Symmetry and Momentum of Bianchi Black Holes

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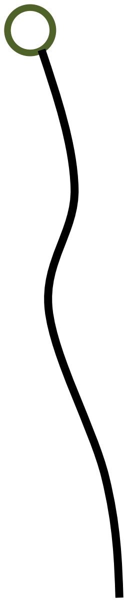
Based on work with Norihiro Iizuka and Kengo Maeda

1312.6124 & 1403.0752

Instability of AdS and singularity theorems

- **Singularity (incomplete causal geodesic)**
  must form under the conditions of

  1. **Convergence (generic & energy conditions)**
  2. **Global structure (causality or Cauchy surface)**
  3. **Strong-gravity (trapped set)**

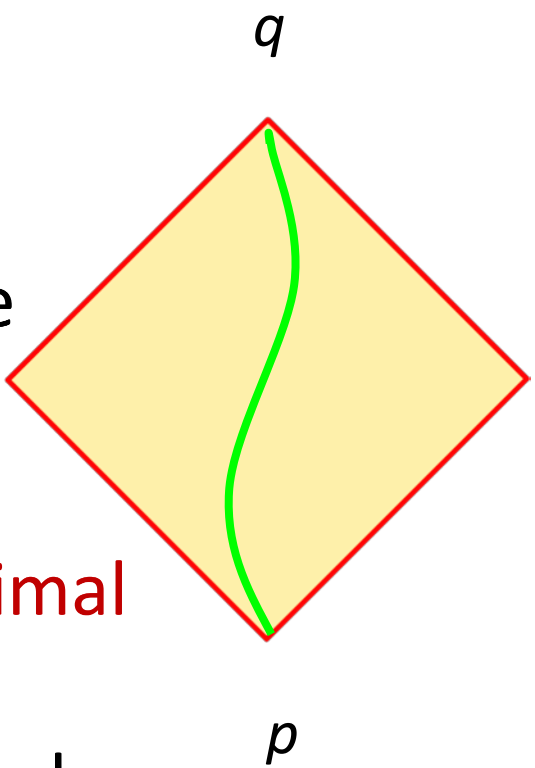


World line of a particle

A key of the proof

- **Globally hyperbolic sub-region that contains an endless timelike curve**

 ← null-convergence & trapped surface



-- guarantee the existence of the maximal length curve among all causal curves from *p* → *q*. and the maxim is attained by a timelike geodesic

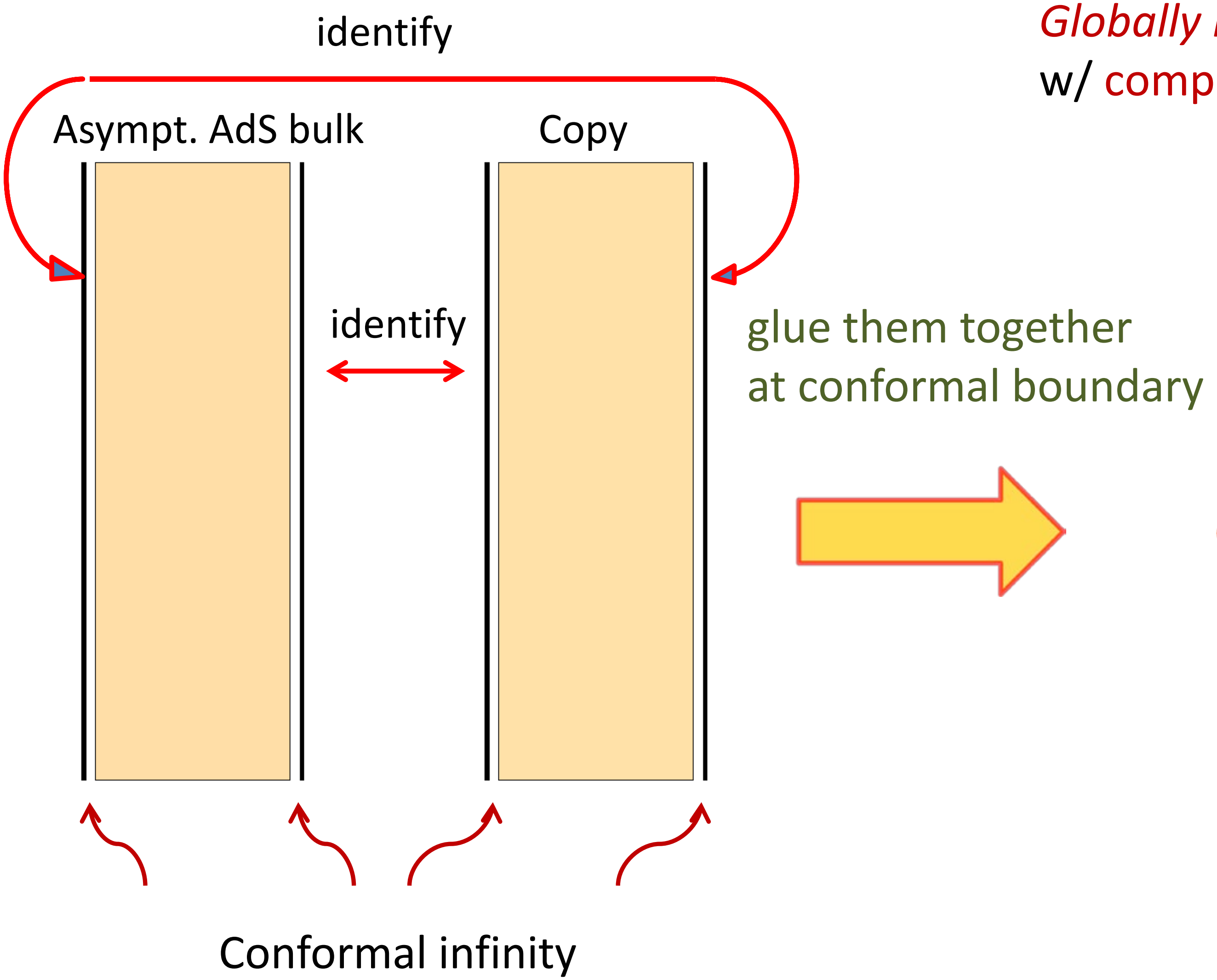
- **AdS is non-globally hyperbolic**

To get a desired maximal length curve, one may think of

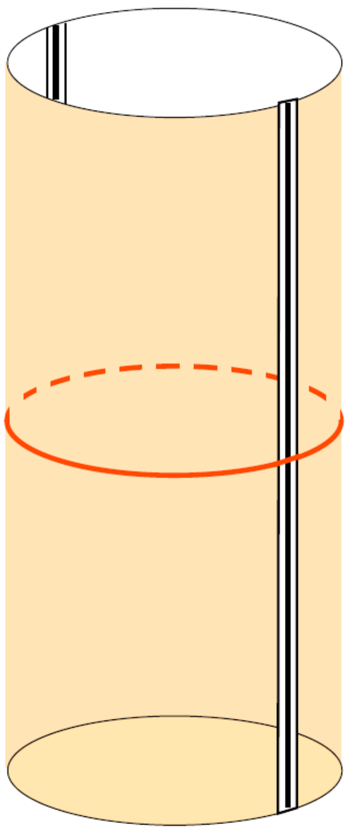
double covering

of the physical, asymptotically AdS spacetime to construct a *globally hyperbolic* unphysical spacetime w/ compact Cauchy surface.

Attempt to show a singularity theorem in the unphysical spacetime rather than in the physical spacetime.



Globally hyperbolic spacetime
w/ compact Cauchy surface

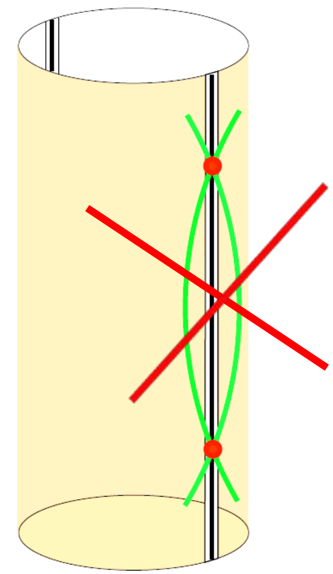


Can one apply the argument of maximum length causal curve?

Under the standard boundary conditions
(e.g. Dirichlet conditions)

isometric to an open set of the Einstein-Static Universe

The convergence (generic) condition is NOT satisfied for timelike geodesics at $\chi = \pi/2$ (AdS infinity)



-- cannot lead a contradiction!

⇩

How big are the stability islands on the turbulent ocean?

Bizon's talk

The answers may also be related to what type of (time-dependent) boundary conditions one considers

Symmetry and Momentum of Bianchi Black Holes

Motivation: AdS/CMP

AdS-black hole ⟷ superconductor

Conductivity calculation ↓

Momentum flow ← persistent current

↑ ? ↓ Realistic model

Translational symmetry broken ← Lattice structure

↓

Recent efforts – break translational invariance

-- lattice structure introduced by chemical potential or scalar field

Hartnoll-Hofman2012, Iizuka-Maeda 2012, Horowitz – Santos –Tong 2012, 2013 … etc

-- wish to construct a holographic superconductor in which a current flows without dissipation in direction of lattice where translational invariance is broken.

In asymptotically flat case, if an *asymmtric* black hole rotates along direction of no symmetry, it emits gravitational waves and settles down to a static spacetime.

In asymptotically AdS case, gravitational radiation will be reflected by AdS boundary, and the geometry could possibly approach

- an equilibrium state with no axisymmetry, or
- state of forever dynamical

See e.g. Maliborski's talks

The event-horizon itself does not rotate but some radiation (or matter fields) outside the horizon may carry the (angular) momentum.

e.g. Dias-Horowitz-Santos 2011

Closely related to the rigidity theorem:

“stationary, rotating implies axisymmetric”

Hawking 72, Hollands-Al-Wald 07

Moncrief-Isenberg 08

Key requirements:

- Weak energy conditions
- Compact Horizon cross-sections
- Analyticity

Claims:

(1) The event horizon is a Killing horizon

(2) if rotating then axisymmetric

Can a black hole have momentum along a direction of *no* translational invariance?

If possible, in what circumstances?

This talk

- We consider 5-dimesional AdS black hole whose horizon cross-sections are given by a Bianchi (homogeneous anisotropic) geometry

Bianchi geometry:

- 3 Killing vectors $\xi_I\,, \quad I = 1,\, 2,\, 3$

$$[\xi_I,\, \xi_J] = C^K{}_{IJ}\xi_K$$

Structure constant $C^K{}_{IJ}$ classifies the Bianchi type

- Invariant 1-forms $\omega^I\,, \quad \pounds_\xi \omega^I = 0$

Type VII0 and helical structure

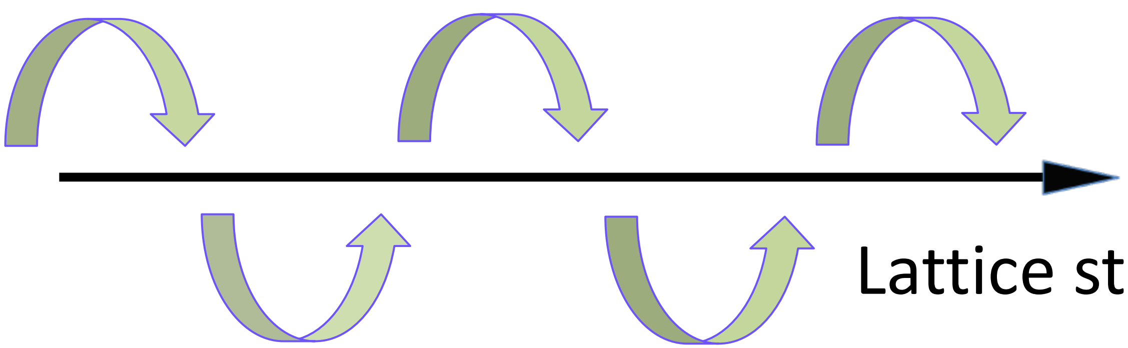
$$\omega_1 = \cos x dy + \sin x dz$$

$$\omega_2 = -\sin x dy + \cos x dz$$

$$\omega_3 = dx$$

No translational invariance along x - direction

Discrete symmetry: $x \to x + 2\pi n. \quad n \in \mathbf{Z}$



Lattice structure is introduced

Donos-Hartnoll 2012, Donos- Guantlett 2012

Metric ansatz

$$ds^2 \;=\; -f(r)dt^2 + \frac{dr^2}{f(r)} + e^{2v_3(r)}(\omega^3 - \Omega(r)dt)^2$$
$$+\, e^{2v_1(r)}(\omega^1)^2 + e^{2v_2(r)}(\omega^2)^2 \,.$$

(i) Einstein equations reduce to a set of ODEs for

$$f(r),\; v_i(r),\; \Omega(r)$$

(ii) Event horizon located at $r = r_H\,,\quad f(r_H) = 0$

(iii) At asymptotic region: impose $v_1 = v_2$

(iv) translational invariance along x- dir. recovered when

$$v_1 = v_2 \qquad e^{2v_1}(\omega^1)^2 + e^{2v_2}(\omega_2)^2 \to e^{2v_1}(dy^2 + dz^2)$$

On the horizon H

Null vector on the horizon:

$$\ell^a = (\partial/\partial t)^a + \Omega_H \underbrace{(\partial/\partial x)^a}$$

NOT Killing unless $v_1 = v_2$

$$R_{ab}\ell^a \ell^b = -2\Omega_H^2 \sinh^2(v_1 - v_2) \leq 0$$

Null convergence (weak energy) condition implies

$\Omega_H = 0$ or $v_1 = v_2$ on the horizon

We seek for solutions of

Case (I) $\Omega_H = 0$ and $v_1 \neq v_2$ on H

Case (II) $\Omega_H \neq 0$ and $v_1 = v_2$ on H

Gravity dual in Case (I) $\Omega_H = 0$ $v_1 \neq v_2$ Lattice on H

$$S = \int R + \frac{12}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}W^2 - |D\Phi|^2 - m^2|\Phi|^2$$

$$F = dA, \quad W = dB$$

$$\Phi = \phi(r)\,, \quad A_\mu dx^\mu = A_{x^1}(r)\,\omega^3 + A_t(r)dt$$

for convenience we set: $m^2L^2 = -15/4$

$B_\mu dx^\mu = b(r)\,\omega^1$: a source for the helical structure

$\xi := v_1 - v_2$

$$\begin{aligned} & f\xi'' + \{f' + f(v_1' + v_2' + v_3')\}\xi' \\ & \quad - 2e^{-2v_3}\left(1 - e^{2v_3}f^{-1}\Omega^2\right)\sinh 2\xi \\ & = \frac{1}{2}e^{-2(v_2+v_3)}\left(1 - e^{2v_3}f^{-1}\Omega^2\right)b^2 - \frac{1}{2}e^{-2v_1}fb'^2 \end{aligned}$$

Asymptotic behaviour

- Parameters:

$$A_{x^1}(r_h), \quad A_t'(r_h), \quad \phi(r_h), \quad \xi(r_h),$$
$$v_1(r_h), \quad v_3(r_h), \quad \kappa, \quad \Omega_h', \quad b(r_h)$$

- Boundary conditions:

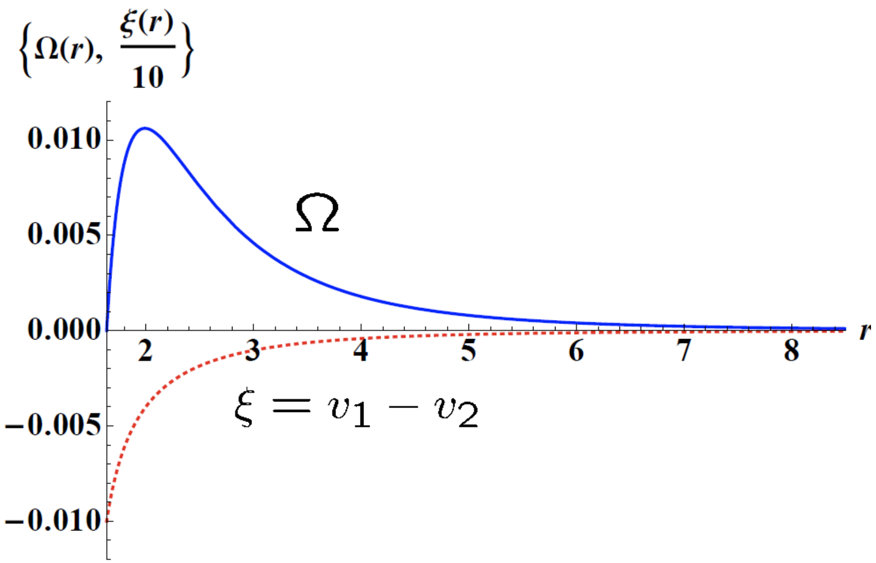
$$\Omega \simeq \Omega_0 + \frac{\Omega_{\rm N}}{r^4} \qquad \phi \simeq C_+ r^{-\frac{5}{2}} + C_- r^{-\frac{3}{2}}$$

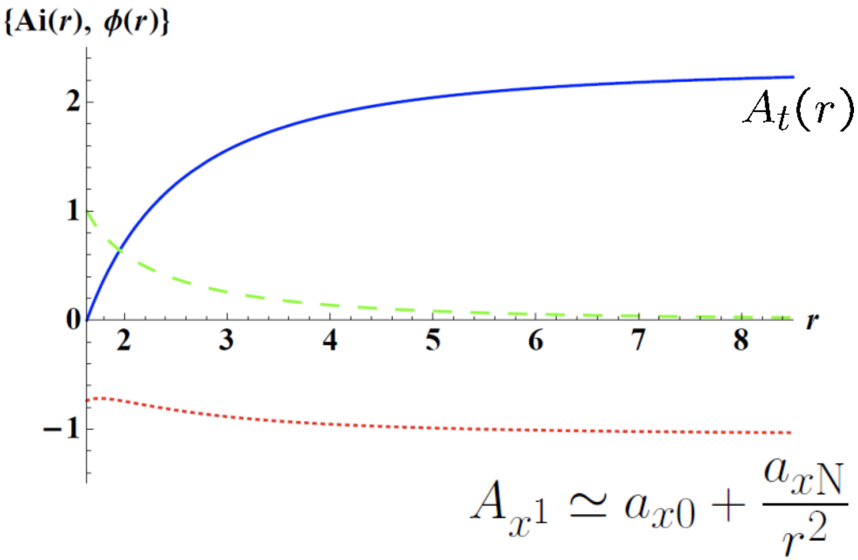
$$\Omega_0 = C_- = \xi = 0$$

AdS at infinity:
$$\begin{cases} f(r) \to \dfrac{r^2}{L^2} + O(1) \\ v_i \to \log r + const. \end{cases}$$

Solutions:

$$q=\kappa=1\,,\ L^2=2\,,\ \xi(r_h)=-0.1\,,$$
$$v_1(r_h)=v_3(r_h)=\phi(r_h)=1\,,\quad \Omega_h'=0.1\,,\ r_h=1.635$$
$$A_{x1}(r_h)=-0.7385\,,\ A_t'(r_h)=2.608\,,\ b(r_h)=7.552\,.$$





Accordingly to AdS/CFT dictionary:

$$\langle\, j_{x1}\,\rangle \sim a_{xN} \qquad \langle\, T_{tx1}\,\rangle \sim \Omega_N$$

$$a_{xN} \simeq 1.7$$
$$\Omega_N \simeq 0.57$$

This solution describes a persistent current/momentum along the direction of lattice, no dissipation

Consistent to Superfluid dynamics by Landau Tisza

$$\frac{T_{tx}}{\mu\, j_x} = 0.5 \pm O(10^{-5})$$

Gravity dual in Case (II): $\Omega_H \neq 0$ $v_1 = v_2$ on H

$$S = \int d^5x\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}W^2 - V(C)\right)$$

$$W = dC$$

$$V(C) = a_0(C - C_0)^2 + a_1(C - C_0)^4$$

$$A_\mu dx^\mu = A_t(r)dr \quad C = [c(r) - c_0]\omega^1$$

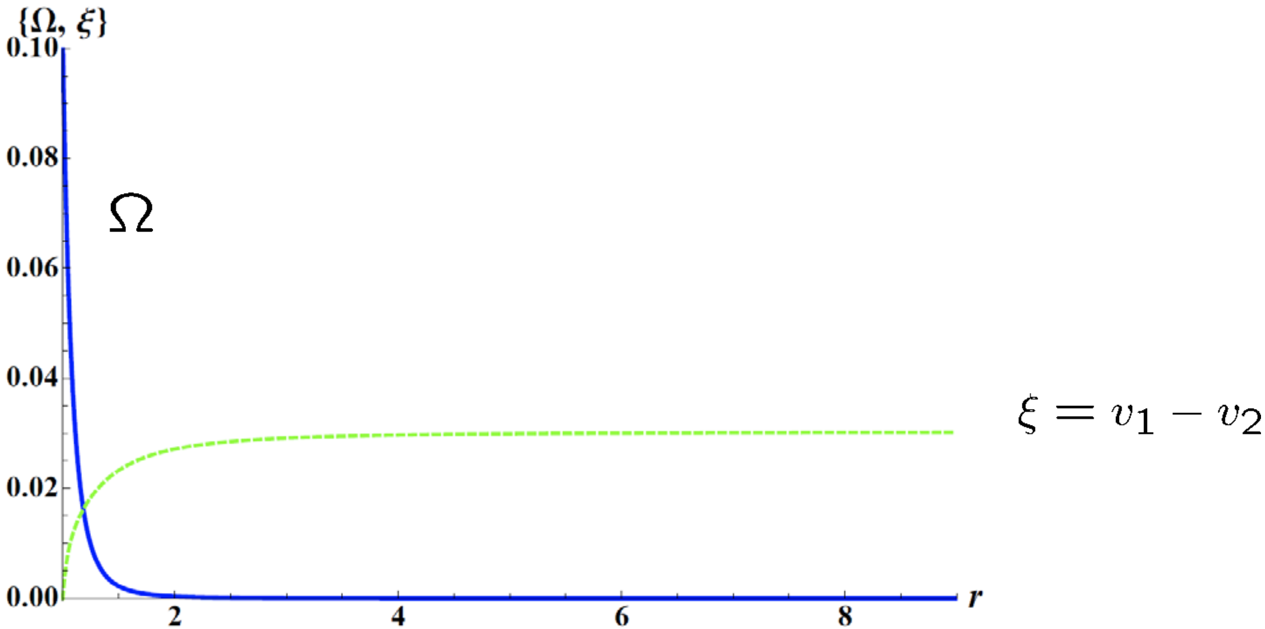
Asymptotic condition:

$$f(r) \to \frac{r^2}{L^2} + O(1) \qquad v_i \simeq \ln r + \frac{1}{2}\ln C_i \qquad \Omega = \frac{\Omega_1}{r^4}$$

$$c(r) \simeq \alpha_+ r^{n_+} + \alpha_- r^{n_-} + \left(1 - \frac{e^{2\xi(\infty)}}{2C_3 a_0} r^{-2}\right) c_0, \quad n_\pm = -1 \mp \sqrt{1 + 4a_0 L^2}$$

with $\alpha_- = 0$ so that it is normalisable

Numerical solutions



Outside the horizon: $v_1 \neq v_2$ symmetry broken

On the horizon: $v_1 = v_2$ symmetry restored

and momentum flows $\Omega_H \neq 0$

- In this model weak energy condition holds and Bianchi VII0 manifold can be compactified

  --- requirements of the rigidity theorem

This solution suggests the possibility of a regular rotating black hole which is *not analytic* at the horizon, thereby evading the rigidity theorem.

c.f. evading the rigidity by considering *non-compact* horizon

See Figueras-Wiseman 2012

c.f. charged Multi-black holes with non-smooth horizons

See, e.g., Welch 1995