec{\nabla} \log M$ *(in analogy to WGC)*

Yukawa scalar force: $\mathcal{L} \supset M^2(\phi)\chi^2 \simeq 2M\partial_\phi M \, \phi\chi^2 + \ldots$ → *scalar charge*

Exponential rate of the tower: $\alpha(\Delta) = \vec{n} \cdot \vec{z}$

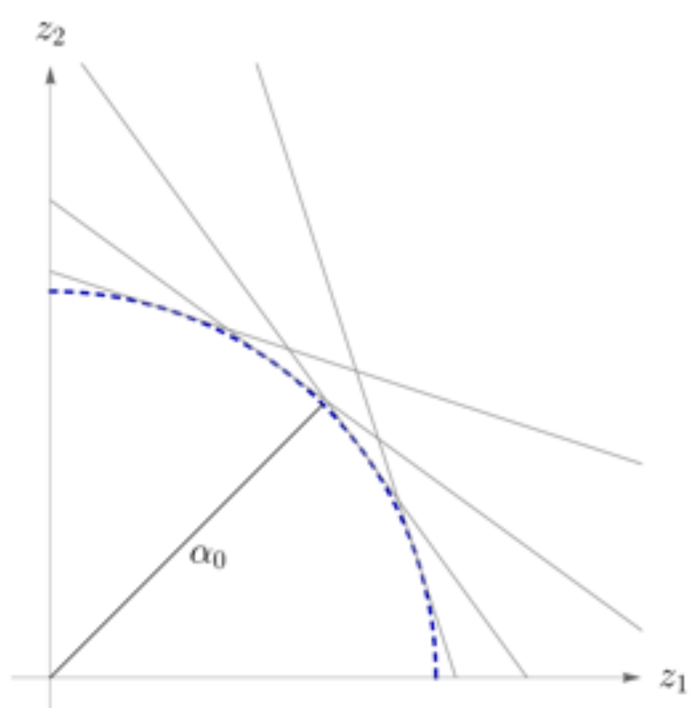
$\mathbb{G}$: *space of possible "charge" directions*

SDC: For every charge direction, there must exist a charged infinite tower of states with $\alpha(\Delta) \geq \alpha_0$

Define "extremal states" as those satisfying

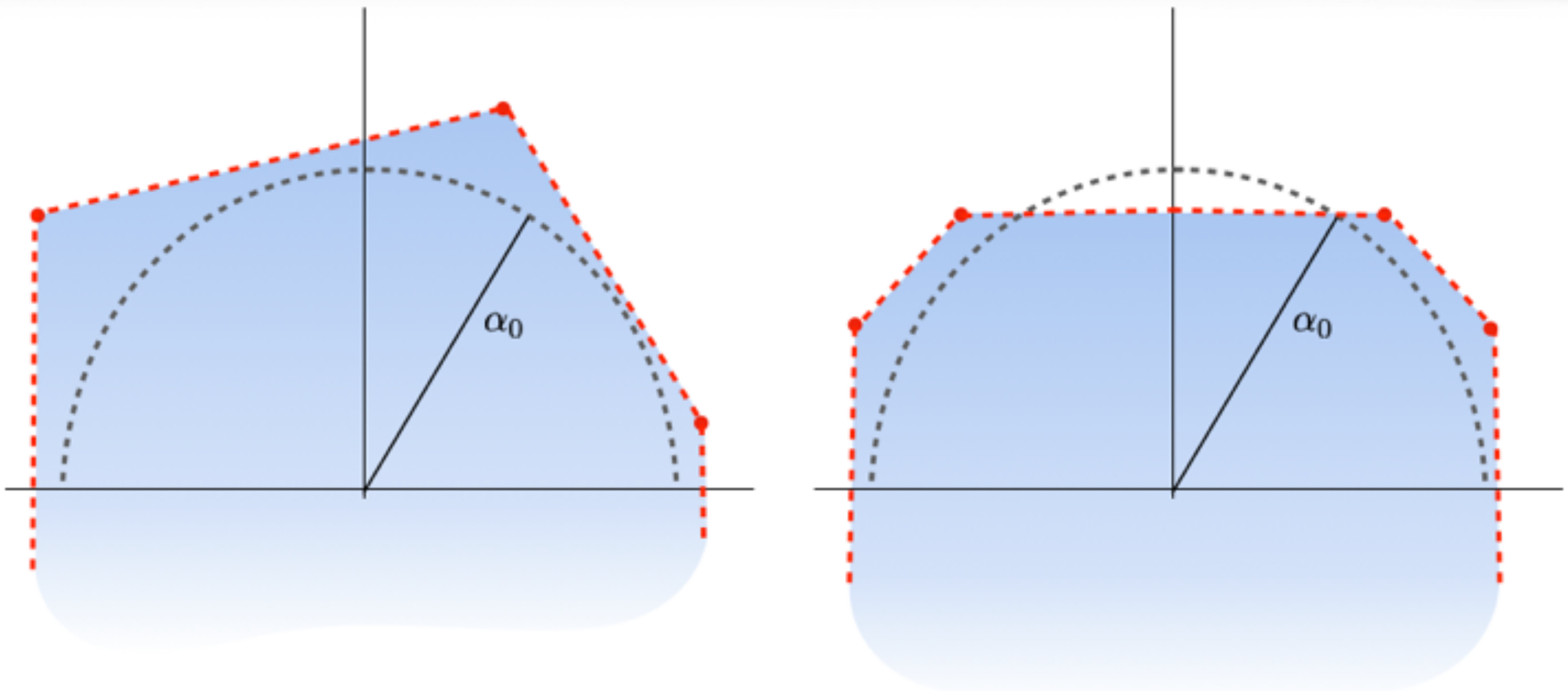
$$\vec{n} \cdot \vec{z} = \alpha_0$$

"extremal region" = ball of radius α_0



Convex Hull SDC

Convex Hull SDC: the SDC is satisfied by any trajectory with exponential rate α_0 if the convex hull of the vectors $\vec{z}_i$ contains the extremal region, namely the unit ball of radius α_0



It resembles a Scalar WGC $|\vec{z}| \geq \mathcal{O}(1)$

[Palti16]

Convex Hull SDC

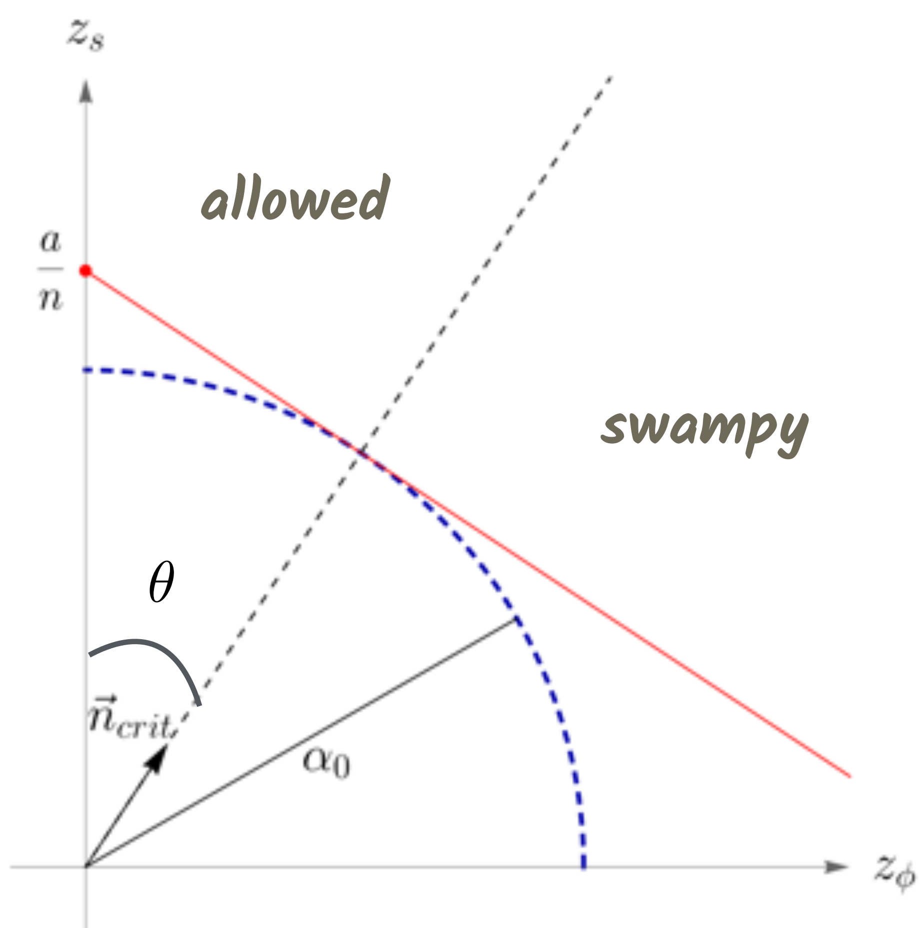
SDC can be used to constrain either:

- the spectra of the theory, by requiring as many towers as needed to satisfy the convex hull condition,
- or the possible trajectories along which the SDC can be satisfied for a fixed set of towers and, therefore, the scalar potentials consistent with quantum gravity.

Example

Hyperbolic moduli space: $\mathcal{L} \supset \frac{n^2}{s^2}\left(\partial_\mu s\, \partial^\mu s + \partial_\mu \phi\, \partial^\mu \phi\right)$

Tower: $M \sim s^{-a} \sim \exp(-\alpha\Delta)\ ,\quad \alpha = \frac{a}{n}$ ➡ $\vec{z} = (0, a/n)$



Critical paths:

$$\phi = f(s)\ ,\ f'(s) \to \beta \equiv \text{const.}$$

$$\vec{n} = \frac{1}{\sqrt{1+\beta^2}}(\beta, 1)$$

Those with $\beta \le \beta_{\max}$ will satisfy the SDC

$$\alpha_{\text{crit.}} = \frac{a}{n\sqrt{1+\beta^2}}$$

$$\beta_{\max} = (\cos\theta)^{-2} - 1 = \left(\frac{a}{n\alpha_0}\right)^2 - 1$$

Evidence in string theory

Calabi-Yau flux compactifications of Type II string theory:

- Asymptotic behaviour of field metric:

$$K = -\log(p_d(s^j) + \mathcal{O}(e^{2\pi i t^j}))$$

$$d\Delta^2 = \sum_i \frac{n_i^2}{(s^i)^2}\left[(ds^i)^2 + (d\phi^i)^2\right] + \ldots$$

Hyperbolic behaviour

- Asymptotic behaviour of flux scalar potential:

$$V(\kappa s^i, \kappa\phi^i) \simeq \kappa^{n_i} V(s^i, \phi^i)$$

$$\partial_{s^i} V = 0 \to s^i = \beta\,\phi^i + \ldots$$

with β a **flux-independent** parameter.

[Grimm,Li,IV'19]

→ *critical path!*

Evidence in string theory

Calabi-Yau flux compactifications of Type II string theory:

→ lead to the most generic potentials allowing for maximum non-geodesicity of the potential valleys while respecting the SDC along them.

$$\Delta\phi \leq \frac{1}{\alpha} \log \frac{M_p}{\Lambda} \qquad \alpha = \frac{\alpha_{\text{geod}}}{\sqrt{1+\beta^2}}$$

confirming backreaction issues found in [Baume,Palti'16] [I.V.,'16]

Hence, the SDC also constraints axionic trajectories!

Open tasks: Check other compactifications

e.g. [McAllister,Silversten,Westphal,Warse"14]

Go beyond parametric control

(3) Sharpening order one factors

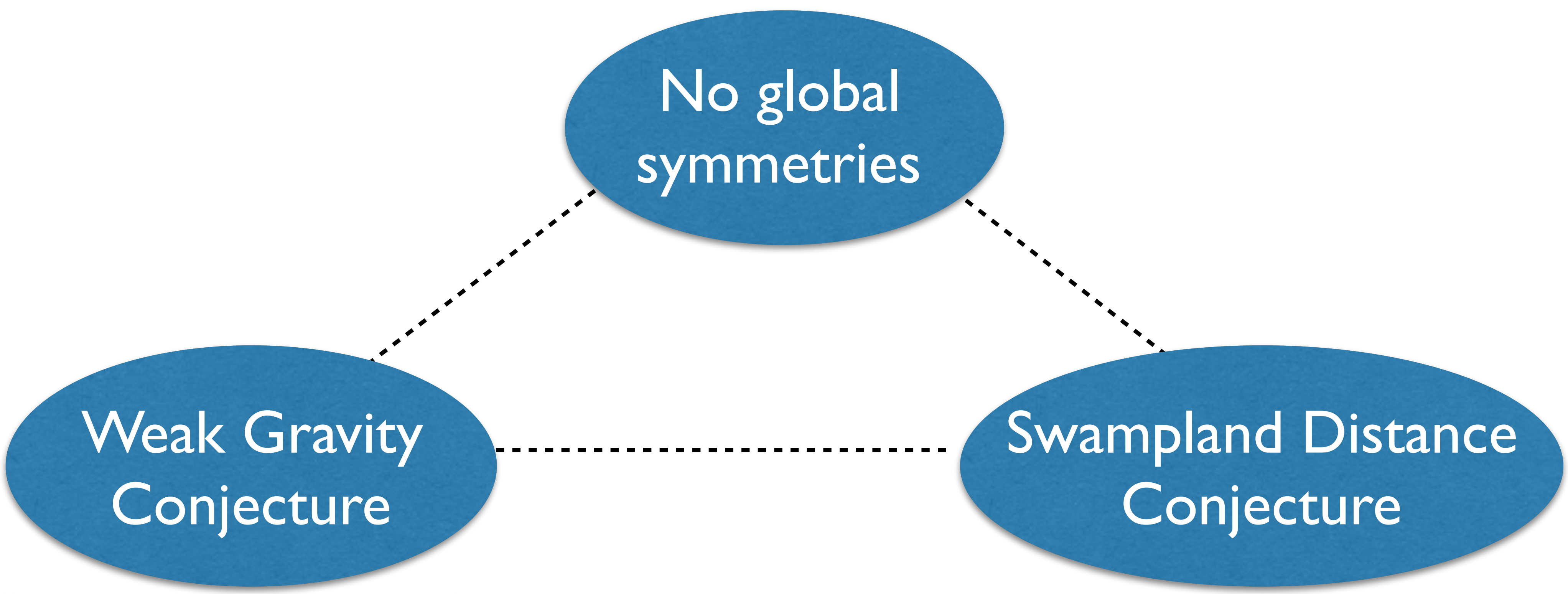
[Lanza, Marchesano, Martucci, IV ’20]

[Gendler, IV ’20]

Sharpening order one factors

So far, the infinite tower of states is always charged under some p-form gauge field that becomes weakly coupled asymptotically.

$$g \to 0 \quad \textbf{at} \quad \Delta\phi \to \infty$$



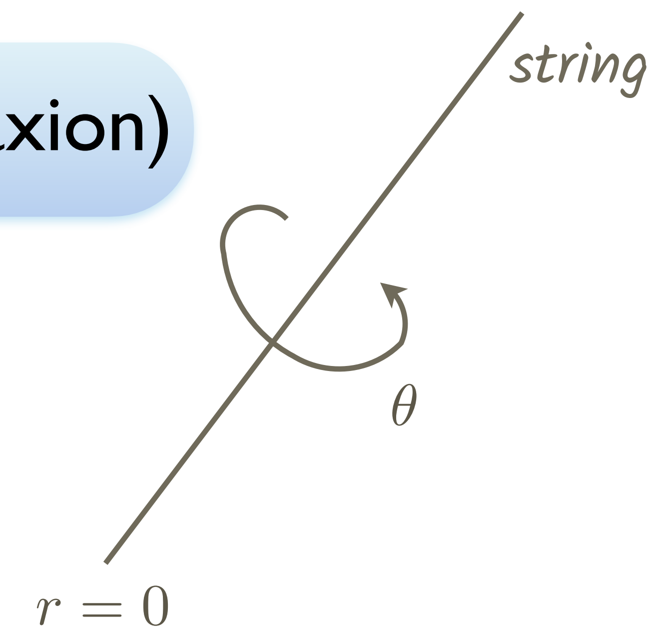
Same tower satisfies the SDC and the WGC and acts as a quantum gravity obstruction to restore a global symmetry

Exponential rate fixed by black hole extremality bound!

N=1 4d EFTs

String compactifications suggest that an approximate axionic shift symmetry emerges at infinite field distances

Consider a BPS string charged under B_2 (dual to the axion)



- Saxions are driven by the string backreaction to infinite distance at the string core!

$$s(r) = s_0 + \frac{e}{2\pi} \log \frac{r}{r_0} \qquad \phi = \phi_0 + \frac{e\theta}{2\pi}$$

- The string becomes weakly coupled and tensionless as $s^i \to \infty$

String Backreaction ⟷ RG flow ⟷ trajectory in field space

[Polchinski'14]

N=1 4d EFTs

Weakly coupled axionic strings ⟷ Infinite field distance limits

(Proposal: All infinite distance limits of a 4d EFT can be realised as an RG flow endpoint of an axionic string) [Lanza, Marchesano, Martucci, IV '20]

If the string satisfies the WGC: $Q \geq \gamma\, T$

$$\Lambda^2_{max} \equiv T_{max} \leq T_0 \exp\left(-\gamma d_{max}\right)$$

The cut-off due to the tower of string modes decreases exponentially with the distance

We derive the SDC!

Exponential rate:

$$m \sim m_0 e^{-\alpha\Delta\phi} \quad \rightarrow \quad \alpha = \frac{\gamma}{2} \geq \frac{1}{\sqrt{2n_i}}$$

using $K = -\log(s_1^{n_1} s_2^{n_2} \dots)$

Summary

- Consistency of SDC at any energy scale implies constraints on non-geodesic trajectories and, therefore, the scalar potentials consistent with quantum gravity.

- The SDC can be formulated as a convex hull condition on the scalar charge to mass ratio of the towers, in analogy to the WGC.

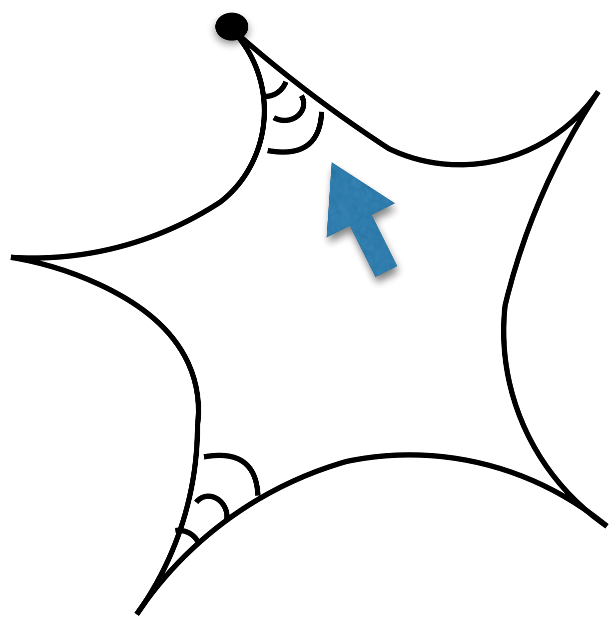
- CY flux string compactifications lead to potentials realising the maximum level of non-geodesicity consistent with the SDC.

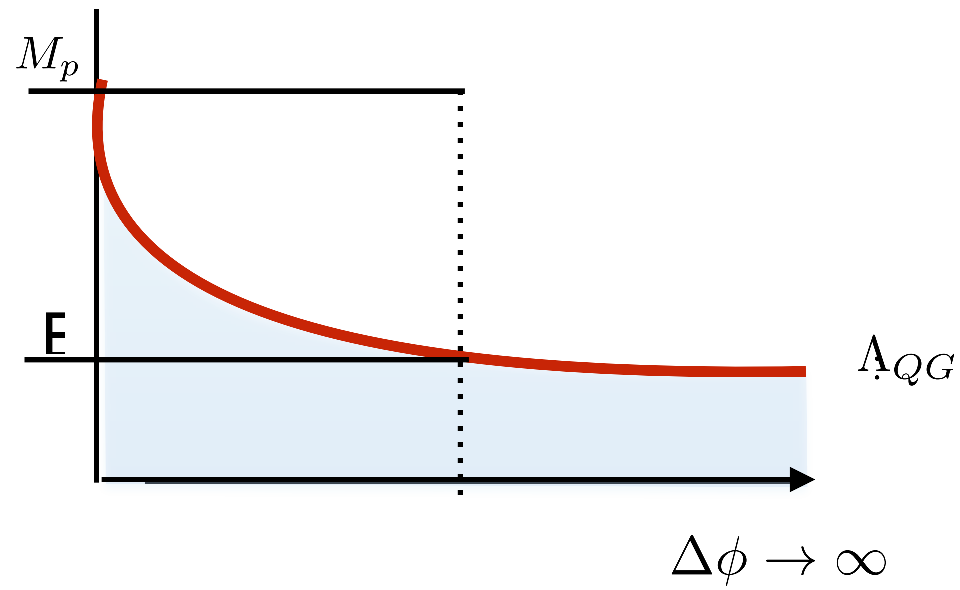
- The exponential rate of the tower is bounded by black hole extremality bound if there is a vanishing gauge coupling asymptotically. Example: towers from BPS strings in N=1 4d EFTs

Thank you!

back-up slides

Asymptotic limits in moduli spaces





These limits seem under control from the point of view of QFT

but still, the EFT must break down when approaching the boundary
by quantum gravity effects

Approximate global symmetries,
Weakly coupled gauge theories,
Large field ranges…

…come at a price.

Swampland conjectures:

- Infinite towers of massless states WGC, SDC
- Runaway potentials dSC

Dual formulation in terms of 2,3-forms

4D N=1 EFT: $$S = \int \left(\frac{M_P^2}{2} R - M_P^2 K_{\alpha\bar\beta}\, d\phi^\alpha \wedge * d\bar\phi^{\bar\beta} - V \right)$$

$$(s^i, a^i) \;\to\; (l_i, B_{2i})$$

$$f_a \;\to\; C_3^a$$

$$-\frac{1}{2}\int G^{ij}\left(M_{\rm P}^2\, d\ell_i \wedge * d\ell_j + \frac{1}{M_{\rm P}^2} H_{3\,i} \wedge * H_{3\,j} \right)$$

$$G_{ij} \equiv \frac{1}{2}\frac{\partial^2 K}{\partial s^i \partial s^j}$$

$$-\int \frac{1}{2} T_{ab} F_a^a * F_4^b$$

$$V = \frac{1}{2} T^{ab} f_a f_b$$

Field metric = 2-form gauge couplings

Potential = 3-form gauge couplings

No force identities

We add BPS charged objects: $-\int d^{p+1}\xi\, T(\phi)\sqrt{-h} + e\int B_{p+1}$

Strings ➡ $T_{\mathbf{e}} = M_{\rm P}^2 |e^i \ell_i|\,, \qquad Q_{\mathbf{e}} = M_{\rm P}\sqrt{G_{ij}e^i e^j}$

Membranes ➡ $T_{\mathbf{q}} = 2M_{\rm P}^3\, e^{\frac{1}{2}K}\, |q_a \Pi^a|\,, \qquad Q_{\mathbf{q}} = M_{\rm P}\sqrt{T^{ab}q_a q_b}$

They satisfy (off-shell):

$$\|\partial T_{\rm str}\|^2 = M_{\rm P}^2 Q_{\mathbf{e}}^2$$

$$\|\partial T_{\rm mem}\|^2 - \frac{3}{2}T_{\rm mem}^2 = M_{\rm P}^2 Q_{\mathbf{q}}^2$$

They look as a no-force condition: (see also [Herraez'20])

$$G^{ij}\partial_i T \partial_j T + \frac{(p+1)(1-p)}{2}T^2 = F^{ab}q_a q_b$$

Interpretation

Low codimension objects → change asymptotic structure of vacuum

How to define T and Q?

Localised operators entering the EFT rather than states of a vacuum

$$-\int d^{p+1}\xi\, T(\phi)\sqrt{-h} + e\int B_{p+1}$$

[Polchinski'14]

Brane couplings should be regarded as defined at the EFT cut-off Λ

Classical back reaction → Classical RG flow $T(\Lambda)$

codim	brane coupling
> 2	irrelevant
2	marginally relevant
1	relevant

Strings

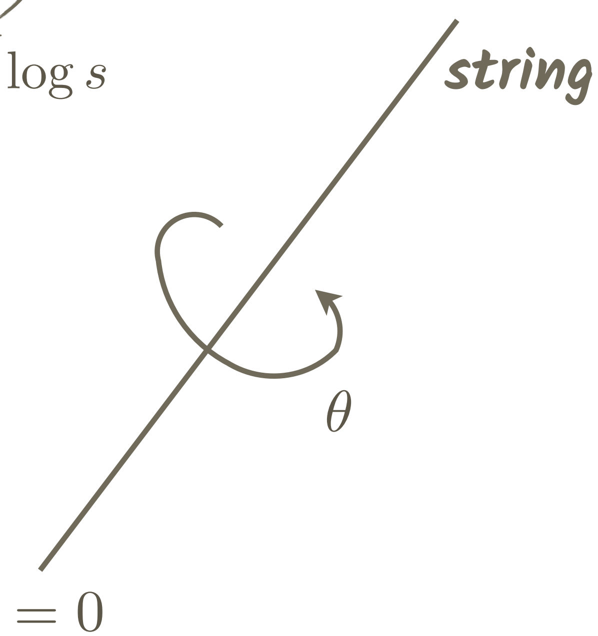
Metric Ansatz: $ds^2 = -dt^2 + dx^2 + e^{2D} dz d\bar{z}$

String induces a flow of the scalars $t = is + a$

$$s(r) = s_0 + \frac{e}{2\pi} \log \frac{r}{r_0} \quad , \qquad a = a_0 + \frac{e\theta}{2\pi}$$

String tension: $T(r) = M_p^2 \, e \, \ell(r) \quad , \quad \ell(r) = -\frac{1}{2}\frac{dK}{ds} = \frac{1}{2s(r)}$

$K = -\log s$



RG flow:

$r_\Lambda = \Lambda^{-1}$

$$\frac{T(\Lambda)}{M_{\rm P}^2} = \frac{e}{2s_0 + \frac{e}{\pi}\log(\Lambda r_0)}$$

String Backreaction ⟷ RG flow

⟷ trajectory in field space

Strings

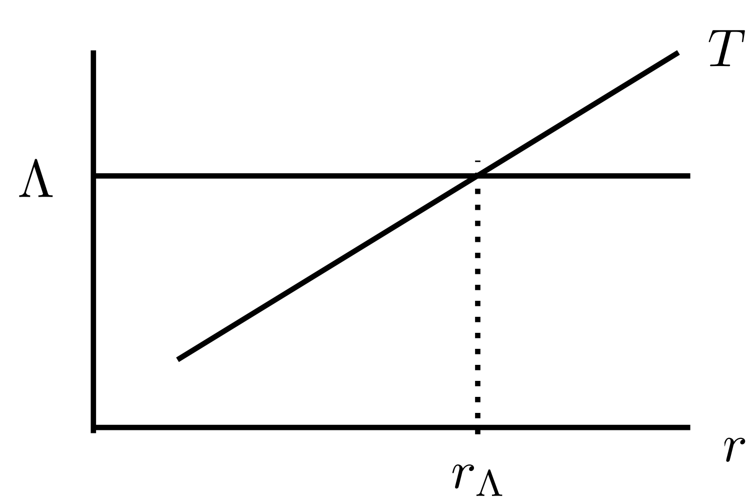
Recall: $$\frac{T(\Lambda)}{M_p^2} = \frac{e}{\frac{M_p^2}{T(\Lambda_0)} + \frac{e}{\pi}\log\frac{\Lambda}{\Lambda_0}}$$

- At $\Lambda \to 0 \ (r \to \infty): \ T(\Lambda) \to \infty$

EFT breaks down at $$\Lambda_{strong} = \Lambda \exp(-\frac{\pi M_p^2}{T(\Lambda)})$$

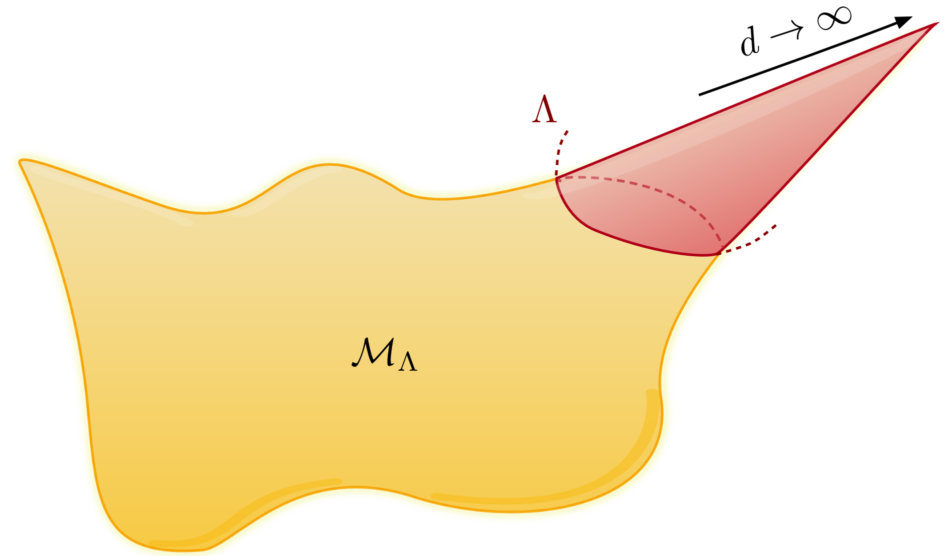
- At $\Lambda \to \infty \ (r \to 0): \ T(\Lambda) \to 0 \ , \ s(r_\Lambda) \to \infty$

EFT breaks down at $$\Lambda_{max} = T(\Lambda_{max})^{1/2}$$



Derivation of SDC

For a given Λ, the moduli space accessible by the EFT is finite since it breaks down when



$$\Lambda_{max}^2 = T(\Lambda_{max})$$

Field distance from r_0 to $r_{max} = \Lambda_{max}^{-1}$:

$$d_{max} = \int_0^{\sigma_{max}} Q_s(\sigma) d\sigma = \frac{1}{M_p} \int_{T_{max}}^{T_0} \frac{1}{Q} dT \leq \frac{1}{\gamma} \log \frac{T_0}{T_{max}}$$

field metric = gauge coupling $G_{ij} = \frac{1}{2}\partial_{ij}^2 K$

RG flow + no-force condition: $Q^2 = -\frac{dT}{d\sigma}$

WGC $QM_p \geq \gamma T$

Max cut-off: $\Lambda_{max}^2 \equiv T_{max} \leq T_0 \exp\left(-\gamma d_{max}\right)$ **SDC!**

WGC for strings → SDC with $\lambda = \gamma/2$

DASC

Weakly coupled axionic strings ⟷ Infinite field distance limits

Distant Axionic String Conjecture (DASC):

All infinite distance limits of a 4d EFT can be realised as an RG flow endpoint of a fundamental axionic string

Evidence from String theory: [Lanza, Marchesano, Martucci, IV'to appear]

Higher dimensional spaces: only a subset of BPS strings are weakly coupled

$$\mathcal{C}_S^{\rm EFT} = \{\mathbf{e} \in N_{\mathbb{Z}} | \langle \mathbf{m}, \mathbf{e} \rangle \geq 0 \ \forall \mathbf{m} \in \mathcal{C}_I\} \quad \subset \quad \mathcal{C}_S$$

instanton charges ($\mathbf{m}$) — *string charges* ($\mathbf{e}$)

If $\mathbf{e} \in \mathcal{C}_S^{\rm EFT}$ ⟶ *weakly coupled string* : *tensionless at infinite distance*

If $\mathbf{e} \in \mathcal{C}_S - \mathcal{C}_S^{\rm EFT}$ ⟶ *strongly coupled string* : *tensionless at finite distance*

WGC for strings

Result: WGC for strings → SDC with $\lambda = \gamma/2$

extremality factor

What strings satisfy WGC?

- All BPS strings satisfy $\|\partial T_{\rm str}\|^2 = M_{\rm P}^2 Q_{\mathbf{e}}^2$
- Satisfying WGC $Q \geq \gamma\, T$ is a non-trivial condition on the Kahler metric

Check: Strings at the asymptotic limits of field space satisfy WGC

(we replace asymptotic behaviour of $K = -\log(s_1^{n_1} s_2^{n_2} \ldots)$ *)*

→ $$\lambda = \frac{1}{\sqrt{2n_i}}$$

(Same type of bound than for N=2)

In terms of extremality factors

Recall, WGC:

$\exists$ *p-dim state with* $QM_p \geq \gamma T$ *and* $\gamma^2 \equiv \left.\frac{Q}{T}\right|_{\text{extr.}} = \frac{p(2-p)}{2} + \frac{|\alpha|^2}{4}$

- SDC tower coming from BPS string in N=1 4d:

$$m \sim m_0 e^{-\lambda \Delta \phi} \quad \rightarrow \quad \lambda = \frac{\gamma}{2} = \frac{|\alpha_s|}{4}$$

- Flux induces N=1 potential dual to the charge of a BPS membrane:

$$|\nabla V| \geq cV \quad \rightarrow \quad c = |\alpha_m|$$

if membrane is extremal

- SDC tower coming from BPS particles in N=2 4d: [Gendler, IV' 20]

$$m \sim m_0 e^{-\lambda \Delta \phi} \quad \rightarrow \quad \lambda = \frac{|\alpha_p|}{2}$$

In terms of geometric data

Recall, WGC:

$\exists$ *p-dim state with* $QM_p \geq \gamma T$ *and* $\gamma^2 \equiv \left.\frac{Q}{T}\right|_{\text{extr.}} = \frac{p(2-p)}{2} + \frac{|\alpha|^2}{4}$

- SDC tower coming from BPS string in N=1 4d:

using $K = -\log(s_1^{n_1} s_2^{n_2} \dots)$

$$m \sim m_0 e^{-\lambda \Delta\phi} \;\Rightarrow\; \lambda = \frac{\gamma}{2} = \frac{|\alpha_s|}{4} \geq \frac{1}{\sqrt{2n_i}}$$

- Flux induces N=1 potential dual to the charge of a BPS membrane:

$$|\nabla V| \geq cV \;\Rightarrow\; c = |\alpha_m| \geq \frac{2}{n_i}$$

if membrane is extremal

$$\frac{|\nabla m|^2}{m^2} \sim \frac{|\nabla V|}{V} \; ?$$

(see also [Andriot et al'20])

- SDC tower coming from BPS particles in N=2 4d: [Gendler, IV' 20]

$$m \sim m_0 e^{-\lambda \Delta\phi} \;\Rightarrow\; \lambda = \frac{|\alpha_p|}{2} \geq \frac{1}{\sqrt{2n_i}}$$

Membranes

Classical backreaction: $$T_{\mathbf{q}}^{\text{eff}}(\Lambda) = \frac{T_{\mathbf{q}}}{1 - \frac{kT_{\mathbf{q}}}{2M_{\text{P}}^2 \Lambda}}$$

The charge parametrises the scalar potential: $Q^2 = T^{ab} q_a q_b = 2V(q)$

$$||\partial T_{\text{m}}||^2 - \frac{3}{2} T_{\text{m}}^2 = M_{\text{P}}^2 Q_{\text{m}}^2$$ *no-force condition*

$$e^K(||DW||^2 - 3W^2) = V(q)$$ *N=1 sugra*

If $\partial_\alpha T_{\mathbf{q}} = K_\alpha \sigma_\alpha T_{\mathbf{q}}$ ➡ $Q_m(\Lambda) M_p = \gamma\, T_m(\Lambda)$ extremal with

$$\gamma = \sqrt{\frac{|\alpha^2|}{4} - \frac{3}{2}}$$

diatonic factor $T_{ab} \sim e^{-\alpha\phi}$

These extremal membranes satisfy $||\partial Q_m^2|| = c\, Q_m^2$

➡ $||\partial V^2|| = c\, V^2$ with $c = |\alpha|$ **dS conjecture!**

Membranes

Result: Saturating WGC → No deSitter conjecture

What membranes saturate WGC?

At the asymptotic limits, we can identify some membranes saturating indeed the WGC with

$$\gamma^2 = \sum_i 2n_i\sigma_i^2 - \frac{3}{2}$$

using $K = -\log(s_1^{n_1} s_2^{n_2} \dots)$

$$\rightarrow \quad c = |\alpha| = 2\sqrt{\sum_i 2n_i\sigma_i^2} \geq \sum_i \frac{2}{n_i}$$

Consistent with no-go theorem for dS at asymptotic limits [Grimm,Li, IV'19]

Recall that in N=2: $\lambda = \alpha/2$

(exp rate of 1-form gauge coupling)

Could $\frac{|\nabla M|^2}{M^2} \sim \frac{|\nabla V|}{V}$? [Andriot et al'20]