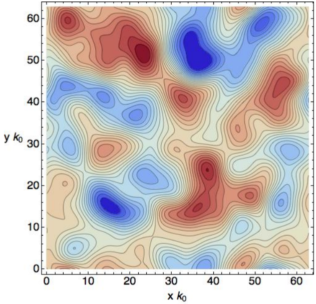
Cold planar horizons are floppy

Jorge E. Santos

New frontiers in dynamical gravity



In collaboration with
Sean A. Hartnoll - arXiv:1402.0872 and arXiv:1403.4612

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- What I am going to describe doesn't happen in such setups.

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$$S = \frac{1}{16\pi G_d} \int \mathrm{d}^d x \sqrt{-g} \left[R + \frac{(d-1)(d-2)}{L^2} - \frac{1}{2} F^{ab} F_{ab} \right] ,$$

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- Focus on $d = 4$, with $\mu(x) = \bar{\mu} \left[1 + A_0 \cos(k_L x)\right]$.
- Moduli space space of solutions is 2D: A_0 and $k_0 \equiv k_L / \bar{\mu}$.

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$$\mathrm{d}s^2 = \frac{L^2}{y^2}\left[-G(y)(1-y)^2\mathrm{d}t^2 + \frac{\mathrm{d}y^2}{G(y)(1-y)^2} + \mathrm{d}x^2 + \mathrm{d}w^2\right],$$
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- Brings line element (1) to

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$$\nu_-(k_L) = \sqrt{\left(\frac{1}{2} - \sqrt{\frac{k_L^2}{3} + 1}\right)^2 - \frac{k_L^2}{6} - \frac{1}{2}} > 0\,.$$

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Breakdown of perturbation theory - resumm perturbation theory.

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- Close to $x = 0$, perturbation theory is saved, however away from $x = 0$ perturbation theory breaks down!

How to decide which is which?

Proceed without any approximation - Numerics.

Ansetzen & Numerics

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- Alternatively, use very, very small $T/\bar{\mu}$.

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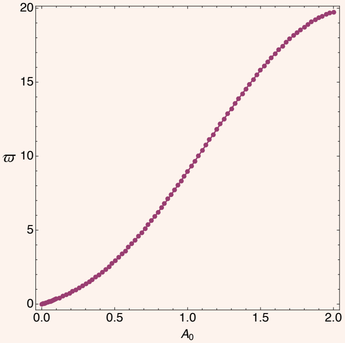

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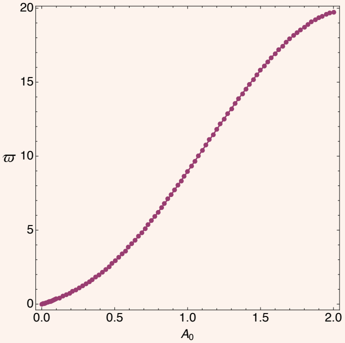

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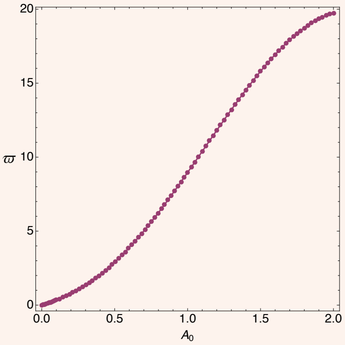
- We repeated this calculation for several values of k_0, and find similar results.



Results:

- To measure deviations from $AdS_2 \times \mathbb{R}^2$:

$$\varpi \equiv \frac{\mathcal{W}_{\max}}{\mathcal{W}_{\min}} - 1\,,$$

where $\mathcal{W} = (\partial_w)^a(\partial_w)_a|_{\mathcal{H}}$.

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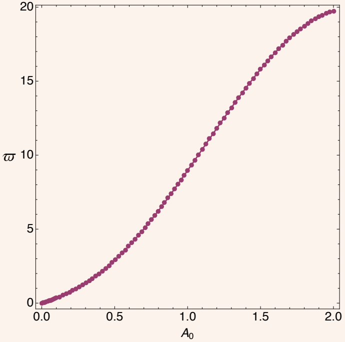
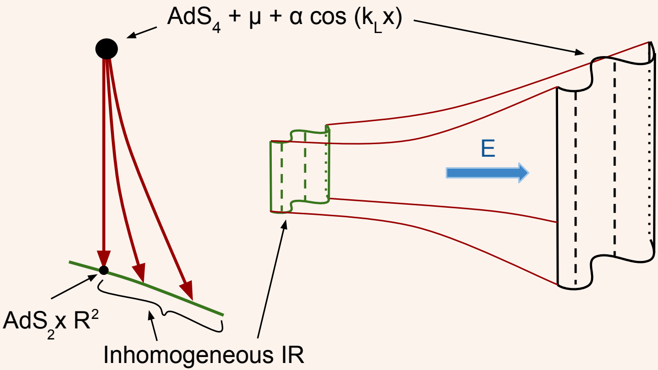


Einstein's equations chose a resummation that renders the IR floppy - broken translational invariance.

Emergent picture:



Periodic potentials in AdS_4

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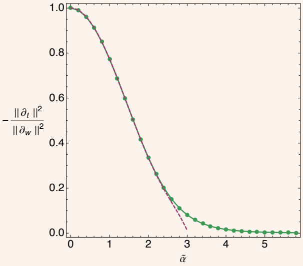

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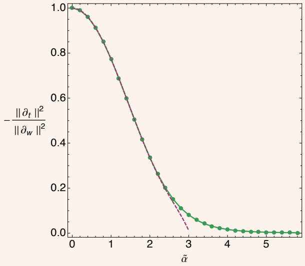

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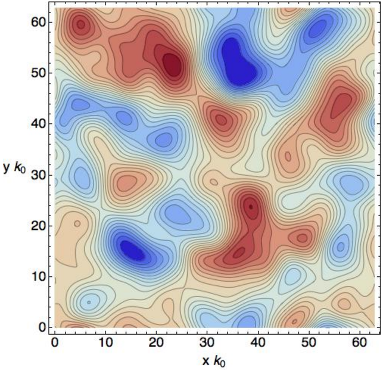

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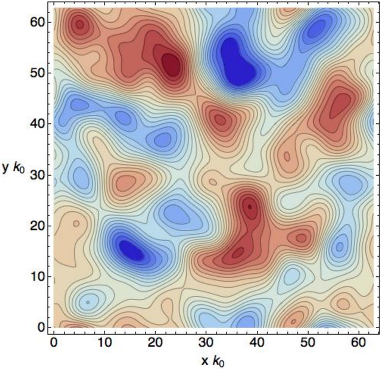

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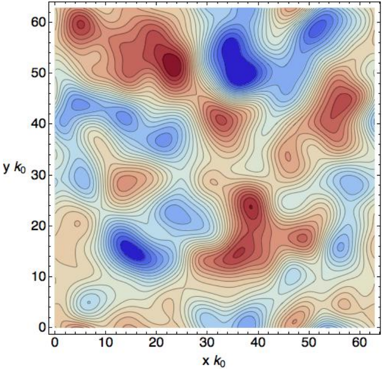

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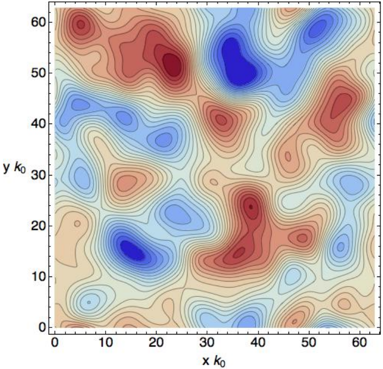

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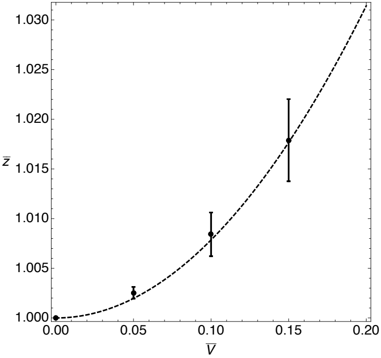

$\langle g_{ab} \rangle_R$ is accurately described by a Lifshitz geometry:

$$\langle \mathrm{d}s^2 \rangle_R = \frac{L^2}{y^2} \left[-\frac{\mathrm{d}t^2}{y^{2(\bar{z}-1)}} + \mathrm{d}x^2 + \mathrm{d}w^2 + \mathrm{d}y^2 \right] .$$

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Outlook:

- Can these new IR geometries affect time dependence?
- Can we make a connection with glassy physics?
- ...