Universal features of black holes in the large D limit

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Why black hole dynamics is hard

Non-decoupling:

BH is an extended object whose dynamics mixes strongly with background

BH’s own dynamics not well-localized, not decoupled

Why black hole dynamics is hard

BHs, like other extended objects, have (quasi-) normal modes

but typically localized at some distance from the horizon

$\sim$ photon orbit in AF

in AdS backgrounds may be further away

$\rightarrow$ hard to disentangle bh dynamics from background dynamics

Why black hole dynamics is hard

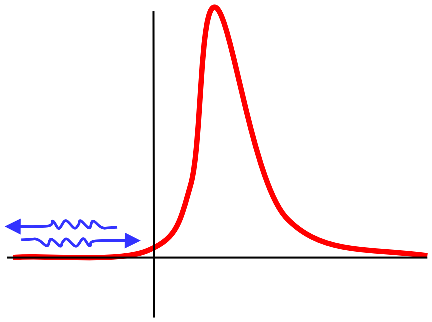
BH dynamics lacks a *generically small* parameter

Decoupling *requires* a small parameter

Near-extremality does it: AdS/CFT-type decoupling

Develop a throat

effective radial potential



Large D limit

Kol et al

RE+Suzuki+Tanabe

1/D as small parameter

Separates bh's own dynamics from background spacetime

- *sharp* localization of bh dynamics

BH near-horizon well defined

- a very special $2D$ bh

Somewhat similar to decoupling limit in ads/cft

Large D limit

Far-region: background spacetime w/ holes

only knows bh size and shape

$\rightarrow$ far-zone trivial dynamics

Near-region:

– non-trivial geometry

– large universality classes eg neutral bhs (rotating, AdS etc)

Large *D* expansion may help for

- calculations: new perturbative expansion
- deeper understanding of the theory (reformulation?)

Universality (due to strong localization) is good for both

Large *D* black holes

Basic solution

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

length scale r_0

Large *D* black holes

r_0 not the only scale

Small *parameter* $1/D \Longrightarrow$ scale hierarchy

$$r_0/D \ll r_0$$

This is the **main feature** of large-D GR

Localization of interactions

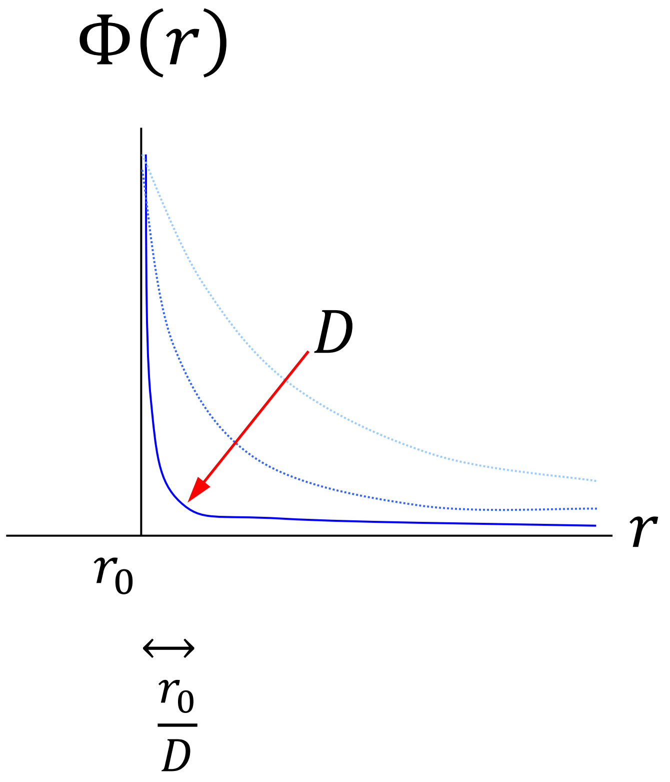
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\frac{r_0}{D} \ll r_0$$



Far zone

Fixed $r > r_0 \quad D \to \infty$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{D-3} \to 1$$

$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{D-2}$$

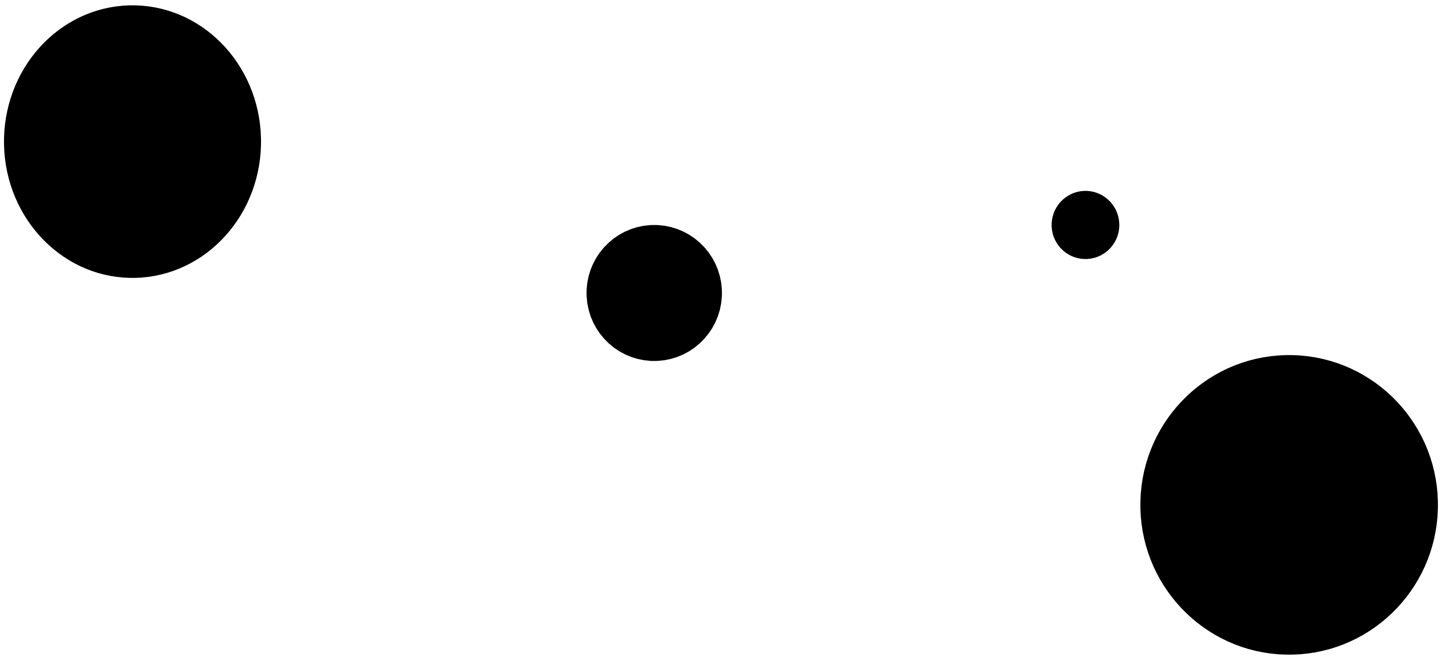
Flat, empty space at $r > r_0$

no gravitational field

Far zone geometry

scale $\mathcal{O}(r_0 D^0)$

Holes cut out in Minkowski space



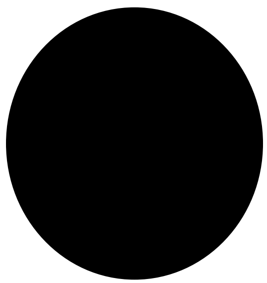
Far zone

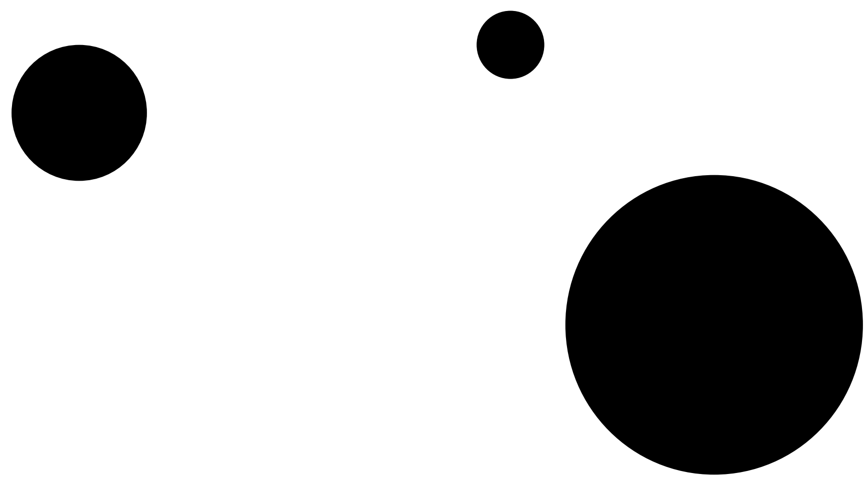
scale $\mathcal{O}(r_0 D^0)$

Holes cut out in Minkowski space

No wave absorption (perfect reflection)

for $D \to \infty$

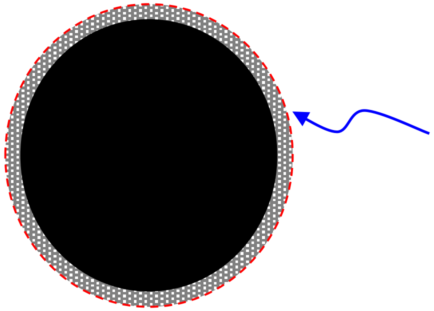




Near zone

Gravitational field appreciable only in *thin* near-horizon region

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \Leftrightarrow r - r_0 < \frac{r_0}{D}$$



$$r - r_0 \sim \frac{r_0}{D}$$

Near zone

Keep non-trivial gravitational field:

Length scales $\sim r_0/D$ away from horizon

Surface gravity $\kappa \sim D/r_0$ finite

Near-horizon coordinate: $R = (r/r_0)^{D-3}$

All remain $\mathcal{O}(1)$ where grav field is non-trivial

Near zone

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0} t \end{aligned}\right\} \text{finite as } D \to \infty$$

Near zone

$$ds_{nh}^2 \to \frac{4r_0^2}{D^2}\underbrace{\left(-\tanh^2\rho \ \ dt_{near}^2 + d\rho^2\right)}_{\text{2d string black hole}} + r_0^2 d\Omega_{D-2}^2$$

2d string black hole

Elitzur et al
Mandal et al
Witten

Soda
Grumiller et al

$$\ell_{string} \sim \frac{r_0}{D}, \qquad \alpha' \sim \left(\frac{r_0}{D}\right)^2$$

Near zone universality: neutral bhs

2d string bh is near-horizon geometry of **all neutral non-extremal bhs**

- rotation appears as a local boost (in a third direction)
- cosmo const shifts 2d bh mass

More near-horizon structure than just Rindler limit

Near zone universality

Charge modifies near-horizon geom

some are 'stringy' bhs

eg, 3d black string *Horne+Horowitz*

but many different solutions possess same near-horizon

universality classes

Large D expansion:

1. BH quasinormal modes

2. Instability of rotating bhs

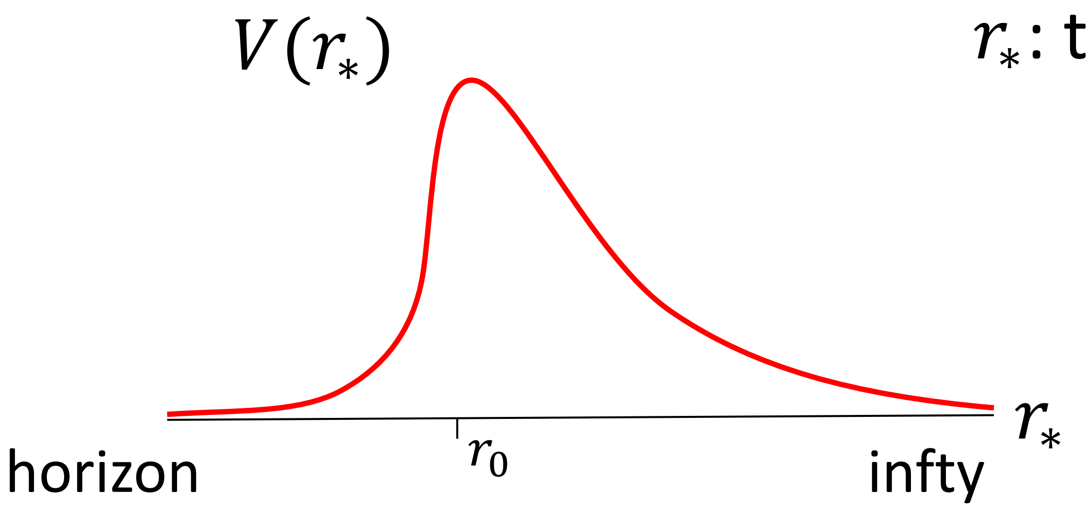
Massless scalar field

$\Box\Phi = 0$

$$\Phi = r^{-\frac{D-2}{2}}\,\phi(r)\,e^{-i\omega t}\,Y_\ell(\Omega)$$

$$\frac{d^2\phi}{dr_*^2} + (\omega^2 - V(r_*))\phi = 0$$

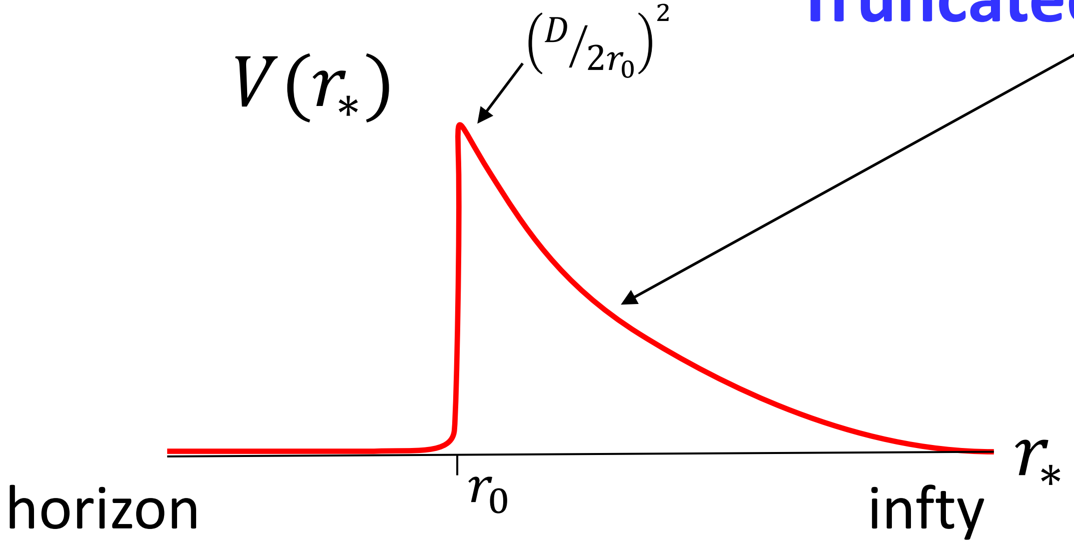
r_*: tortoise coord



Massless scalar field

$$D \to \infty \qquad\qquad V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$

Truncated flat-space barrier

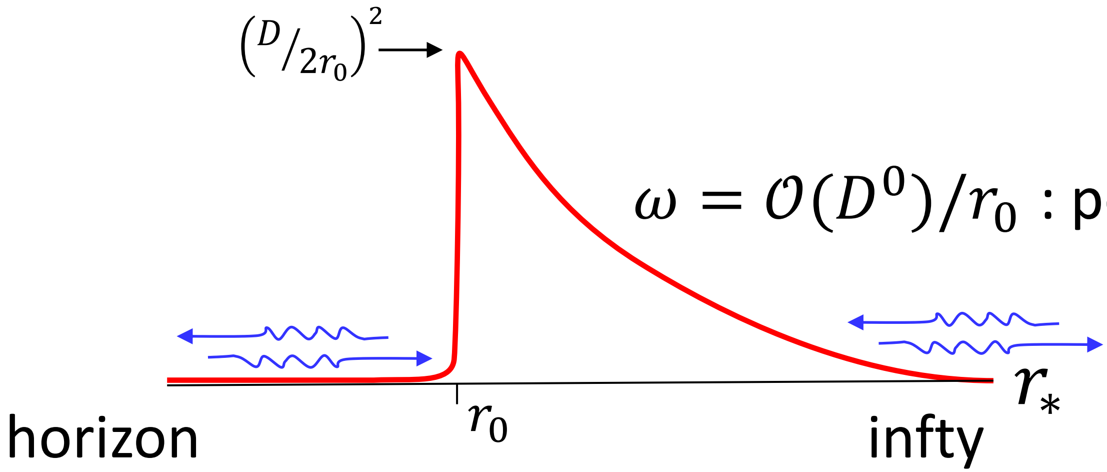


Massless scalar field

$$V(r_*) \rightarrow \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$

$\omega > \frac{D}{2r_0}$: perfectly absorbed

$\omega = \mathcal{O}(D^0)/r_0$: perfectly reflected



Schwarzschild bh grav perturbations

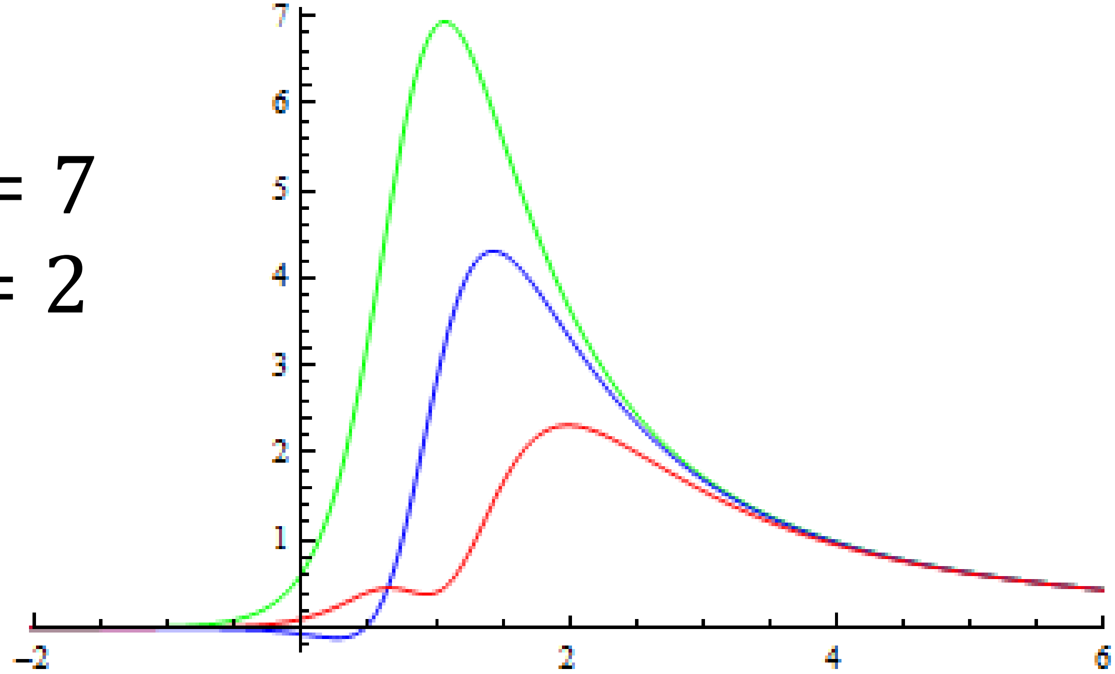
Kodama+Ishibashi

Gravitational scalar, vector, tensor modes

$SO(D-1)$ reps

$V(r_*)$

$D = 7$
$\ell = 2$

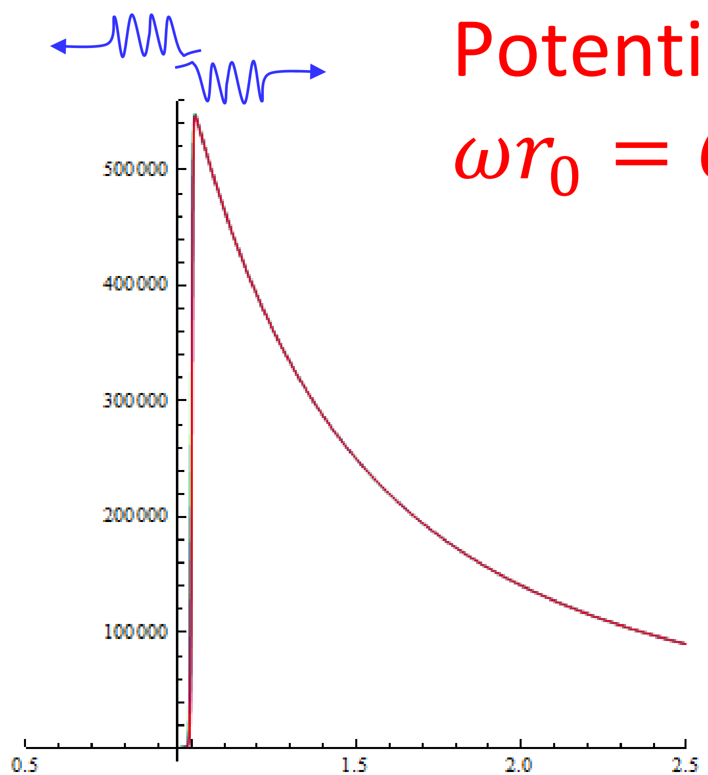


r_*

Schwarzschild bh grav perturbations

scalar vector tensor

$D = 500$
$\ell = 500$



Potential seen by
$\omega r_0 = \mathcal{O}(D)$

Schwarzschild bh grav perturbations

scalar vector tensor

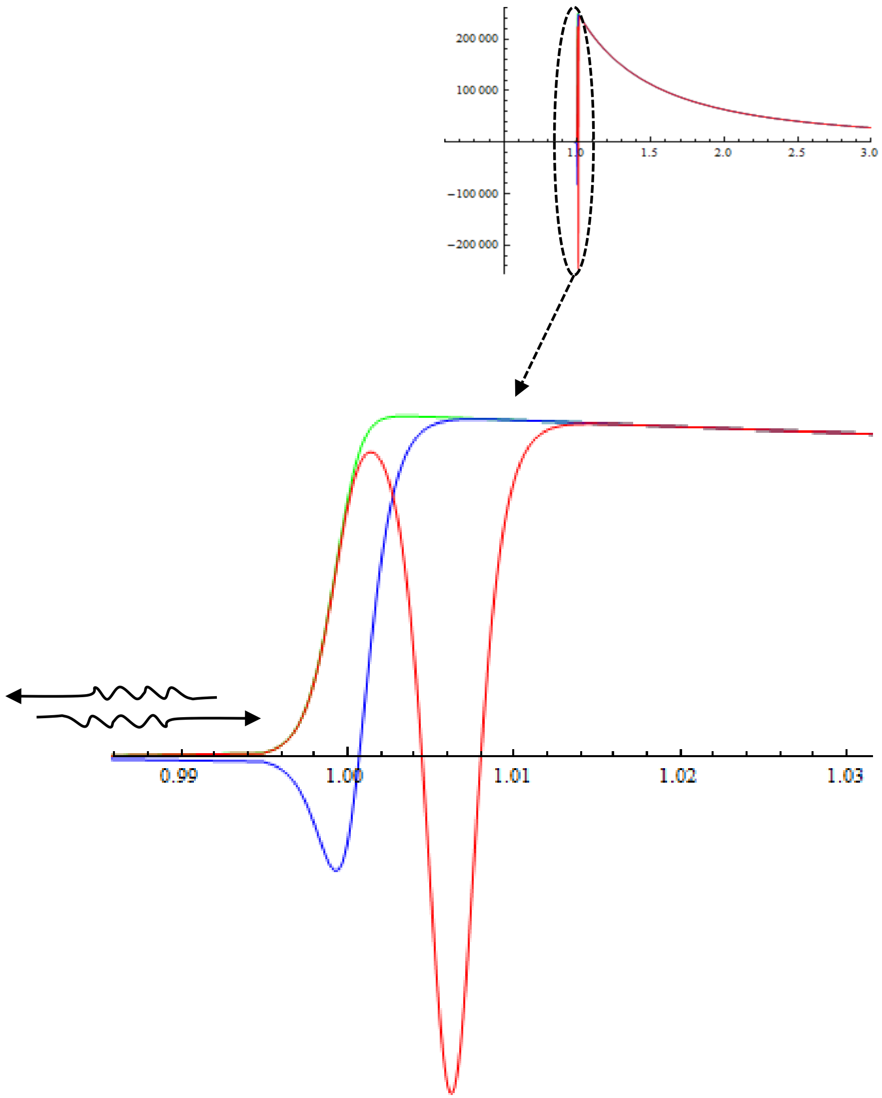
Potential seen by

$\omega r_0 = \mathcal{O}(1)$

$\ell = \mathcal{O}(1)$

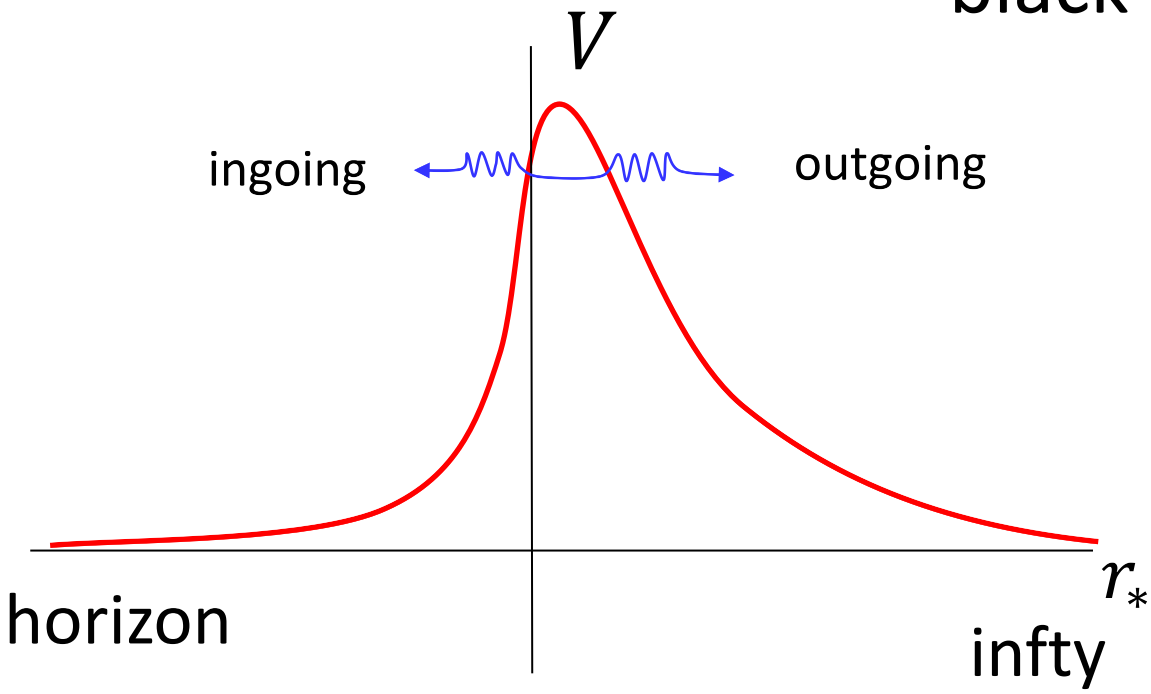
$D = 1000$

$\ell = 2$



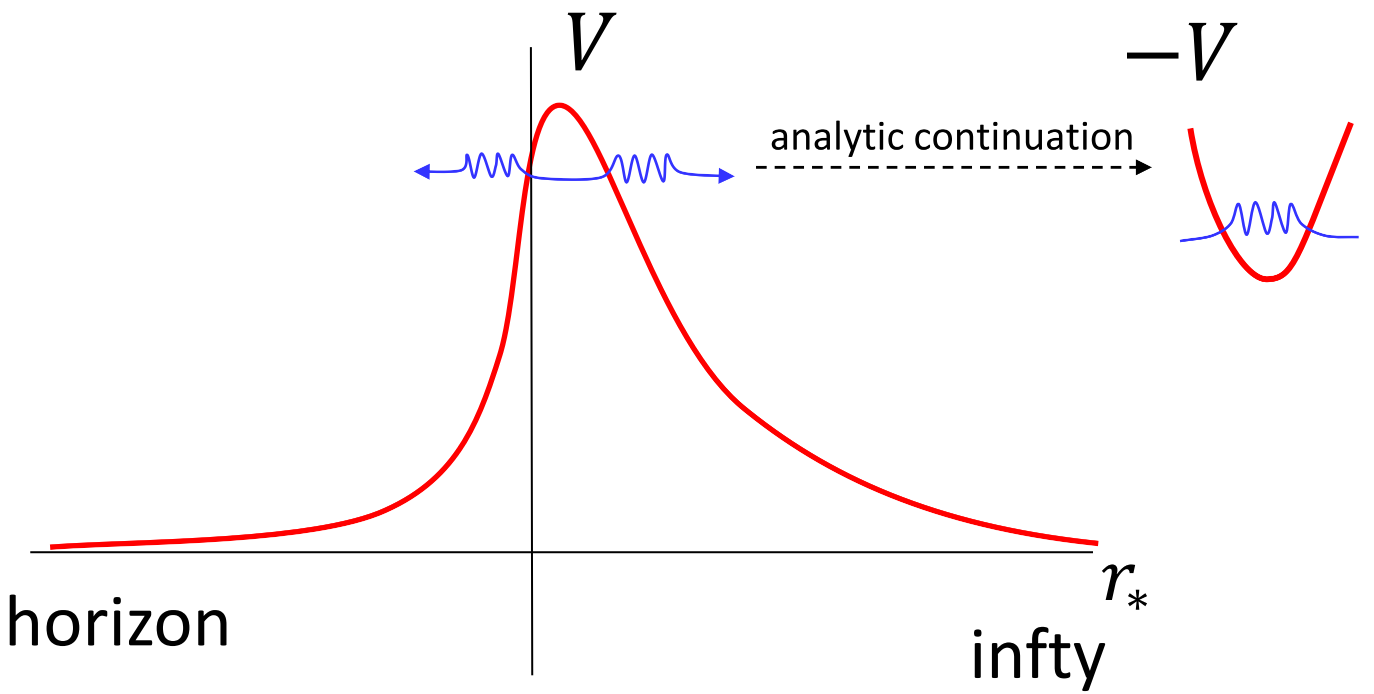
Quasinormal modes

Free, damped oscillations of black hole



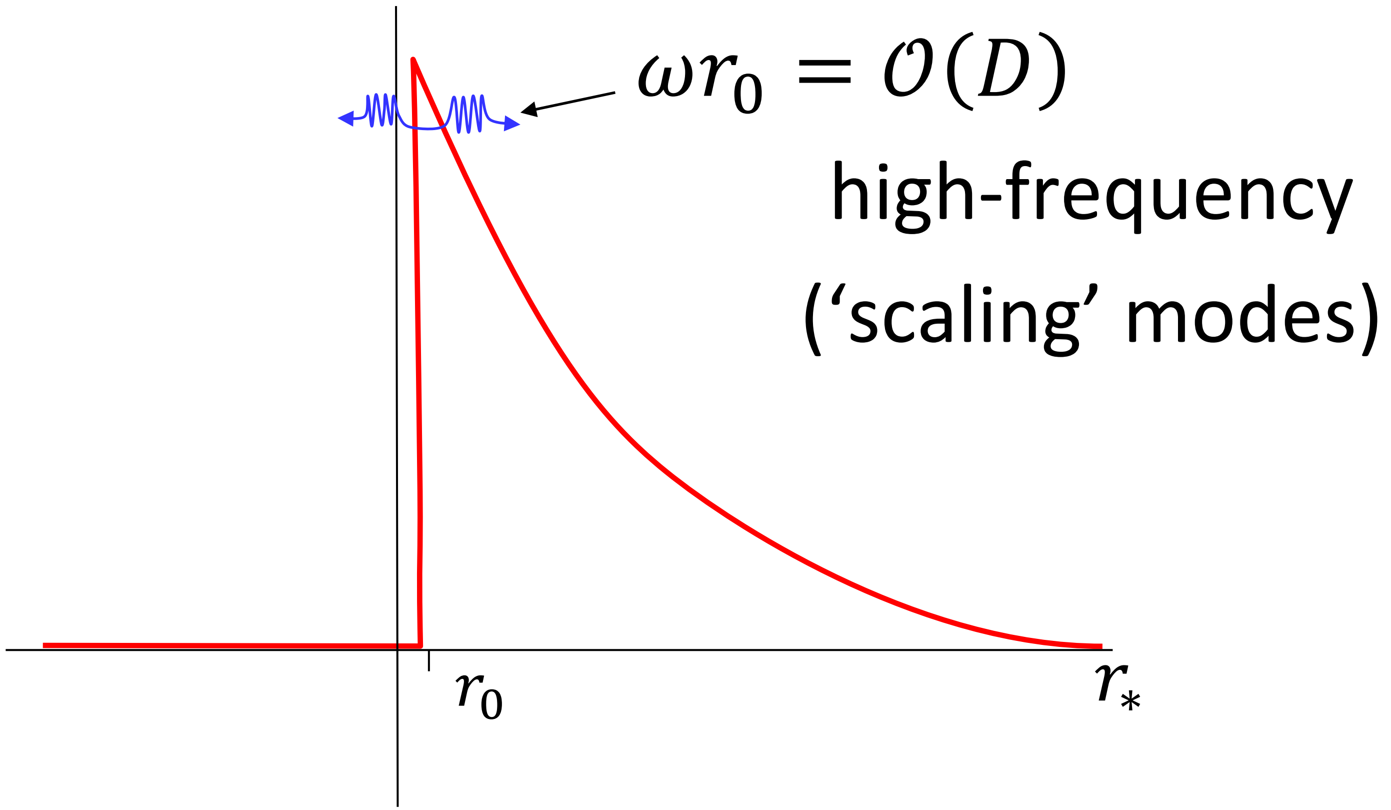
Quasinormal modes

QNMs as bound states in inverted potential



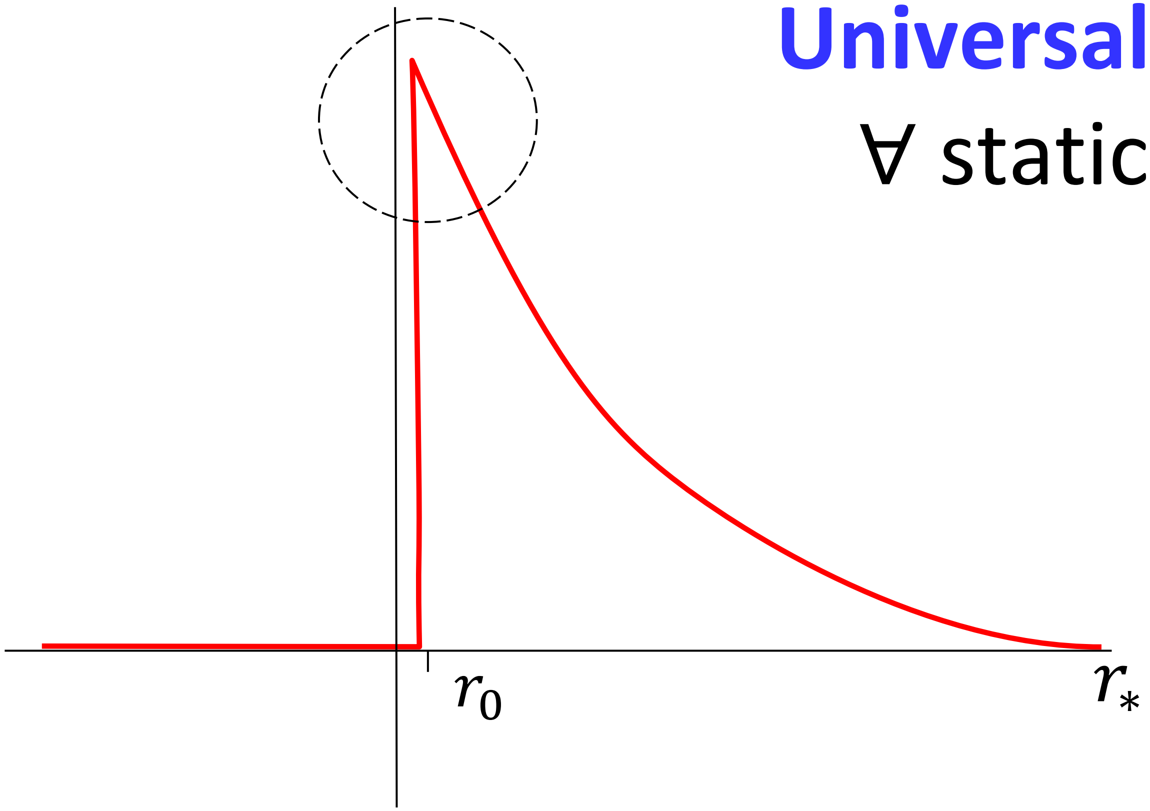
$\omega r_0 = \mathcal{O}(D)$ QNMs

$$V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$



$\omega r_0 = \mathcal{O}(D)$ QNMs

$$V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$

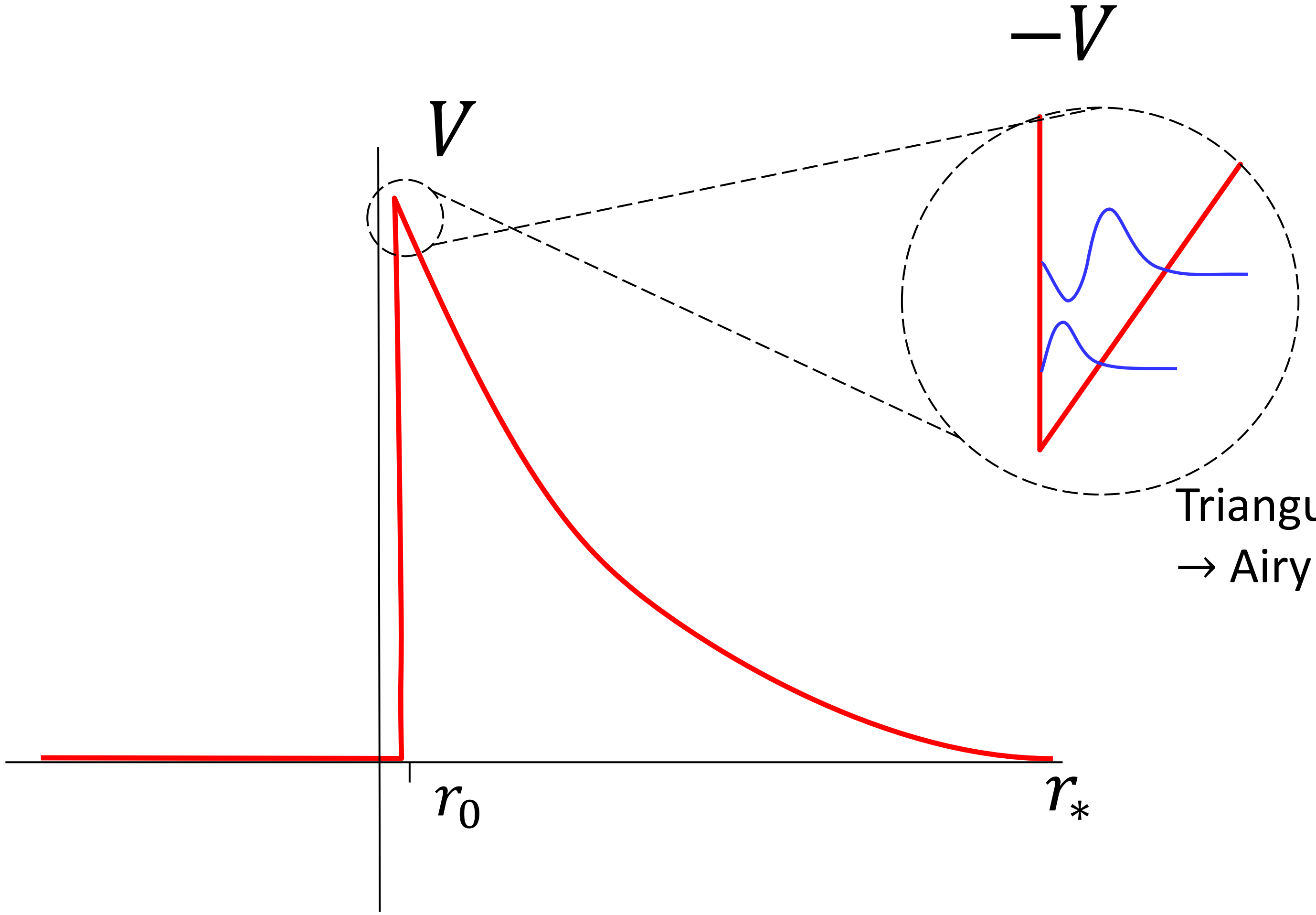


Holes in flat space

Universal structure

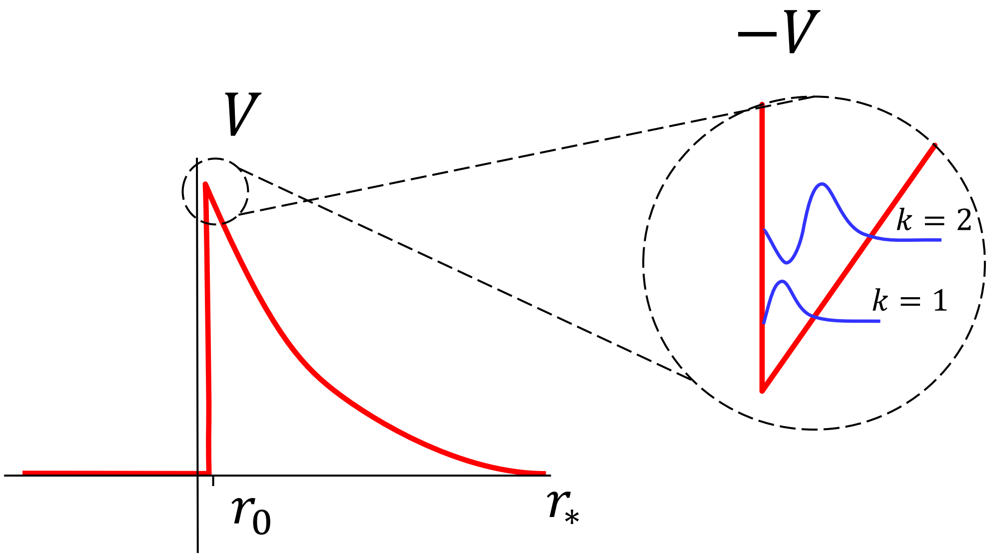
∀ static, AF bhs

$\omega r_0 = \mathcal{O}(D)$ QNMs



Triangular well
→ Airy wavefns

$\omega r_0 = \mathcal{O}(D)$ QNMs



Airy zeroes

$$\Rightarrow \omega_{(\ell,k)}\, r_0 = \frac{D}{2} + \ell - \left(\frac{e^{i\pi}}{2}\left(\frac{D}{2}+\ell\right)\right)^{\frac{1}{3}} a_k$$

Universal spectrum @ large D

$$\omega_{(\ell,k)} r_0 = \frac{D}{2} + \ell - \left(\frac{e^{i\pi}}{2} \left(\frac{D}{2} + \ell \right) \right)^{\frac{1}{3}} a_k$$

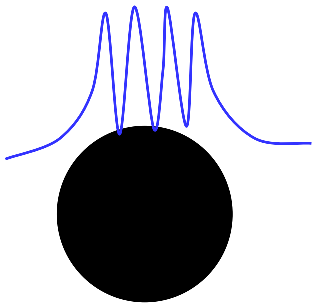
Depends **only on** bh radius r_0

Same spectrum for:

- any charges, dilaton coupling etc
- scalar, vector, tensor perturbations

Universal spectrum @ large D

$$\omega_{(\ell,k)} r_0 = \frac{D}{2} + \ell - \left(\frac{e^{i\pi}}{2} \left(\frac{D}{2} + \ell \right) \right)^{\frac{1}{3}} a_k$$



spectrum of scalar
oscillations of a hole
in space

$$\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} \sim D^{-2/3} \to 0:$$

sharp resonances

'normal modes' of bh

$\omega r_0 = \mathcal{O}(1)$ QNMs

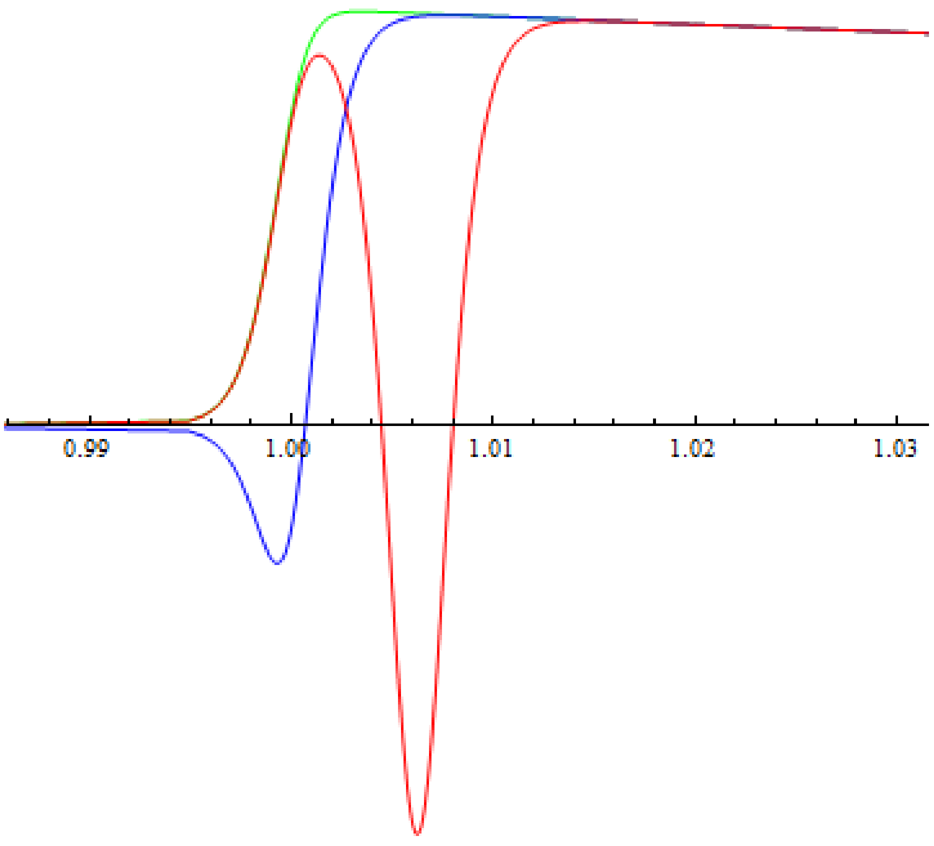
More complicated wave eqn

but we've solved it up to

D^{-3} for vectors

D^{-2} for scalars

(no tensors)



Quantitative accuracy

$\boldsymbol{\omega r_0 = \mathcal{O}(1)}$ **modes**

Vector mode (purely imaginary)

- At $D = 100$:

$\ell = 2$ mode $\quad \mathrm{Im}\, \omega r_0 =$ -1.01044742 (analytical)

-1.01044741 (numerical *Dias et al*)

Quantitative accuracy

$\omega r_0 = \mathcal{O}(1)$ modes

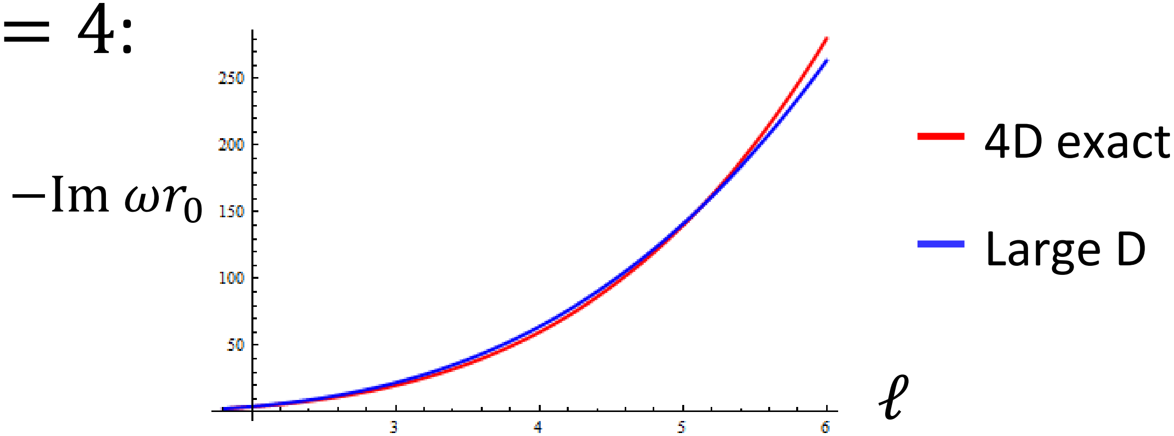
Vector mode (purely imaginary)

- At $D = 100$:

$\ell = 2$ mode $\quad \mathrm{Im}\, \omega r_0 =$ -1.01044742 (analytical)

-1.01044741 (numerical *Dias et al*)

- At $D = 4$:

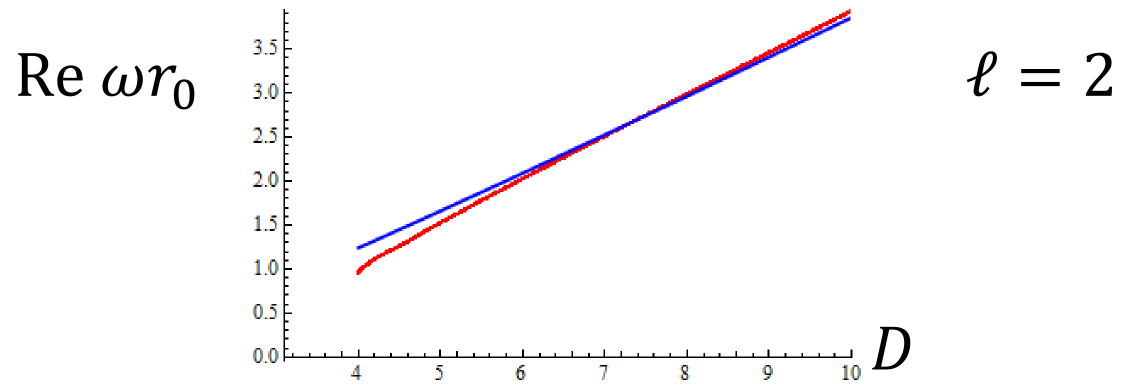


'algebraically special' mode

Quantitative accuracy

$\boldsymbol{\omega r_0 = \mathcal{O}(D)}$ modes

$\mathrm{Re}\,\omega r_0 = \frac{D}{2} + \ell$: good at moderate D



$\mathrm{Im}\,\omega r_0 \sim D^{1/3}$: only good at *very* high D

$D \gtrsim 300$ (!)

Instability of rotating bhs

Hi-D bhs have *ultra-spinning* regimes

Expect instabilities:

- axisymmetric
- non-axisymmetric (at lower rotation)

Confirmed by numerical studies *Dias et al*

Hartnett+Santos

Shibata+Yoshino

Analytically solvable in ${}^{1}/_{D}$ expansion thanks to universality features – *also in AdS*

Equal-spin, odd-D, Myers-Perry black holes

→ only radial dependence → ODEs

But equations are coupled

– analytically hopeless

$$-\frac{2}{g^2}\left(\left(\frac{f'}{f}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)\Bigg]f_{00}$$

$$+2i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{01}+2\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{\mathcal{N}h\Omega'}{fg^2r}\right]f_{02}$$

$$+\left[\left(\frac{h\Omega'}{fg}\right)^2-\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{f'}{fg}\right)\right]f_{11}-\frac{2}{g^2}\left[\frac{f'h'}{fh}+\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right]f_{22}-\frac{4\mathcal{N}f'}{fg^2r}\mathcal{H}_L=-k^2f_{00}, \qquad (72a)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(2\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2+\frac{2\mathcal{N}}{r^2}\right)+\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{f'}{fg}\right)\Bigg]f_{01}$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{3}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(-\frac{f'}{f}+\frac{h'}{h}+\frac{\mathcal{N}}{r}\right)\right]f_{12}$$

$$+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right](f_{00}+f_{11})-i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{02}$$

$$+\frac{1}{r^2g\sqrt{\lambda}}\left[\mathcal{W}^+(\lambda-2m\mathcal{N})+\mathcal{W}\ \ (\lambda+2m\mathcal{N})\right]=-k^2f_{01}, \qquad (72b)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{f'}{f}-\frac{h'}{h}\right)^2-2\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2\mathcal{N}h^2}{r^4}\Bigg]f_{02}$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{\mathcal{N}}{r}\right)\right]f_{00}+i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{01}$$

$$-\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)-\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}-\frac{h'}{h}\right)\right]f_{11}+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{12}$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{\mathcal{N}h\Omega'}{fg^2r}\right]f_{22}$$

$$-\frac{ih}{r^3\sqrt{\lambda}}\left[\mathcal{W}^+(\lambda-2m\mathcal{N})-\mathcal{W}\ \ (\lambda+2m\mathcal{N})\right]-\frac{2\mathcal{N}h\Omega'}{fg^2r}\mathcal{H}_L=-k^2f_{02}, \qquad (72c)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(\mathcal{N}+1)-2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{f'}{f}-\frac{1}{r}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2h^2}{r^4}\Bigg]\mathcal{W}^+$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{(\mathcal{N}+1)h\Omega'}{fg^2r}\right]\mathcal{Z}^+$$

$$+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}-\frac{2h^2\Omega'}{fgr^2}\right]\mathcal{X}^++\frac{2\sqrt{\lambda}}{r^2g}f_{01}+\frac{i2h\sqrt{\lambda}}{r^3}f_{02}=-k^2\mathcal{W}^+, \qquad (72d)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(\mathcal{N}+1)+2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{f'}{f}-\frac{1}{r}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2h^2}{r^4}\Bigg]\mathcal{W}$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{(\mathcal{N}+1)h\Omega'}{fg^2r}\right]\mathcal{Z}$$

$$+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}+\frac{2h^2\Omega'}{fgr^2}\right]\mathcal{X}\ \ +\frac{2\sqrt{\lambda}}{r^2g}f_{01}-\frac{i2h\sqrt{\lambda}}{r^3}f_{02}=-k^2\mathcal{W}\ \ , \qquad (72e)$$

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$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{2}{g^2}\left(\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2+\frac{2\mathcal{N}}{r^2}\right)\Bigg]f_{11}$$

$$-2\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{f'}{fg}\right)-\frac{1}{2}\left(\frac{h\Omega'}{fg}\right)^2\right]f_{00}+2i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{01}$$

$$+2\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)-\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}-\frac{h'}{h}\right)\right]f_{02}-2i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{12}$$

$$+2\left[\frac{1}{g}\frac{\partial}{\partial r}\left(\frac{h'}{hg}\right)+\frac{1}{2}\left(\frac{h\Omega'}{fg}\right)^2\right]f_{22}+\frac{2}{r^2g\sqrt{\lambda}}\left[\mathcal{X}^+(\lambda-2m\mathcal{N})+\mathcal{X}\ \ (\lambda+2m\mathcal{N})\right]$$

$$-\frac{4\mathcal{N}}{g^2r}\left(\frac{g'}{g}+\frac{1}{r}\right)\mathcal{H}_L=-k^2f_{11}, \qquad (72f)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{f'}{f}\right)^2+2\left(\frac{h'}{h}\right)^2-2\left(\frac{h\Omega'}{f}\right)^2+\frac{2\mathcal{N}}{r^2}\right)-\frac{2\mathcal{N}h^2}{r^4}+\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{h'}{hg}\right)\Bigg]f_{12}$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}-\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{2f'}{f}+\frac{\mathcal{N}}{r}\right)\right]f_{01}$$

$$+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{m\Omega'}{fg}\right]f_{02}+i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right](f_{11}-f_{22})$$

$$-\frac{ih}{r^3\sqrt{\lambda}}\left[\mathcal{X}^+(\lambda-2m\mathcal{N})-\mathcal{X}\ \ (\lambda+2m\mathcal{N})\right]+\frac{1}{r^2g\sqrt{\lambda}}\left[\mathcal{Z}^+(\lambda-2m\mathcal{N})+\mathcal{Z}\ \ (\lambda+2m\mathcal{N})\right]$$

$$=-k^2f_{12}, \qquad (72g)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(\mathcal{N}+1)-2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2+\frac{2\mathcal{N}+3}{r^2}\right)-\frac{2h^2}{r^4}-\frac{2g'}{g^3r}\Bigg]\mathcal{X}^+$$

$$+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{h\Omega'}{fg}\left(\frac{m}{h}-\frac{2h}{r^2}\right)\right]\mathcal{W}^++i\left[-\frac{h\Omega'(\omega-m\Omega)}{f^2g}+\frac{2mh'}{h^2g}-\frac{4h}{r^2g}\left(\frac{h'}{h}-\frac{2}{r}\right)\right]\mathcal{Z}^+$$

$$+\frac{1}{r^2g\sqrt{\lambda}}\left[\left(\lambda-4(\mathcal{N}+1)-2m(\mathcal{N}+2)\right)\mathcal{H}^{++}+\frac{(\mathcal{N}-1)(\lambda+2m\mathcal{N})}{\mathcal{N}}\mathcal{H}^+\ \ -2\lambda\mathcal{H}_L\right]$$

$$+\frac{2\sqrt{\lambda}}{r^2g}f_{11}+\frac{i2h\sqrt{\lambda}}{r^3}f_{12}=-k^2\mathcal{X}^+, \qquad (72h)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(\mathcal{N}+1)+2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{f'}{f}\right)^2+\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2+\frac{2\mathcal{N}+3}{r^2}\right)-\frac{2h^2}{r^4}-\frac{2g'}{g^3r}\Bigg]\mathcal{X}$$

$$+i\left[\frac{2f'(\omega-m\Omega)}{f^2g}+\frac{h\Omega'}{fg}\left(\frac{m}{h}+\frac{2h}{r^2}\right)\right]\mathcal{W}\ \ +i\left[-\frac{h\Omega'(\omega-m\Omega)}{f^2g}+\frac{2mh'}{h^2g}+\frac{4h}{r^2g}\left(\frac{h'}{h}-\frac{2}{r}\right)\right]\mathcal{Z}$$

$$+\frac{1}{r^2g\sqrt{\lambda}}\left[\left(\lambda-4(\mathcal{N}+1)+2m(\mathcal{N}+2)\right)\mathcal{H}\ \ \ +\frac{(\mathcal{N}-1)(\lambda-2m\mathcal{N})}{\mathcal{N}}\mathcal{H}^+\ \ -2\lambda\mathcal{H}_L\right]$$

$$+\frac{2\sqrt{\lambda}}{r^2g}f_{11}-\frac{i2h\sqrt{\lambda}}{r^3}f_{12}=-k^2\mathcal{X}\ \ , \qquad (72i)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

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$$-\frac{2}{g^2}\left(\left(\frac{h'}{h}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{4\mathcal{N}h^2}{r^4}\Bigg]f_{22}$$

$$-\frac{2}{g^2}\left[\frac{f'h'}{fh}+\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right]f_{00}+2\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{\mathcal{N}}{r}\right)\right]f_{02}$$

$$+\left[\frac{2}{g}\frac{\partial}{\partial r}\left(\frac{h'}{hg}\right)+\left(\frac{h\Omega'}{fg}\right)^2\right]f_{11}+2i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}\right]f_{12}$$

$$-\frac{i2h}{r^3\sqrt{\lambda}}\left[\mathcal{Z}^+(\lambda-2m\mathcal{N})-\mathcal{Z}\ \ (\lambda+2m\mathcal{N})\right]+4\mathcal{N}\left(\frac{h'}{hg^2r}-\frac{2h^2}{r^4}\right)\mathcal{H}_L=-k^2f_{22}, \qquad (72j)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(\mathcal{N}+1)-2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{h'}{h}-\frac{1}{r}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2(\mathcal{N}+3)h^2}{r^4}\Bigg]\mathcal{Z}^+$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{\mathcal{N}-1}{r}\right)\right]\mathcal{W}^+$$

$$+i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}+\frac{4h}{r^2g}\left(\frac{h'}{h}-\frac{2}{r}\right)\right]\mathcal{X}^+$$

$$-\frac{ih}{r^3\sqrt{\lambda}}\left[\left(\lambda-4(\mathcal{N}+1)-2m(\mathcal{N}+2)\right)\mathcal{H}^{++}-\frac{(\mathcal{N}-1)(\lambda+2m\mathcal{N})}{\mathcal{N}}\mathcal{H}^+\ \ +2\lambda\mathcal{H}_L\right]$$

$$+\frac{2\sqrt{\lambda}}{r^2g}f_{12}+\frac{i2h\sqrt{\lambda}}{r^3}f_{22}=-k^2\mathcal{Z}^+, \qquad (72k)$$

$$-\Bigg[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-2(\mathcal{N}+1)+2m}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}$$

$$-\frac{1}{g^2}\left(\left(\frac{h'}{h}-\frac{1}{r}\right)^2-\frac{1}{2}\left(\frac{h\Omega'}{f}\right)^2\right)-\frac{2(\mathcal{N}+3)h^2}{r^4}\Bigg]\mathcal{Z}$$

$$+\left[\frac{h\Omega'}{fg^2}\frac{\partial}{\partial r}+\frac{1}{2g}\frac{\partial}{\partial r}\left(\frac{h\Omega'}{fg}\right)+\frac{h\Omega'}{fg^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{\mathcal{N}-1}{r}\right)\right]\mathcal{W}$$

$$+i\left[\frac{h\Omega'(\omega-m\Omega)}{f^2g}-\frac{2mh'}{h^2g}-\frac{4h}{r^2g}\left(\frac{h'}{h}-\frac{2}{r}\right)\right]\mathcal{X}$$

$$+\frac{ih}{r^3\sqrt{\lambda}}\left[\left(\lambda-4(\mathcal{N}+1)+2m(\mathcal{N}+2)\right)\mathcal{H}\ \ \ -\frac{(\mathcal{N}-1)(\lambda-2m\mathcal{N})}{\mathcal{N}}\mathcal{H}^+\ \ +2\lambda\mathcal{H}_L\right]$$

$$+\frac{2\sqrt{\lambda}}{r^2g}f_{12}-\frac{i2h\sqrt{\lambda}}{r^3}f_{22}=-k^2\mathcal{Z}\ \ , \qquad (72l)$$

$$\frac{1}{\sqrt{\lambda}}\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-4(\mathcal{N}+1+m)}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right]\mathcal{H}^{++}$$

$$-\frac{4}{r^2g}\mathcal{X}^+-\frac{i4h}{r^3}\mathcal{Z}^+=k^2\frac{1}{\sqrt{\lambda}}\mathcal{H}^{++}, \qquad (72m)$$

$$\frac{1}{\sqrt{\lambda}}\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-4(\mathcal{N}+1-m)}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}\right]\mathcal{H}$$

$$-\frac{4}{r^2g}\mathcal{X}\ \ +\frac{i4h}{r^3}\mathcal{Z}\ \ =k^2\frac{1}{\sqrt{\lambda}}\mathcal{H}\ \ , \qquad (72n)$$

$$\frac{1}{\sqrt{\lambda}}\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-4\mathcal{N}}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}+\frac{4}{r^2}\left(1-\frac{2h^2}{r^2}\right)\right]\mathcal{H}^+$$

$$-\frac{2}{r^2g}(\mathcal{X}^++\mathcal{X}\ \ )+\frac{2ih}{r^3}(\mathcal{Z}^+-\mathcal{Z}\ \ )=k^2\frac{1}{\sqrt{\lambda}}\mathcal{H}^+\ \ , \qquad (72o)$$

$$-\left[\frac{1}{g}\frac{\partial}{\partial r}\frac{1}{g}\frac{\partial}{\partial r}+\frac{1}{g^2}\left(\frac{f'}{f}+\frac{h'}{h}+\frac{2\mathcal{N}}{r}\right)\frac{\partial}{\partial r}-\frac{\lambda-4(\mathcal{N}+1)}{r^2}-\frac{m^2}{h^2}+\frac{(\omega-m\Omega)^2}{f^2}-\frac{4\mathcal{N}}{g^2r^2}-\frac{8h^2}{r^4}\right]\mathcal{H}_L$$

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Dias, Figueras, Monteiro, Reall, Santos 2010

Equations *do decouple* for rotation=0

Large D expansion:

Leading large D near-horizon: rotating bh is just a boost of Schw

→ rotating eqns decouple

can be solved analytically

Beyond leading order, MP metric is not boosted Schw, but LO boost allows to decouple eqns

Analytical computation of QNMs

- *Axisymmetric* instability for

$$a > \frac{\sqrt{3}}{2}\, r_+$$

- *Non-axisymmetric* instability for

$$a > \frac{1}{\sqrt{2}}\, r_+ = .71\, r_+$$

Comparison to numerical:

D=5: $a > .81 r_+$, *D*=15: $a > .73 r_+$ *Hartnett+Santos*

Outlook

Any problem that can be formulated in arbitrary D is amenable to large D expansion

simpler, even analytically solvable

Universal features

Far: *empty space* $\forall$ *bhs*

Near: *2D string bh* $\forall$ *neutral bhs*

BH dynamics splits into:

$\omega r_0 = \mathcal{O}(D)$: non-decoupled modes

- scalar field **oscillations of a hole** in space
- universal *normal* modes

$\omega r_0 = \mathcal{O}(D^0)$: decoupled modes

- localized in near-horizon region