Some nonstandard gravity setups in AdS/CFT

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M. Heller, RJ, P. Witaszczyk, 1103.3452, 1203.0755
RJ, J. Jankowski, P. Witkowski, work in progress

Outline

Introduction: Global AdS versus Poincare Patch

Outer boundary conditions (in the bulk) – freezing the evolution

Subtleties with ADM at the AdS boundary

Dirac δ-like boundary conditions

Conclusions

Global AdS versus Poincare Patch

Global AdS_5

$$ds^2 = -\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2$$

Poincare patch

$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$

- Poincare patch covers only a part of the global Anti-de-Sitter spacetime
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  – in the global AdS case it is $\mathbb{R} \times S^3$
  – for the Poincare patch it is $\mathbb{R}^{1,3}$
- This provides a quite different physical interpretation on the gauge theory side:
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- Sometimes one can *interpret* results from both perspectives...
- ... however a natural physical configuration/problem in one perspective may be bizarre (or not very natural) in the other perspective
- Moreover some natural initial conditions in the Poincare context do not extend to smooth configurations in the global context (e.g. periodic configurations)
- There are fascinating questions in both contexts!

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Three examples of complications/stumbling blocks in various setups within the Poincare patch context...

as encountered by an outsider in NR...

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Physical context: Study the evolution of a strongly coupled plasma system from various initial conditions until a hydrodynamic description becomes accurate...

Method: Describe the time dependent evolving strongly coupled plasma system through a dual 5D geometry — given e.g. by

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

i) use Einstein's equations for the time evolution

$$R_{\alpha\beta} - \frac{1}{2} g^{5D}_{\alpha\beta} R - 6\, g^{5D}_{\alpha\beta} = 0$$

ii) read off $\langle T_{\mu\nu}(x^\rho) \rangle$ from the numerical metric $g_{\mu\nu}(x^\rho, z)$

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$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x^\rho) + \ldots \qquad \langle T_{\mu\nu}(x^\rho) \rangle = \frac{N_c^2}{2\pi^2} \cdot g^{(4)}_{\mu\nu}(x^\rho)$$

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Various initial geometries correspond *intuitively* (at weak coupling) to preparing the initial plasma system with various momentum distributions of gluons...

Question: What kind of initial conditions to consider?

- What kind of initial geometries on the initial slice are acceptable?
- One possibility would be to consider only geometries regular until the 'center of AdS'...
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Locate an apparent horizon, then use whatever techniques are used to excise the rest of spacetime...

However we encounter a problem due to our 'kinematics'...

Boost-invariant flow Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.

At $\tau = 0$, the initial hypersurface intersected with the boundary is light-like

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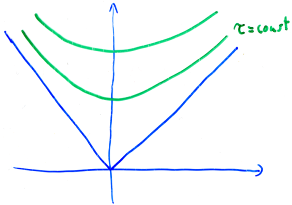

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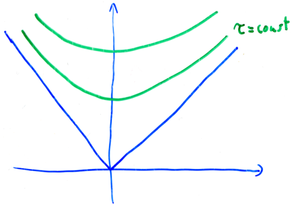

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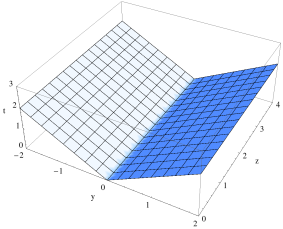

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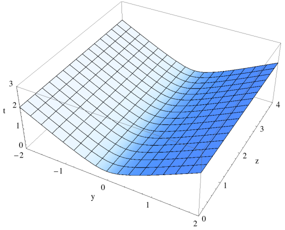

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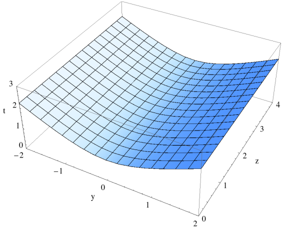

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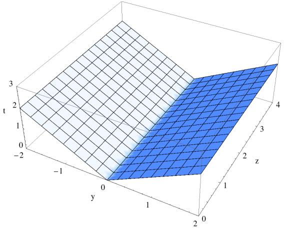

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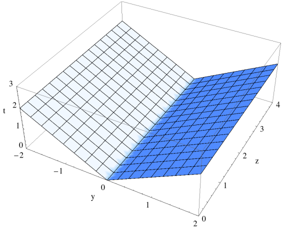

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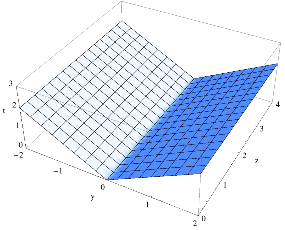

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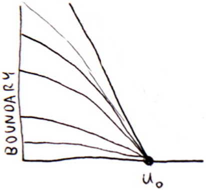

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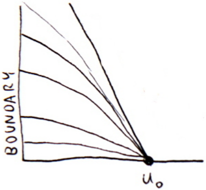

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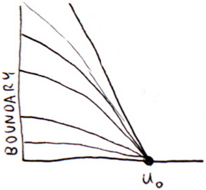

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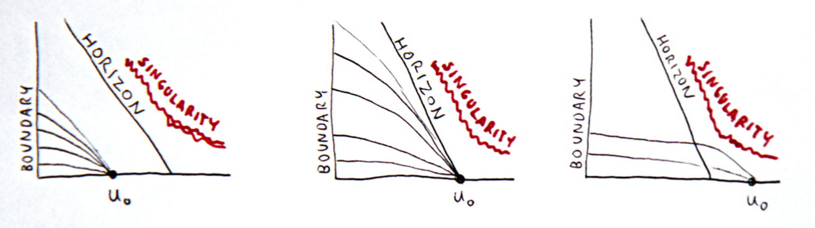

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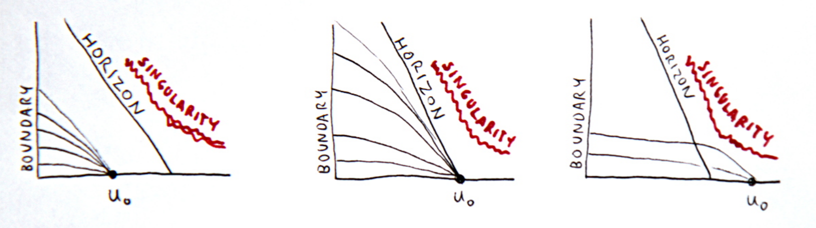

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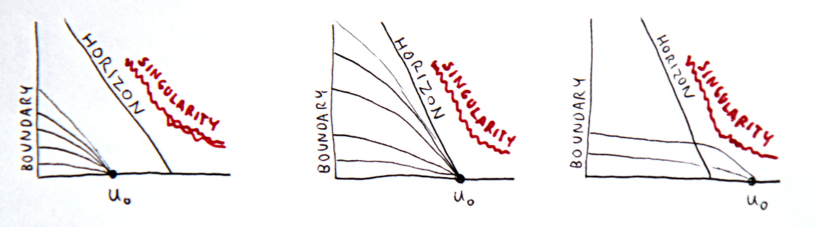

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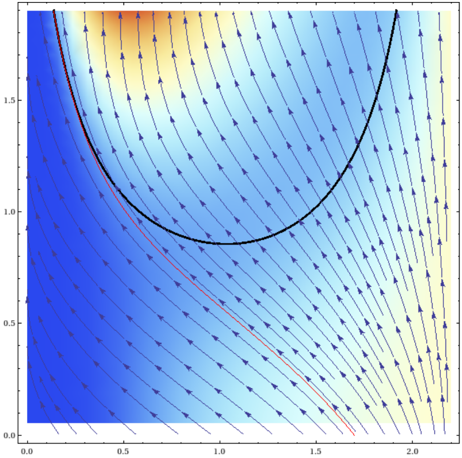

black line – dynamical horizon, arrows – null geodesics, colors represent curvature

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Physical condition in the AdS/CFT context:

The asymptotic form of the metric at the AdS boundary should be Minkowski..

i.e.

at the boundary ($z \sim 0$) it should be possible to write the metric as

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- $b(t,u)$, $c(t,u)$, $d(t,u)$ are the dynamical metric coefficients. $u = 0$ is the boundary, $u > 0$ is the bulk.
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  and obtain an ADM metric with **nontrivial** boundary values of $b(t,u)$, $c(t,u)$, $d(t,u)$
- One has to work out the conditions under which the boundary metric is related to Minkowski through such a boundary diffeomorphism
- This leads to quite nontrivial constraints in the ADM formulation...

III. Dirac δ-like boundary conditions

RJ, J.Jankowski, P. Witkowski, work in progress

Recall

- Within the AdS/CFT correspondence (gravity and matter) fields in the bulk – e.g. a scalar field $\phi(x^\mu, z)$ – correspond to some particular local operators on the gauge theory side – $\mathcal{O}(x^\mu)$
- Suppose that the scalar field $\phi(x^\mu, z)$ has the near-boundary expansion

$$\phi(x^\mu, z) \sim \underbrace{z\,\phi_0(x^\mu)}_{non-normalizable} + \underbrace{z^2\,\phi_1(x^\mu)}_{normalizable} + \ldots$$

- Then $\phi_0(x^\mu)$ is a source for deforming the field theory action

$$e^{-S+\int d^n x\, \phi_0(x^\mu)\,\mathcal{O}(x^\mu)}$$

while $\phi_1(x^\mu)$ is the corresponding expectation value

$$\langle \mathcal{O}(x^\mu) \rangle = \phi_1(x^\mu)$$

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Question:

What happens for **local** point-like sources like

$$\phi_0(x) = \eta\,\delta^2(x)$$

or line-like sources

$$\phi_0(x, y) = \eta\,\delta(x)$$

Motivation:

- Horowitz, Santos, Tong use $\cos(kx)$ source to mimick a lattice...
- Try to construct a lattice from $\sum_n \delta(x - na)$...
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Develop techniques to numerically solve Einstein's equations (with matter fields) with b.c. $\phi(x^\mu, z) \to z\delta(x)$

What is known for discontinuous boundary conditions

- $\theta(x)$ with $m^2 = 0$: Janus solutions (analytical, 1D ODE)
  Bak, Gutperle, Hirano
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- $\theta(x)$ with $m^2 = 0$ at $T \neq 0$: Janus BH – numerical PDE and analytical in 2+1 dimensions Bak, Gutperle, RJ
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Linearized solutions

- It is easy to obtain analytically *linearized* solutions around empty AdS_4 e.g.

$$\phi_0(x, z) = \frac{\eta}{\pi} \frac{z^2}{x^2 + z^2}$$

  for a line defect
- Numerical generalization to finite temperature/chemical potential (i.e. a background charged black hole solution) is fairly easy
- For the $m^2 = -2$ field and a line defect, the linearized solution is scale invariant!

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Backreacted solution at $T = 0$?

- Assume that the full solution will be a function of α
- The metric will have an AdS slicing

$$ds^2 = \frac{1}{A^2(\alpha)}\left[d\alpha^2 + \frac{dt^2 + dy^2 + dr^2}{r^2}\right] \qquad \phi = \phi(\alpha)$$

- The AdS boundaries on both sides of the defect are at $\alpha = \pm\alpha_0$ ($\alpha_0 \neq \pi/2$)
- We performed both a numerical and an analytical perturbative solution...
- **Problem:** The *backreacted* scalar field neccessarily has a nonzero source at AdS boundaries!

$$\eta\,\delta(x) + (\eta^3 + \ldots)\frac{1}{|x|}$$

Conclusion: The original $\delta(x)$ source has a dynamically generated scale (logs !?!) — no reduction to an ODE!

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- This can already be used for studying various physical questions...
- Numerical problem — very large gradients!!

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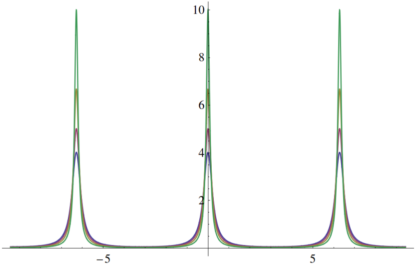

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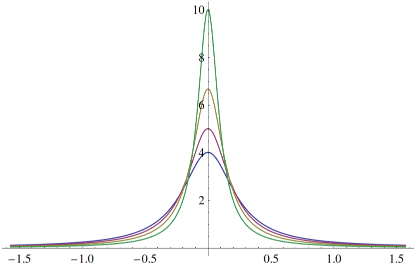

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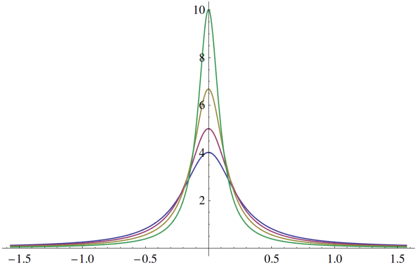

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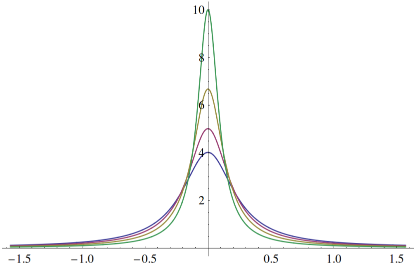

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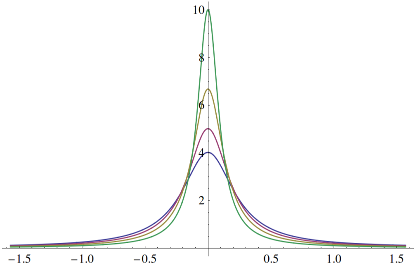

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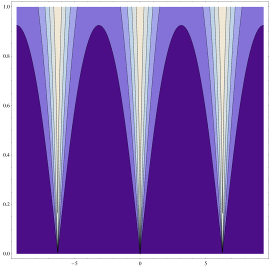

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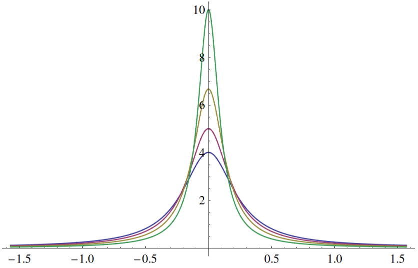

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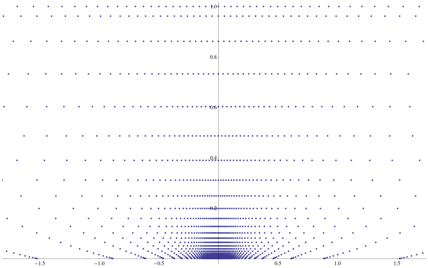

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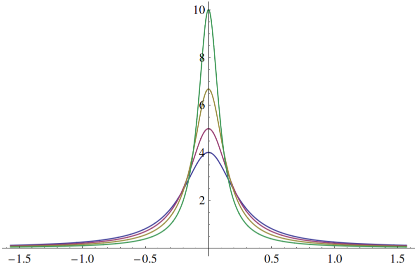

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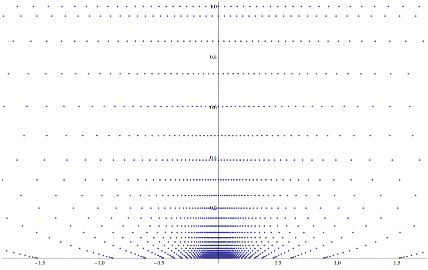

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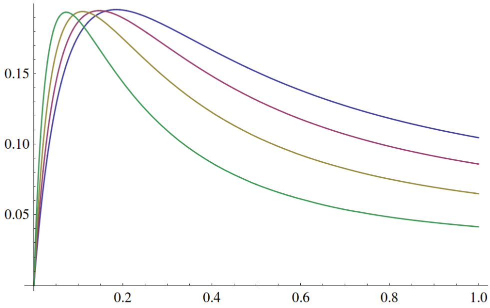

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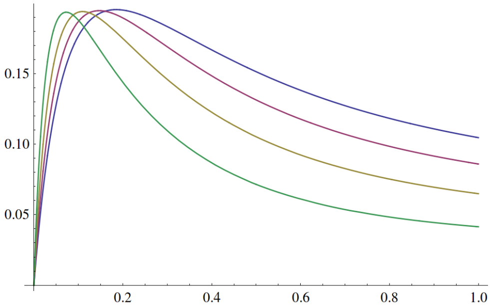

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Conclusions

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- These problems arise both in 'global AdS' and 'Poincare patch' contexts
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