Numerical solutions of AdS gravity:

new lessons about dual equilibration processes at strong coupling

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Introduction

Numerical holography

numerical relativity + holography = new window on far-from-equilibrium physics.

Why interesting? *ab initio* calculations in a class of *interacting quantum field theories*.

My main motivation will be the creation of quark-gluon plasma in Heavy Ion Collisions

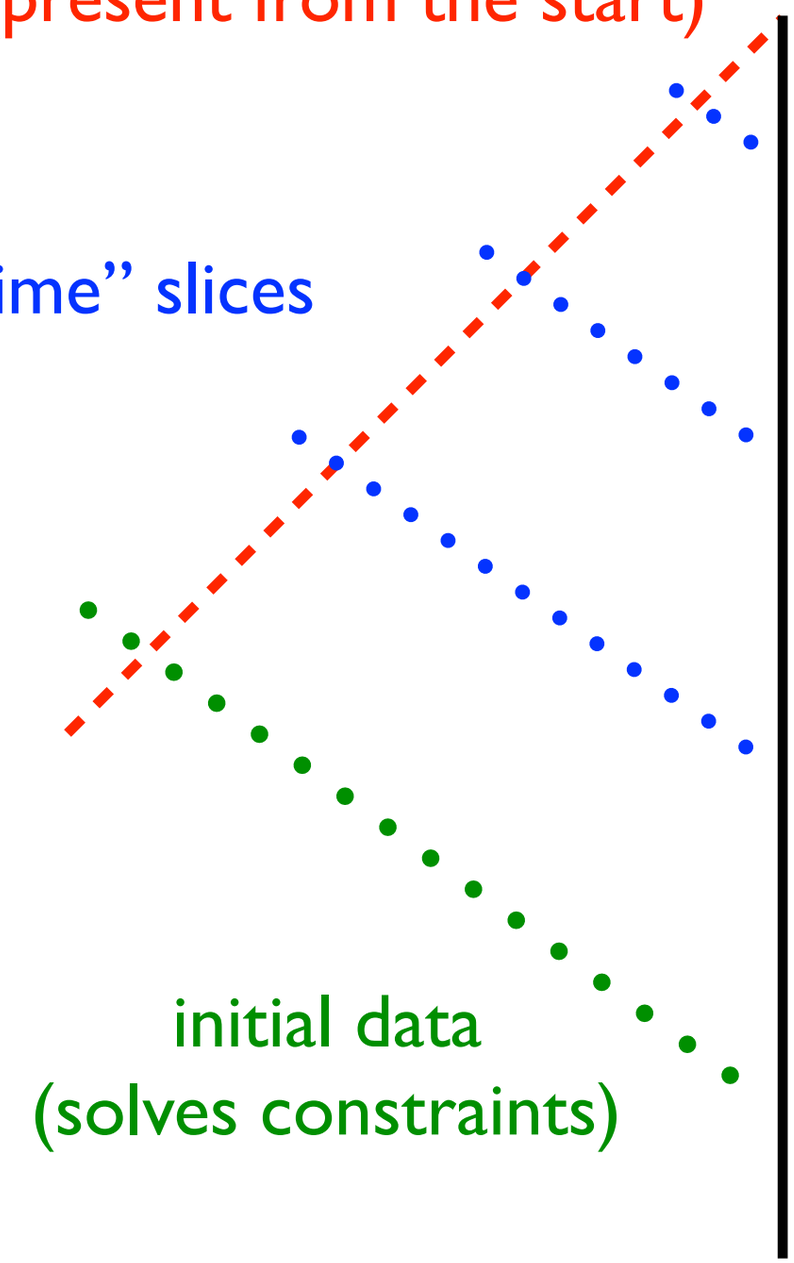
Hence I will consider *the Poincare patch of AdS$_{4+1}$*

states in a large-N_c CFT at strong coupling

In this talk I will discuss 3 solutions of $R_{ab} - \frac{1}{2} R\, g_{ab} - \frac{6}{L^2}\, g_{ab} = 0$ with planar horizons.

In common: applications to HIC & using the ingoing Eddington-Finkelstein coordinates.

Setups



boundary; we demand here that if

$$ds^2_{4+1} = \frac{L^2}{u^2}\left(du^2 + h_{\mu\nu}(u,x)\,dx^\mu dx^\nu\right)$$

then

$$h_{\mu\nu}(u,x) = \eta_{\mu\nu} + \frac{4\pi G_N}{L^3}\langle T_{\mu\nu}(x)\rangle \cdot u^4 + \dots$$

$$= \frac{2\pi^2}{N_c^2} \text{ (for } N\text{=4 SYM)}$$

note that in the setups I am going to study $\int d^3x\,\langle T_{tt}\rangle = \infty$.

Why the ingoing Eddington-Finkelstein coordinates?

Manifestly regular on the horizon + attractive integrations scheme (*if no caustics*).

fluid-gravity duality++ **1309.1439 [hep-th]** Chesler & Yaffe

Isotropization at strong coupling

1202.0981 [hep-th] PRL 108 (2012) 191601: MPH, D. Mateos, W. van der Schee & D. Trancanelli
1304.5172 [hep-th] JHEP 1309, 026 (2013): MPH, D. Mateos, W. van der Schee & M. Triana

Holographic isotropization

1202.0981 [hep-th] PRL 108 (2012) 191601: MPH, D. Mateos, W. van der Schee & D. Trancanelli

One of the simplest equilibration processes to study holographically is described by

$$\langle T_{\mu\nu}\rangle = \text{diag}\left(\mathcal{E}, \frac{1}{3}\mathcal{E} - \frac{2}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t)\right)_{\mu\nu}$$

It is *identically* traceless and conserved. EOMs are $R_{ab} - \frac{1}{2}R\, g_{ab} - \frac{6}{L^2}\, g_{ab} = 0$.

Symmetries of the stress tensor lead to a general metric ansatz

$$ds^2 = -f_{tt}dt^2 + 2f_{tr}dtdr + f_{rr}dr^2 + \Sigma^2 e^{-2B}dx_1^2 + \Sigma^2 e^{B}(dx_2^2 + dx_3^2)$$

We fix almost all the gauge freedom by adopting (the ingoing EF coordinates)

$$ds^2 = 2dtdr - Adt^2 + \Sigma^2 e^{-2B}dx_1^2 + \Sigma^2 e^{B}(dx_2^2 + dx_3^2)$$

We can solve Einstein's equations near the boundary and obtain*

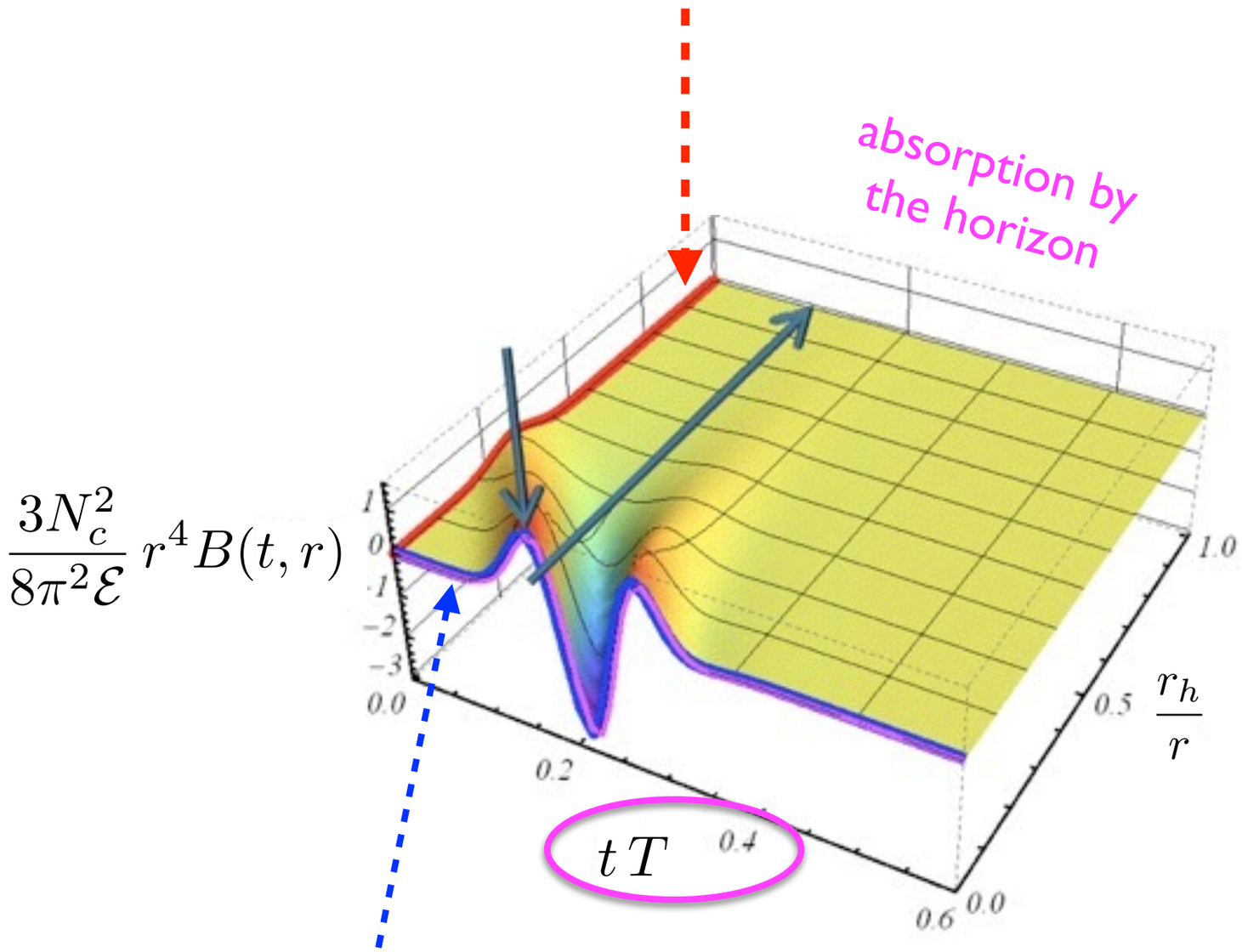
$$B = \frac{1}{r^4}\left\{b_4(t) + \frac{1}{r}b_4'(t) + \frac{2}{12r^6}b_4''(t) + \frac{1}{4r^3}b_4^{(3)}(t) + \ldots\right\} \quad \text{with} \quad \Delta P(t) = \frac{3}{8\pi^2}N_c^2 b_4(t)$$

Equilibration dynamics

1202.0981 [hep-th] PRL 108 (2012) 191601: MPH, D. Mateos, W. van der Schee & D. Trancanelli

$$ds^2 = 2dtdr - Adt^2 + \Sigma^2 e^{-2B} dx_1^2 + \Sigma^2 e^{B}(dx_2^2 + dx_3^2)$$

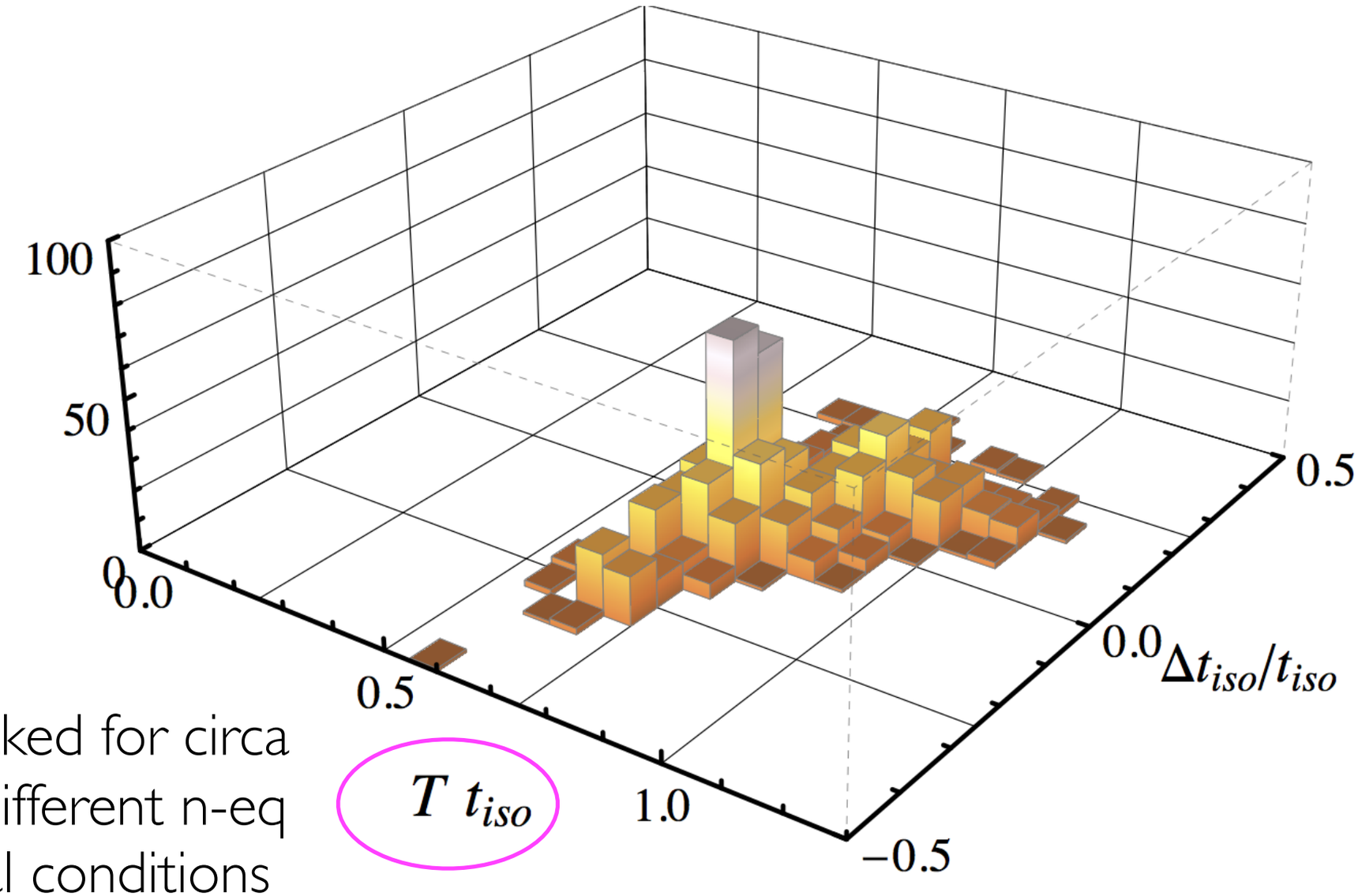
initial data: $\langle T_{00}\rangle = \mathcal{E}$ and $B(t=0,r)$



$$\langle T_{\mu\nu}\rangle = \text{diag}\left(\mathcal{E}, \frac{1}{3}\mathcal{E} - \frac{2}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t), \frac{1}{3}\mathcal{E} + \frac{1}{3}\Delta\mathcal{P}(t)\right)_{\mu\nu}$$

Fast relaxation

$$t_{iso} : \left| \frac{\Delta \mathcal{P}(t \geq t_{iso})}{\mathcal{E}} \right| \leq 0.1$$



Checked for circa
10^3 different n-eq
initial conditions

(RHIC c=0-5%: $0.25\,\mathrm{fm} \times 500\,\mathrm{MeV} = 0.63$)

0801.4361 [nucl-th] W. Broniowski et al.

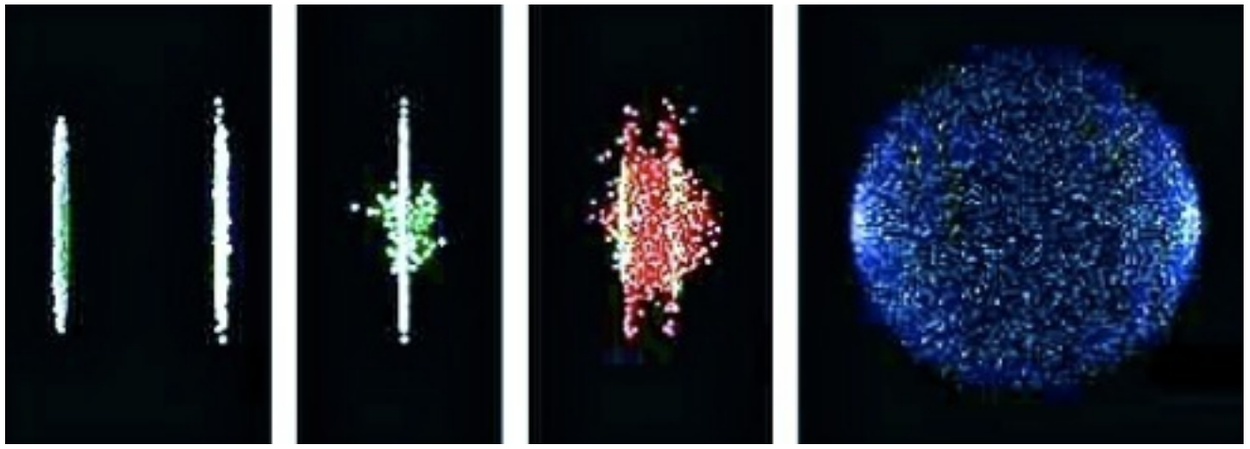
Shock wave collisions and hydrodynamization

1305.4919 [hep-th] PRL 111 (2013) 181601: J. Casalderrey-Solana, MPH, D. Mateos & W. van der Schee

1312.2956 [hep-th]: J. Casalderrey-Solana, MPH, D. Mateos & W. van der Schee

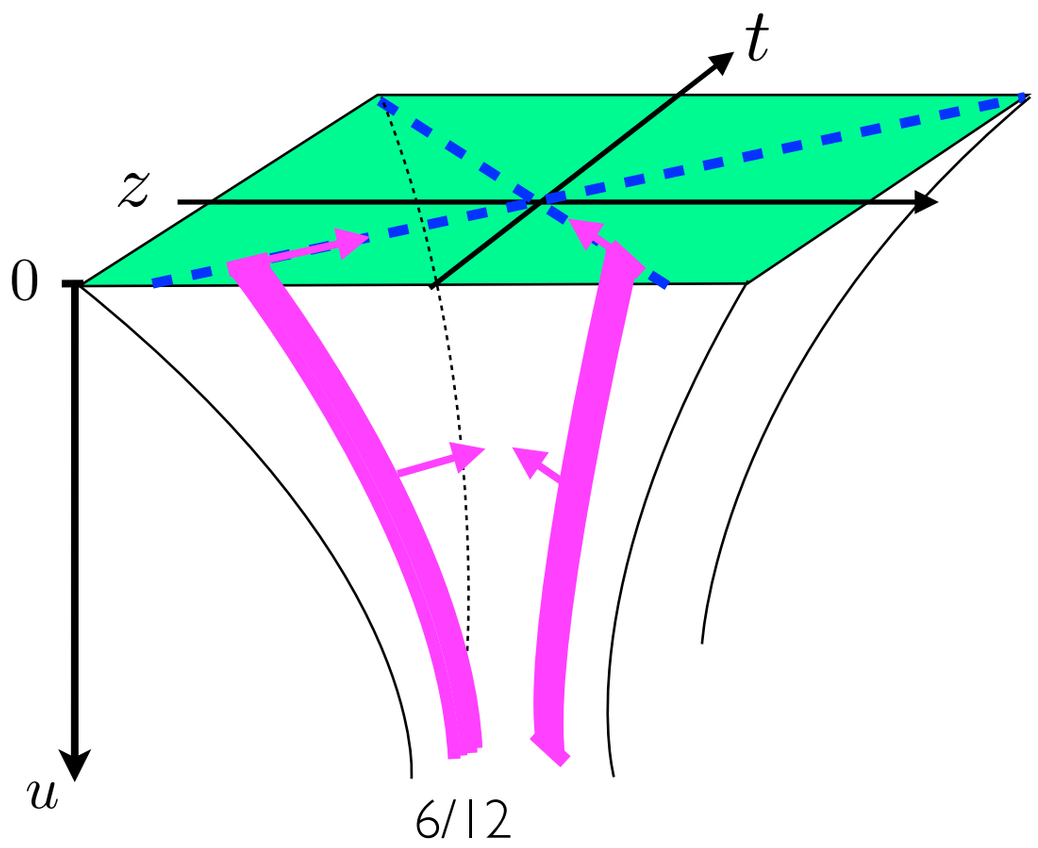
Towards a holographic „heavy ion collision”

general issue: which holographic initial conditions are closest to the experiment?



practical viewpoint: collide two lumps of matter moving at relativistic speeds.

[hep-th/0512162] R. Janik & R. Peschanski **1011.3562 [hep-th]** P. Chesler & L. Yaffe



Gravitational shock wave solutions

Janik & Peschanski **[hep-th/0512162]** Chesler & Yaffe **1011.3562 [hep-th]**

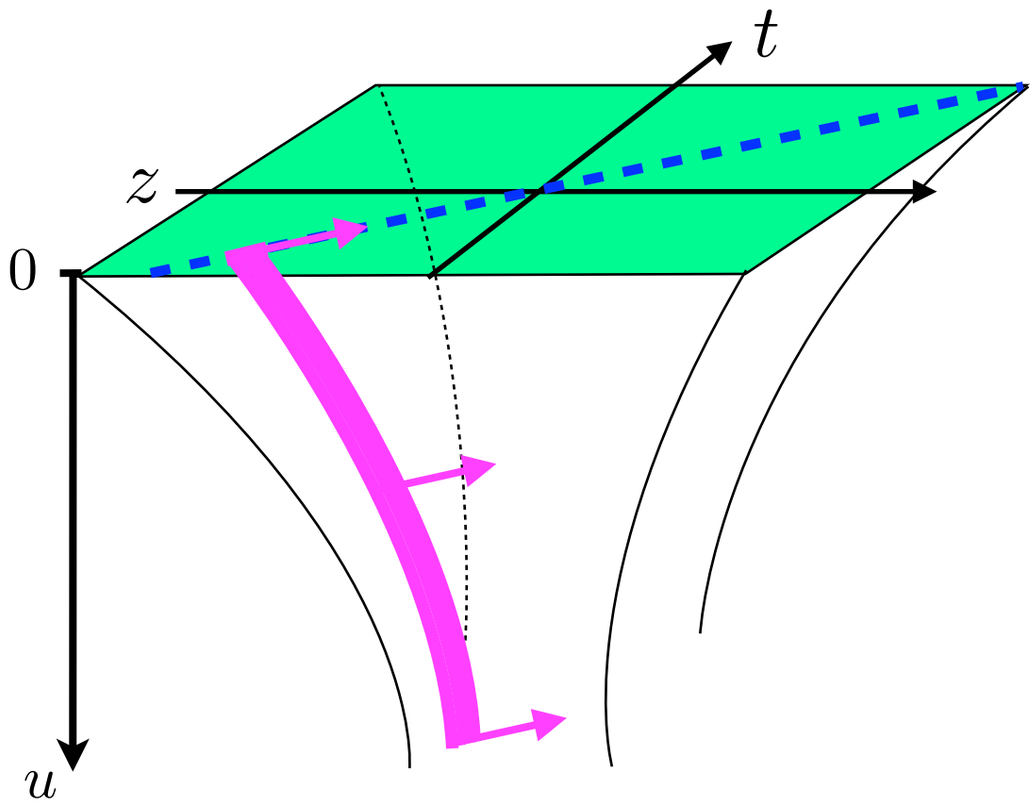
dual stress tensor: $\langle T^{tt}\rangle = \langle T^{zz}\rangle = \pm\langle T^{tz}\rangle = \dfrac{N_c^2}{2\pi^2}h(t\mp z)$

shock wave disturbance
moving with the speed of light

$$ds^2 = \frac{1}{u^2}(du^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + u^2 h(x_-)dx_-^2$$

Poincare patch
vacuum AdS

Solution of Einstein's equations with the negative CC
for <u>any</u> longitudinal profile $h(x_-)$



Let's consider now $h(t\pm z) = \rho^4 \exp\left[-(t\pm z)^2/2\sigma^2\right]$. But, in a CFT, what matters is:

$e = \rho\,\sigma$ **(in real HIC $e \sim \gamma^{-1/2}$ and $e_{CY} \approx 0.64$ corresponds to Pb at RHIC)**

Dynamical crossover

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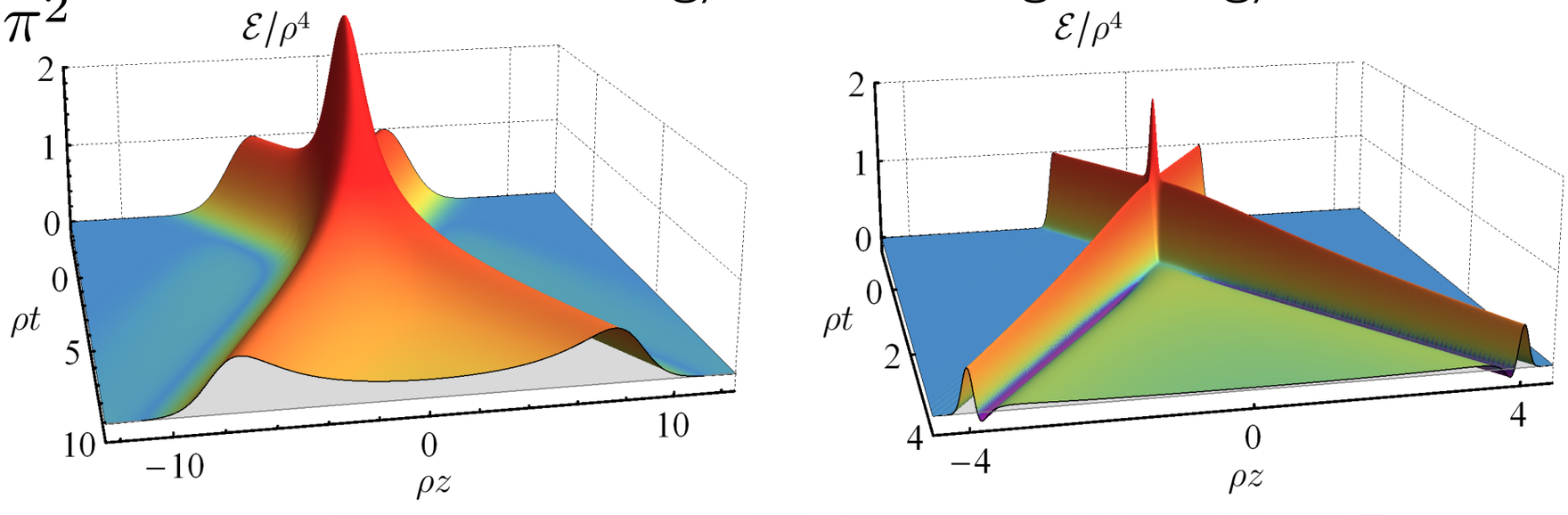
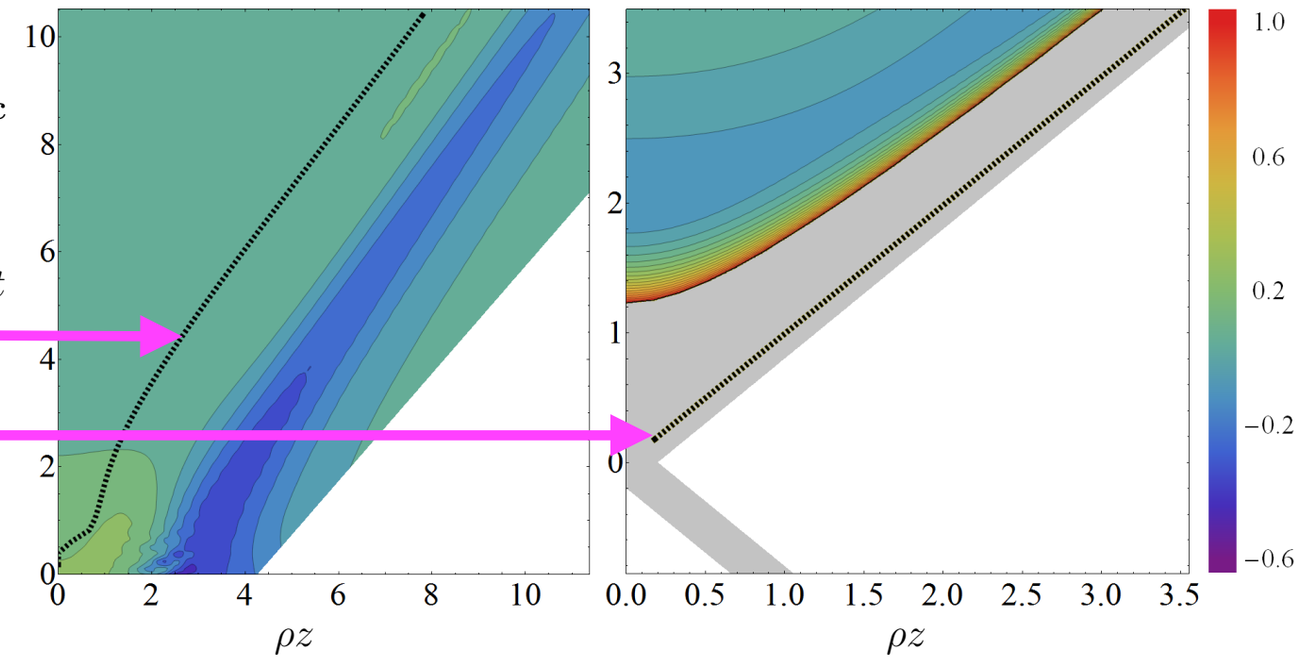
J. Casalderrey-Solana, MPH, D. Mateos & W. van der Schee

$$\langle T^{tt}\rangle = \frac{N_c^2}{2\pi^2}\mathcal{E}$$

$e_{left} = 2\, e_{CY}$ „low energy"

$e_{right} = 0.125\, e_{CY}$ „high energy"





maximum of the energy flux

deviation from viscous hydro

shocks coalesce and explode hydro-dynamically (similar to the Landau picture)

hydro applicable only at mid-rapidities and late enough!!!

Dispels the myth that strong coupling necessarily leads to immediate stopping*

Hydrodynamization in a shock wave collision

1305.4919 [hep-th] PRL 111 (2013) 181601: J. Casalderrey-Solana, MPH, D. Mateos & W. van der Schee

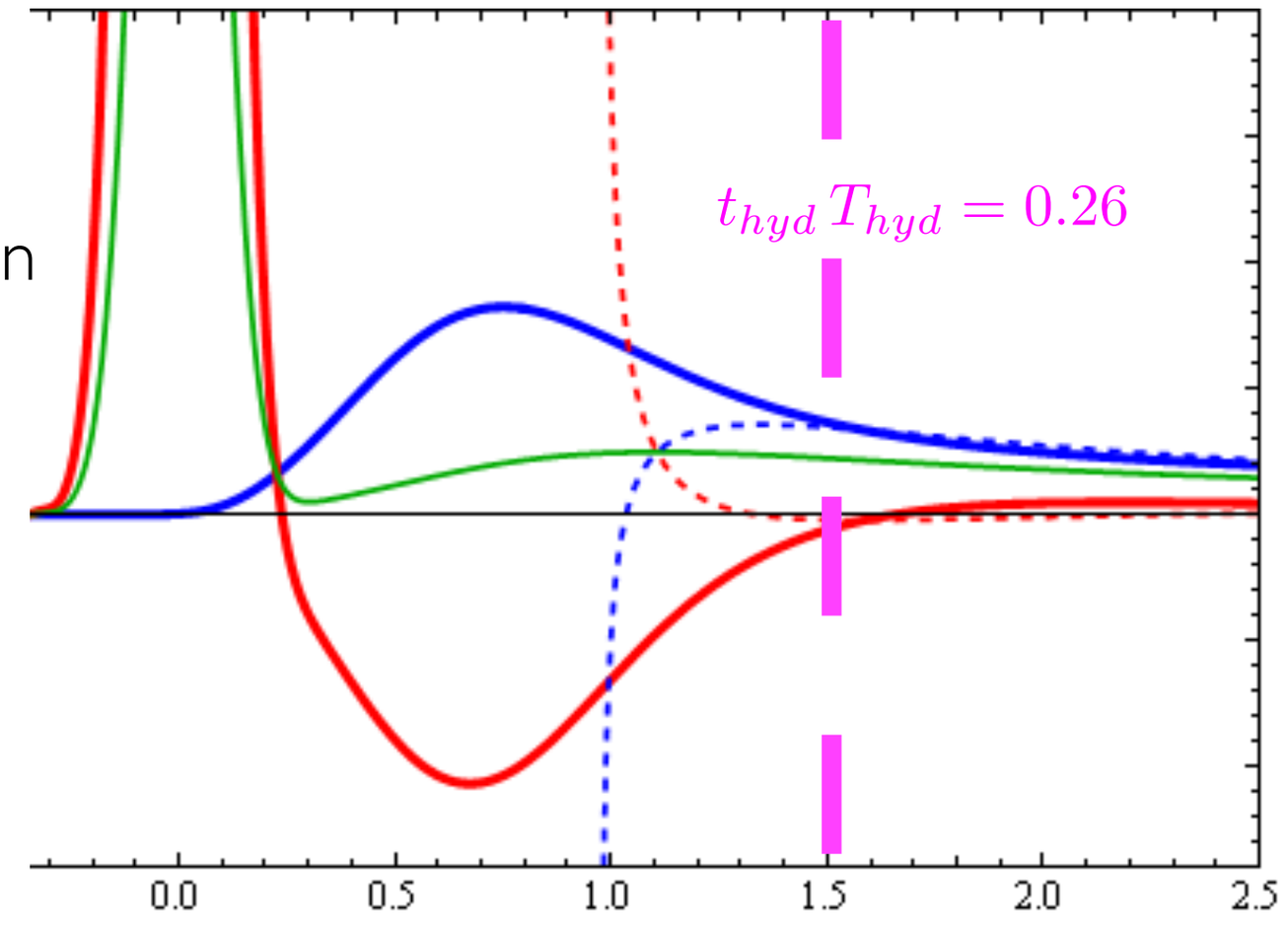
Hydrodynamics: $\langle T^{\mu\nu}\rangle = \{\mathcal{E}+\mathcal{P}(\mathcal{E})\}\,u^\mu u^\nu + \mathcal{P}(\mathcal{E})\,\eta^{\mu\nu} + \Pi^{\mu\nu}\left(\big|_{CFT} = -\eta\,\sigma^{\mu\nu}+\ldots\right)$

perfect fluid dissipative

We use $\langle T^\mu{}_\nu\rangle u^\nu = -\mathcal{E}\,u^\mu$ and compare $\langle T^{zz}\rangle$ and $\langle T^{\perp\perp}\rangle$ with hydro prediction.

$\frac{\langle T^{tt}\rangle}{3\,\mathcal{E}_0}$ $\frac{\langle T^{zz}\rangle}{\mathcal{E}_0}$ $\frac{\langle T^{\perp\perp}\rangle}{\mathcal{E}_0}$

dotted: hydro prediction



at the collision axis (z = 0)

Surprise: large anisotropy at the onset of hydrodynamics due to the shear tensor!

see also **0906.4426 & 1011.3562** Chesler & Yaffe and **1103.3452** MPH, Janik & Witaszczyk

The nature of hydrodynamics

1302.0697 [hep-th] PRL 110 (2013) 211602: MPH, R. A. Janik & P. Witaszczyk

Hydrodynamic series at high orders

1302.0697 [hep-th] PRL 110 (2013) 211602: MPH, R. A. Janik & P. Witaszczyk

Question: what is the nature of hydrodynamic gradient expansion?

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\Pi\left[\langle D\sigma^{\mu\nu}\rangle + \frac{1}{d-1}\sigma^{\mu\nu}(\nabla\cdot u)\right] + \kappa\left[R^{\langle\mu\nu\rangle} - (d-2)u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right] + \lambda_1\sigma^{\langle\mu}{}_\lambda\sigma^{\nu\rangle\lambda} + \lambda_2\sigma^{\langle\mu}{}_\lambda\Omega^{\nu\rangle\lambda} + \lambda_3\Omega^{\langle\mu}{}_\lambda\Omega^{\nu\rangle\lambda} + \dots$$

Idea: use the fluid-gravity duality to compute subsequent gradient terms *on-shell*.

To make it operational, we used the boost-invariant flow. Why?

$$t = \tau\cosh y \quad \text{and} \quad z = \tau\sinh y$$

$$u^\mu\partial_\mu = \partial_\tau \quad \text{and} \quad \langle T^{\tau\tau}\rangle = \mathcal{E}(\tau) = \frac{3}{8}N_c^2\pi^2T(\tau)^4 \longrightarrow \frac{1}{T(\tau)}\nabla_\mu u_\nu \sim \frac{1}{T(\tau)}\frac{1}{\tau}$$

Gradient expansion → solving ODEs in the bulk → go to 240th order in grads.

Hydrodynamic series at high orders

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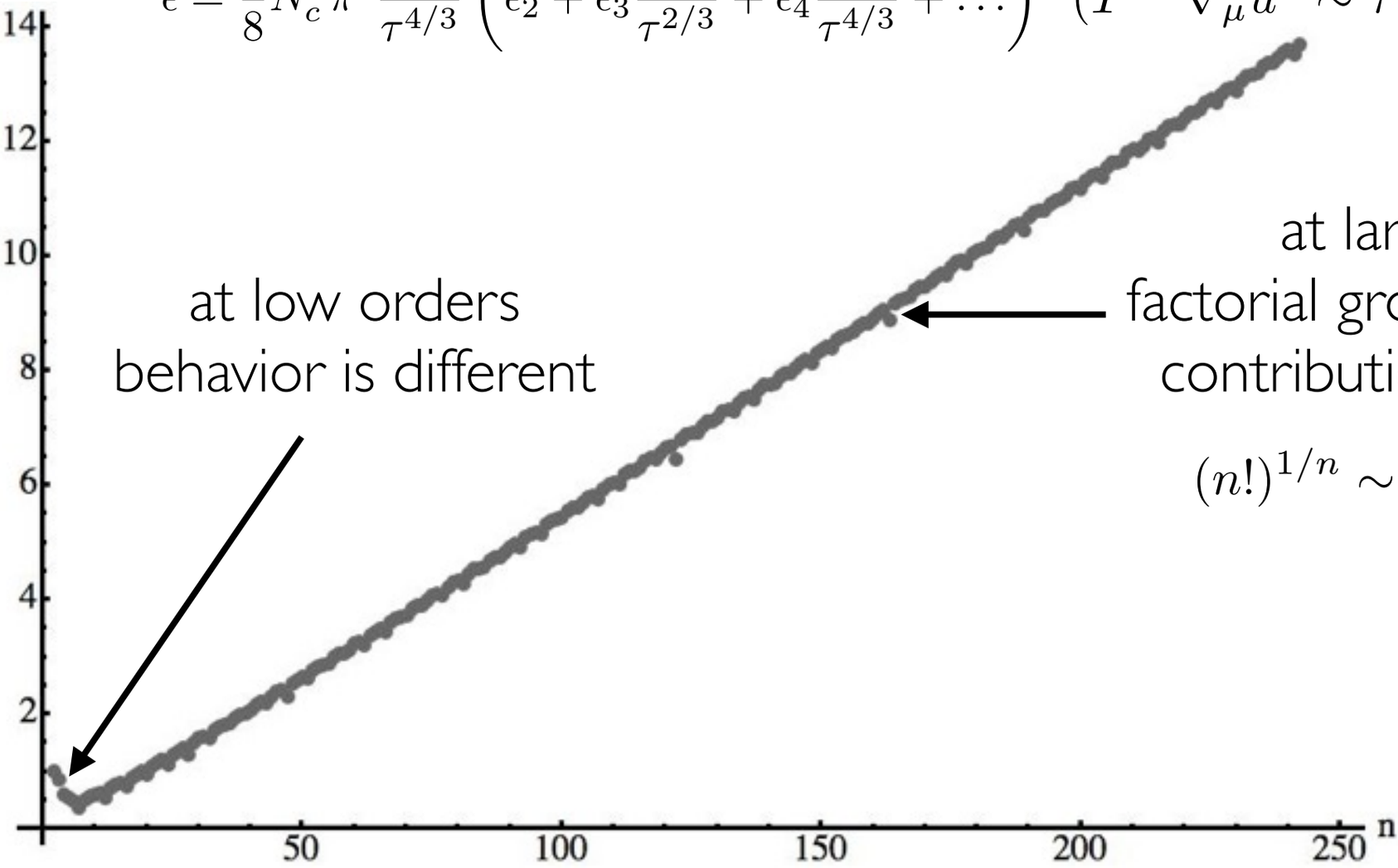
$$\left|\frac{\epsilon_n}{\epsilon_2}\right|^{1/n}$$

$$\epsilon = \frac{3}{8}N_c^2\pi^2\frac{1}{\tau^{4/3}}\left(\epsilon_2 + \epsilon_3\frac{1}{\tau^{2/3}} + \epsilon_4\frac{1}{\tau^{4/3}} + \dots\right) \quad (T^{-1}\nabla_\mu u^\nu \sim \tau^{-2/3})$$



at low orders
behavior is different

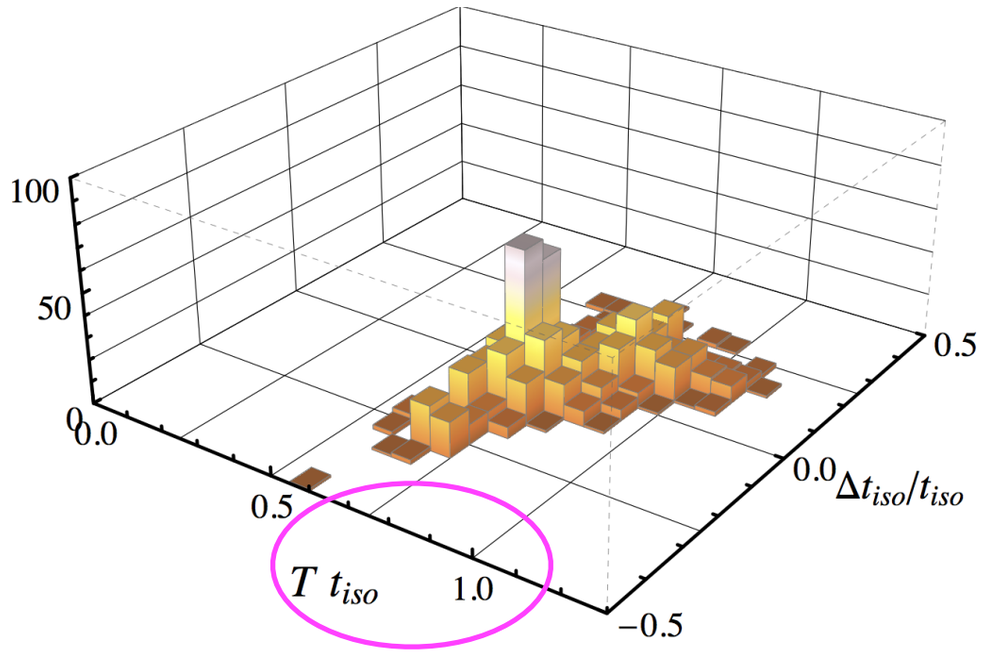
at large orders
factorial growth of gradient
contributions with order

$$(n!)^{1/n} \sim \frac{(2\pi n)^{1/2n}}{e}\cdot n$$

First evidence that hydrodynamic expansion has a zero radius of convergence!

Summary

Summary



1202.0981 [hep-th] PRL 108 (2012) 191601:
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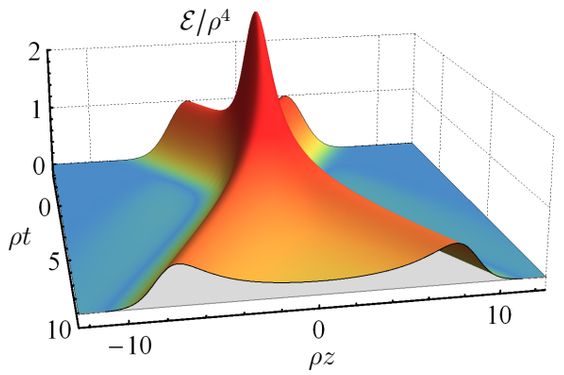
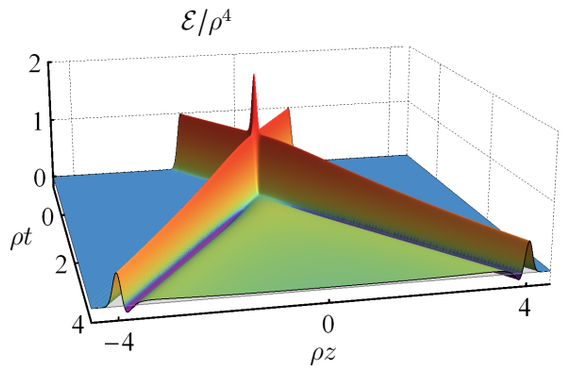
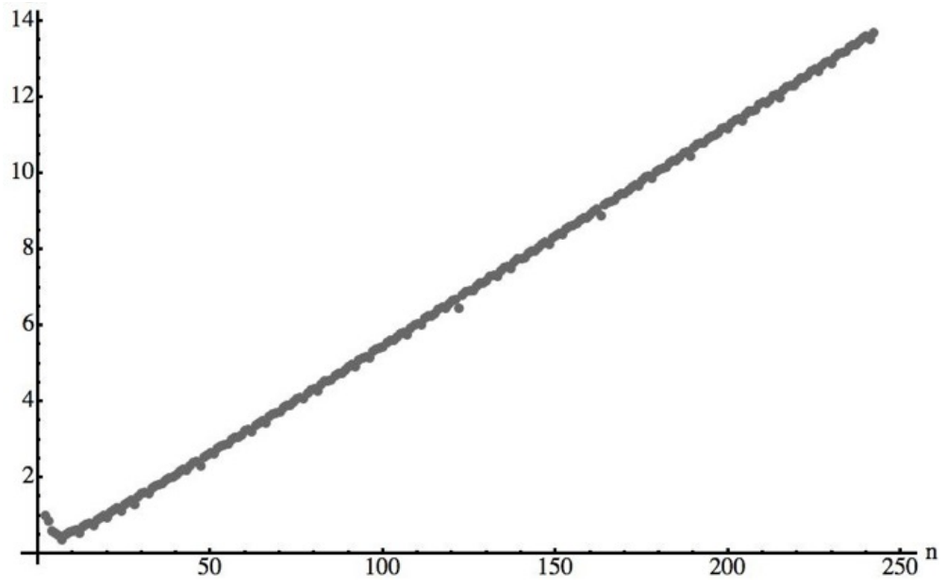




+ hydrodynamization

1305.4919 [hep-th] PRL 111 (2013) 181601:
J. Casalderrey-Solana, MPH, D. Mateos & W. van der Schee

$$\left|\frac{\epsilon_n}{\epsilon_2}\right|^{1/n}$$



1302.0697 [hep-th] PRL 110 (2013) 211602:
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CERN-CKC TH Institute on Numerical Holography

8-18 December 2014
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Gauge/gravity duality (or holographic duality) relates the dynamics of certain strongly interacting quantum field theories to the dynamics of classical gravity in one higher dimension. Over the last decade holographic duality has emerged as a powerful tool for studying strongly interacting systems. A significant part of these developments has been motivated by experimentally explored strongly coupled systems ranging from the quark-gluon plasma produced in ultra-relativistic heavy ion collisions at RHIC and at the LHC to various condensed matter systems including superfluidity, quantum phase transitions, and non-Fermi liquids. One of the assets of holography is that it can yield insight into the real-time dynamics of strongly coupled quantum field theories via classical partial differential equations in the gravity dual. To date, the vast majority of work in holographic duality is focused on problems which are either static, or sufficiently close to equilibrium that analytic methods can be used to solve the required differential equations. However, in recent years there has been increasing attention focused on challenging problems which require greater use of numerical techniques. These include the question of how strongly coupled systems far from equilibrium relax, how turbulent behavior can be studied in the gravity dual description of fluids or superfluids, and how ground states of strongly interacting matter with broken translational invariance can be found. This workshop aims at an in-depth exchange on the class of challenging problems that are now under study with numerical techniques in holography.

Starts 8 Dec 2014 08:00
Ends 18 Dec 2014 18:00
Europe/Zurich

CERN
TH Conference Room

Heller, Michal P.
Wiedemann, Urs
Chesler, Paul
Hong, Deog-Ki
Yaffe, Larry
Herzog, Christopher

No material yet

→ http://goo.gl/BBYh7J

extra