Testing General Relativity at Cosmological Scales

$$G^a_b + \Lambda\delta^a_b = \kappa T^a_b$$

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi\rho_{DE}(\frac{1}{3} + w)$$

$$S_{(5)} = \frac{1}{2}M^3_{(5)}\int d^4x\,dy\sqrt{-g_{(5)}}R_{(5)} + \frac{1}{2}M^2_{(4)}\int d^4x\sqrt{-g_{(4)}}R_{(4)} + S_{matte}$$

$$P_\kappa(l) = \frac{9}{4}H_o^4\Omega_m^2\int_0^{\chi_H}\frac{g^2(\chi)}{a^2(\chi)}P_{3D}\left(l/\sin_K(\chi),\chi\right)d\chi$$

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Possible Causes to Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
  - I) A dark energy component, vacuum energy/cosmological constant, quintessence field …
  - II) Modified Gravity: A modification to general relativity at cosmological scales; a different theory of gravity
  - ~~III) Apparent acceleration due to different rates of expansion in the universe (excluded by ksz and also the absence of growth suppression)~~
  - IV) A completely unexpected explanation

An important question:
Distinguishing between possibility I (i.e. dark energy) or possibility II (i.e. modified gravity) from using cosmological data

- the growth rate of large scale structure can be used to distinguish between the two competing alternatives

- At least two methods have been used in the literature so far:

    - Method-1) Looking for inconsistencies in the dark energy parameter spaces (e.g. Lue, Scocciamarro, 2004; Song, 2005; MI, Upadhye, and Spergel, 2005,2006 and others)

    - Method-2) Constraining the growth of structure parameters (e.g. Linder, 2005; Kayam, 2006; Polarski & Gannouji, 2008; Bean, Tagmatithan, 2010; Dossett, MI, Moldenhauer, 2011) and many other papers and authors. See e.g. 74 of them in Dossett et al. 2011.

Trying to
find clues
from observations

Method IIa: based on parameterization of the Growth rate of large scale structure: the growth index, γ

e.g. Linder, 2005, Gong, MI, Wang 2009; MI, Dossett, 2009;

- large scale matter density perturbation, $\delta = \Delta\rho_m / \rho_m$, satisfies the ODE:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m \delta = 0$$

- The ODE can be written in terms of the logarithmic growth rate $f = d\ln\delta / d\ln a$ as:

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{eff}}{G}\Omega_m$$

where the underlying gravity theory is expressed via the expression for G_{eff} , *$H(z)$ and $\Omega_m(z)$*.

A constant growth rate index parameter

- The growth function f *can be* approximated using the ansatz

$$f = \Omega_m^{\gamma}$$

where γ is the growth index parameter

- It was found there that

$$f(z) = \Omega_m^{0.6} \qquad f = \Omega_m^{4/7}$$

were good approximations for matter dominated models.

Growth rate index parameter and [General Relativity + Dark Energy] models

- [L. Wang and Steinhardt, 1998] considered Dark Energy models with slowly varying *w* and derived

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^2(1-12w/5)}(1-\Omega_m) \qquad f = \Omega_m^{\gamma}$$

- with the asymptotic early value $\gamma_\infty = \frac{3(1-w)}{(5-6w)}$
- [see also for example Linder and Cahn, 2007; Mortonson, Hu, Huterer, 2009; Zhang et al. 2007; Gong, 2008; Polarski and Ganouji, 2008, Gong, MI, A. Wang 2009 …]
- The approximation provides a fit of about 1% to the growth function f as numerically integrated from the ODE

Redshift dependent growth rate index

[Polarski and Gannouji, PLB, 2008; MI and Dossett, PRD 2009; Gong, MI, Wang, PRD 2009]

- A parameterization that interpolates between a small/intermediate redshift expression and an asymptotic constant value at high redshifts:

$$\gamma(a) = \tilde{\gamma}(a)\, \frac{1}{1+(a_{ttc}/a)} + \gamma_{early}\, \frac{1}{1+(a/a_{ttc})}$$

or

$$\gamma(z) = \tilde{\gamma}(z)\, \frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_{early}\, \frac{1}{1+\frac{1+z_{ttc}}{1+z}}$$

where z_{ttc} is a transition resdhift from an early-time, almost constant value, to the following redshift dependent form . It gives fit to 0.01% to the ODE.

$$\gamma(a)_{late} = \tilde{\gamma}(a) = \gamma_0 + (1-a)\gamma_a$$

or

$$\gamma(z)_{late} = \tilde{\gamma}(z) = \gamma_0 + \left(\frac{z}{1+z}\right)\gamma_a$$

The growth index parameter as a discriminator for Gravity Theories

- The asymptotic constant growth index parameter takes distinctive value for distinct gravity theories

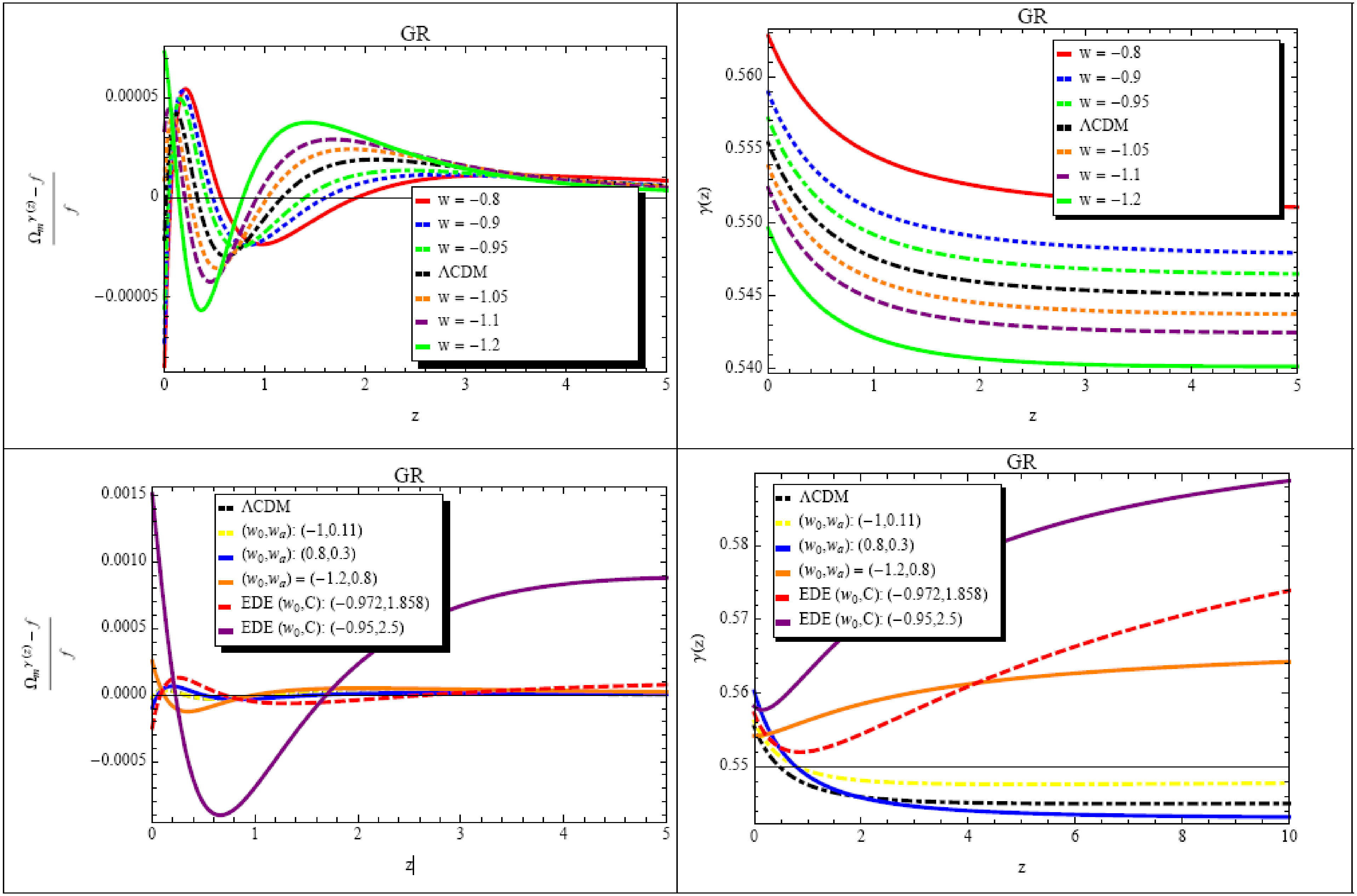
- Thus, can be used to probe the underlying gravity theory and the cause of cosmic acceleration

- γ=6/11=0.545 for the Lambda-Cold-Dark-Matter model. (i.e. for w=-1), i.e. General Relativistic Models.

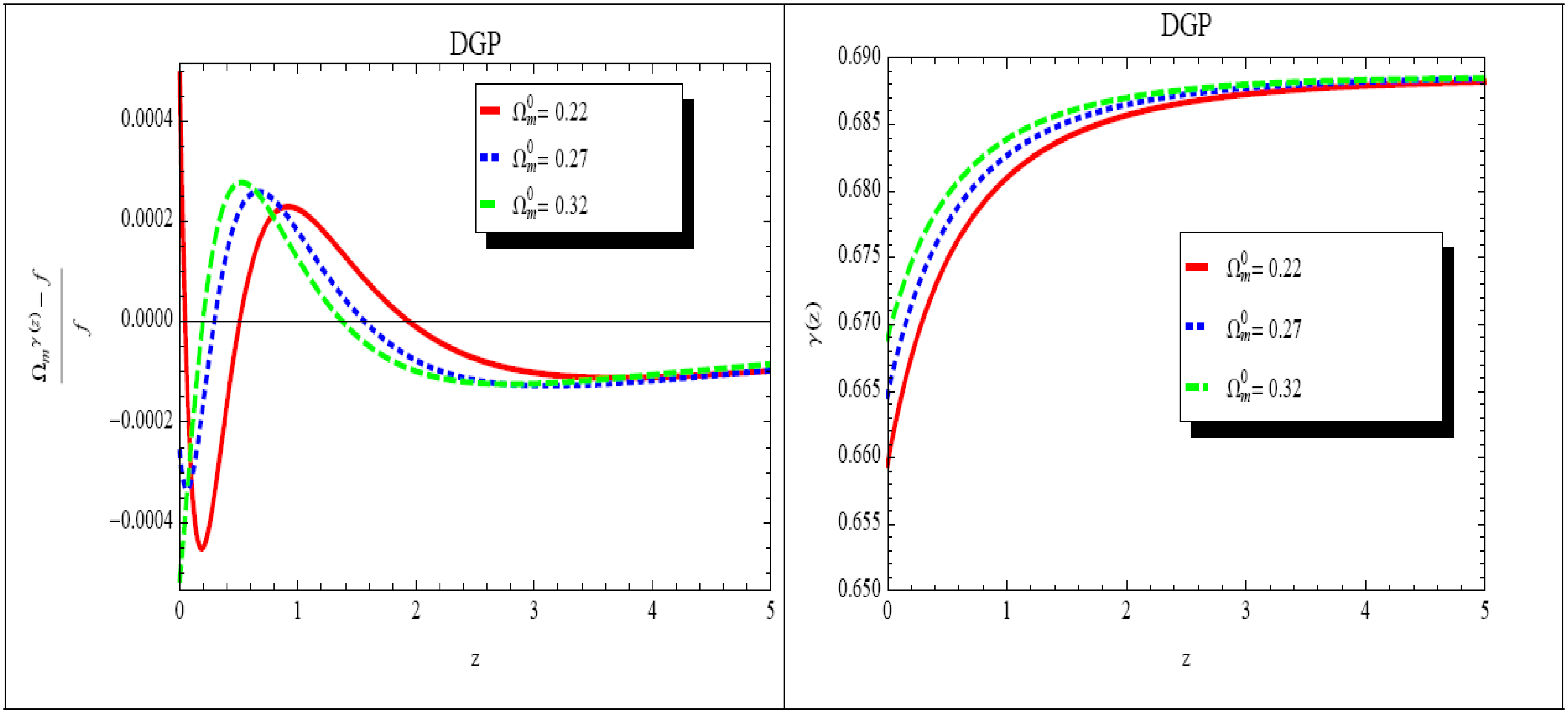
- γ=11/16=0.687 for the flat DGP modified gravity model [e.g. Linder and Cahn, 2007; Gong 2008].

- The slope of $\gamma(z)$ can also act as a discriminant (MI & Dossett, 2009; Fu et al, 2009)

Growth index parameter for GR + Dark Energy models. LEFT: Very precise parameterization. RIGHT: Very little dispersion around the γ=6/11=0.545



Growth index parameter for DGP models.
LEFT: Very precise parameterization.
RIGHT: Very little dispersion around the γ=11/16=0.687
<u>and far enough from the 0.545 of the LCDM</u>



The effect of spatial curvature: degeneracy

(Huterer, Linder, Hu, PRD, 2008; Gong, MI, Wang, PRD 2009)

For the curved dark energy model with constant equation of state w, we have

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\Omega_k - \frac{3}{2}[1 + w(1 - \Omega_m - \Omega_k)]. \tag{3}$$

The energy conservation equation tells us that

$$\Omega_m' = 3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k. \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2), we get

$$[3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k]\frac{df}{d\Omega_m} + f^2 + \left[\frac{1}{2} + \frac{1}{2}\Omega_k - \frac{3}{2}w(1 - \Omega_m - \Omega_k)\right] f = \frac{3}{2}\Omega_m. \tag{5}$$

Plugging $f = \Omega_m^\gamma$ into Eq. (5), we get

$$[3w(1-\Omega_m-\Omega_k)-\Omega_k]\Omega_m \ln\Omega_m \frac{d\gamma}{d\Omega_m} + \left(\gamma - \frac{1}{2}\right)[3w(1-\Omega_m-\Omega_k)-\Omega_k] + \Omega_m^\gamma - \frac{3}{2}\Omega_m^{1-\gamma} + \frac{1}{2} = 0. \tag{6}$$

Method IIb: Using modified growth parameters that enter the perturbed Einstein's Equations

GROWTH EQUATIONS IN GR

Perturbed FLRW Metric.

$$ds^2 = a(\tau)^2[-(1+2\psi)d\tau^2 + (1-2\phi)\gamma_{ij}dx^i dx^j]$$

where

$$\gamma_{ij} = \delta_{ij}\left[1 + \frac{K}{4}\left(x^2+y^2+z^2\right)\right]^{-2}$$

Applying Einstein's Equations

$$\left(k^2 - 3K\right)\phi = -4\pi G a^2 \sum_i \rho_i \Delta_i \qquad \text{Poisson Eqn.}$$

$$k^2(\psi - \phi) = -12\pi G a^2 \sum_i \rho_i (1+w_i)\sigma_i \qquad \text{Anisotropy Eqn.}$$

where

- $$\Delta_i = \delta_i + 3\mathcal{H}\frac{q_i}{k}$$

Method IIb: Modified growth parameters (MG). Various notations but 2 parameters in general. Here we used P, Q that take the value 1 in GR but deviate from it in modified gravity models.

Modified Growth Equations

$$\left(k^2 - 3K\right)\phi = -4\pi G a^2 \sum_i \rho_i \Delta_i Q$$

$$k^2(\psi - R\phi) = -12\pi G a^2 \sum_i \rho_i (1 + w_i)\sigma_i Q,$$

$$k^2(\psi + \phi) = \frac{-8\pi G a^2}{1 - 3K/k^2} \sum_i \rho_i \Delta_i \mathcal{D} - 12\pi G a^2 \sum_i \rho_i (1 + w_i)\sigma_i Q.$$

$$\mathcal{D} = Q(1 + R)/2$$

- See, for example, Bean, Tagmatithan, 2010, Dossett, MI, Moldenhauer, PRD, 2011)
- See also IsitGR software package at http://www.utdallas.edu/~jdossett/isitgr/

CAMB AND SYNCHRONOUS GAUGE VARIABLES

- CAMB is written in the synchronous gauge.

$$ds^2 = a(\tau)^2[-d\tau^2 + (\gamma_{ij} + h_{ij})dx^i dx^j],$$

$$h_{ij} = \frac{h}{3}\gamma_{ij}G + (h + 6\eta)(k^{-2}G_{|ij} + \frac{1}{3}\gamma_{ij}G)$$ where $\nabla^2 G(\vec{k},\vec{x}) = -k^2 G(\vec{k},\vec{x})$.

Einstein's Equations give:

$$(k^2 - 3K)(\eta - \mathcal{H}\alpha) = -4\pi G a^2 \sum_i \rho_i \Delta_i,$$

$$k^2(\dot{\alpha} + 2\mathcal{H}\alpha - \eta) = -12\pi G a^2 \sum_i \rho_i (1 + w_i)\sigma_i$$

where

$$\alpha = (h + 6\eta)/2k^2.$$

CAMB AND SYNCHRONOUS GAUGE VARIABLES CONT'D

Can relate perturbations by

$$\begin{aligned}\phi &= \eta - \mathcal{H}\alpha,\\ \psi &= \dot{\alpha} + \mathcal{H}\alpha\end{aligned}$$

Two useful variables

$$\sigma_{CAMB} \equiv k\alpha = \frac{k(\eta - \phi)}{\mathcal{H}},$$

$$\mathcal{Z} \equiv \frac{\dot{h}}{2k} = \sigma_{CAMB} - 3\frac{\dot{\eta}}{k}.$$

Now just need $\dot{\eta}$

$$\dot{\eta} = \frac{-1}{2f_Q}\Bigg\{2(\mathcal{H}^2 - \dot{\mathcal{H}})k^2\alpha + \sum_i \tilde{\rho}_i(a)\Big[\left(2\mathcal{H}\left[\mathcal{D} - Q\right] + \dot{Q}\right)\Delta_i - Q(1+w_i)k^2\alpha - Qf_1\frac{q_i}{k}\Big]\Bigg\},$$

$$f_1 = k^2 + 3(\mathcal{H}^2 - \dot{\mathcal{H}}) \qquad f_Q = k^2 + \frac{3Q}{2K_{f1}}\sum_i \tilde{\rho}_i(1+w_i)$$

$$K_{f1} = 1 - 3K/k^2$$

These growth equations and parameters enter into observables and data sets

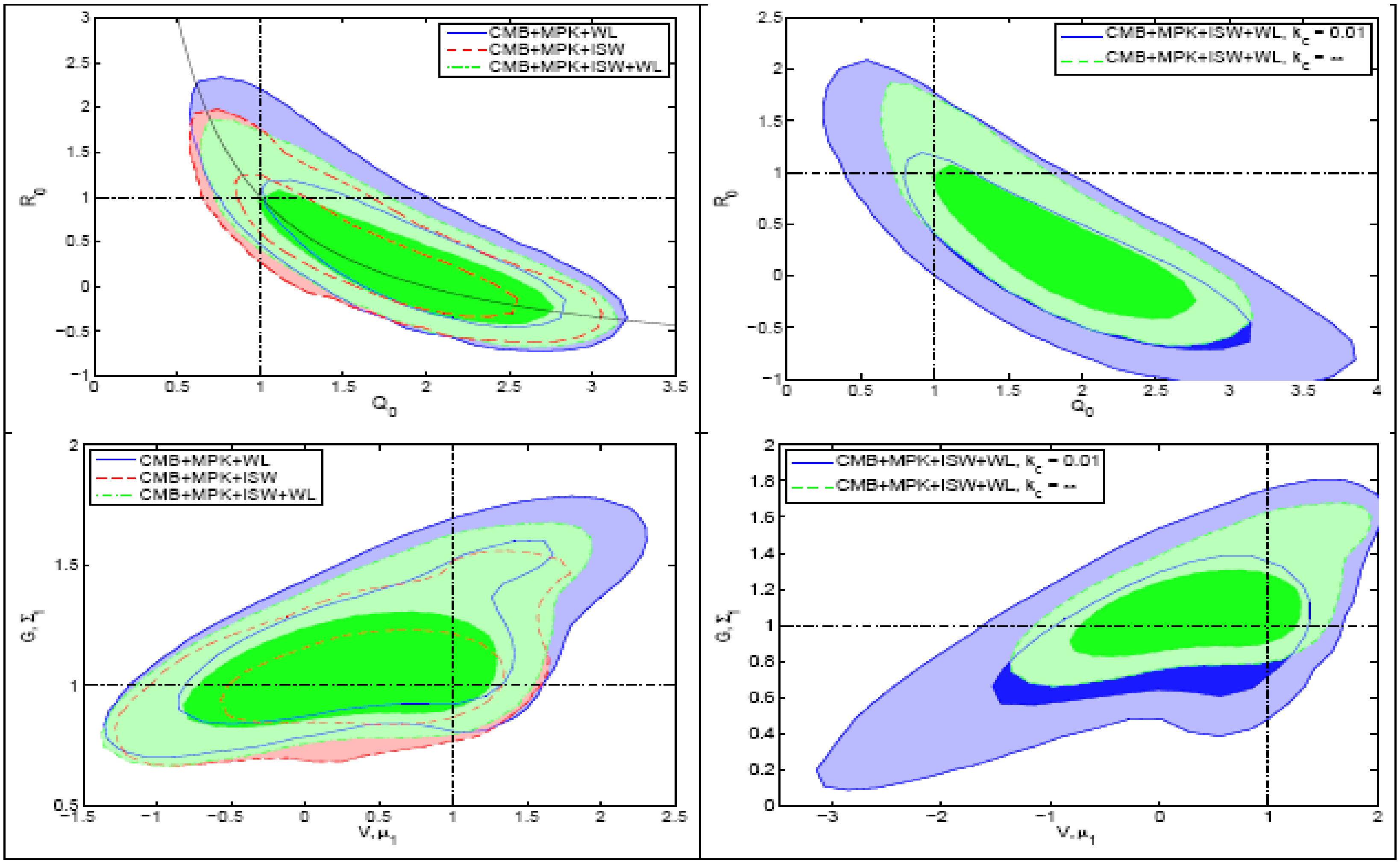
- So we used various combinations of data sets to constrain these MG parameters (J. Dossett, J. Moldenhauer, MI, Phys.Rev.D84:023012,2011):
  - WMAP 7 year temperature and polarization spectra
  - Union 2 Supernovae Data
  - BAO from Two-Degree Field and SDSS-DR7
  - Matter Power Spectrum (MPK) from SDSS-DR7
  - ISW-galaxy cross-correlations (SDSS-LRG, 2MASS, NVSS)
  - Refined HST COSMOS 3D weak lensing tomography.

(some data sets are particularly sensitive to the growth and other used in order to break degeneracies)

Using the latest cosmological data sets including refined COSMOS 3D weak lensing (Jason Dossett, Jacob Moldenhauer, MI)

Phys.Rev.D84:023012,2011

No apparent deviation from GR using current data from using functional or binned parameterizationa



Findings

- There are some tensions between the preferred MG parameter values of different datasets.
- The tensions are more pronounced using some particular parameterization.
- The parameter values for general relativity are within 95% confidence level contours for all data sets.

ISiTGR:
Integrated Software in Testing General Relativity

Version 1.1

Developed by **Jason Dossett**, **Mustapha Ishak**, and **Jacob Moldenhauer**.

What is ISiTGR?

ISiTGR is an integrated set of modified modules for the software package CosmoMC for use in testing whether observational data is consistent with general relativity on cosmological scales. This latest version of the code has been updated to allow for the consideration of non-flat universes. It incorporates modifications to the codes: CAMB, CosmoMC, the ISW-galaxy cross correlation likelihood code of Ho et al, and our own weak lensing likelihood code for the refined COSMOS 3D weak lensing tomography of Schrabback et al to test general relativity.

A detailed explanation of the modifications made to these codes allowing one to test general relativity are described in our papers: arXiv:1109.4583 and arXiv:1205.2422.

How to get ISiTGR

Two versions of ISiTGR are available. The normal version of ISiTGR uses a functional form to evolve the parameters used to test general relativity and is available here. ISiTGR_BIN, on the other hand, gives you two options to evovle the parameters used to test general relativity. The first option is to bin the parameters in two redshift and two scale bins, alternatively one can use the hybrid evolution method, as seen in our paper, where scale dependence evolves monotonically, but redshift dependence is binned. That code can be downloaded here.

Download Here: ISiTGR ISiTGR_BIN

The original (flat only) verison of ISiTGR as well as builds for other versions of CosmoMC are available here (**this version is for** CosmoMC **01/2012**).

Our most recent analysis: Effects of Dark Energy Perturbations on the Tests (J. Dossett, MI, 2013)

- Tests must be robust.
  - Can a more complicated dark energy model mimic a modified gravity model?
  - Will we be able to say for sure that a detected deviation in the MG parameter space is due to a departure from GR.

Dark Energy density and anisotropic shear perturbations

$$\dot{\delta} = -(1+w)(\theta - 3\dot{\phi}) + 3\mathcal{H}(w - \frac{\delta P}{\delta\rho})\delta$$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta + \frac{\delta P/\delta\rho}{1+w}k^2\delta + k^2\left(\psi - \sigma\right)$$

Define an effective sound speed of perturbations

$$\frac{\delta P}{\delta\rho}\delta \equiv \frac{\delta P}{\rho} = c_s^2\delta + 3\mathcal{H}(1+w)(c_s^2 - c_a^2)\frac{\theta}{k^2}$$

Adiabatic sound speed

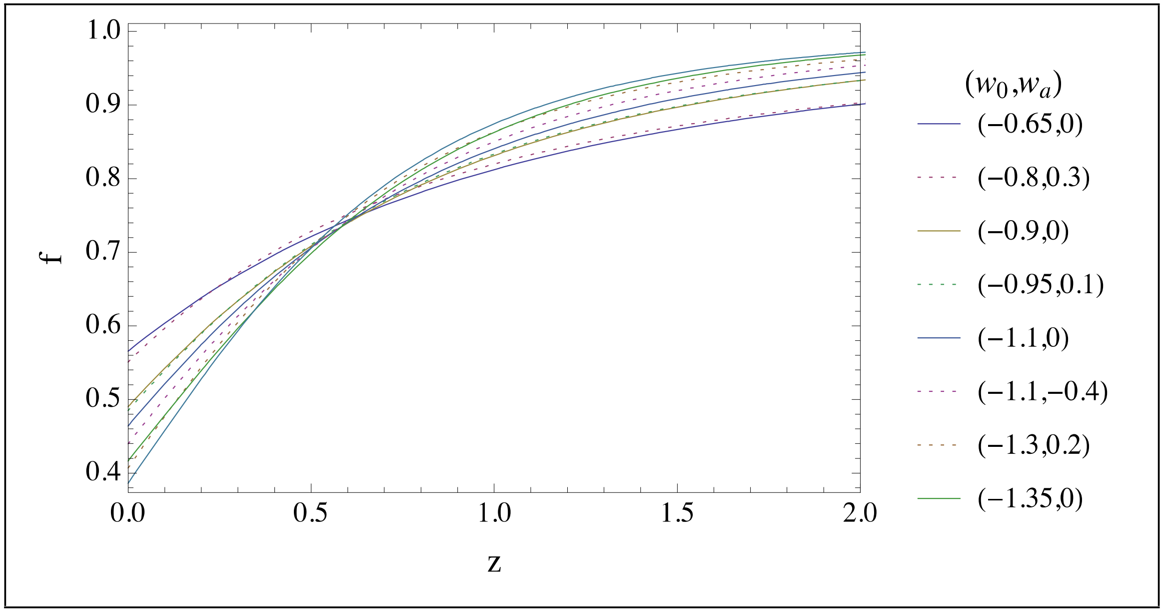
$$c_a^2 = \frac{\dot{P}}{\dot{\rho}} = w - \frac{\dot{w}}{3\mathcal{H}(1+w)}$$

$$\dot{\delta} = -(1+w)\left\{\left[k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)\right]\frac{\theta}{k^2} - 3\dot{\phi}\right\} + 3\mathcal{H}(w - c_s^2)\delta$$

$$\dot{\theta} = (3c_s^2 - 1)\mathcal{H}\theta + k^2\frac{c_s^2\delta}{1+w} + k^2\left(\psi - \sigma\right)$$

Effect on the growth index

We plot the predicted logarithmic growth rate for various dark energy models with perturbations.

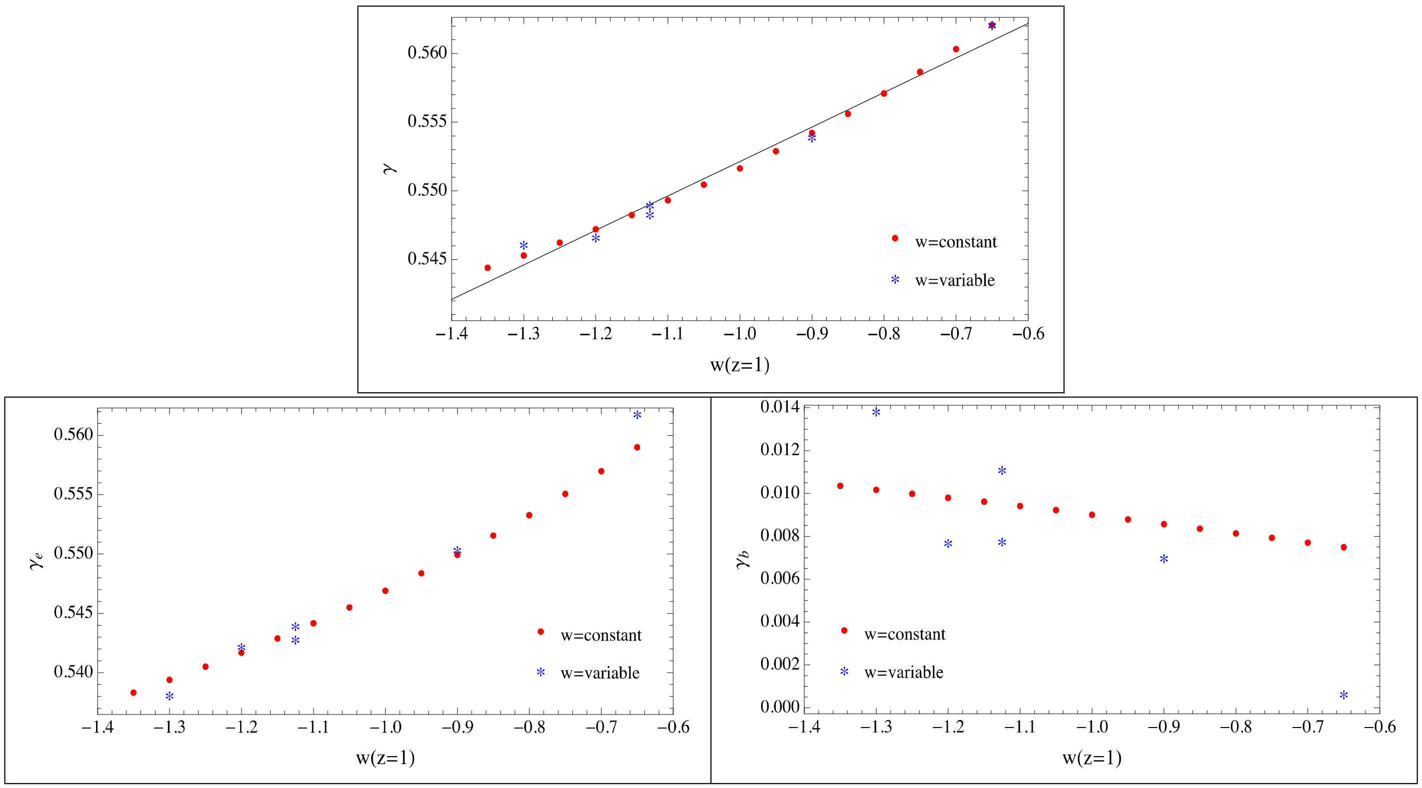


We use a dark energy equation of state

$$w(a) = w_0 + w_a(1 - a)$$

Effect on Growth index cont’d

We fit the growth index parameters for the previously shown dark energy models and plot them as a function of w(z=1).



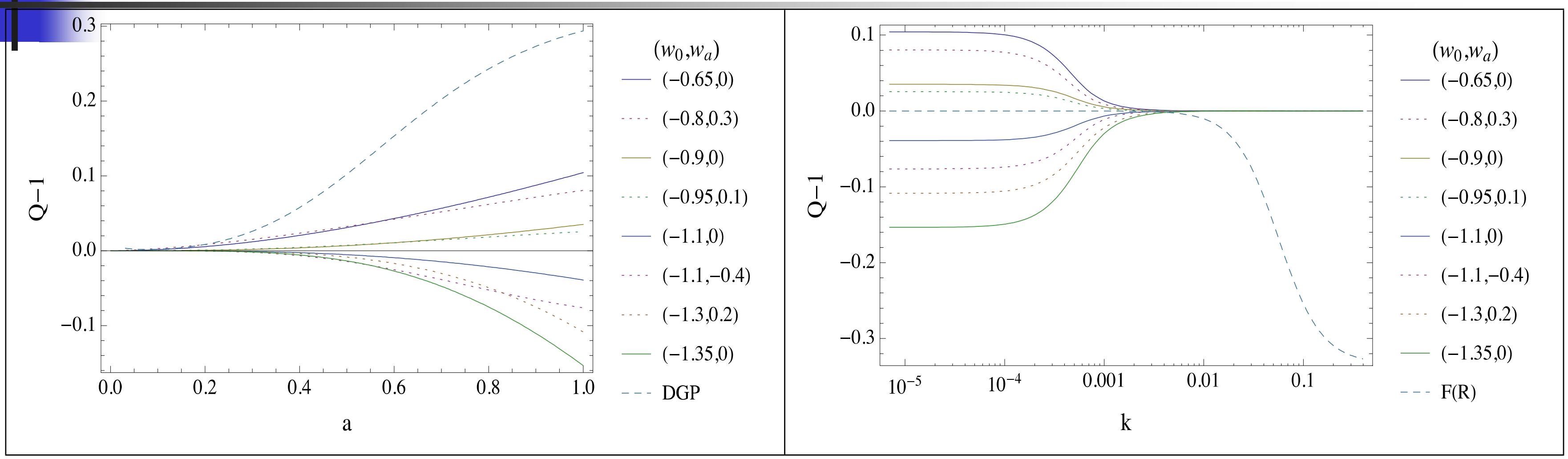
$$\gamma = 0.552 + 0.025(1 + w(z = 1))$$

Effect on the MG parameter Q

We combined the modified and unmodified growth equations

$$
\begin{aligned}
-Q\, 4\pi G a^2 \sum_{i \neq DE} \rho_i \Delta_i &= -4\pi G a^2 \sum_{i \neq DE} \rho_i \Delta_i - 4\pi G a^2 \rho_{DE} \Delta_{DE} \\
\Rightarrow Q &= 1 + \frac{\rho_{DE} \Delta_{DE}}{\sum_{i \neq DE} \rho_i \Delta_i}
\end{aligned}
$$

Effect on the MG parameter Q cont'd



The equation of state, w, for the Dark Energy is chosen to be over 4-sigmas deviation from, e.g., the WMAP9 value. The deviations in Q for the DGP and f(R) models remain larger and the deviation for f(R) happens at a different scale.

$$Q_{DGP} = \frac{4 + 2\Omega_m(a)^2}{3 + 3\Omega_m(a)^2}$$

For DGP Models

$$Q = \frac{1 + \frac{2}{3}\lambda_1^2 k^2 \beta(a) a^{s1}}{1 + \lambda_1^2 k^2 \beta(a) a^{s_1}}$$

$$\beta(a) = [\Omega_m/(1 - \Omega_m) + 4a^3]^{(s_1+2)/3}$$

For f(R) Models

Effect on the MG parameter R

Again, we combining the modified and unmodified growth equations

$$k^2\psi = -\sum_{i\neq DE} \tilde{\rho}_i \left[w_i \Pi_i + \frac{\Delta_i}{2} \right] - \tilde{\rho}_{DE} \left[w_{DE} \Pi_{DE} + \frac{\Delta_{DE}}{2} \right]$$

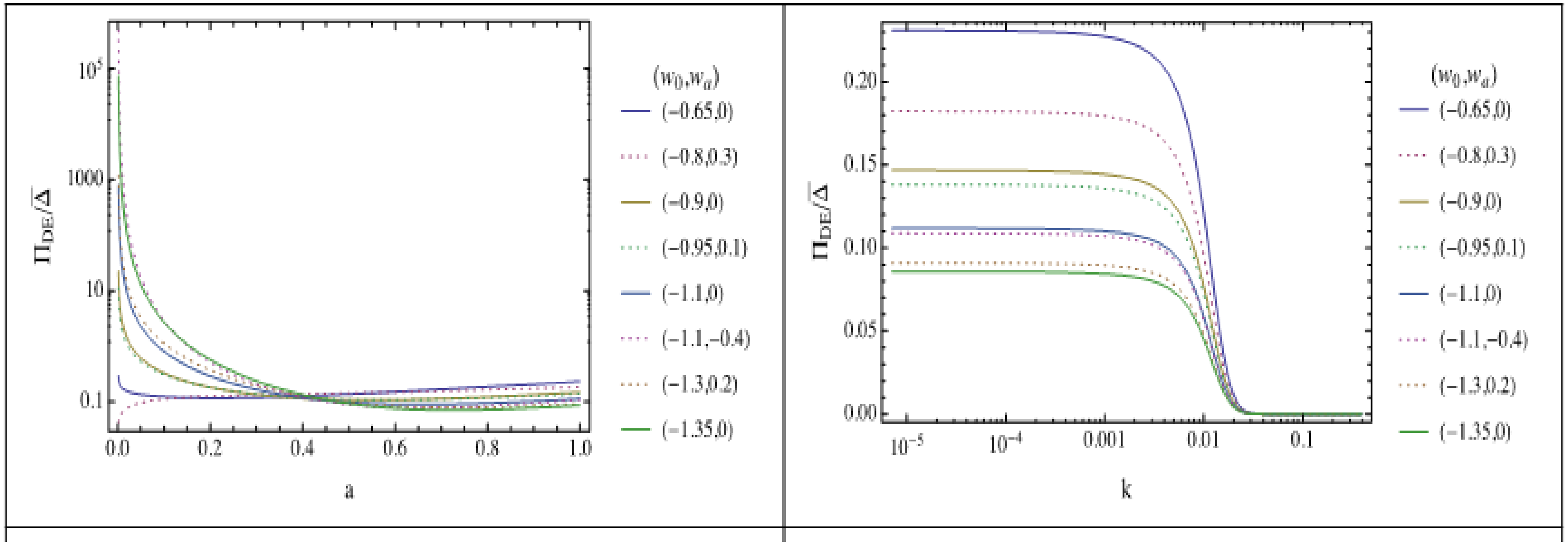
$$k^2\psi = -Q \sum_{i\neq DE} \tilde{\rho}_i \left[w_i \Pi_i + R\frac{\Delta_i}{2} \right],$$

$$R = \frac{3\left[\rho_{DE} w_{DE} \Pi_{DE} - \frac{\rho_{DE}\Delta_{DE}}{\sum_{i\neq DE} \rho_i \Delta_i} \sum_{i\neq DE} \rho_i w_i \Pi_i \right]}{\sum_{i\neq DE} \rho_i \Delta_i + \rho_{DE}\Delta_{DE}} + 1$$

Shear is related to the anisotropic stress perturbation by

$$\sigma_\alpha = \tfrac{2}{3}\Pi_\alpha w_\alpha / (1 + w_\alpha)$$

Effect on the MG parameter R cont'd



For present times, we found that for the very largest scales, the magnitude of the DE shear perturbations must approach an unrealistic 20% of the value of the total mass averaged overdensity in order for the parameter R to deviate by only 20% from its GR value (R~0.8). Again, the values of w are taken to be more than 4 sigmas away from the WMAP9 value. More in finalization stage.

Summary/conclusions

- Current data is found consistent with General Relativity but the allowed parameter space I still too large (future data awaited).
- New frameworks to test other models than GR are needed (because the assumed underlying theory matters for the conclusiveness of the tests).
- We have explored correlations between MG parameters and cosmological parameters. We found ignoring curvature can cause apparent deviations from GR.
- The growth index is found very robust to Dark Energy perturbations
- Though the MG parameters show some deviation for DE models with perturbations, this is not nearly as significant as those given by modified gravity models
- We conclude that DE models that do not include some exotic interactions with ordinary matter cannot mimic a modified gravity model.
- These tests should be able to distinguish between dark energy and modified gravity models.