Revisiting Scalar Collapse in AdS

New Frontiers in Dynamical Gravity
DAMTP, Cambridge

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Instability of Scalar Field in spher. symm. aAdS

Evolution of a Scalar field in sph. symm., asympt. AdS (aAdS)
[Bizoń -Rostworowski,2011]

- **Fully nonlinear:**
  - Consider Gaussian-type initial data w/ amplitude ϵ and width σ
  - Choptuik-type critical behavior
  - However, sub-critical eventually collapses as well
- **Perturbative about pure AdS:**
  - At linear order, uncoupled modes: *oscillon*
  - Resonance at $\mathcal{O}(\epsilon^3)$: $j_r = j_1 + j_2 - j_3$
  - Single mode stable, multiple modes unstable
- **Conjecture:**
  - AdS generically unstable to collapse via weakly nonlinear turbulent cascade

Instability in light of AdS/CFT Correspondence

- Holographic duality between
  $(d+1)$-dimensional global AdS (the bulk)
  and
  conformal field theory (CFT) on $d-1$-dim. boundary $(S^{d-1} \times R)$
- Dictionary translates between bulk quantities of aAdS spacetime and quantum operators of CFT
- Interpretation of instability:
  initial data generically thermalizes by BH formation

...but are there non-thermalizing initial configurations in the CFT?

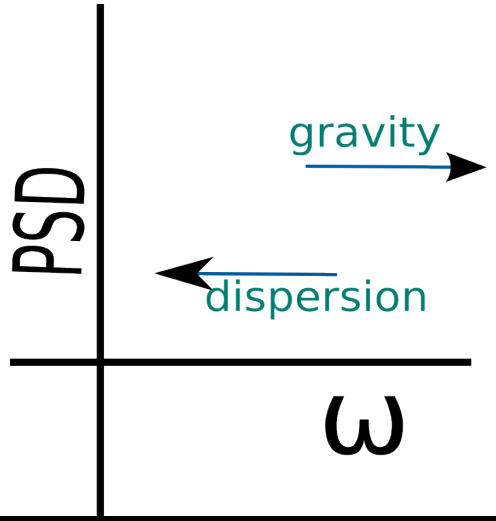
Paths to Stability

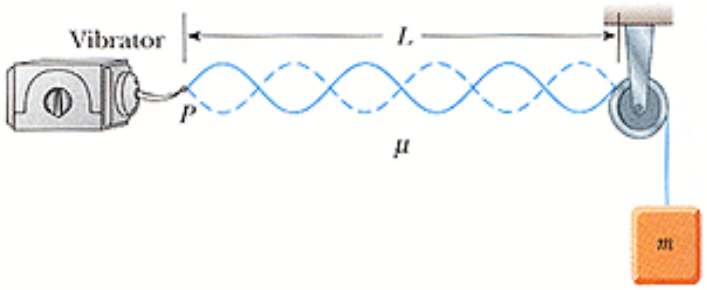
- **Perturbative analysis** showing stable solutions
  [Dias,Horowitz,Marolf,Santos,1208.5772]
  - Argue perturbatively for nonlinear stability
  - Geons and boson stars, not necessarily spher. symm.
- **Excite all modes**
  [Buchel,Lehner,SLL,1210.0890]
  - Perturbative argument for stability at $\mathcal{O}(\epsilon^3)$
  - $\omega_{j_r} \to \omega_{j_r} + \epsilon^2 \sum_{\{j_1,j_2,j_3\}} \frac{A_{j_1} A_{j_2} A_{j_3}}{A_{j_r}} C_{j_1 j_2 j_3 j_r}$, where the triple sum is over all the resonance channels $\omega_{j_1} + \omega_{j_2} = \omega_{j_r} + \omega_{j_3}$
- **Time-periodic** solutions
  [Maliborski,Rostworowski,1303.3186]
  - Construct time-periodic solutions
  - Argue for nonlinear stability
- **Frustrated resonance**
  [Buchel,SLL,Lehner,1304.4166]

Frustrated Resonance

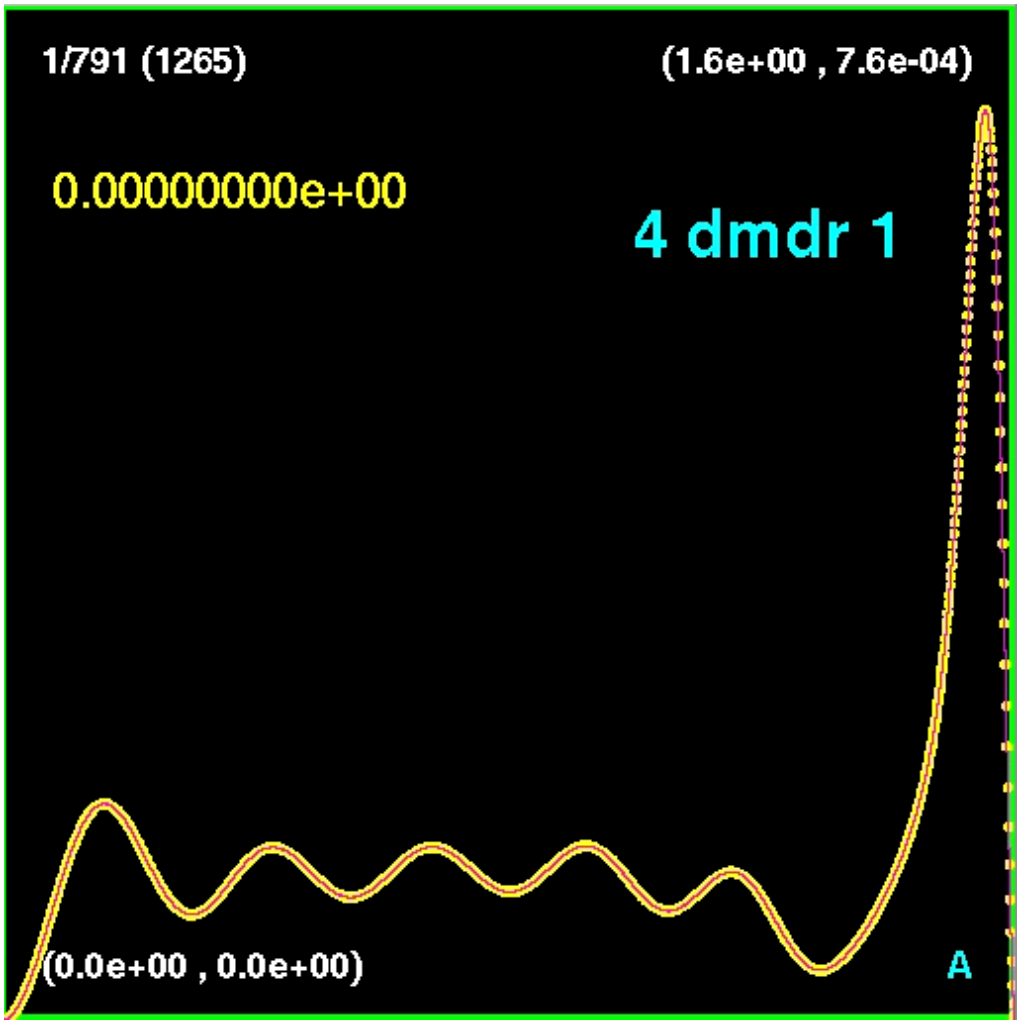
[Buchel,SLL,Lehner,1304.4166]

- Broadly distrib. energy perturbs AdS & introduces dispersion
- Dispersion competes with nonlinear sharpening
- BR data: increasing σ increases distribution of energy
- Issues with σ-parameterization:
  [Maliborski,Rostworowski,1307.2875]
  - Large-σ ceases to be broadly distributed (us and [Abajo-Arrastia,Silva,Lopez,Mas,Serantes,1403.2632])
  - "window" in σ shrinks for higher dims but other ID stable





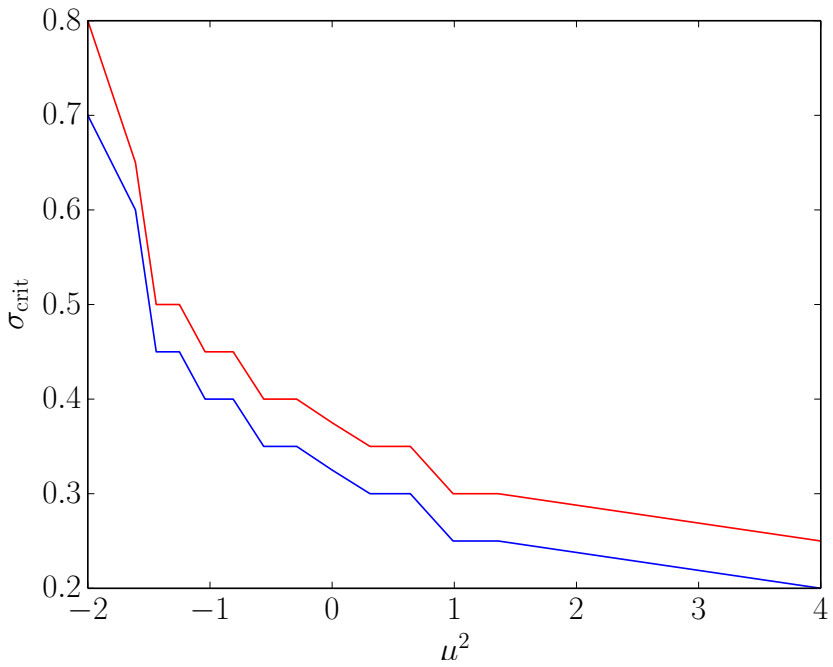
Perturbed Boson Star



4dmdr1.mpg ▸ Perturbed BS

Effect of Mass [Balasubramanian,Buchel,Green,Lehner,SLL,in prep]

- **Motivation:** explore CFT operators of different weight
  ...mass changes decay rate of SF at boundary
- Introduce mass term $-\mu^2|\phi|^2$
- No dispersion at linear order
- Mass changes location of transition σ_{crit}



Open Questions

Among others, just two here:

- What's stable and what's unstable?
  ...in other words, can we identify whether initial data will collapse for any amplitude a priori?
- For ID that appears unstable, can we be sure whether it extends to $\epsilon \to 0$?

...using "unstable" as ID that collapses for $\epsilon \to 0$ but $\epsilon \neq 0$

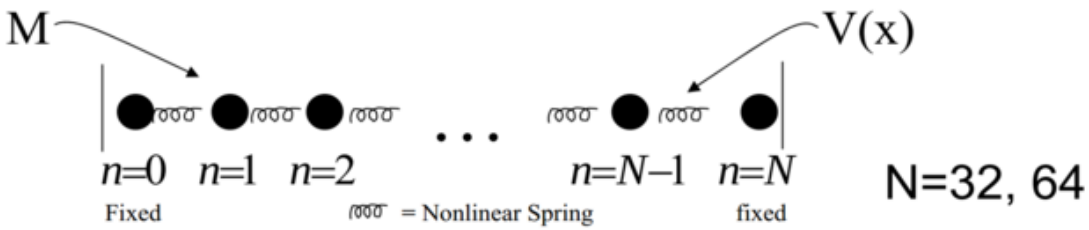
Two-Time Formalism (TTF)

- Dynamics characterized by two time scales:
  - fast time t–generally $t < \pi$ where π is time for a bounce off boundary
  - slow time τ–scale over which energy transfers among oscillons, $\tau \equiv \epsilon^2 t$
- Allow mode amplitudes $A_j(t)$ to be functions of both times $A_j(t, \tau)$
- Enforce at $\mathcal{O}(\epsilon^3)$ the absence of secular terms in the scalar field
- **Advantages:**
  - Goes beyond initial transfer of energy (...to time $t > 1/\epsilon^2$)
  - Conserves energy
  - Both direct and inverse cascades
- Solve coupled, cubic, ODEs in complex mode amplitudes A_j
- Resembles FPUT paradox

TTF and Fermi-Pasta-Ulam-Tsingou (FPUT, 1953)

- Model 1D atoms in a crystal by masses linked by springs with nonlinear term
- At linear order, Fourier modes decouple
- Nonlinear system may not **not approach equipartition**, as predicted by classical stat. mech.

  ...apparently still debated more than 50 years later!
- For $N \to \infty$ (nonlinear string) and small energies, system is similarly **resonant**

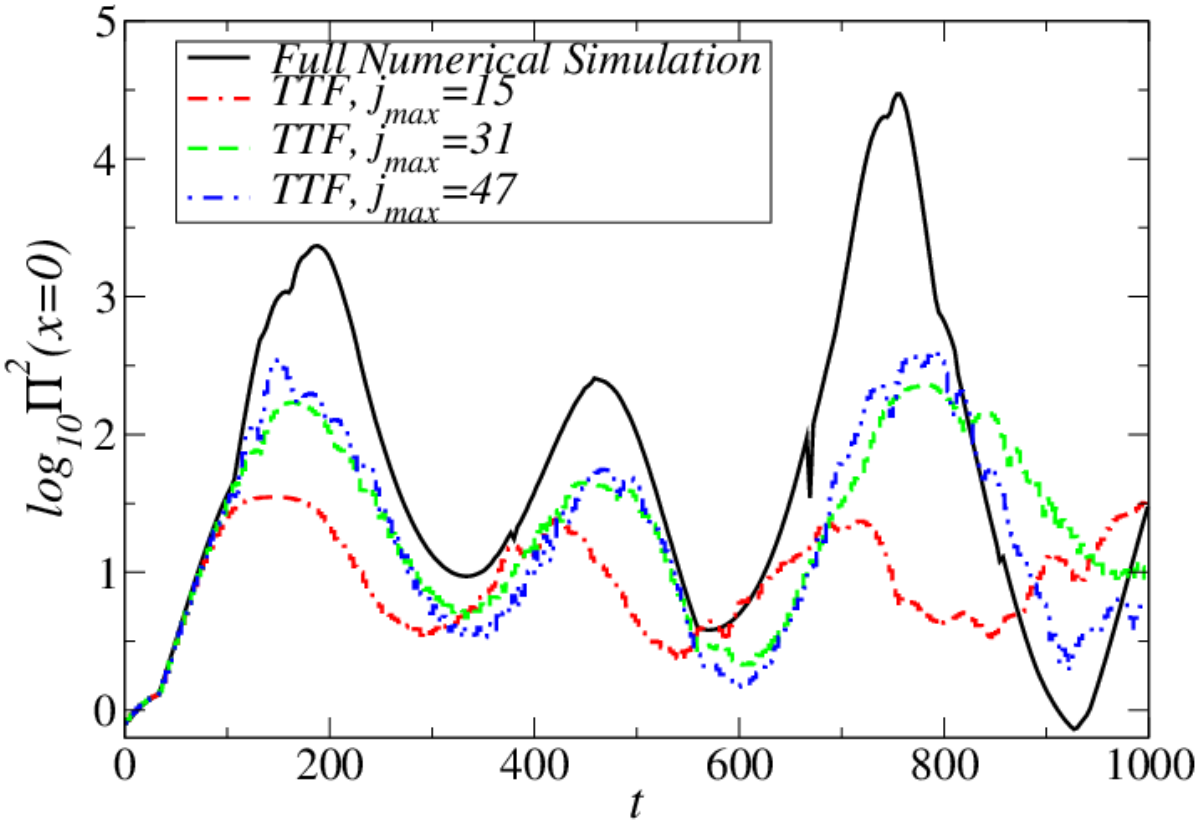


$$V(x) = \tfrac{1}{2} kx^2 + \alpha/3\, x^3 + \beta/4\, x^4$$

[D. Campbell's APS 2010 talk]

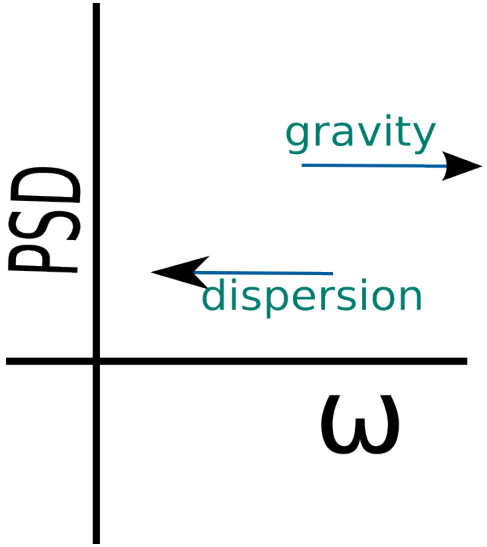
TTF and Fully Nonlinear...two-mode ID

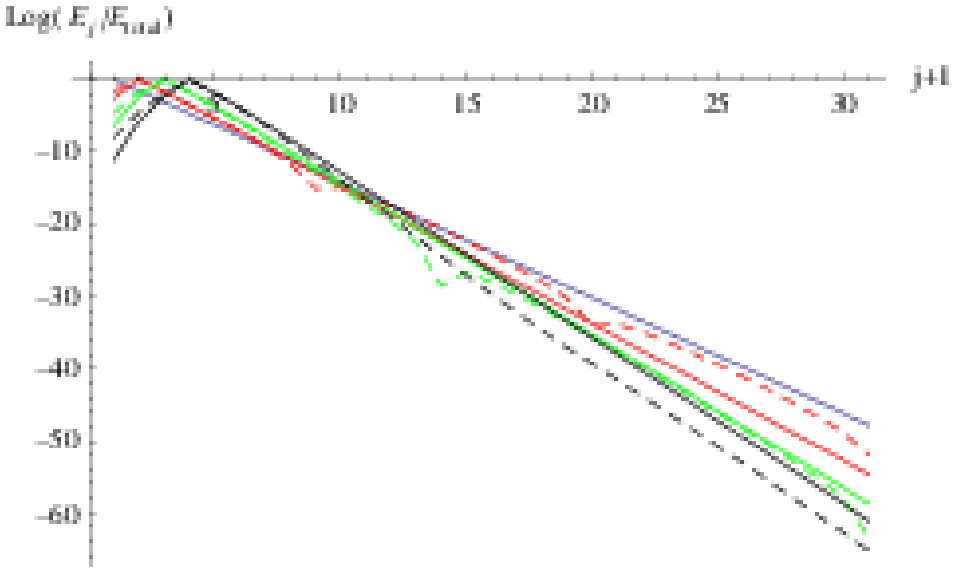
- Similar "evolutions"...**both direct and inverse cascades**
- TTF convergent with increasing $j_{\max}$
- In $\epsilon \to 0$ and $j_{\max} \to \infty$ limits, TTF & NL should converge



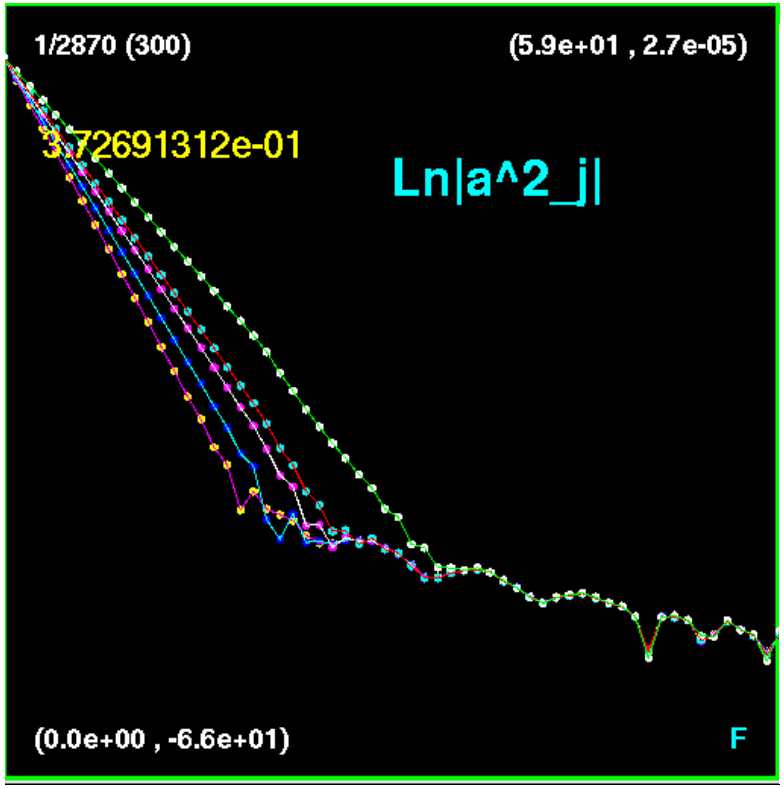
TTF and Quasi-Periodic (QP) Solutions

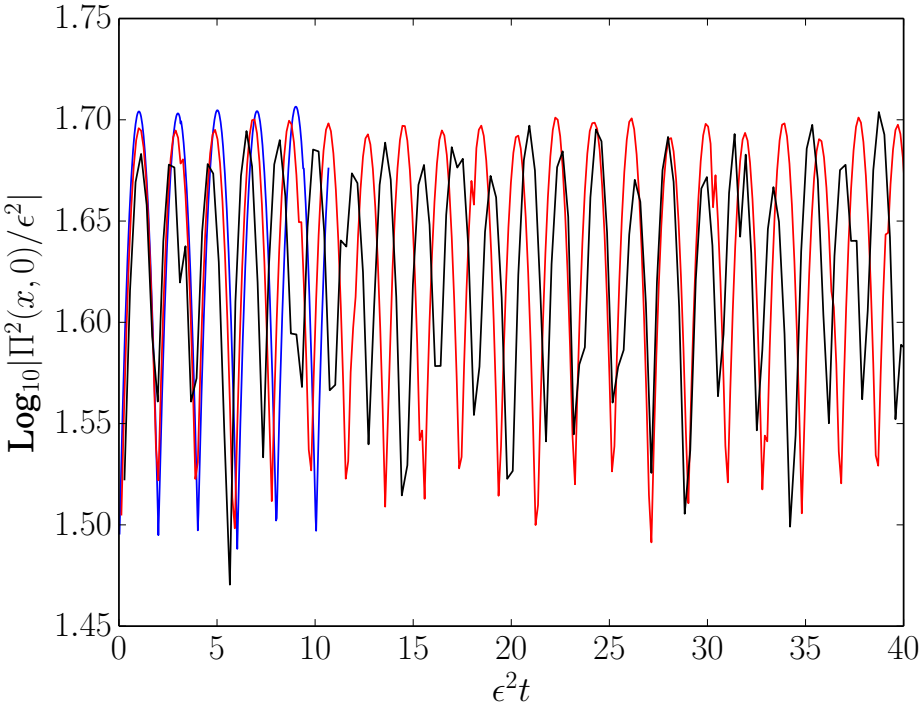
- Specify $A_j^{\rm qp}(\tau) = \alpha_j e^{-i\beta_j \tau}$
- Solutions branching from dominant mode j_r...two branches for $j_r > 0$
- One-parameter generalizations of MR time-periodic solutions
- Such solutions balance direct and inverse transfers...no energy transfer among modes $\dot{E}_j = 0$
- Approximated by exponential spectrum $E_j = e^{-\mu j}$





(Approximate) QP Solution Evolved Fully Nonlinearly





lnasqj_qp.mpg ▸ QP NL

Engineered Initial Data (ID)

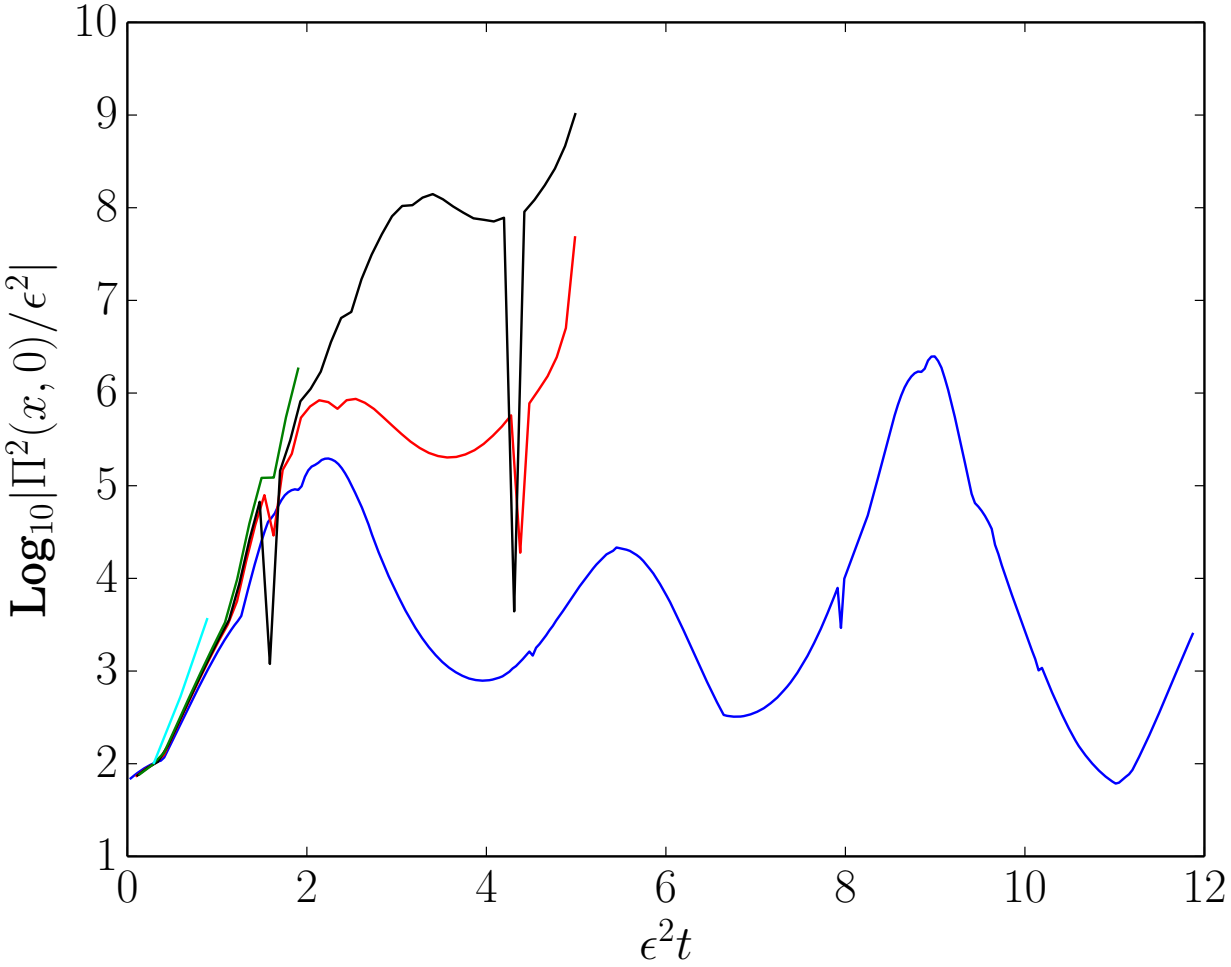
- Form of ID for fully nonlinear evolutions
- Specify amplitudes c_j of oscillons present in ID:

$$\begin{aligned} \Pi(x,0) &= 0 \\ \phi(x,0) &= \Sigma_j \left(c_j e_j(x)\right) \end{aligned}$$

- Examples:
  - Equal energy 2-mode ID: $c_j = \delta^i_j/(3+2i) + \delta^k_j/(3+2k)$
  - Exponential amplitude ID: $c_j = e^{-\alpha j}$
  - Exponential energy ID: $c_j = e^{-\alpha j}/(3+2j)$

Two-mode Stable Solutions

Equal-Energy Two-Mode Initial Data: Modes 0 and 1: $c_j = e_0/3 + e_1/5$



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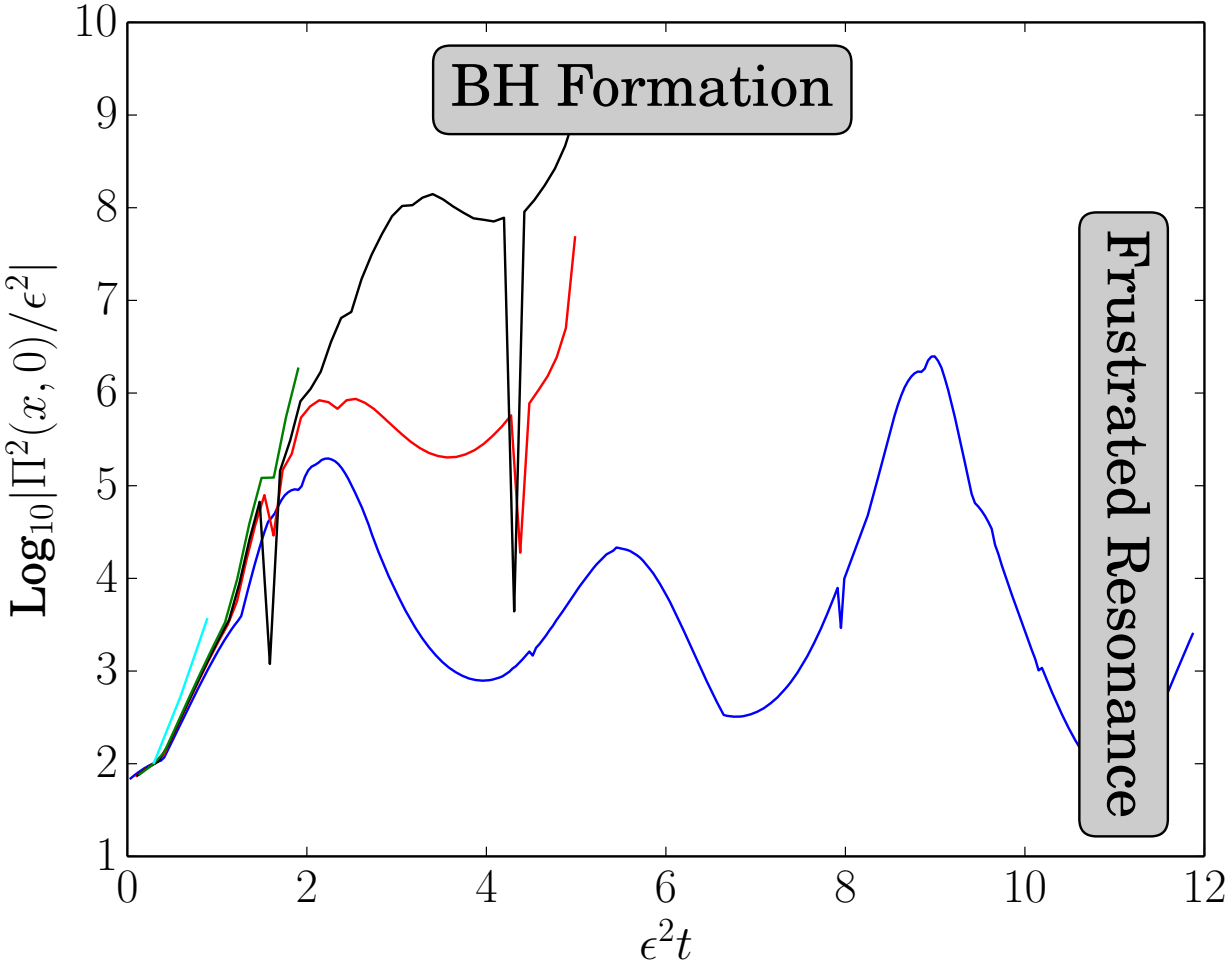

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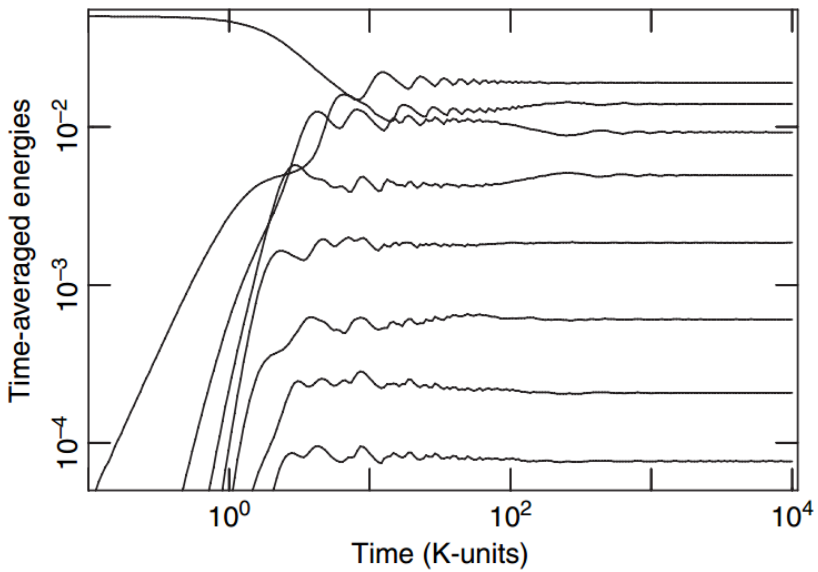
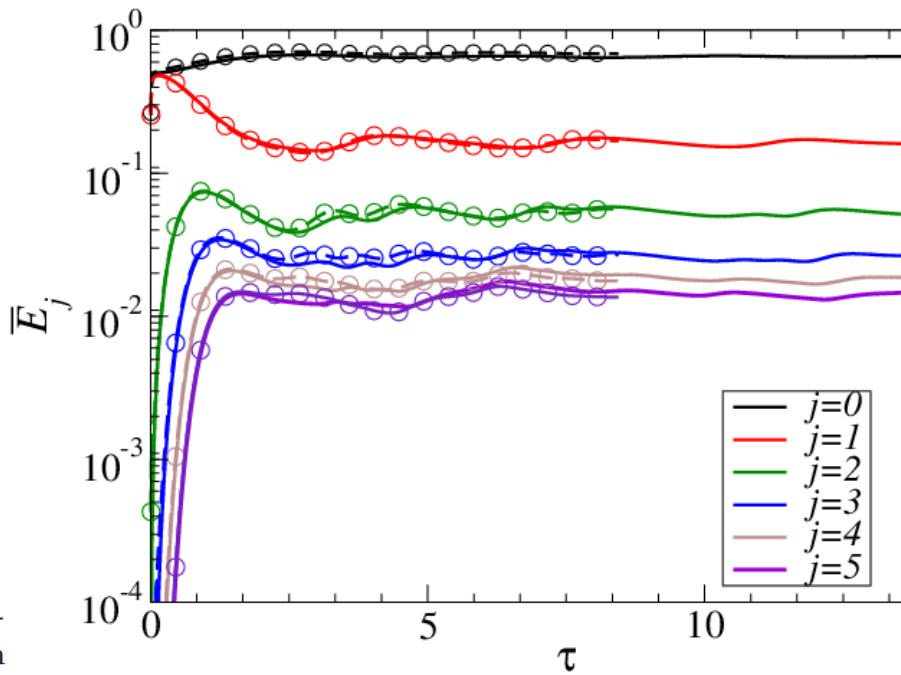


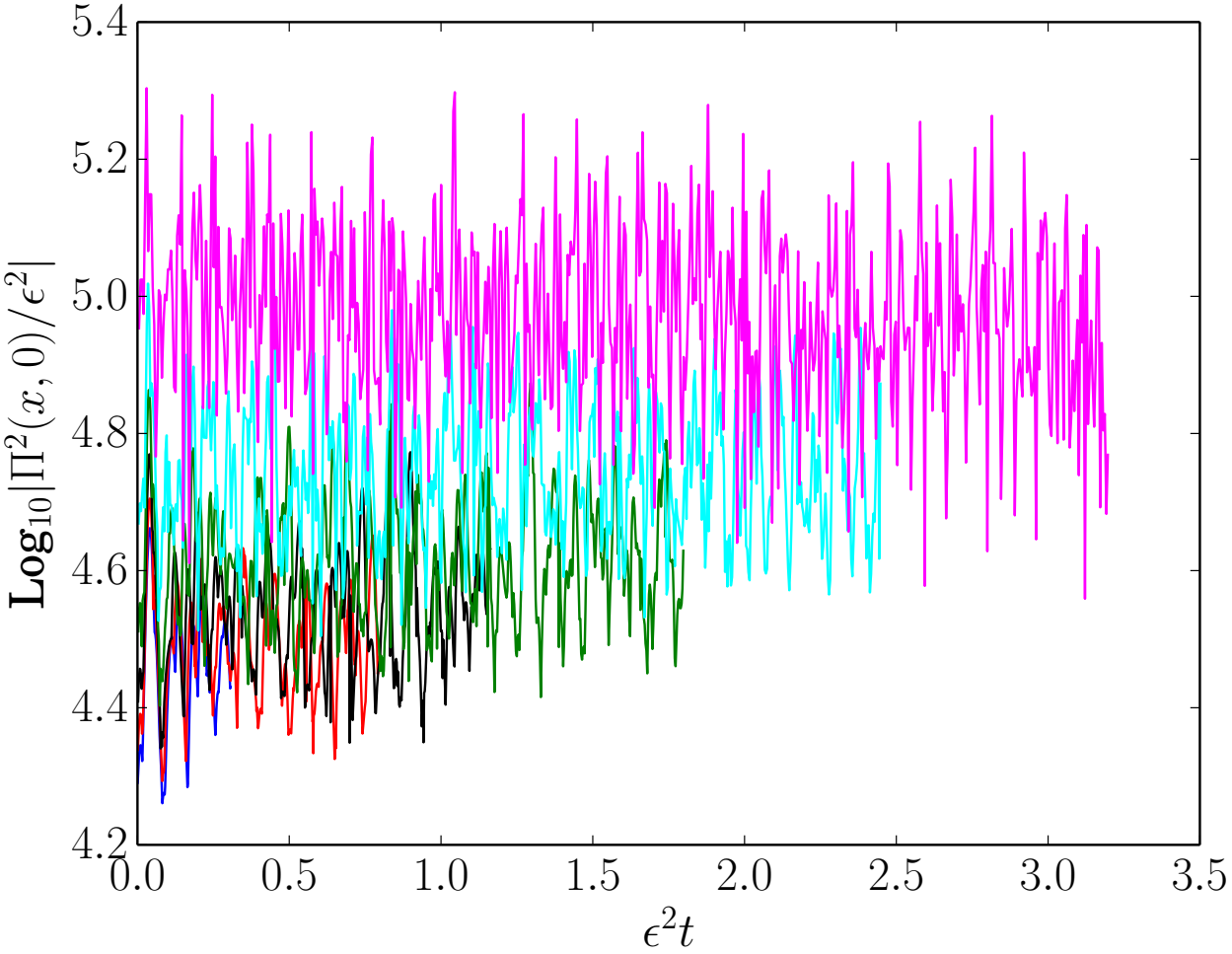
4.3. The FPU phenomenon: exhibition of the apparent stabilization. Time-
ged harmonic energies $\overline{E}_k$ versus time in log–log scale, for a time interval much



Left from: Benettin,Carati,Galgani,Giorgilli, 2008]

Other stable solutions

Three-Mode Initial Data: Modes 1, 3, and 8: $c_j = e_1/10 + e_3 + e_8/10$



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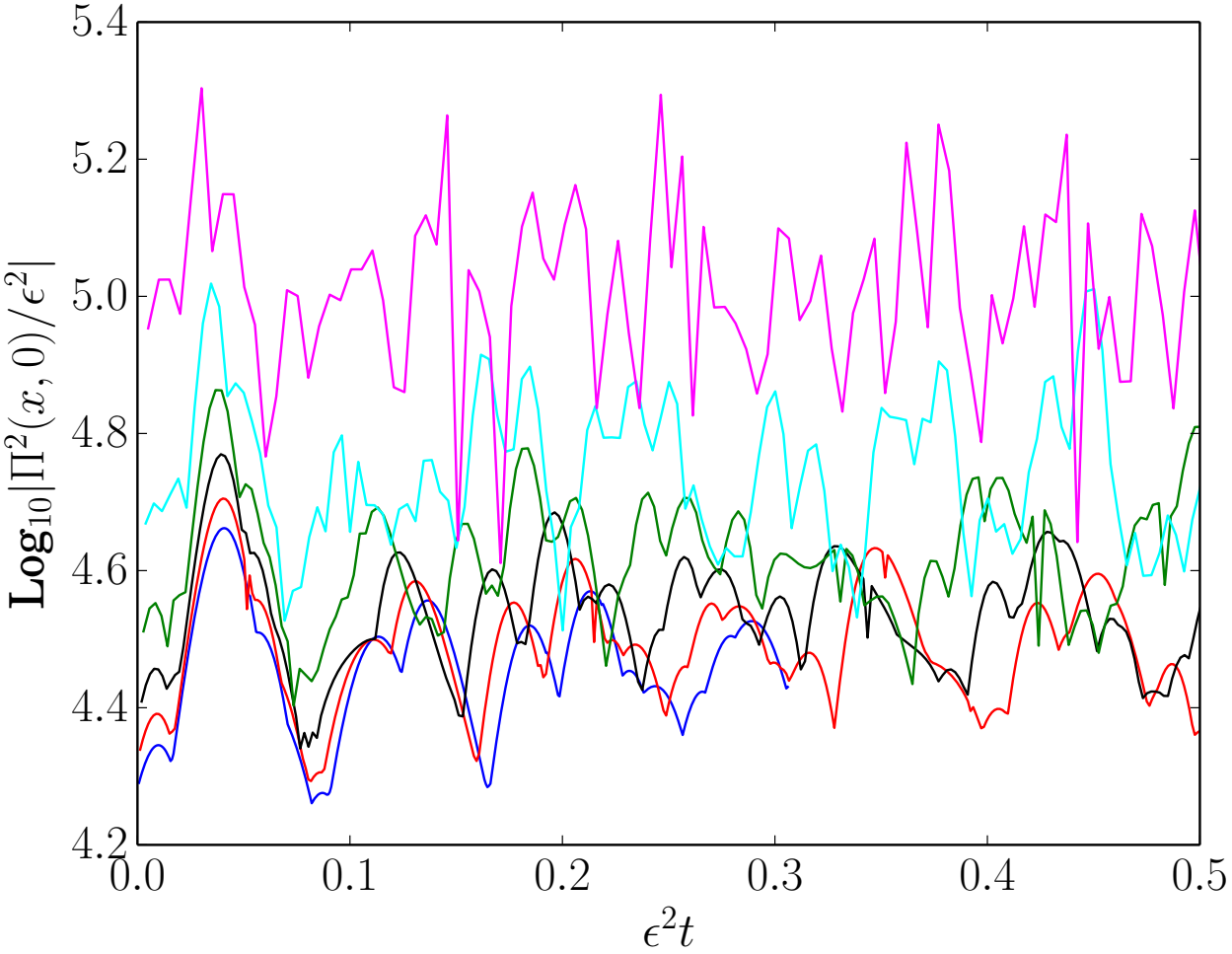
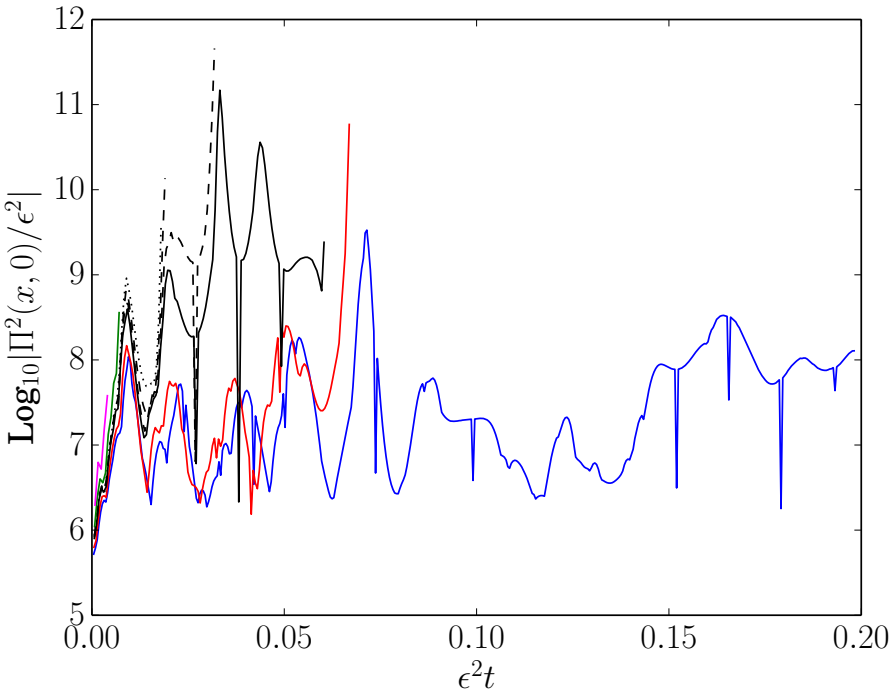
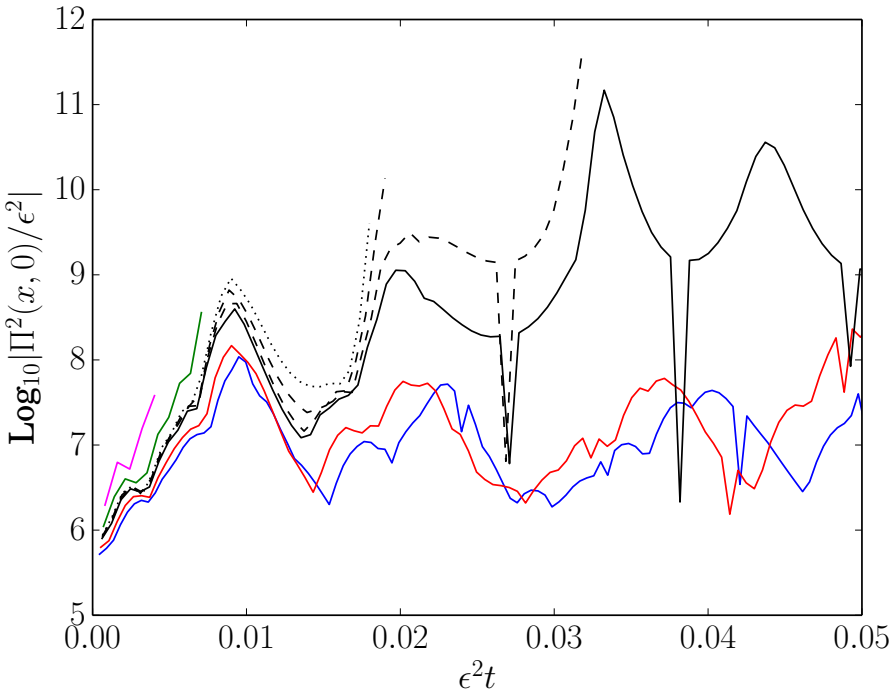


Other (possibly) stable solutions

Three-Mode Initial Data: Modes 1, 3, and 8: $c_j = e_1 + e_3 + e_8$

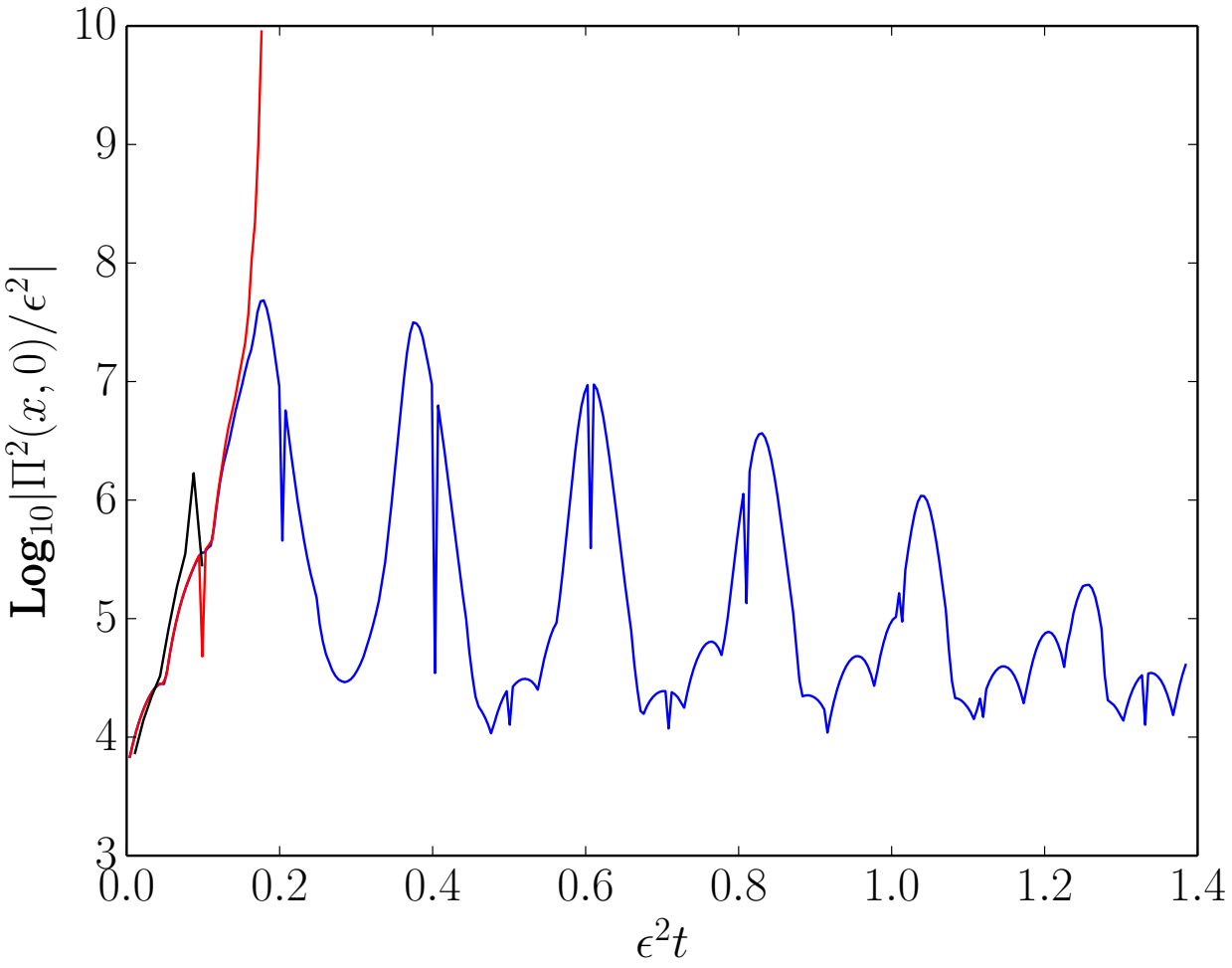
not one-mode dominant





Other (possibly) stable solutions

Exponential Amplitude: $c_j = e^{-\alpha j}$ for $\alpha = 0.575$



Take-home Points

- **TTF formalism** most useful, perturbative approach
- System demonstrates both direct- **and indirect** cascades
- Similarity to **FPUT** system...identical structure of equations as TTF; resonant and nonresonant regimes
- Stability regions
  - Existence of stable, **quasi-periodic solutions**
  - ID w/ broadly distributed energy immune **frustrated resonance**...(regardless of deficiencies in large-σ parameterization)
  - Frustrated resonance continues to **higher dimensions**
  - Frustrated resonance extends to **massive** case (qualitatively similar)
  - Other, **not just one-mode dominant** solutions...equal energy, two-mode ID
  - Poses questions about thermalization/equilibration in CFT

Uncanny resemblance!
FPUT and 2-Mode 0-1 Equal Energy FNL

Plot of energy in each mode $E_j(t)$

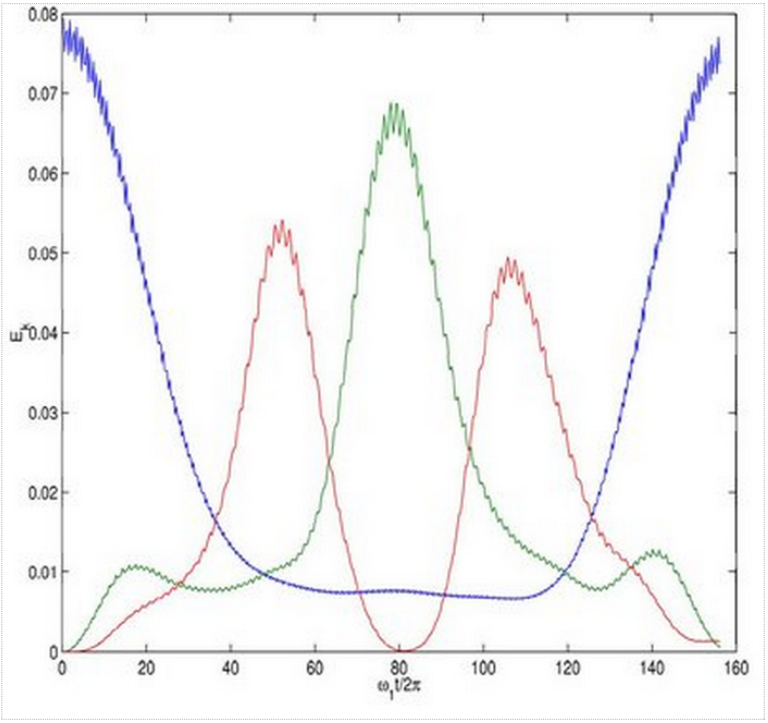


Figure 2: FPU recurrence for a FPU-α model with $N = 32$ masses and fixed ends. The plot shows the time evolution of the energy (kinetic + potential) $E_k = (\dot{A}_k^2 + \omega_k^2 A_k^2)/2$ of each of the three lowest normal modes,