Holographic Probes of Collapsing Black Holes

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Based on work w/ H. Maxfield, M. Rangamani, & E. Tonni:
VH&HM: 1312.6887 + VH, HM, MR, ET: 1306.4004 + VH: 1203.1044

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AdS/CFT correspondence

String theory ($\ni$ gravity) $\Longleftrightarrow$ gauge theory (CFT)

"in bulk" asymp. AdS × K *"on boundary"*

Invaluable tool to:

- Use gravity on AdS to learn about strongly coupled field theory
  (as successfully implemented in e.g. AdS/QCD & AdS/CMT programs)
- Use the gauge theory to define & study quantum gravity in AdS

Pre-requisite: Understand the AdS/CFT 'dictionary'...
esp. how does spacetime (gravity) emerge?

One Approach: Consider natural (geometrical) bulk constructs which have known field theory duals eg. Extremal surfaces

(We can then use these CFT `observables' to reconstruct part of bulk geometry.)

Motivation

Gravity side:

- Black holes provide a window into quantum gravity
  - e.g. what resolves the curvature singularity?
- Study in AdS/CFT by considering a black hole in the bulk
- Can we probe it by extremal surfaces?
  - Not for static BH [VH '12]
  - Certainly for dynamically evolving BH (since horizon is teleological) [VH '02, Abajo-Arrastia,et.al. '06]

$\Rightarrow$ use rapidly-collapsing black hole in AdS $\rightarrow$ Vaidya-AdS

& ask how close to the singularity can extremal surfaces penetrate?

CFT side:

- Important question in physics: thermalization (e.g. after global quantum quench)

$\Rightarrow$ use AdS/CFT…

(recall: BH = thermal state)

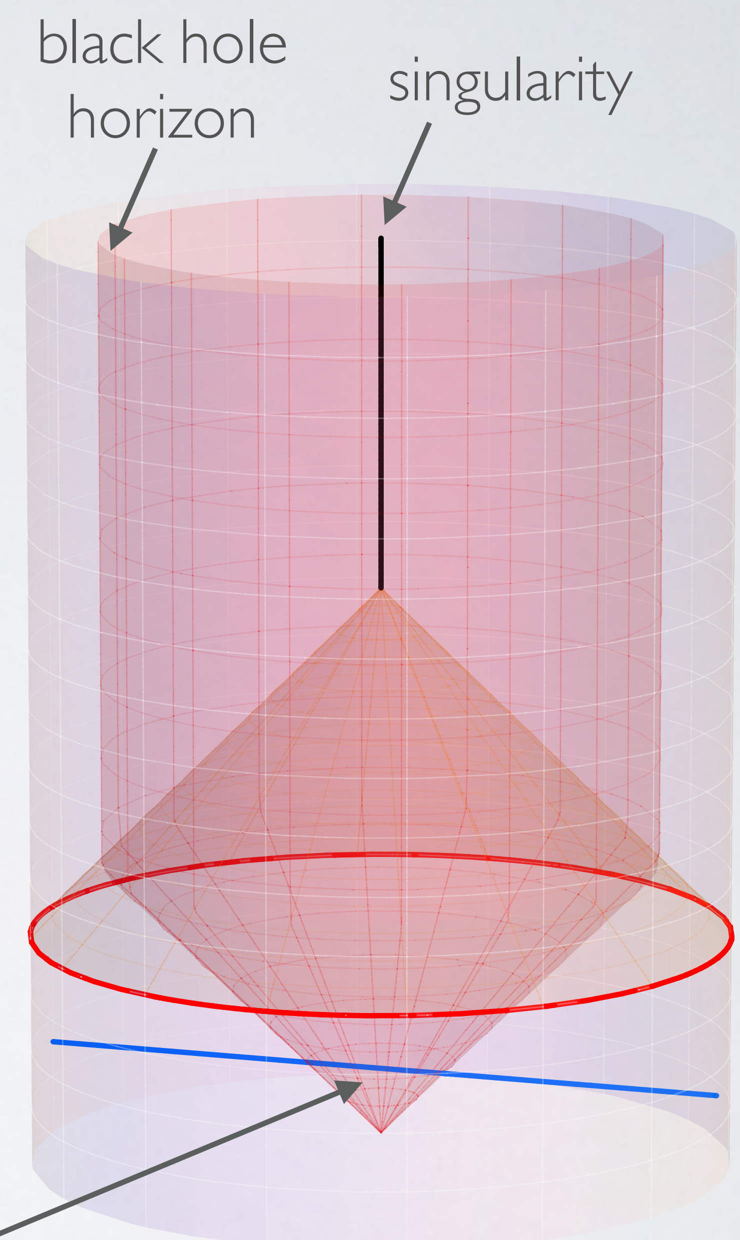
[VH,Rangamani,Takayanagi; Abajo-Arrastia,Aparacio,Lopez '06; Balasubramanian et.al.; Albash et.al.; Liu&Suh; …]

Practical aspect for numerical GR:

- what part of bulk geometry is relevant? (can't stop at apparent horizon!)

Building up Vaidya-AdS

- start with vacuum state in CFT
  = pure AdS in bulk
- at t=0, create a short-duration disturbance in the CFT (global quench)
- this will excite a pulse of matter (shell) in AdS which implodes under evolution
- gravitational backreaction: collapse to a black hole ⇒ CFT 'thermalizes'
- large CFT energy ⇒ large BH
- causality ⇒ geodesics (& extremal surfaces) can penetrate event horizon [VH '02]



Choice of spacetime & probes

Bulk spacetime: Vaidya-AdS

- $d+1$ dimensions qualitatively different for $d=2$ & higher ⇒ choose $d=2, 4$
- null thin shell ⇒ maximal deviation from static case

  ⇒ extreme dynamics in CFT (maximally rapid quench)
- spherical geometry ⇒ richer structure: can go around BH

  ⇒ explore finite-volume effects in CFT

Bulk probes:

- monotonic behaviour in dimensionality ⇒ choose lowest & highest dim.
- spacelike geodesics anchored on the boundary w/ endpoints @ equal time
  ⇒ 2-point fn of high-dimensions operators in CFT (modulo caveats…)
- co-dimension 2 spacelike extremal surfaces (anchored on round regions)
  ⇒ entanglement entropy

Naive expectations

These are ALL FALSE!

- At late times, BH has thermalized sufficiently s.t. extremal surfaces anchored at late time cannot penetrate the horizon.
- There can be at most 2 extremal surfaces anchored on a given region (one passing on either side of the black hole).
- Geodesics with both endpoints anchored at equal time on the boundary are flip-symmetric.
- Length of geodesic with fixed endpoint separation should monotonically increase in time from vacuum to thermal value.

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Vaidya-AdS

Vaidya-AdS$_{d+1}$ spacetime, describing a null shell in AdS:

$$ds^2 = -f(r,v)\,dv^2 + 2\,dv\,dr + r^2\left(d\theta^2 + \sin^2\theta\,d\Omega_{d-2}^2\right)$$

where $\quad f(r,v) = r^2 + 1 - \vartheta(v)\,m(r)$

with $\quad m(r) = \begin{cases} r_+^2 + 1 \quad , & \text{in AdS}_3 \quad \text{i.e. } d{=}2 \\ \frac{r_+^2}{r^2}\,(r_+^2+1) \quad , & \text{in AdS}_5 \quad \text{i.e. } d{=}4 \end{cases}$

and $\quad \vartheta(v) = \begin{cases} 0 \quad , & \text{for } v < 0 \\ 1 \quad , & \text{for } v \geq 0 \end{cases}$ → pure AdS / → Schw-AdS (or BTZ)

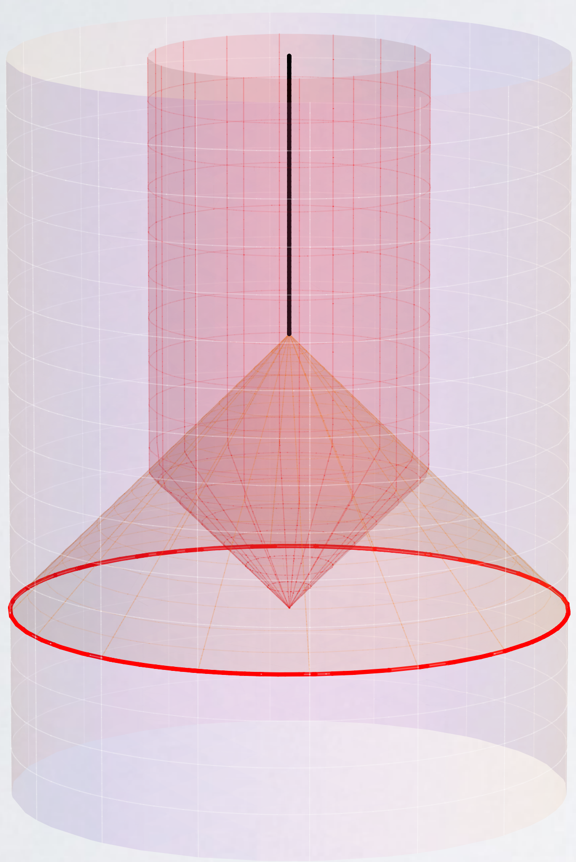
we can think of this as $\delta \to 0$ limit of smooth shell with thickness δ :

$$\vartheta(v) = \frac{1}{2}\left(\tanh\frac{v}{\delta} + 1\right)$$

Graphical representations

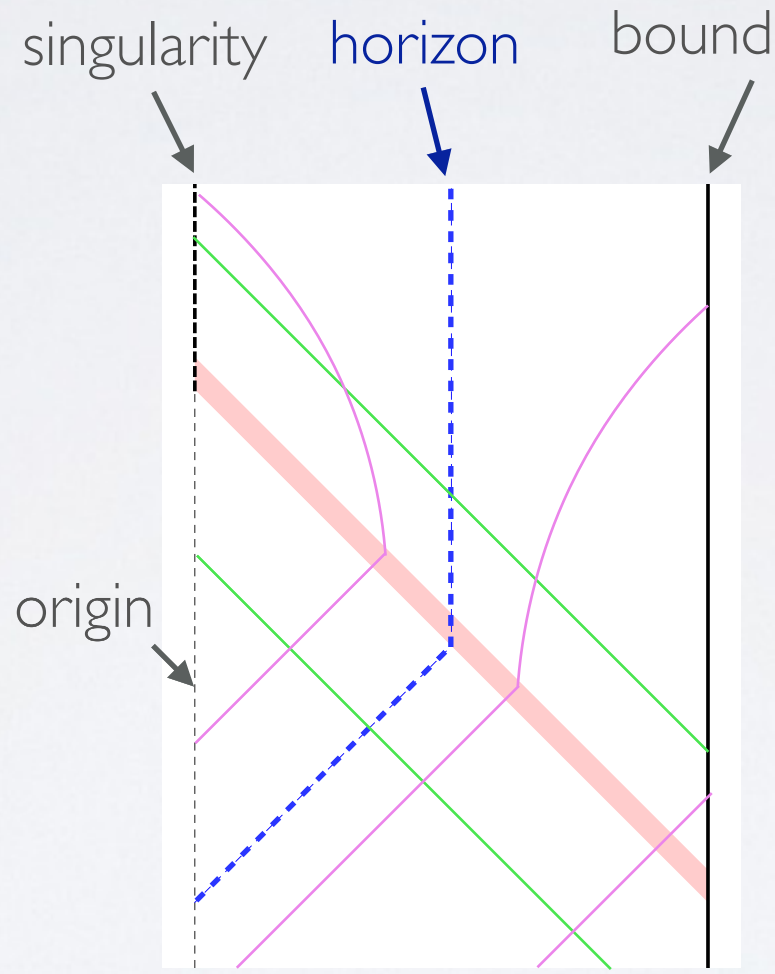
3-d

slice of geometry:



2-d (t,r)

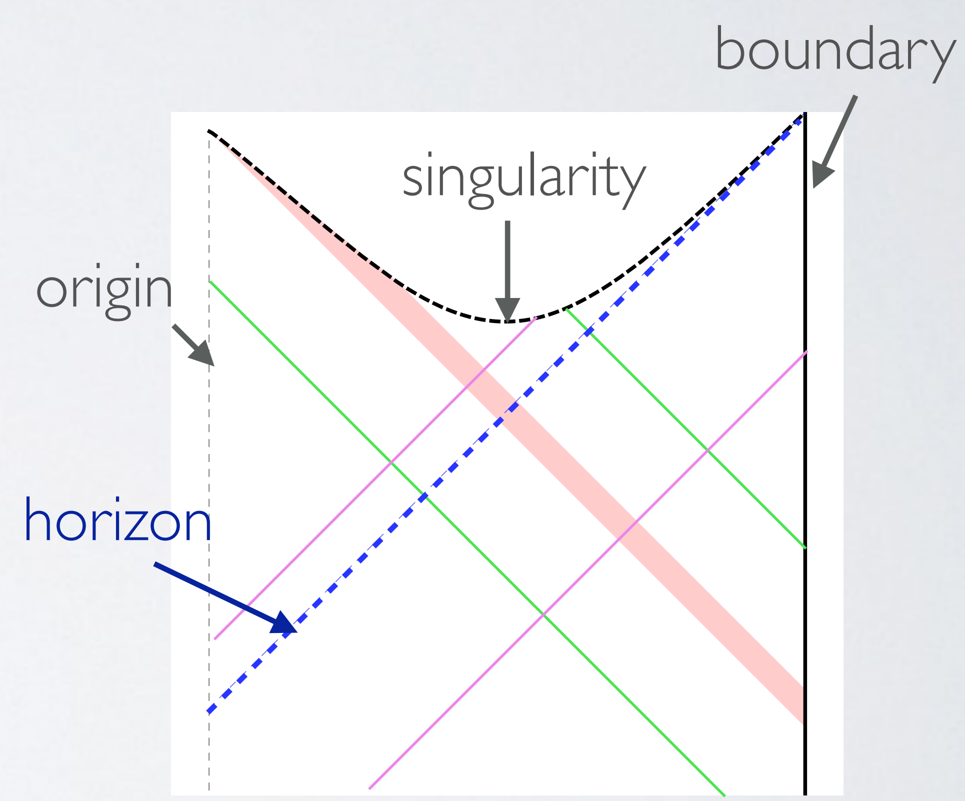
Eddington diagram:



ingoing light rays at 45°

outgoing light rays curved

Penrose diagram:



ingoing light rays at 45°

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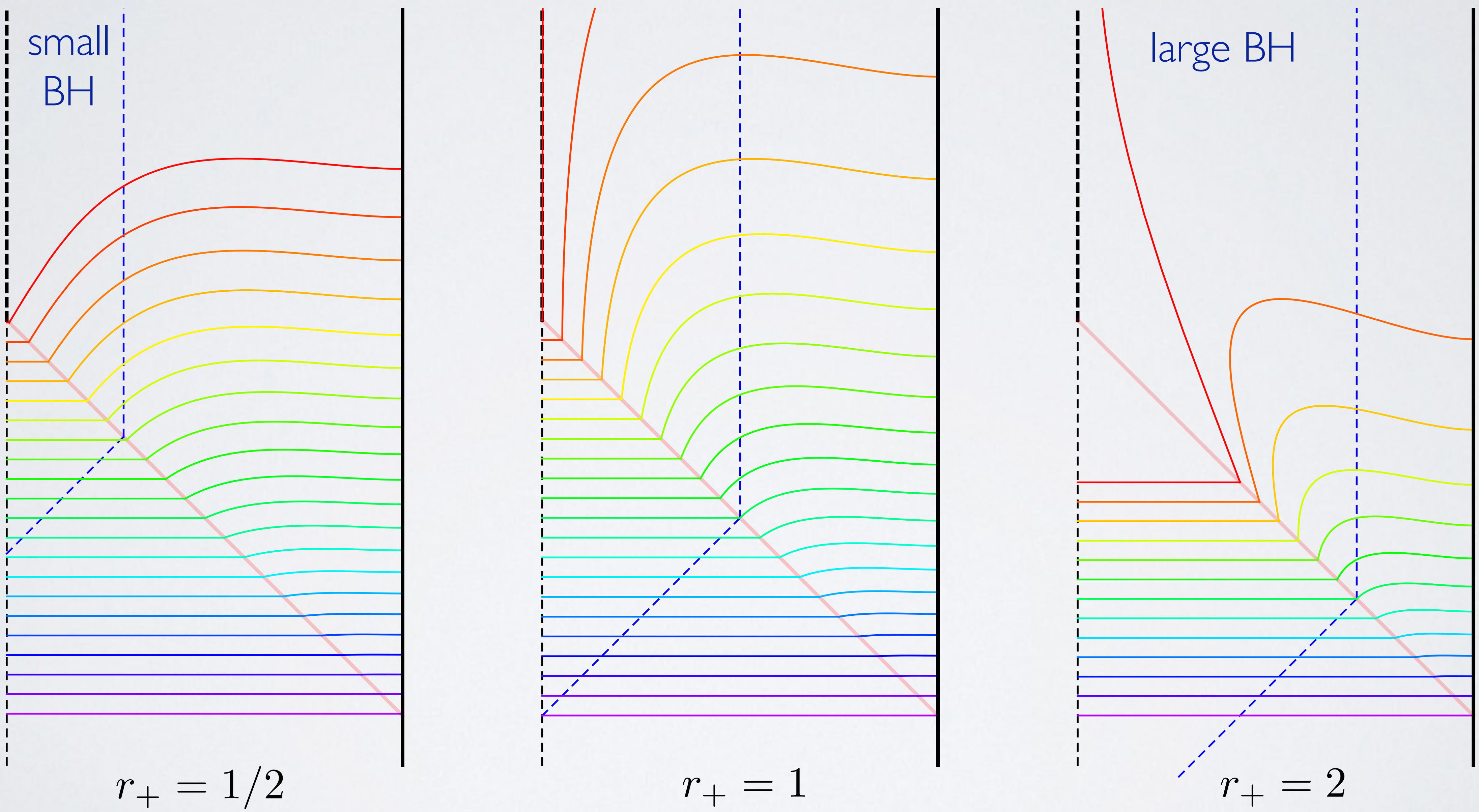
Radial geodesics in Vaidya-AdS3

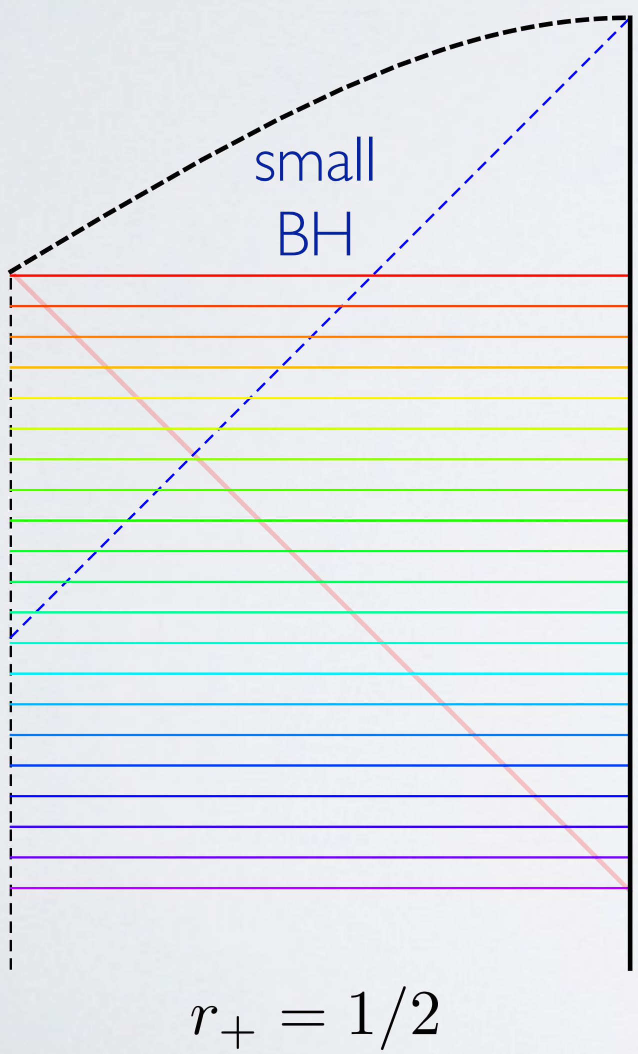
- Qualitatively different behaviour for small vs. large BTZ black holes:



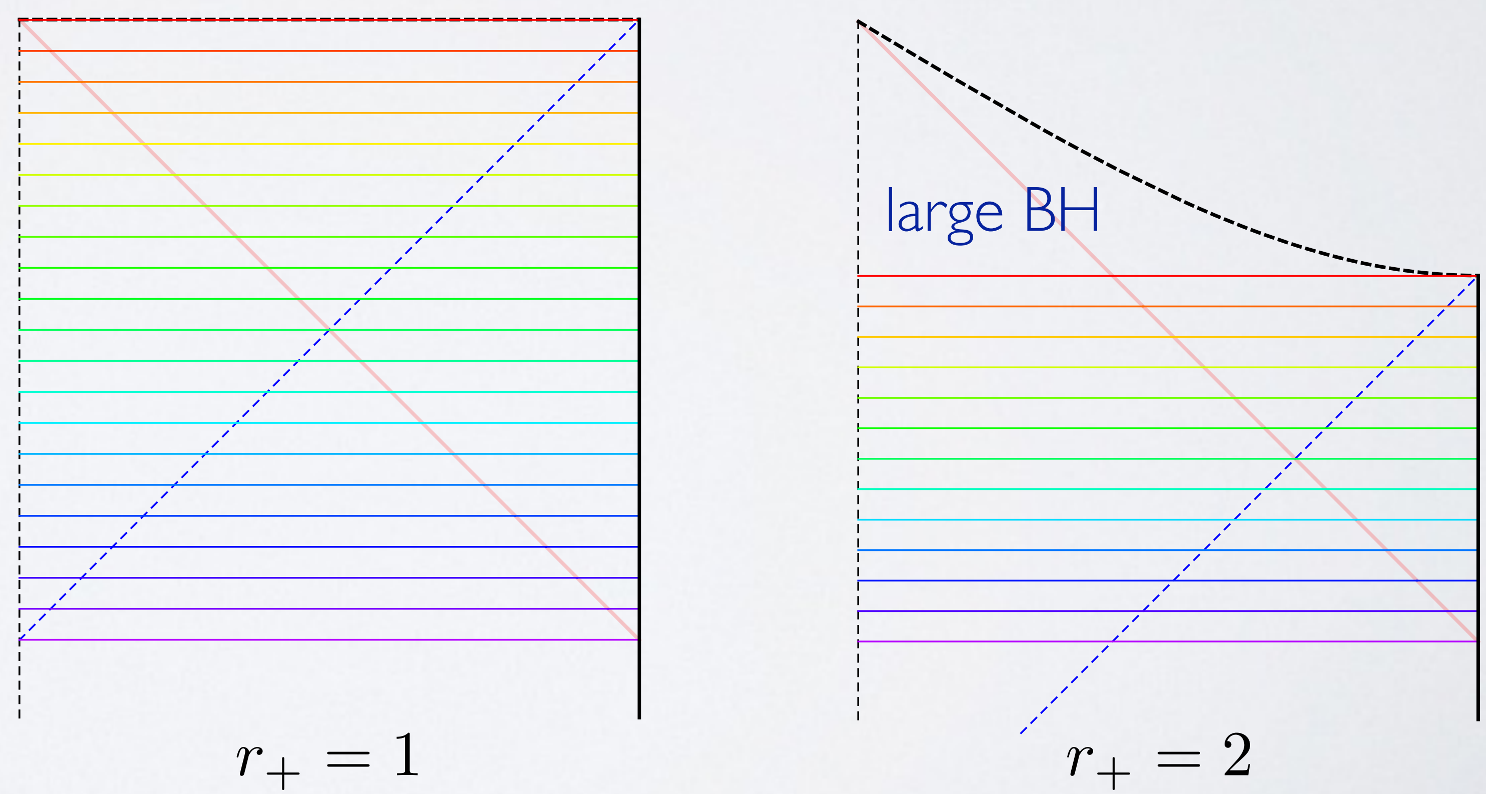
Spacelike radial geodesics on Eddington diagram

Radial geodesics in Vaidya-AdS$_3$

- Geodesic behaviour better seen on the Penrose diagram:
  - Radial spacelike geodesics are horizontal lines
  - For non-radial spacelike geodesics (not shown), BTZ segment bends up



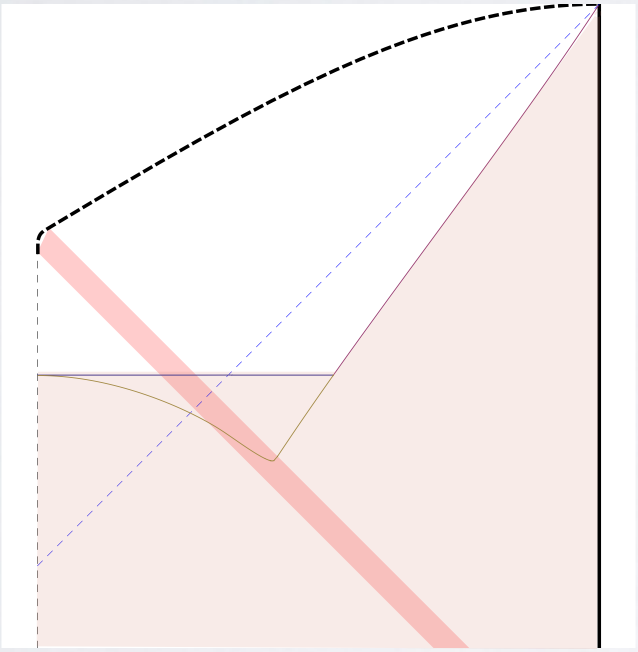
$\Rightarrow$ can probe arb. close to singularity for arb. late time
for small BH, but not for large BH



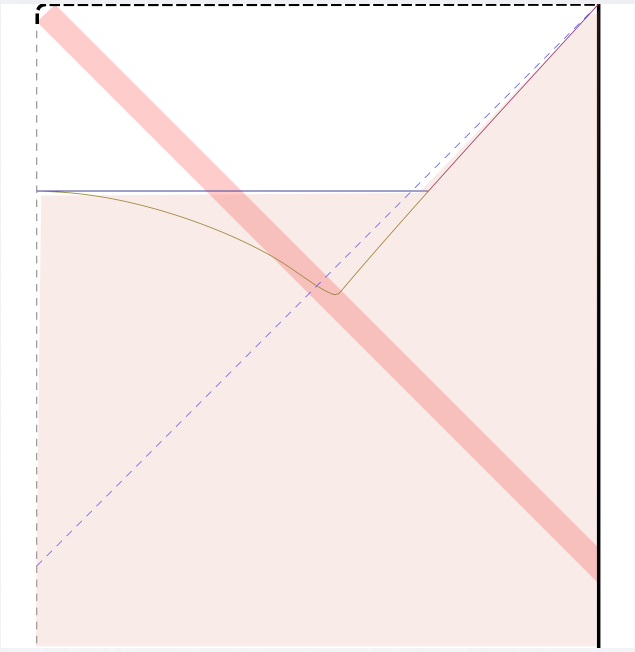
Spacelike radial geodesics on Penrose diagram

Region probed by shortest geodesics

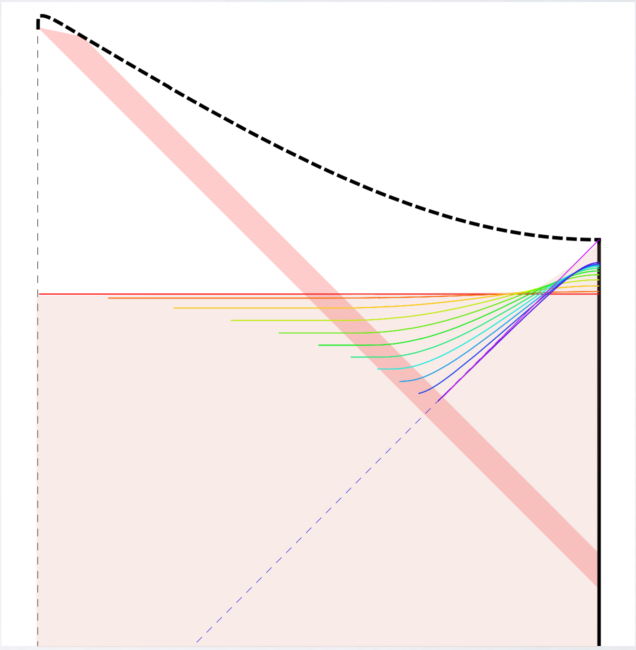
- In all cases, shortest geodesics remain bounded away from the singularity
- For small BHs, shortest geodesics can't even probe very near the horizon



$r_+ = 1/2$



$r_+ = 1$



$r_+ = 2$

Main results (for geods in Vaidya-AdS$_3$)

- Region of spacetime probed depends on BH size:
  - $r_+ = 1$: entire ST probed by radial ($L=0$) geods
  - $r_+ < 1$: entire ST probed by all ($L \geq 0$) geods
  - $r_+ > 1$: only part of ST probed;
    - central region near shell inaccessible to any boundary-anchored geod
    - maximal possible coverage achieved by radial geods
- In all cases, ∃ geods which approach arbitrarily close to late-time singularity region; but bounded curvature since ~ AdS
- Restriction to shortest geods bounds them away from entire singularity & late-time horizon

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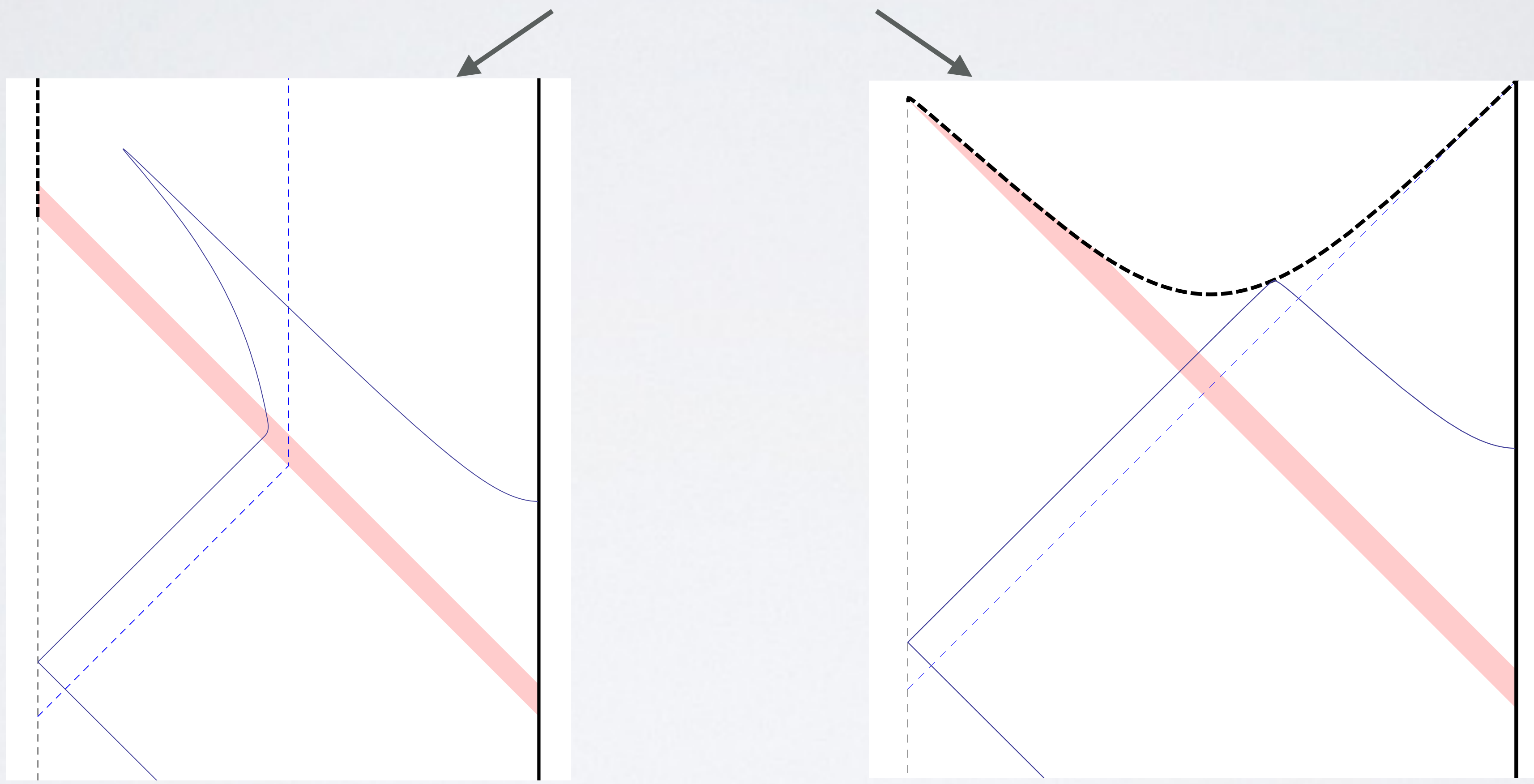
Region probed by geodesics

- Note: for boundary-anchored spacelike geodesics without restriction on equal-time endpoints, this constitutes the entire spacetime!

e.g. of Spacelike radial geodesic on Eddington & Penrose diagram



Since for $d>2$, radial spacelike geodesics are repelled by the curvature singularity

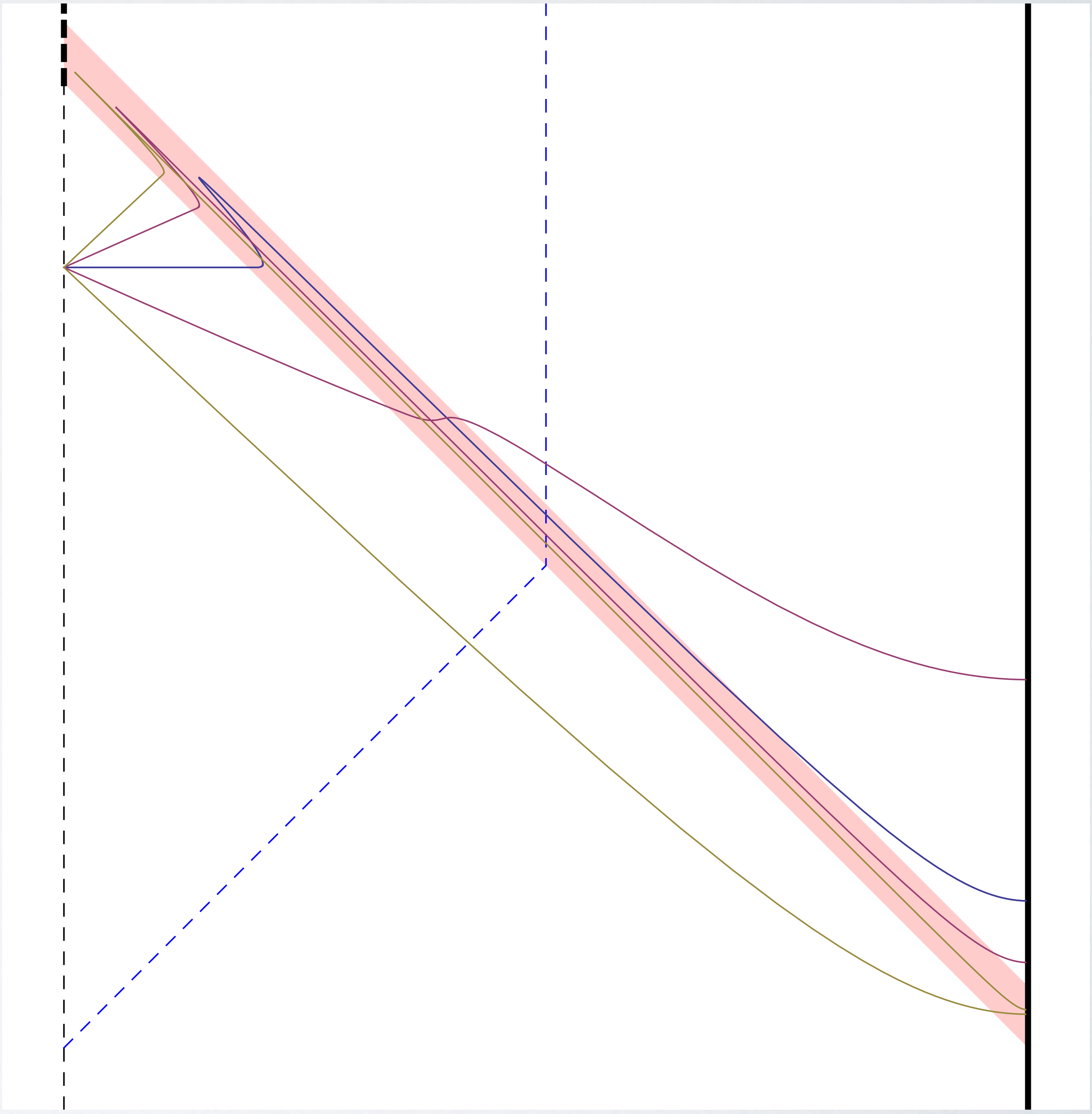
[cf. eternal BH case: Fidkowski,VH,Kleban,Shenker '03, …]

⇒ restrict to geods w/ both endpoints @ equal time on bdy

Interesting observation:

- geodesics with equal-time endpoints need not be symmetric (under flipping the endpoints)

  - symmetric geodesic guaranteed to have equal time endpoints
  - increasing energy separates endpoints
  - but interaction with shell has countering effect; in d>2 these can be balanced

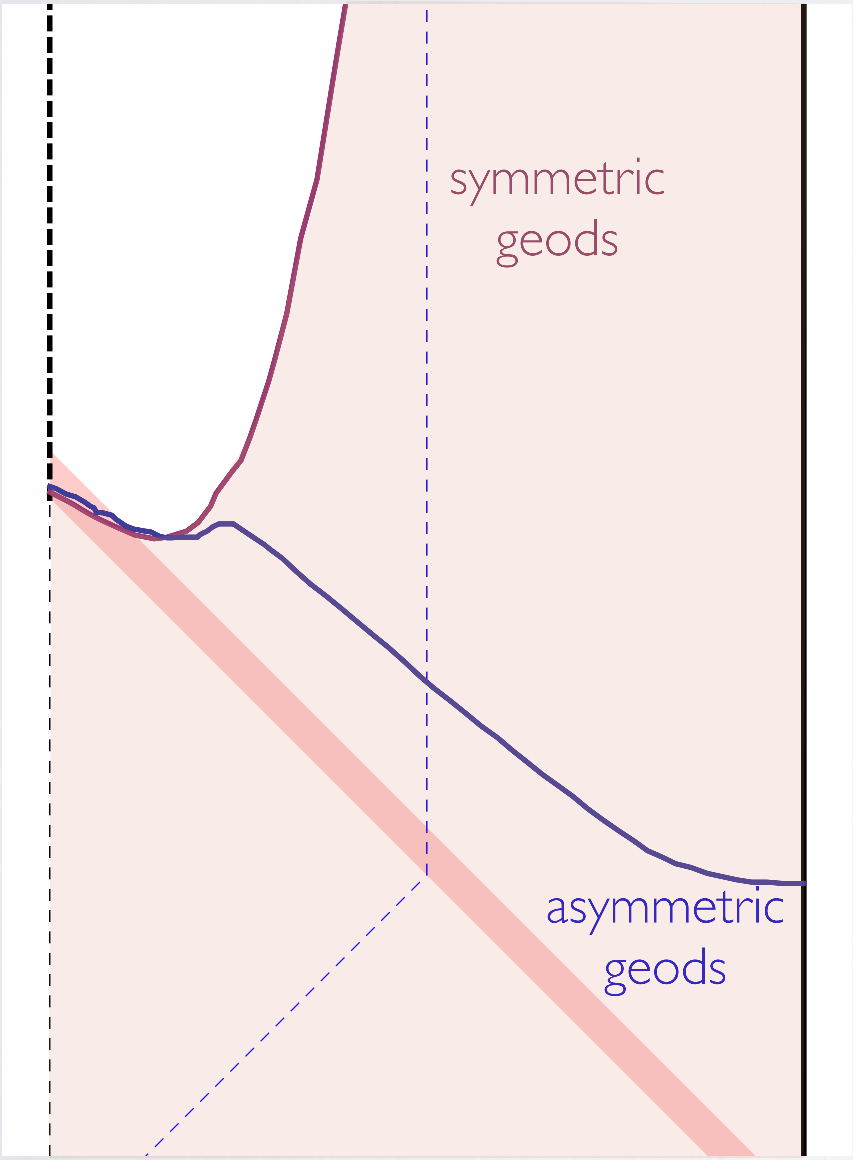


- asymmetric geodesics probe closest to singularity and are shortest (among all geods anchored at antipodal points soon after shell)

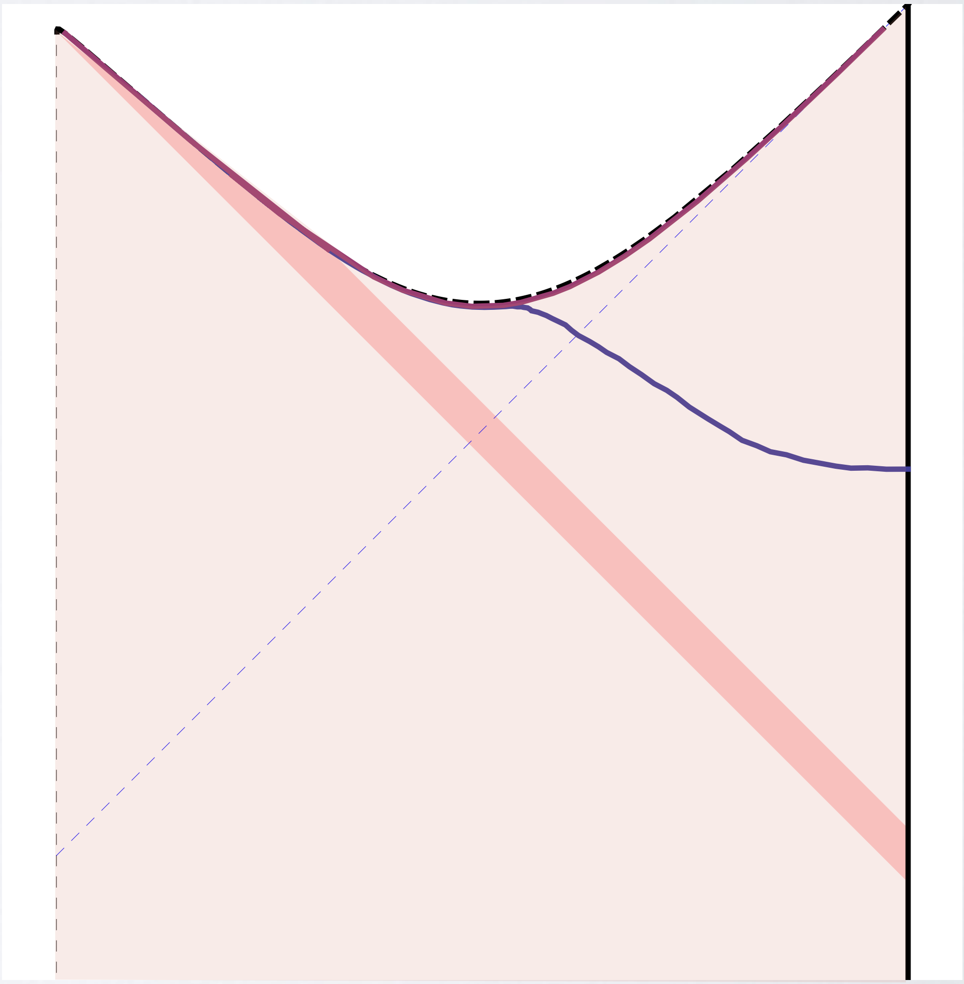
Region probed by geodesics

- ∃ symmetric spacelike geodesics anchored at arbitrarily late time which penetrate past the event horizon. (But the bound recedes to horizon as $t \to \infty$)

Eddington diagram

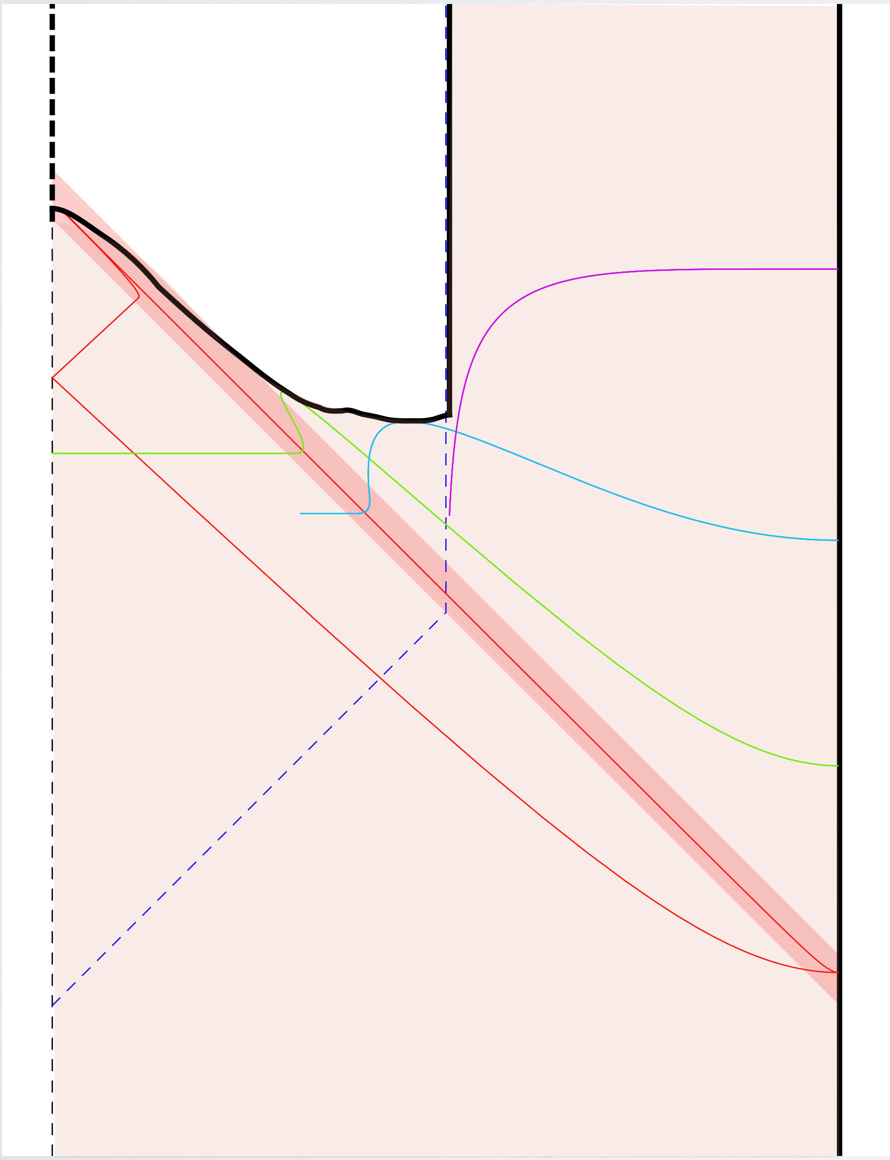


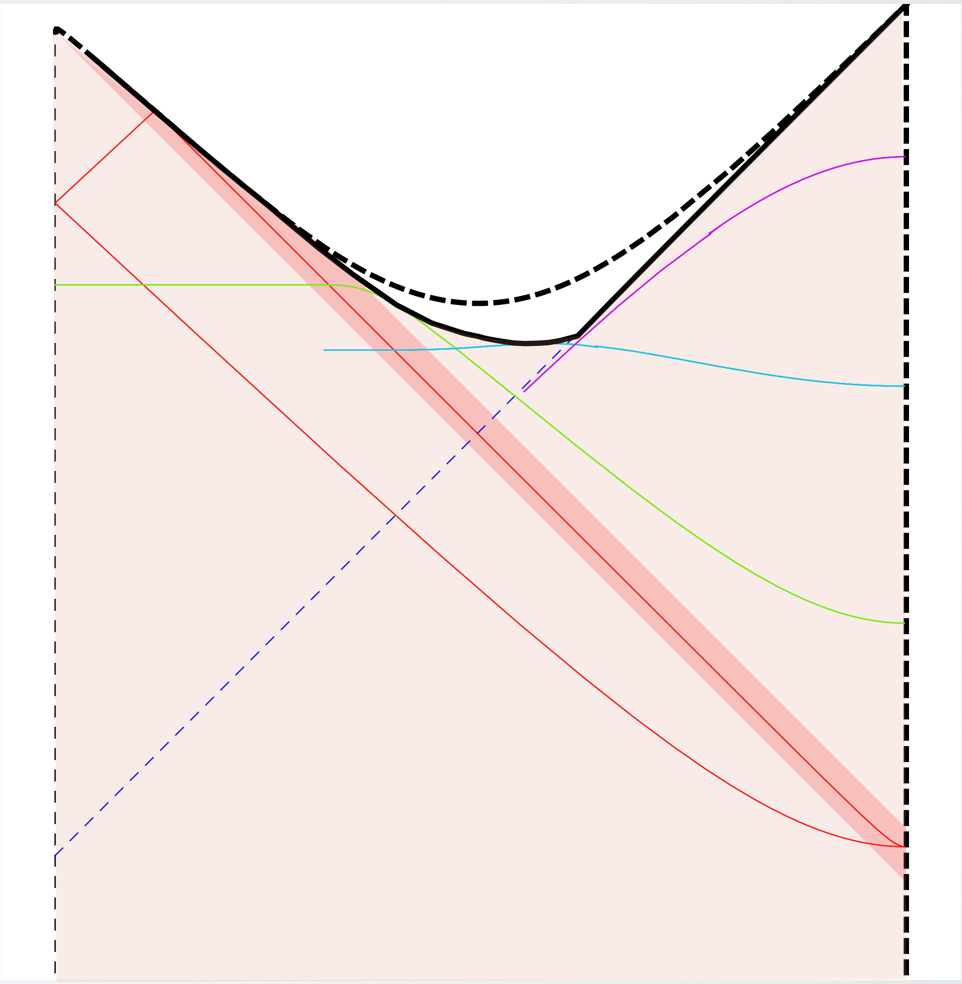
unprobed region hard to see
on the Penrose diagram



Region probed by shortest geodesics

- shortest geodesics anchored at given t are more restricted: they penetrate past the event horizon only for finite t after shell.
- However, they reach arbitrarily close to the curvature singularity.





Main results (for geods in Vaidya-AdS$_5$)

- Shortest geodesics can probe arbitrarily close to singularity (at early post-implosion time and antipodal endpoints), but cannot probe inside BH at late t.
- General geodesics can probe past horizon for arbitrarily late t.
- For nearly-antipodal, early-time endpoints, geodesics can be asymmetric (and in fact dominate), but apart from near-singularity region, their coverage is more limited.

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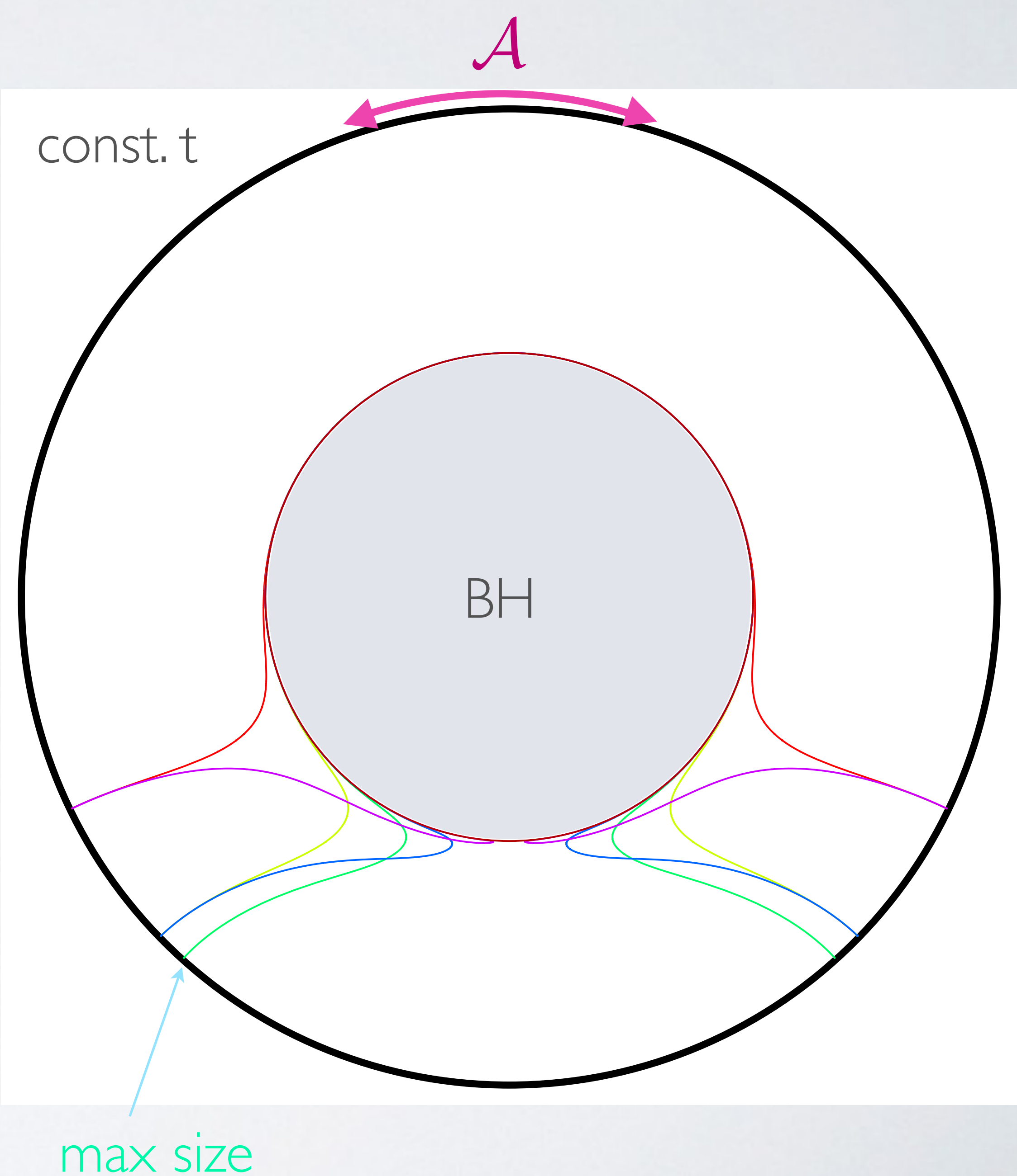
Multitudes of surfaces

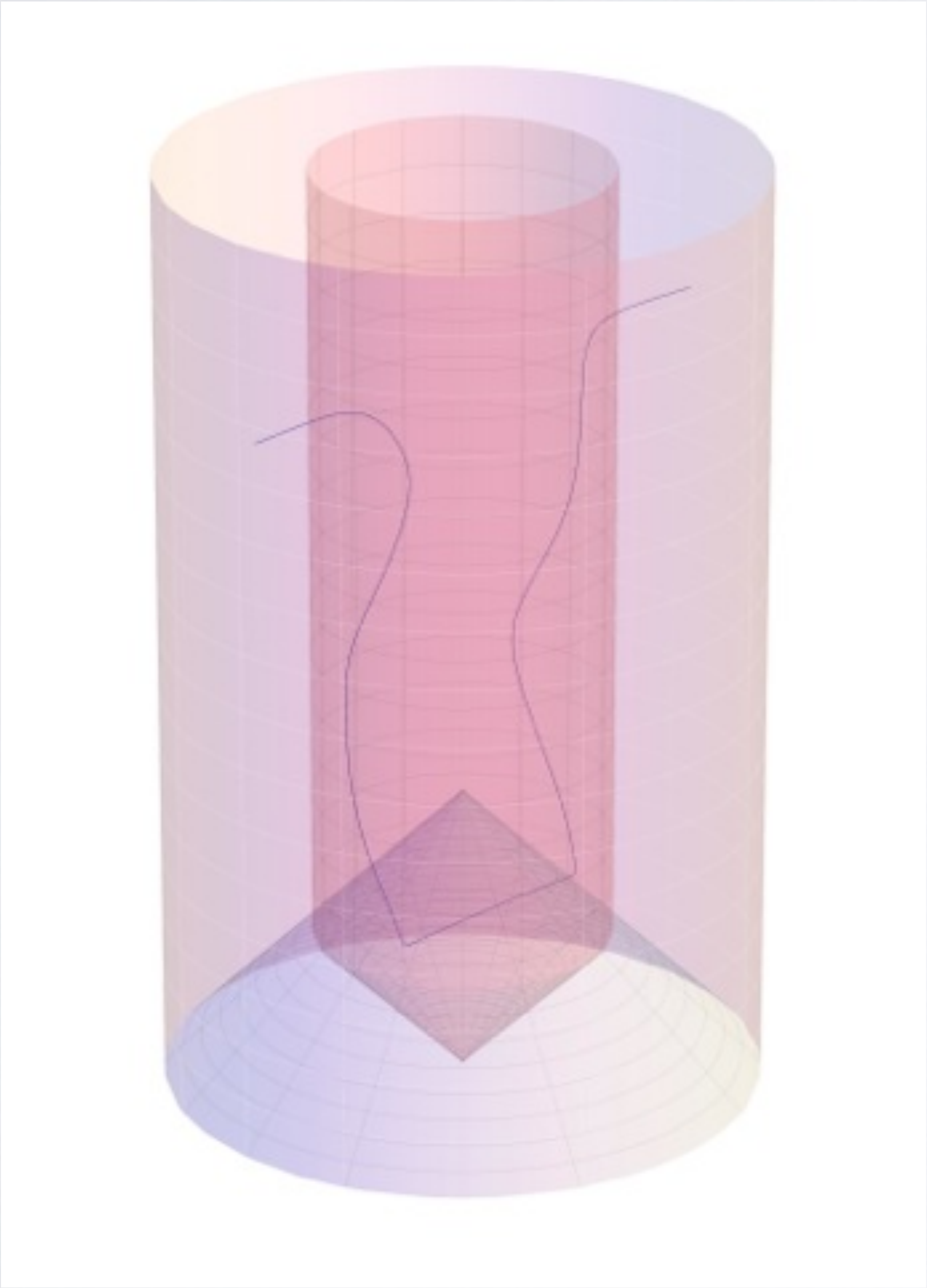
- Already for the static Schw-AdS$_{d+1}$, there is surprisingly rich structure of extremal surfaces: [VH,Maxfield,Rangamani,Tonni]

  - For sufficiently small (or sufficiently large) region $\mathcal{A}$, only a single surface exists.
  - For intermediate regions (shown), there exists infinite family of surfaces
  - These have increasingly more intricate structure (with many folds), exhibiting a self-similar behavior.
  - The nonexistence of extremal & homologous surface for large $\mathcal{A}$ is robust to deforming the state, and follows directly from causal wedge arguments.

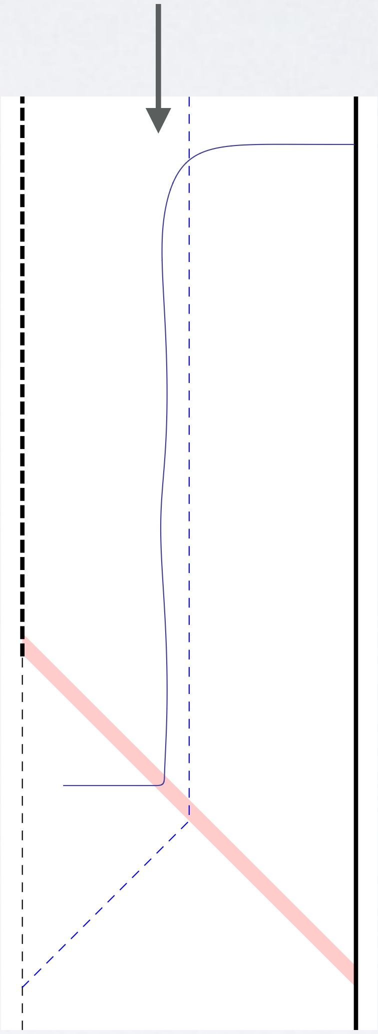


Static surface inside BH

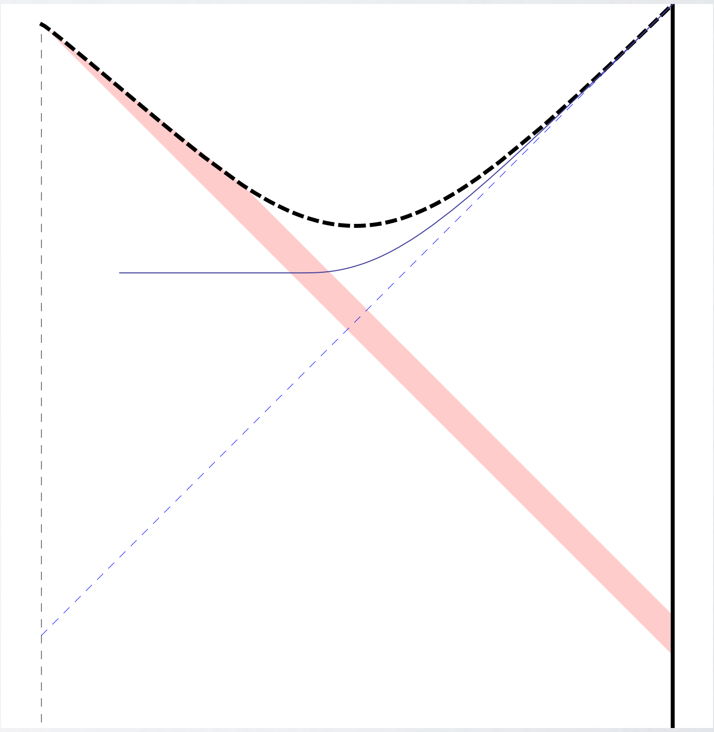
- surface can remain inside the horizon for arb. long



critical radius at which static Schw-AdS admits
a const-r extremal surface, extended in t.
[cf. Hartman & Maldacena, Liu & Suh]

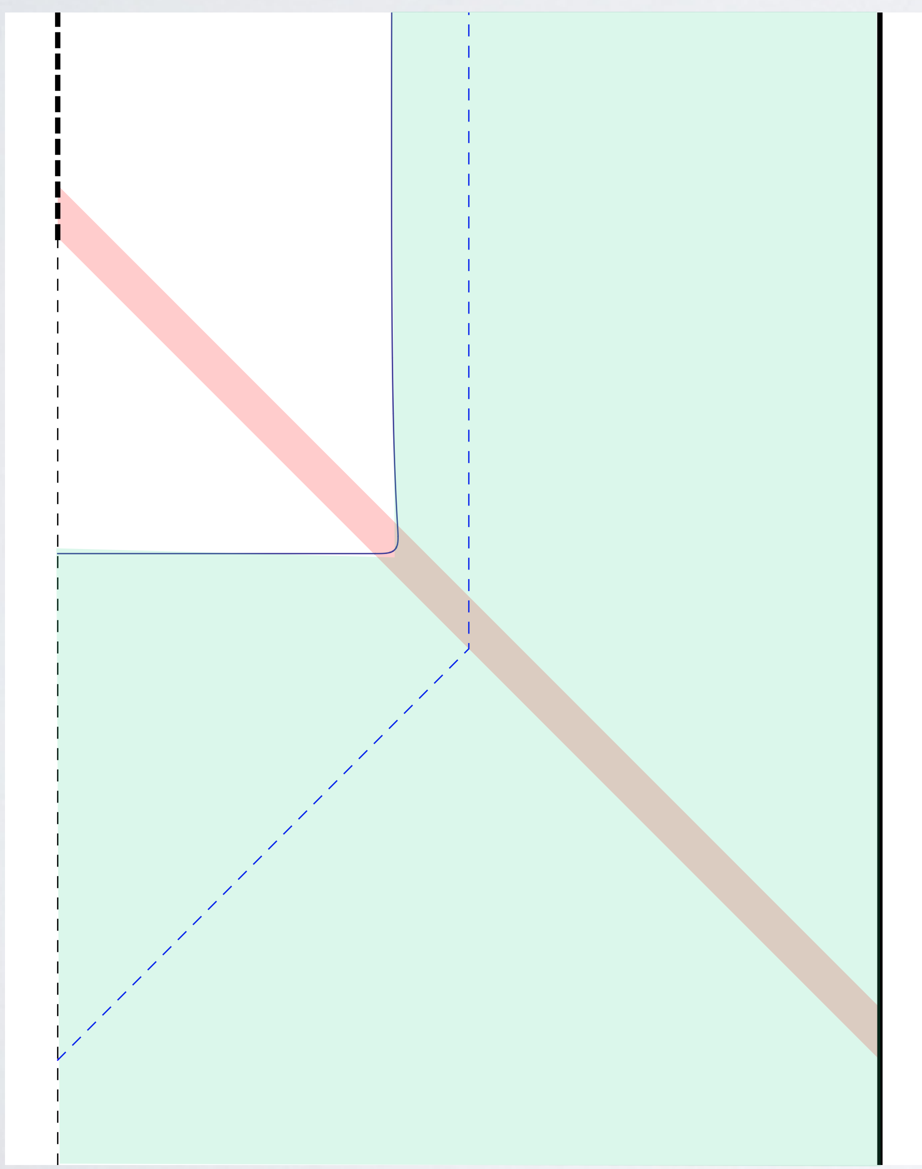


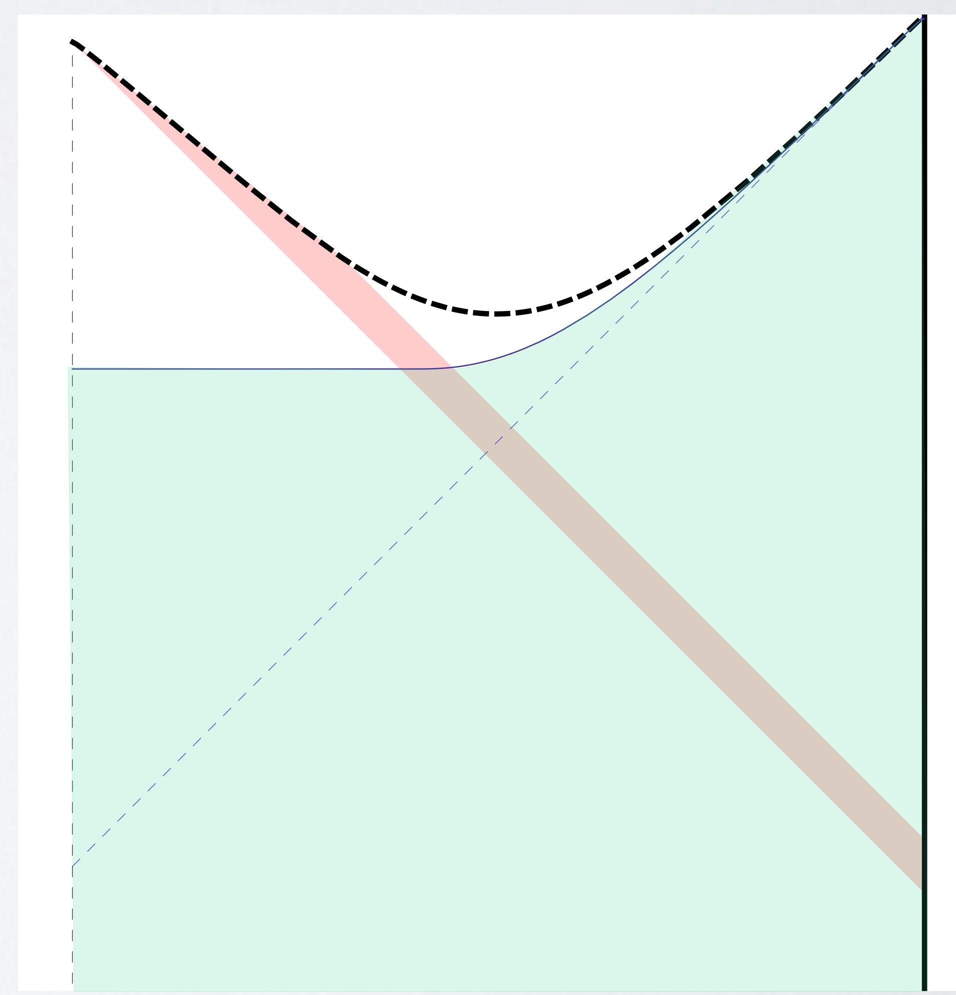
on Penrose diagram:



Region probed by such surfaces

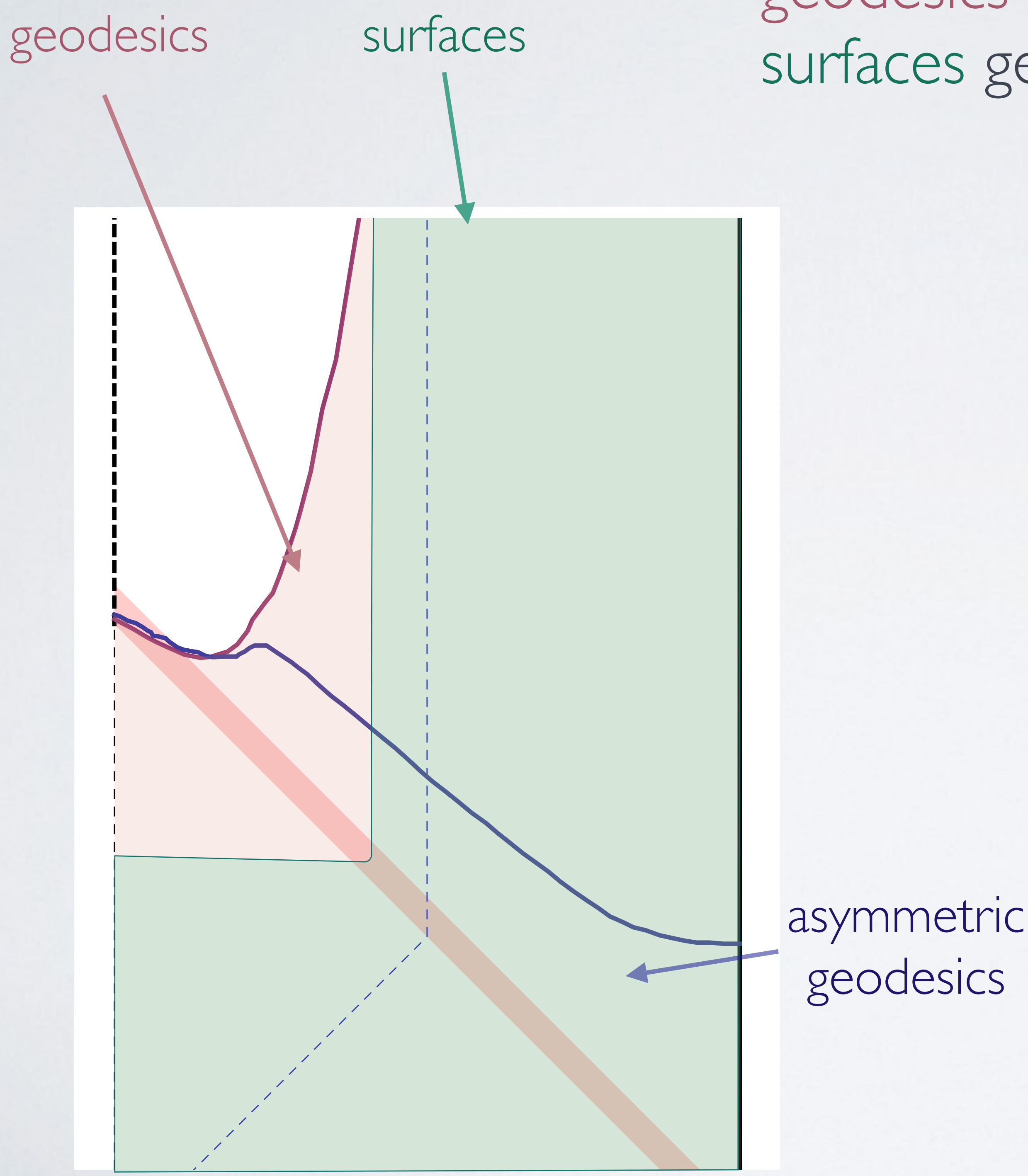
- Any extremal surface anchored at t cannot penetrate past the critical-r surface inside the BH.
- Hence these necessarily remain bounded away from the singularity.

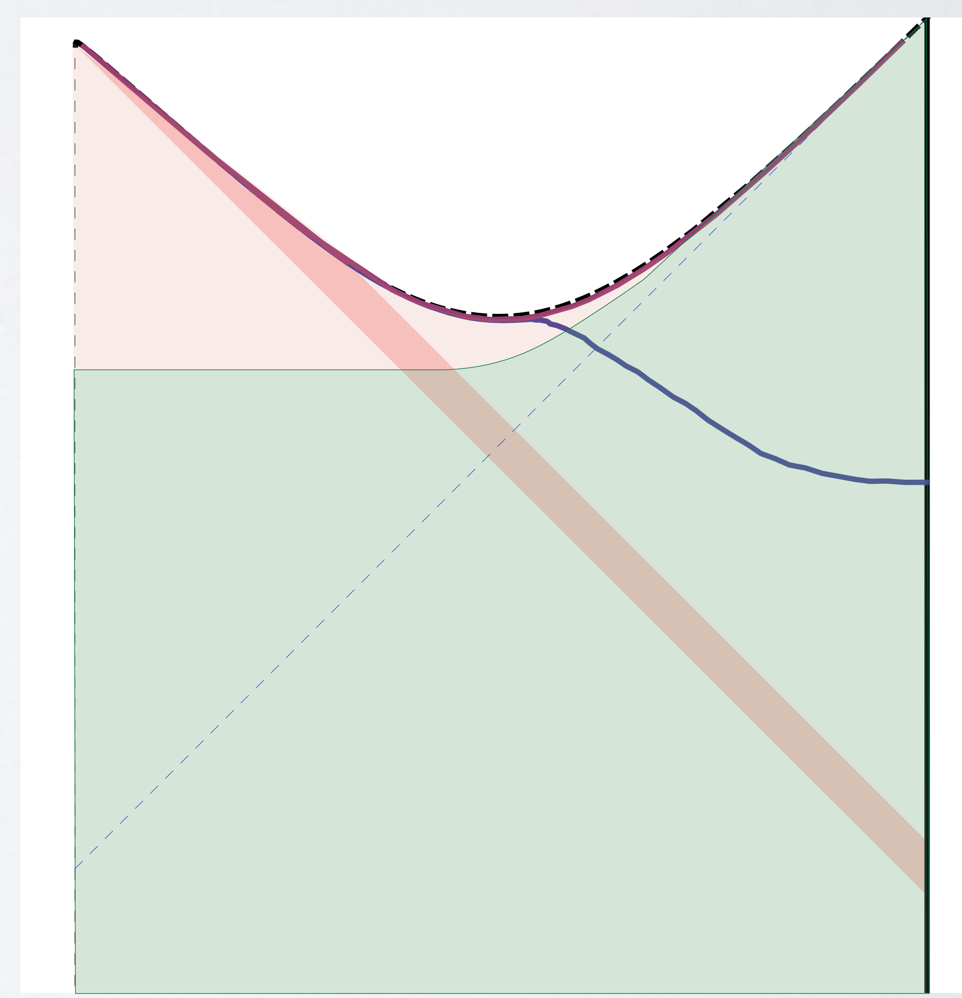




Cf. reach of geods vs. surfaces

geodesics get closer to singularity, but surfaces get further into the BH at late t.

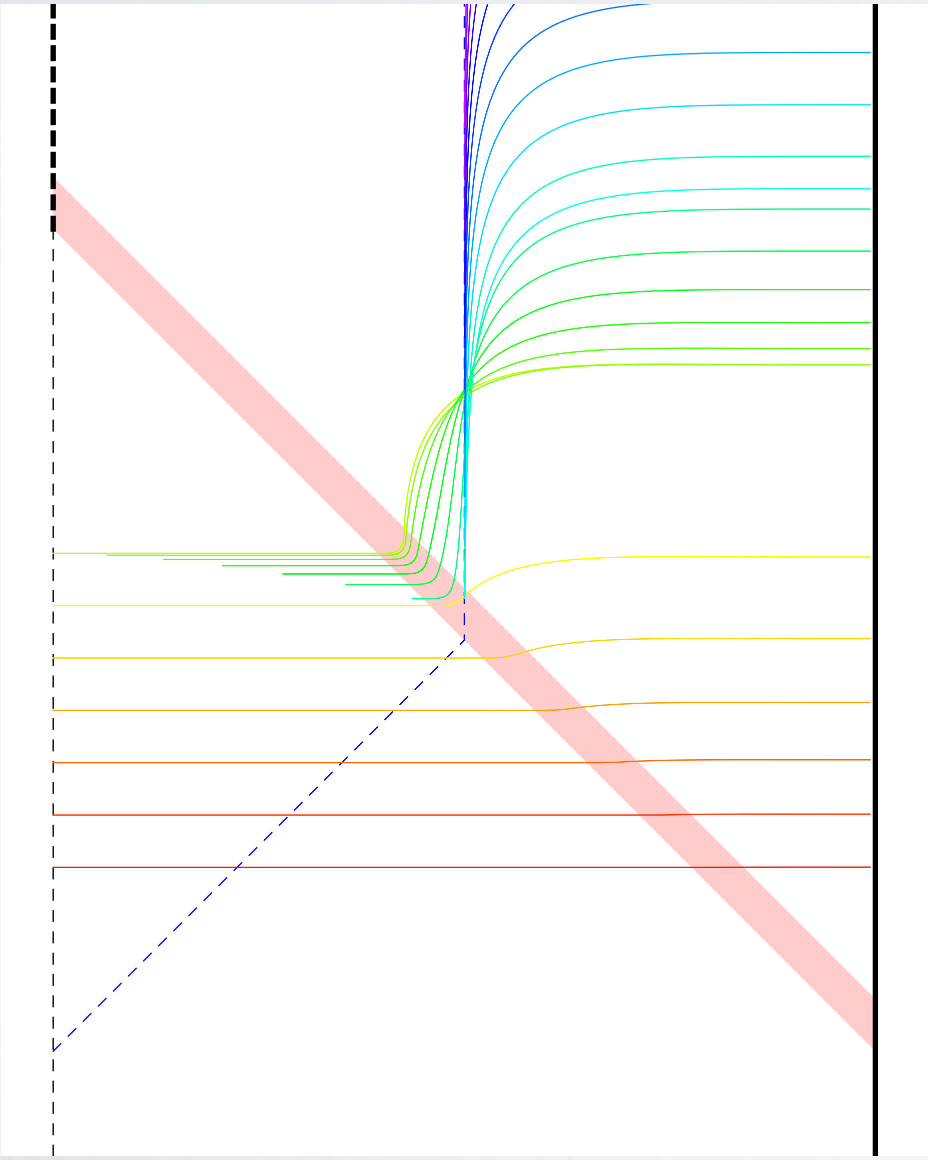


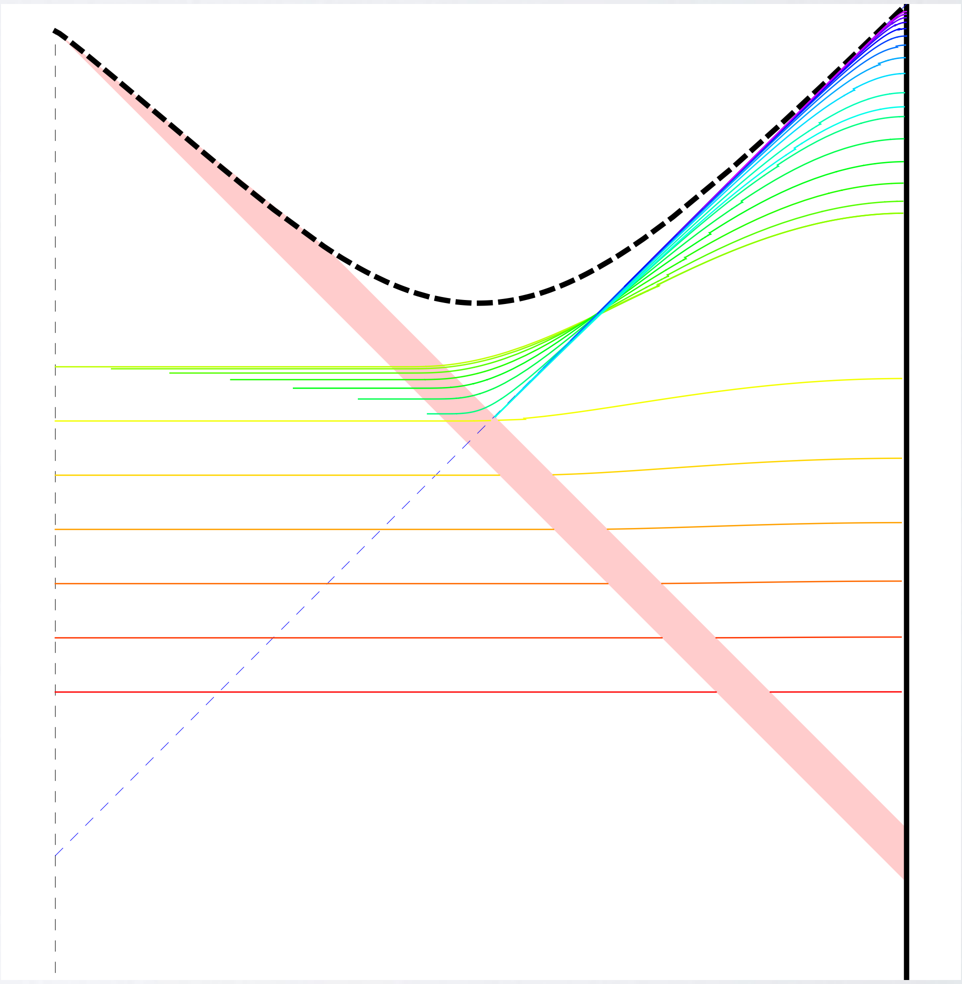


Region probed by smallest surfaces

smallest area 3-d extremal surfaces in Vaidya-AdS$_5$ ($r_+ = 1$)

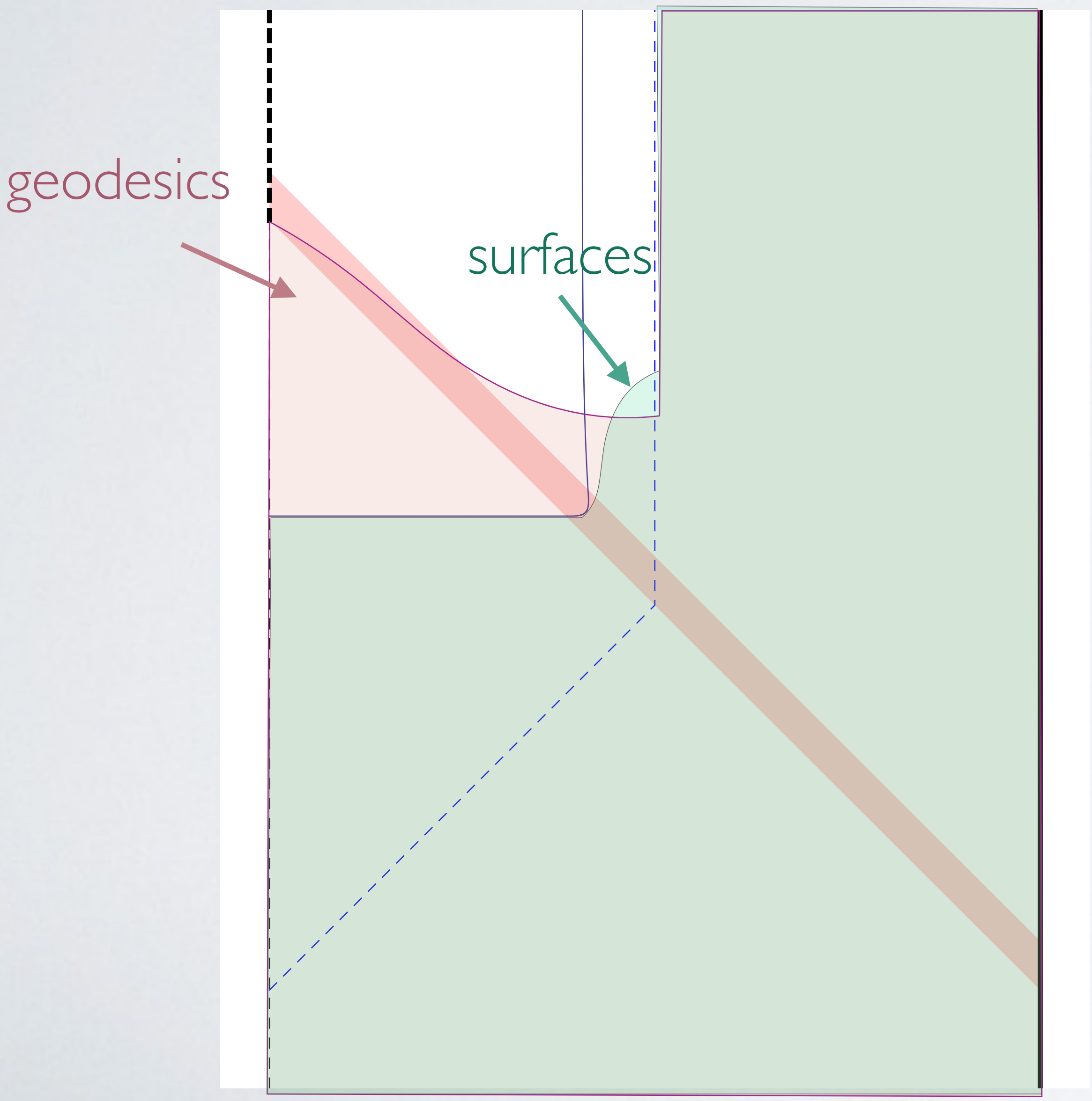
penetrate the black hole only for finite time after the shell

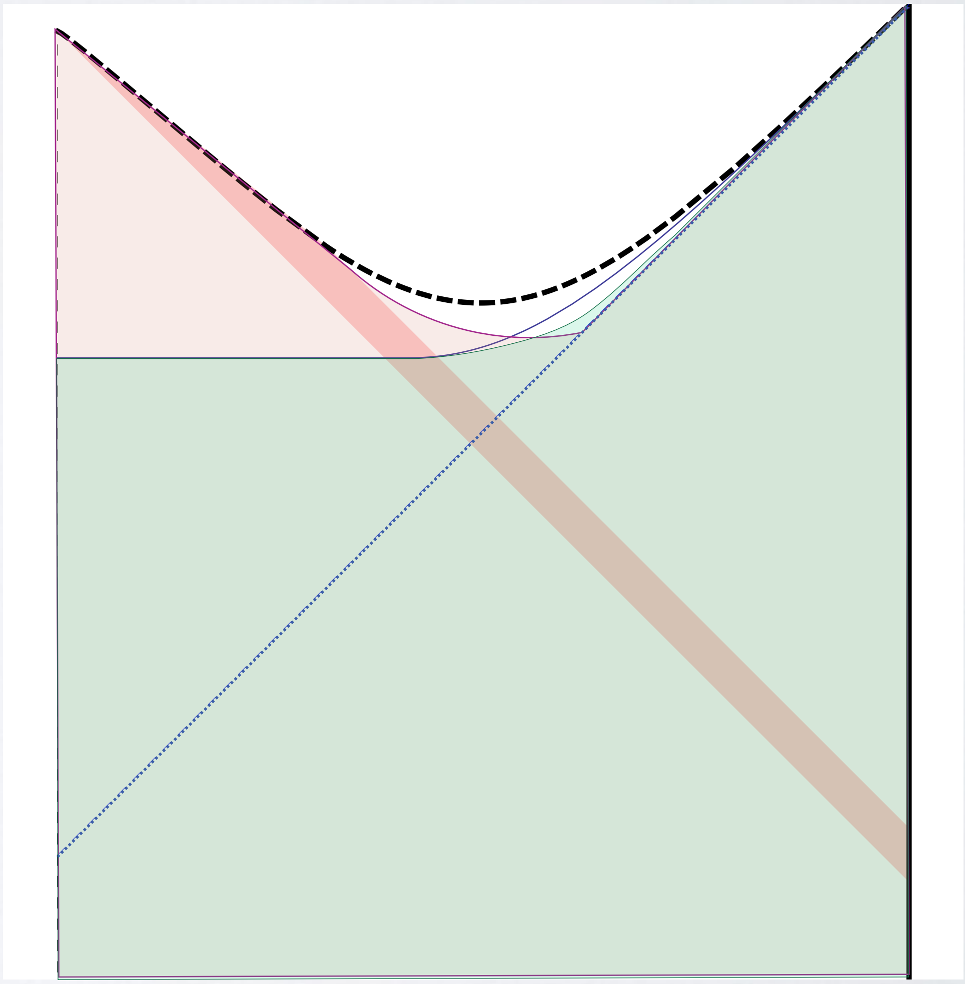




Cf. reach of 'dominant' geods vs. surfaces

shortest geodesics get closer to singularity, but
smallest area surfaces get inside BH till slightly later time.





Main results (for surfaces in Vaidya-AdS$_5$)

- Extremal surfaces exhibit very rich structure.
- Eg. already static Schw-AdS has infinite family of surfaces anchored on the same boundary region (for sufficiently large regions).
- $\exists$ surfaces which penetrate to $r \sim r_c < r_+$ inside BH, for arbitrarily late times.
- However, surfaces cannot penetrate deeper (to $r < r_c$) in the future of the shell. Hence they remain bounded away from the singularity.
- Smallest area surfaces can only reach inside the BH for finite t.

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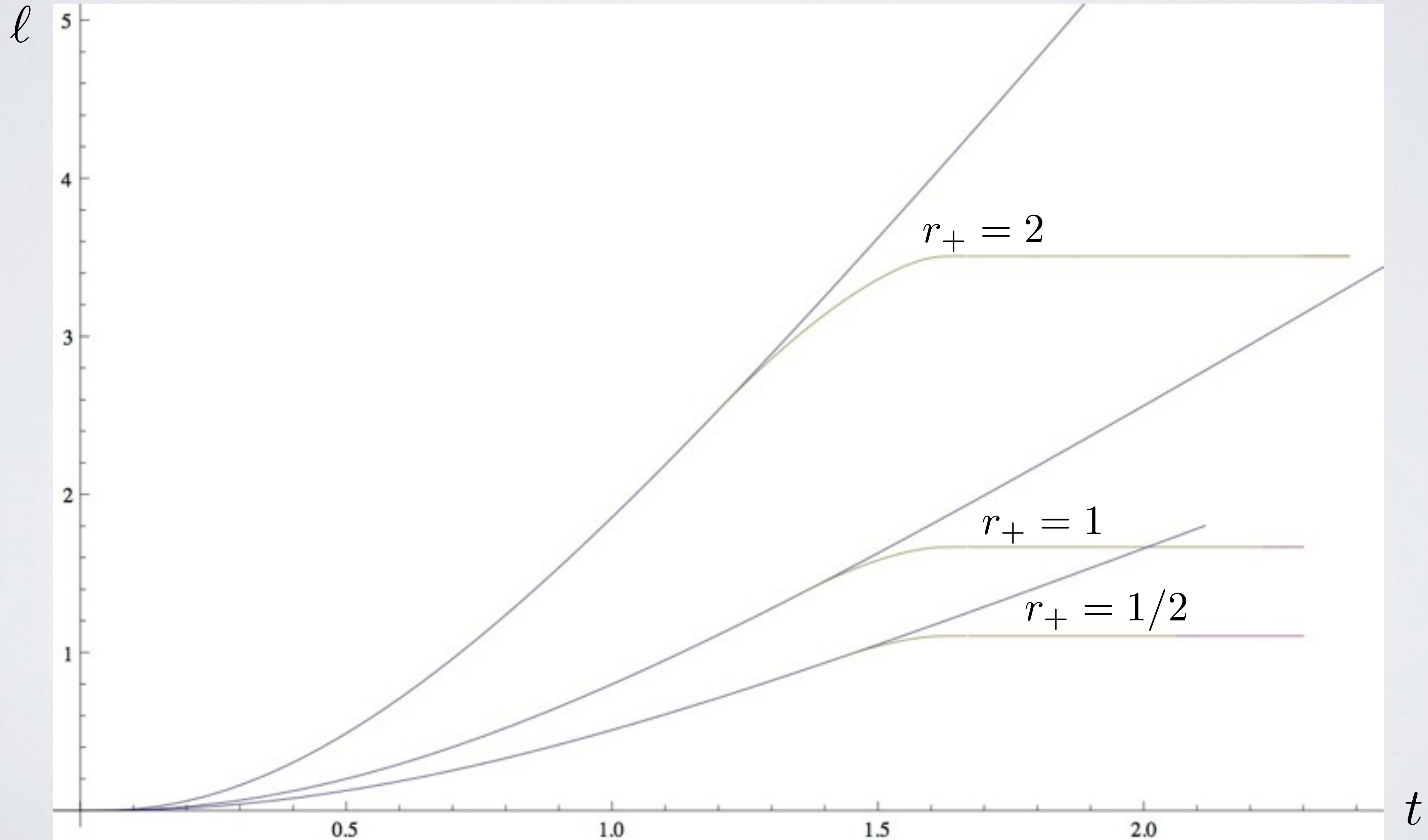
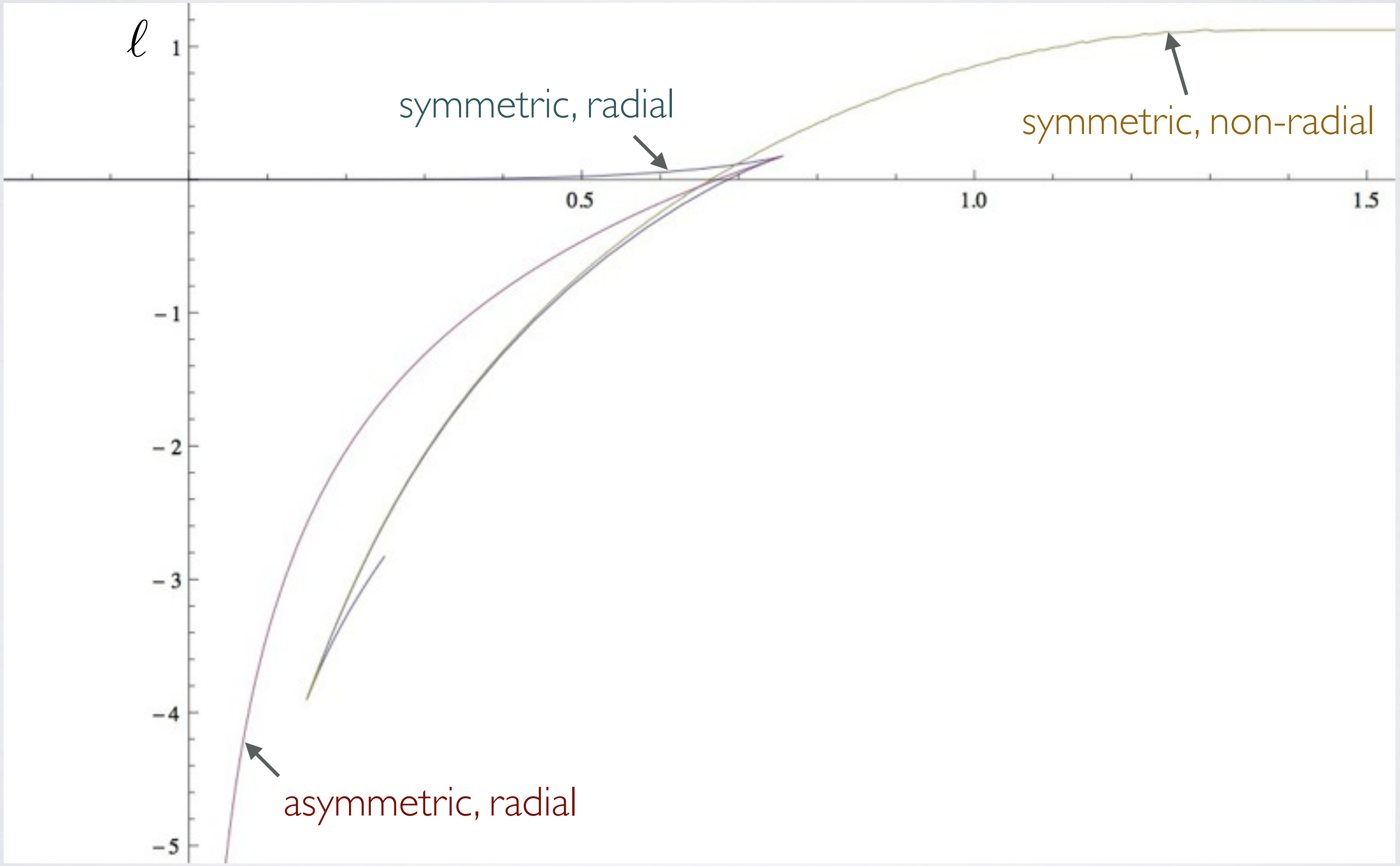
geodesic lengths in Vaidya-AdS$_3$

- Thermalization is continuous and monotonic



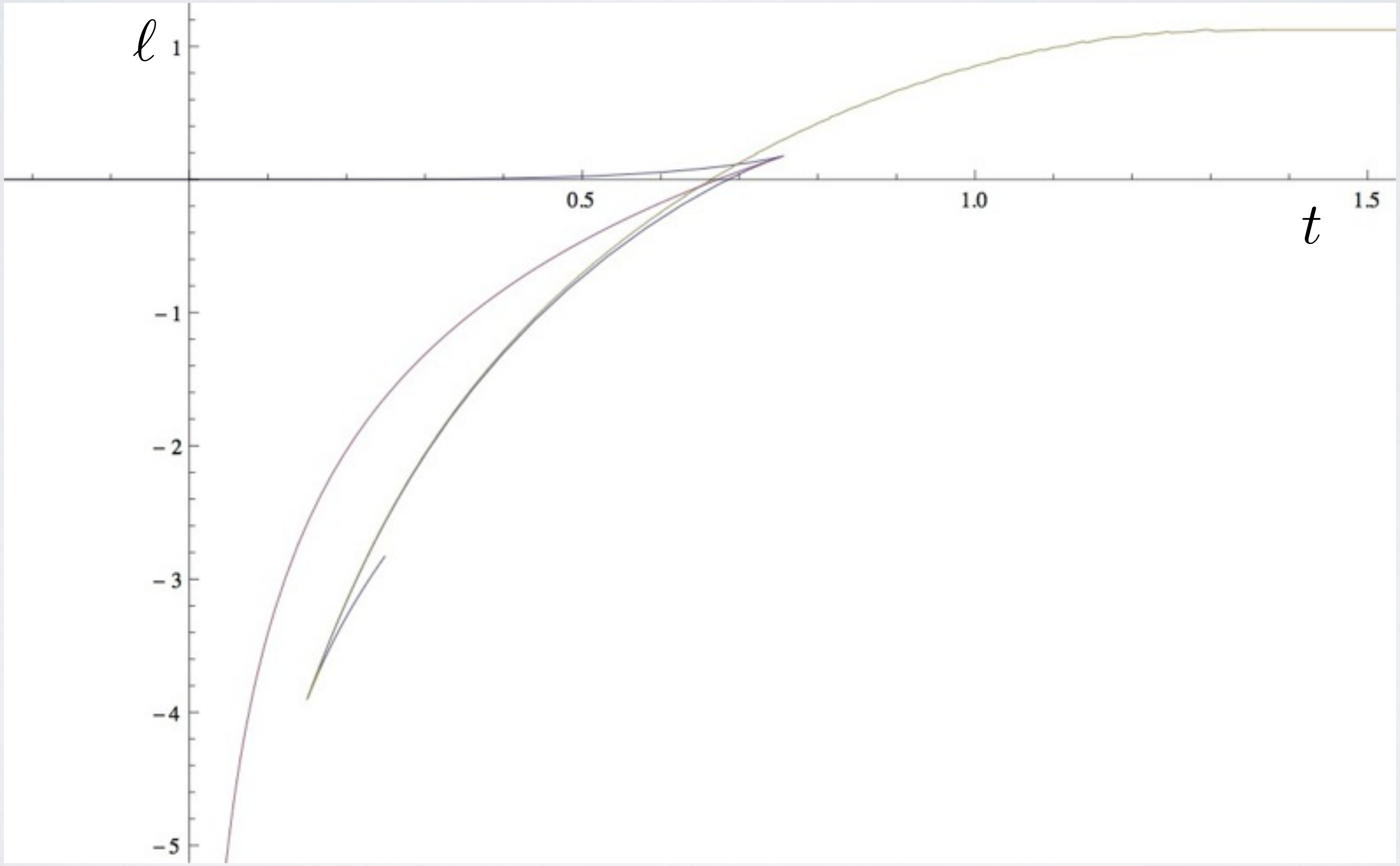
geodesic lengths in Vaidya-AdS$_5$

- Thermalization appears discontinuous and non-monotonic!



geodesic lengths in Vaidya-AdS$_5$

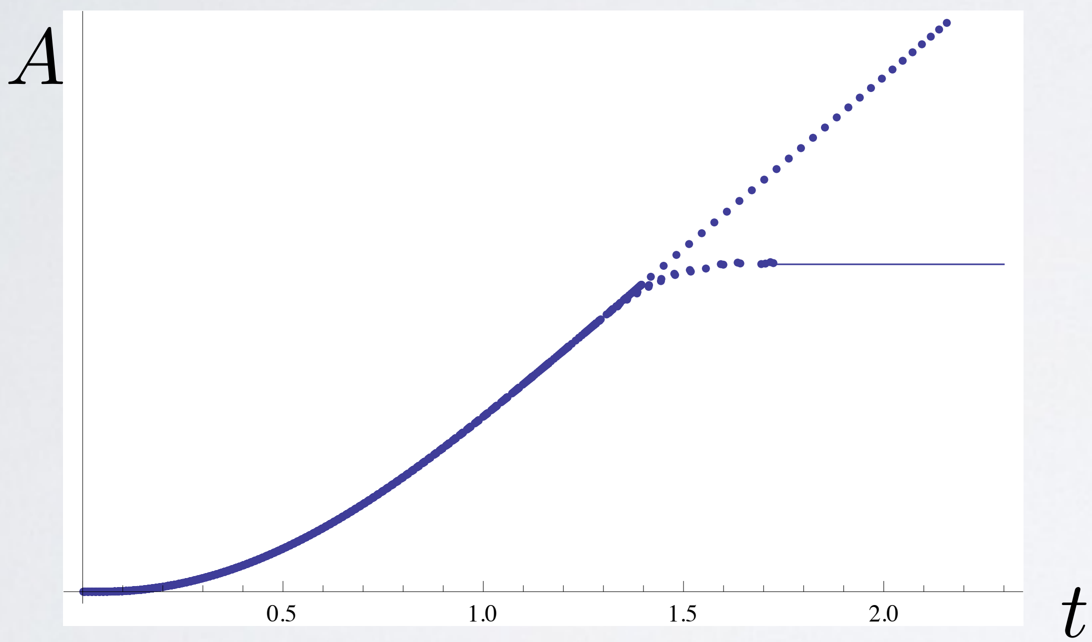
- Puzzle 1: What does this imply for the CFT correlators?



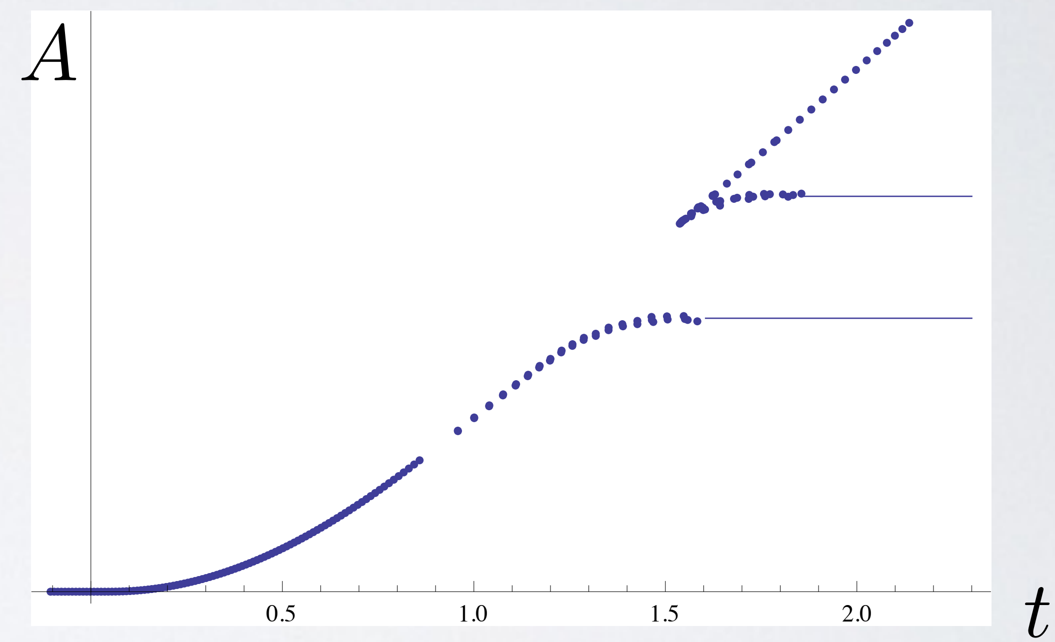
surface areas in Vaidya-AdS_5

- Thermalization is again continuous and monotonic

hemispherical region



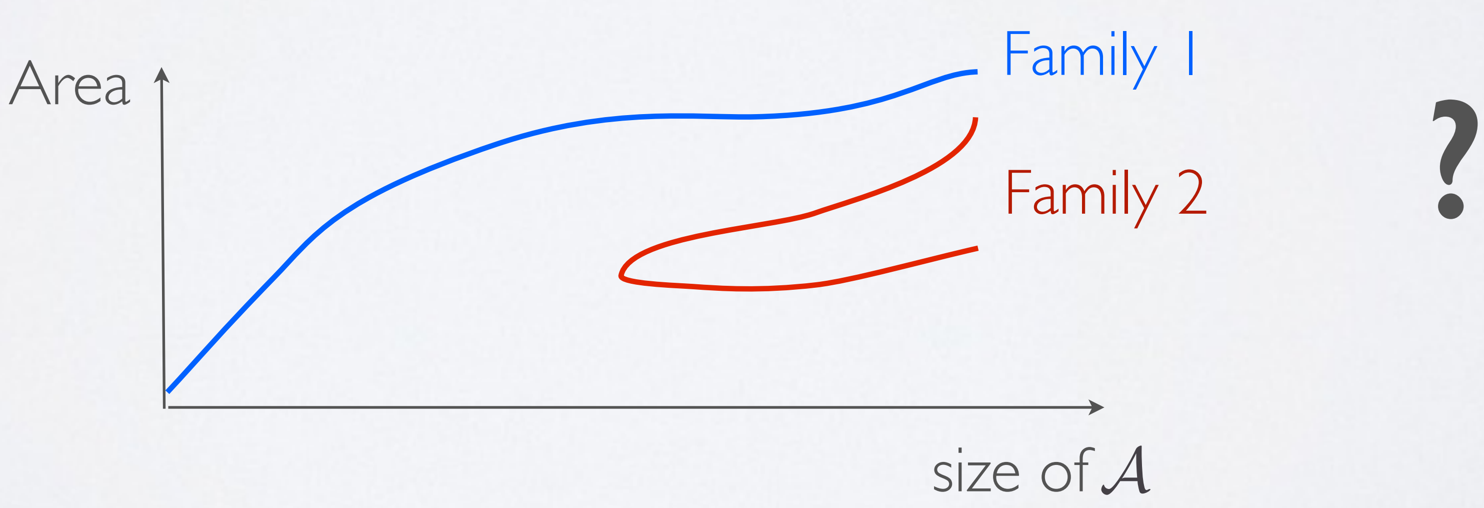
sub-hemispherical region



- Puzzle 2: Was this guaranteed?

Continuity of entanglement entropy?

- RT prescription (EE given by area of *minimal* surface) naturally implies continuity [VH, Maxfield, Rangamani, Tonni; Headrick]
- However, open question whether continuity is upheld by HRT (EE given by area of *extremal* surface).
- New families of extremal surfaces can appear, but is the following situation possible:



Thank you

Appendices

BTZ vs. Schw-AdS

- BTZ = locally AdS, so the geometry does not become highly curved near the singularity
- Correspondingly, spacelike geodesics do not get "repelled" off the singularity for BTZ, but do get repelled in higher dimensions
- This can be seen from the effective potential for the radial problem: