The Big Crunch/Big Bang Transition

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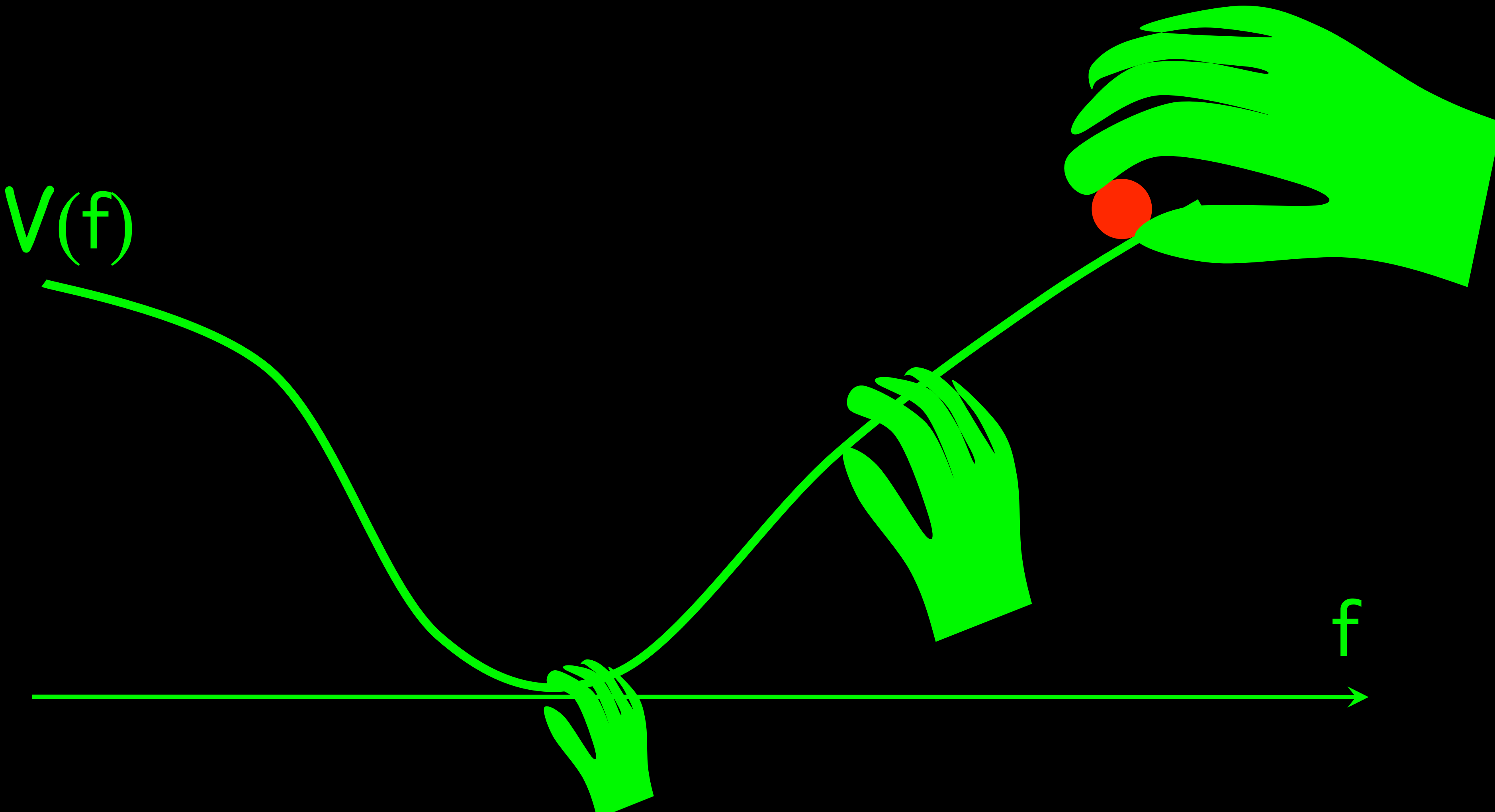
1. Measure for inflation
2. Passing through singularities
   - “no beginning” proposal


To
Stephen
Hawking


Happy
Birthday

inflation

* initial conditions
* fine-tuned potentials
* $L \sim 10^{-120}$; $L_I \sim 10^{-10}$



Does inflation succeed in its claims?

Generic initial conditions ->
* big
* expanding
* flat
* FRW universe
* w/ scale-free perts

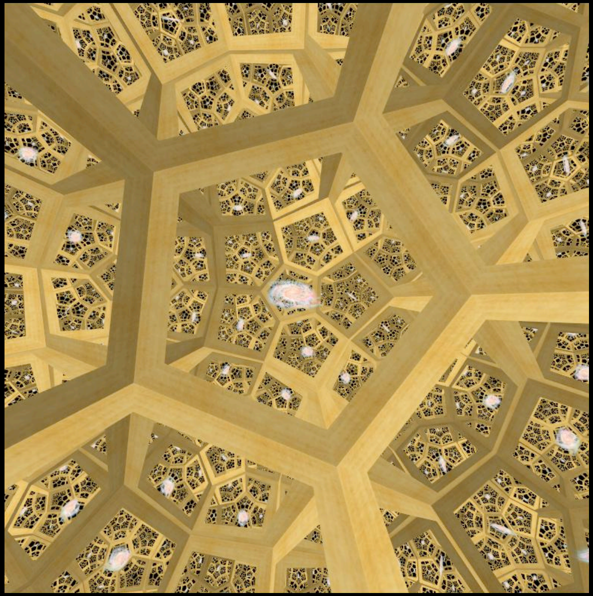
Need a measure on space
of cosmologies
("multiverse")

Focus on a very simple setup where sensible measure (invariant under all symmetries) exists and gives an unambiguous result.

Example:

slow-roll inflation, with V_{min}=0 (eg m^2f^2)

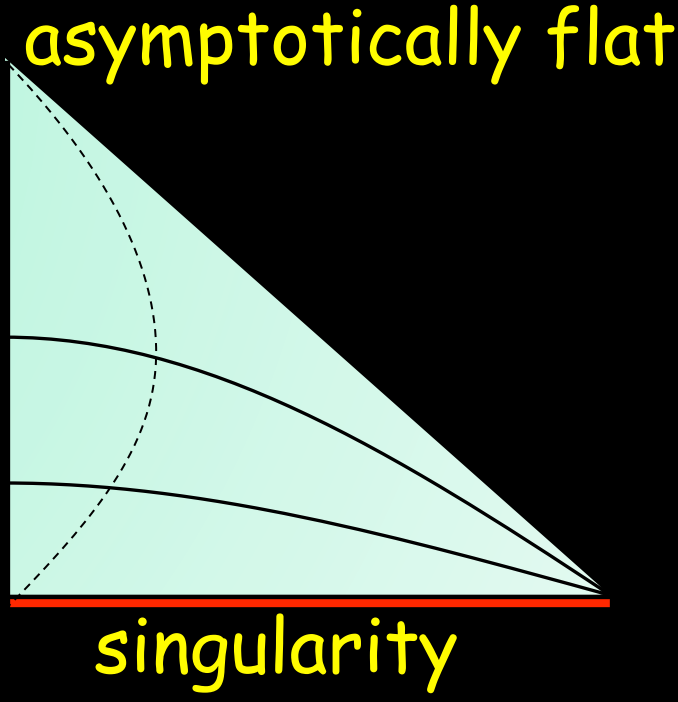
k=-1 FRW, 3-space compactified to U



* a mathematical device to keep everything finite: the results do not depend on the compactification volume U
* but actually has been advocated as a particularly natural setting for chaotic inflation

I will be generous to inflation and just assume homogeneity+isotropy

2-parameter family of solutions with an initial singularity:



Hamiltonian, time reversal invariant

$$\mathrm{H}\,(p_a, a, p_\phi, \phi) = -\frac{p_a^2}{12Ua} + 3Ua + \frac{p_\phi^2}{2Ua^3} + Ua^3 V(\phi)$$

Canonical measure

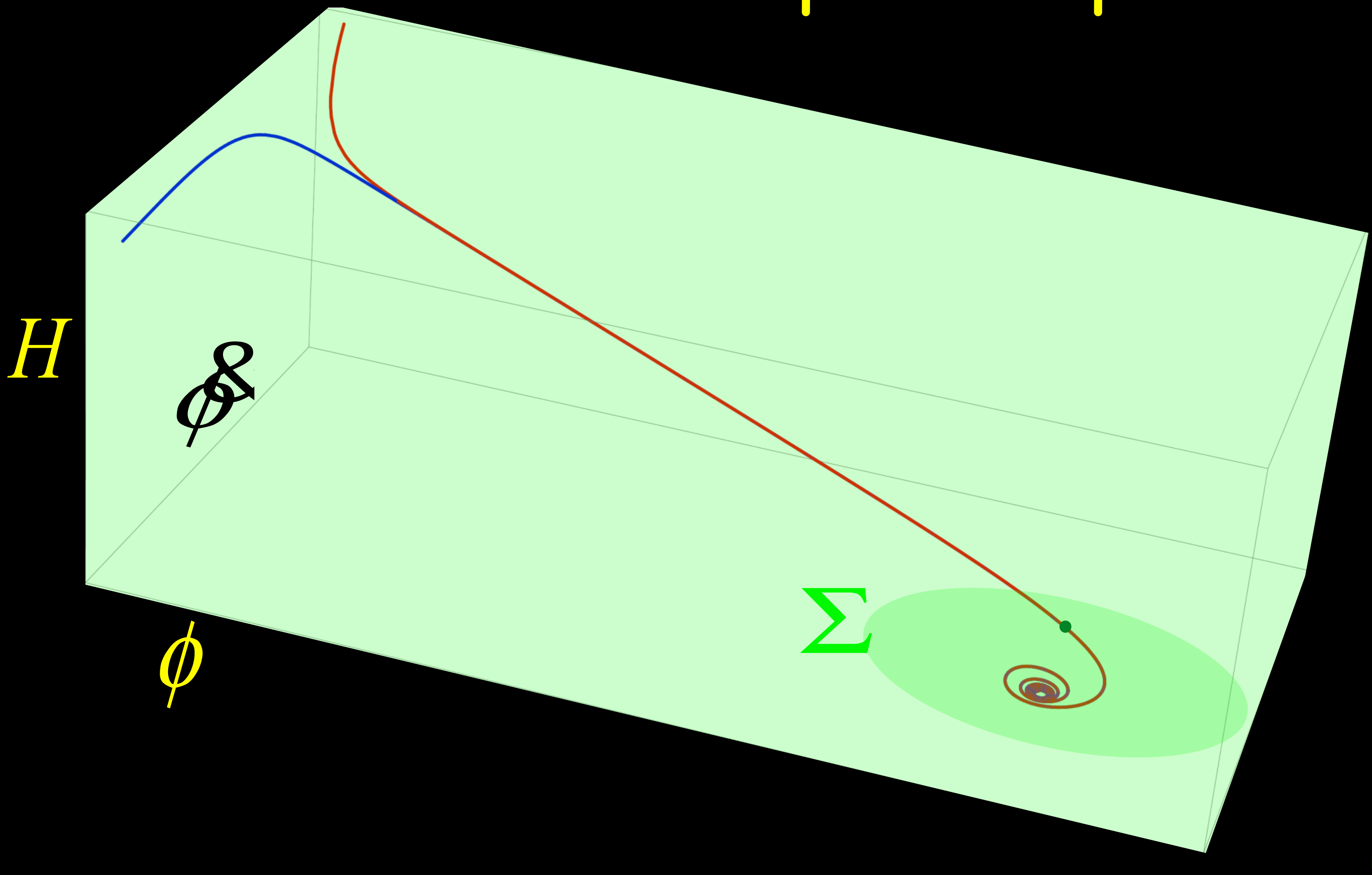
$$\omega_c = dp_a \wedge da + dp_\phi \wedge d\phi$$

$$\int_\Sigma \omega_c \Big|_{\mathrm{H}=0}$$

Liouville
Gibbons, Hawking, Stewart
Hawking, Page
Hollands, Wald
Kofman, Linde, Mukhanov
Gibbons, NT
Carroll, Tan

with Σ pierced once by every trajectory

Universes = curves in phase space



Recall: flat space Gibbs ensemble

Maximise entropy $S = -\sum_i p_i \ln p_i$

subject to $E = \sum_i p_i E_i \quad \text{(assuming } E_i \text{ bb)}$

But in GR, H=0 on all physical states so cannot constrain its expectation value

With no constraint, surface Σ is not compact

What do we do? Need to impose some restriction on universes

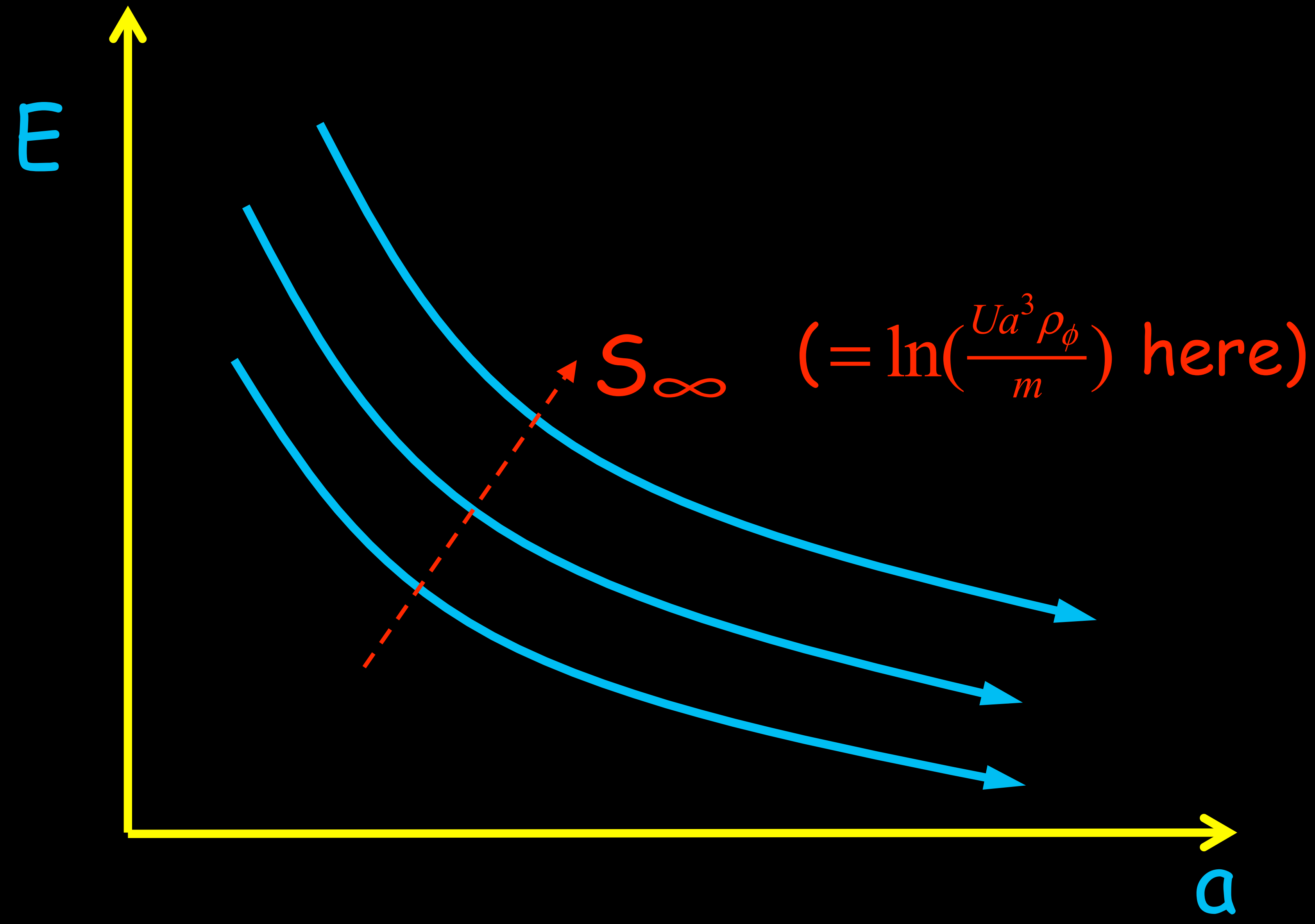
I'll focus on cosmologies which are asymptotically flat to the future (cf holographic measures)

Hence consider k=-1, zero L: at large a, matter diluted away -> gravity becomes negligible, expansion becomes adiabatic

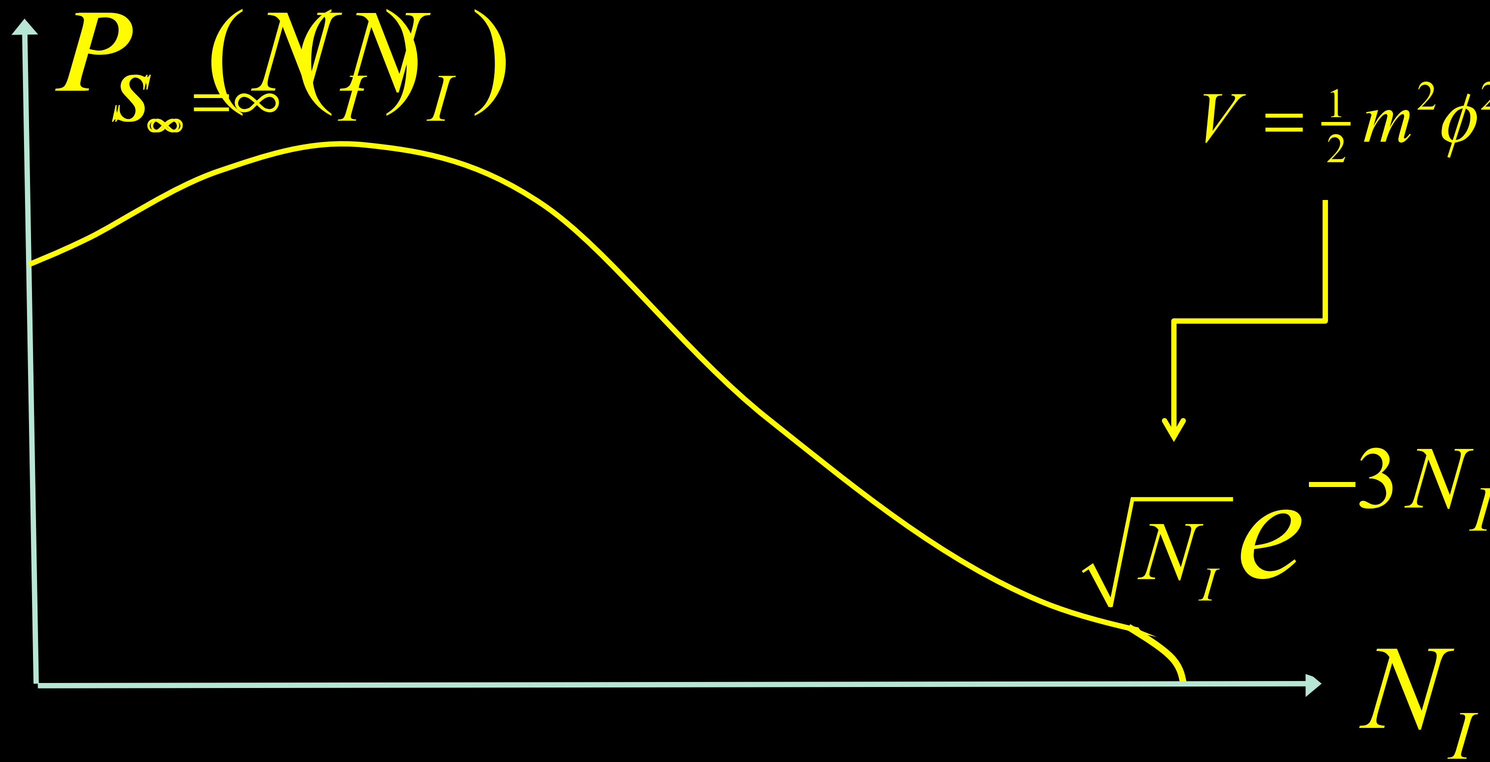
Every trajectory ends on adiabat S(E,a)= const

Natural to label an ensemble of cosmologies by their asymptotic entropy S_∞

Meaningful quantity is $P_{S_\infty}(N_I)$


E
S_∞ $(= \ln(\frac{Ua^3\rho_\phi}{m})$ here)
a

Canonical measure for inflation



$$N_{I,\max} = \tfrac{1}{3} S_\infty - \tfrac{2}{3} \ln\left(\tfrac{m_{Pl}}{m}\right)$$

* Because statistical ensemble defined in asymptotic region where gravity becomes unimportant: the future, inflationary "attractor" becomes "repeller"

* with this canonical measure, slow-roll/`chaotic' inflation cannot be considered an explanation for the observed flatness of the cosmos

* makes precise a problem identified by Penrose long ago

* N fields (eg N-flation) makes problem worse

* can be extended to landscape with inclusion of complex solutions describing tunneling: false vacua don't help

Ways out (other suggestions welcome!):

1) Inhomogeneities (!)

2) Modify measure with unobservable volume factors (!)

3) Evolve forwards from some initial ensemble:
   i) "chaotic"
   ii) asymp. flat region in the past
      -> solve singularity problem

What if the singularity was a bounce?

A cyclic scenario becomes feasible, in which inflation may be entirely unnecessary

Remainder of this talk: long wavelength features may be calculable using only 4d effective theory

I. Bars, S-H Chen, P. Steinhardt, NT arXiv: 1112.2470 [hep-th]
NT to appear (2012)

Example: Einstein-scalar gravity

$$S = \int d^4x\sqrt{-g}[\frac{1}{2\kappa^2}R(g) - \frac{1}{2}(\partial\sigma)^2 - V(\sigma)]$$

Initial conditions:
contracting, perturbed flat FRW universe

If V(s) bounded below (eg inflation, ekpyrotic), then it becomes negligible as singularity nears

KE of scalar s dominates, removes mixmaster chaos, ensures smooth ultralocal (locally Kasner) dynamics

Belinski+Khalatnikov+Lifshitz, Anderson+Rendall

Near singularity, Einstein eqns reduce ultralocally to:

Parameterises radiation

$$\frac{\dot a_E^2}{a_E^4} = \frac{\kappa^2}{3}\left[\frac{\dot\sigma^2 + \overbrace{\dot\alpha_1{}^2 + \dot\alpha_2{}^2}^{\text{Anisotropy}}}{2a_E^2} + \frac{\rho_r}{a_E^4}\right],$$

$$\ddot\sigma + 2\frac{\dot a_E}{a_E}\dot\sigma = 0; \qquad \ddot\alpha_i + 2\frac{\dot a_E}{a_E}\dot\alpha_i = 0$$

following from the effective action:

$$\int d\tau \left\{\frac{1}{2e}\left[-\frac{6}{\kappa^2}\dot a_E^2 + a_E^2(\dot\sigma^2 + \dot\alpha_1^2 + \dot\alpha_2^2)\right] - e\rho_r\right\}$$

canonical momenta for s,a_1,a_2 conserved

“lift” to a Weyl-invariant theory

$$\int d^4x\sqrt{-g}\left[\frac{1}{2}\left((\partial\phi)^2-(\partial s)^2\right)+\frac{1}{12}(\phi^2-s^2)R\right]$$

i.e. two conformally coupled scalars w/opp sign L

- scalar ghost removed by gauge symmetry:

  $g_{mn} \to W^2 g_{mn}$, $f \to W^{-1} f$, $s \to W^{-1} s$
- gravitational trace anomaly cancels
- global O(1,1) symmetry: $f'^2 - s'^2 = f^2 - s^2$

  cf classical shift symmetry in string theory at tree level in g_s, but to all orders in a′

Gauges:

1.Einstein gauge $f^2 - s^2 = 6k^{-2}$:

$$\phi_E = \pm(\sqrt{6}/\kappa)\cosh(\kappa\sigma/\sqrt{6})$$
$$s_E = (\sqrt{6}/\kappa)\sinh(\kappa\sigma/\sqrt{6})$$

2. "Supergravity-like" gauge $f = f_0 = \text{const}$:

$$\int d^4x\sqrt{-g}\left[-\frac{1}{2}(\partial s)^2 + \frac{1}{12}(\phi_0^2 - s^2)R\right]$$

cf N=1 SUGRA models

3. "g–gauge": Detg = -1:

$$\int d\tau\left[-\frac{1}{2}\dot{\phi}_\gamma^2 + \frac{1}{2}\dot{s}_\gamma^2 + \frac{\kappa^2}{12}(\phi_\gamma^2 - s_\gamma^2)(\dot{\alpha}_1^2 + \dot{\alpha}_2^2)\right]$$

4. "String frame" gauge

$$\phi - s = (\phi + s)^2; \quad \phi + s = e^{-2\varphi/3}$$

$$g_s = e^{\varphi}$$

$$d = 10 : \mathcal{S} = \int \sqrt{-g} e^{-2\varphi} \left(R + 4(\partial\varphi)^2 \right)$$

Special quantity: Weyl and O(1,1)-invariant:

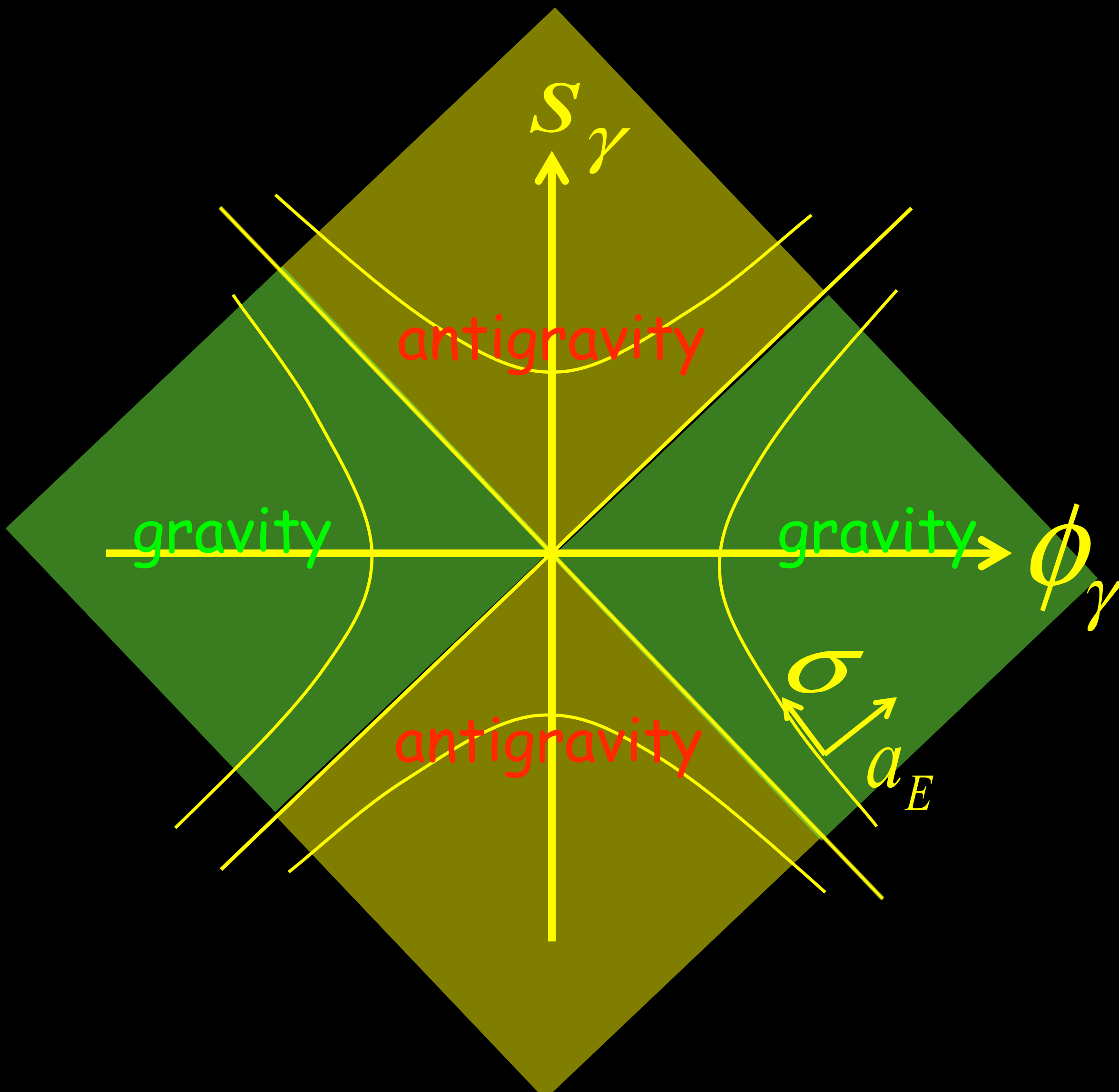
$$\chi \equiv \frac{\kappa^2}{6}(-g)^{\frac{1}{4}}(\phi^2 - s^2)$$ ($a_E^2 = |\chi|$)

- obeys Friedmann-like equation:

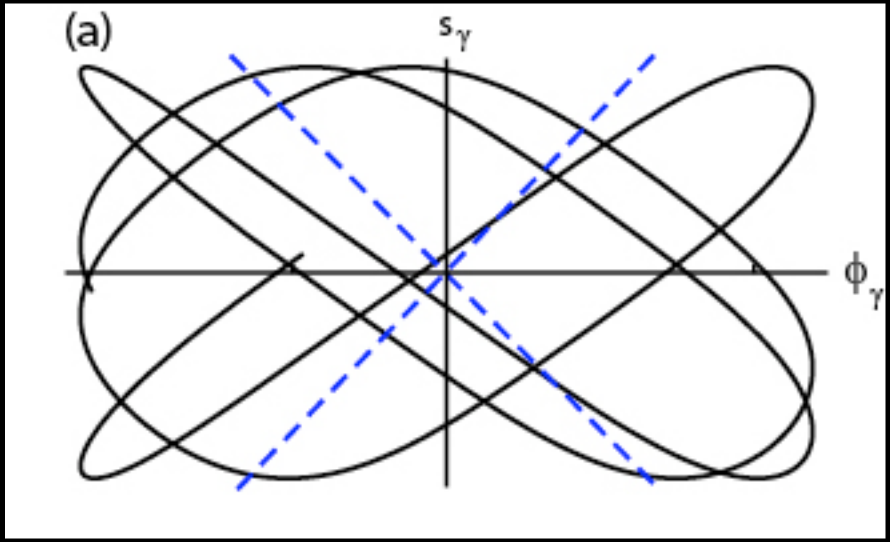
$$\dot{\chi}^2 = \frac{2\kappa^2}{3}\left(p^2 + 2\rho_r \chi\right)$$ $$p \equiv \sqrt{p_\sigma^2 + p_1^2 + p_2^2}$$

- analytic at generic cosmic singularities

Weyl- extended superspace

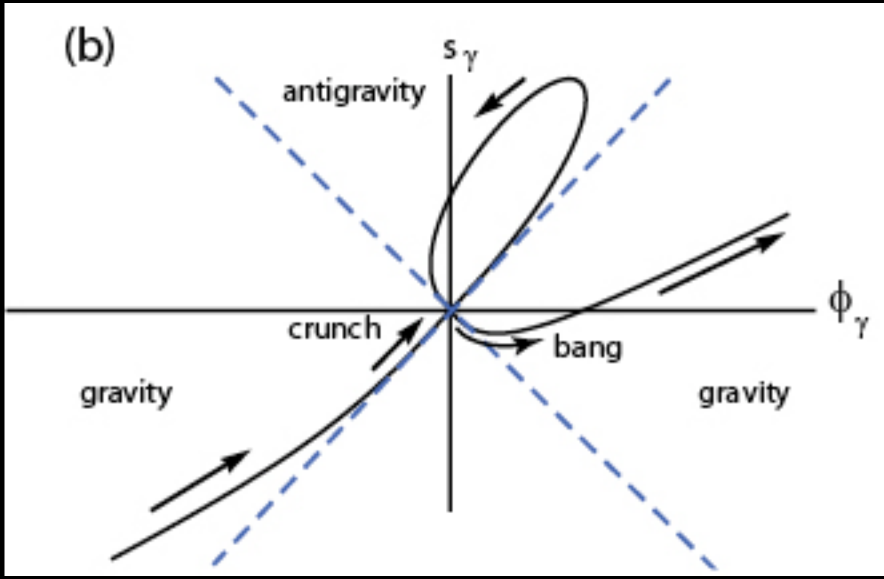


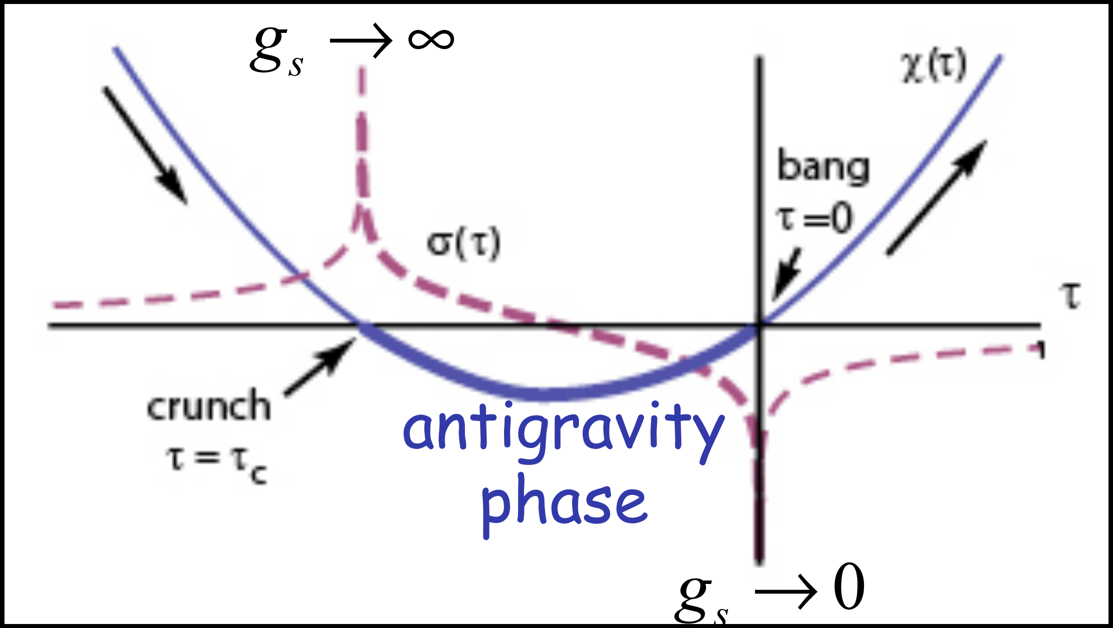
Isotropic case:
$a_1=a_2=0$



Generic case w/anisotropy:

Weyl symm restored at gravity/antigravity transition





$$\chi(\tau) = 2\overline{\tau}(p + \rho_r \overline{\tau})$$

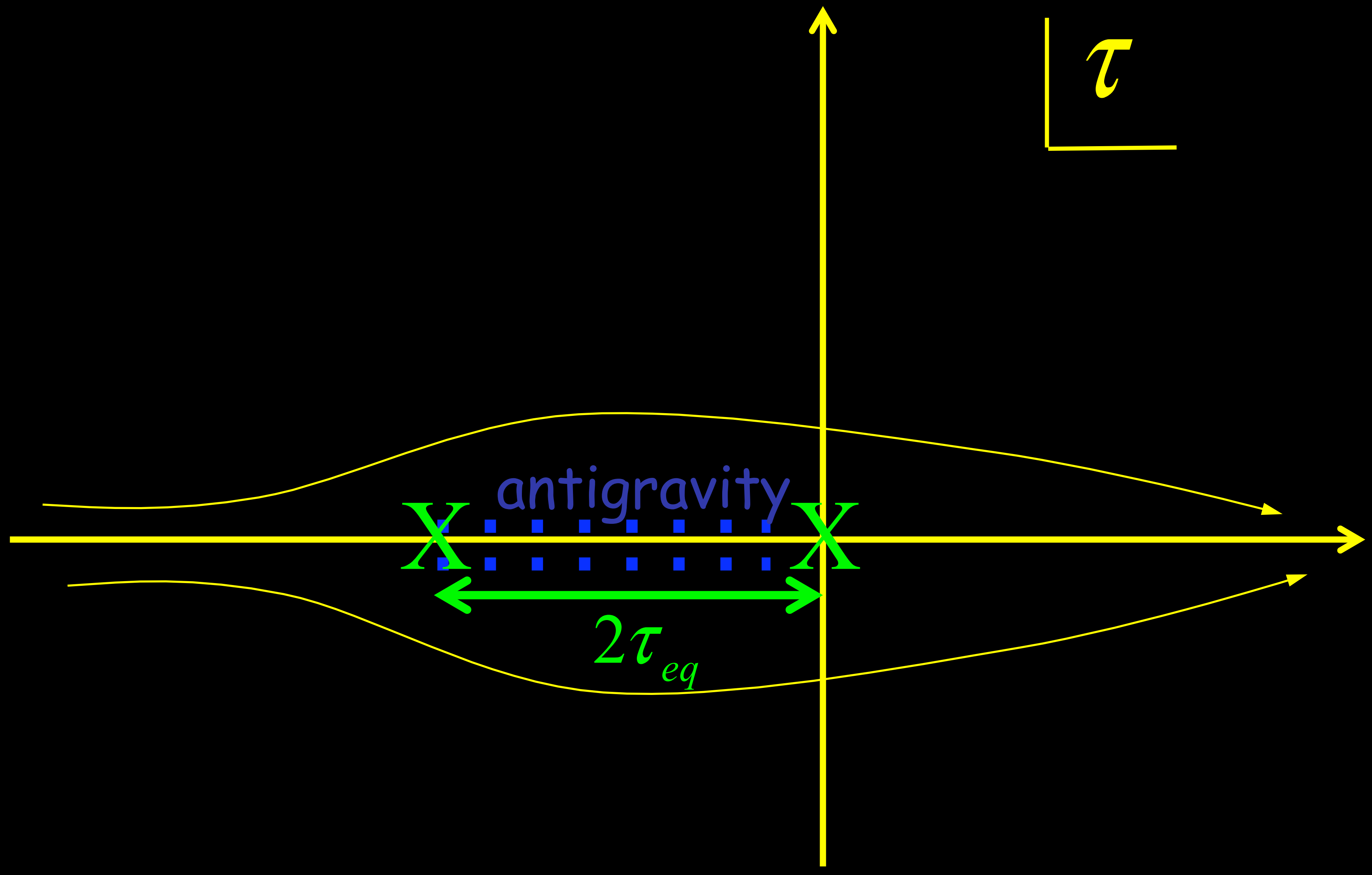
$$\overline{\tau} = \kappa\tau/\sqrt{6}$$

$$\frac{\kappa}{\sqrt{6}}\sigma(\tau) = \frac{p_\sigma}{2p}\ln\left|\frac{\overline{\tau}}{T(p+\rho_r\overline{\tau})}\right|$$

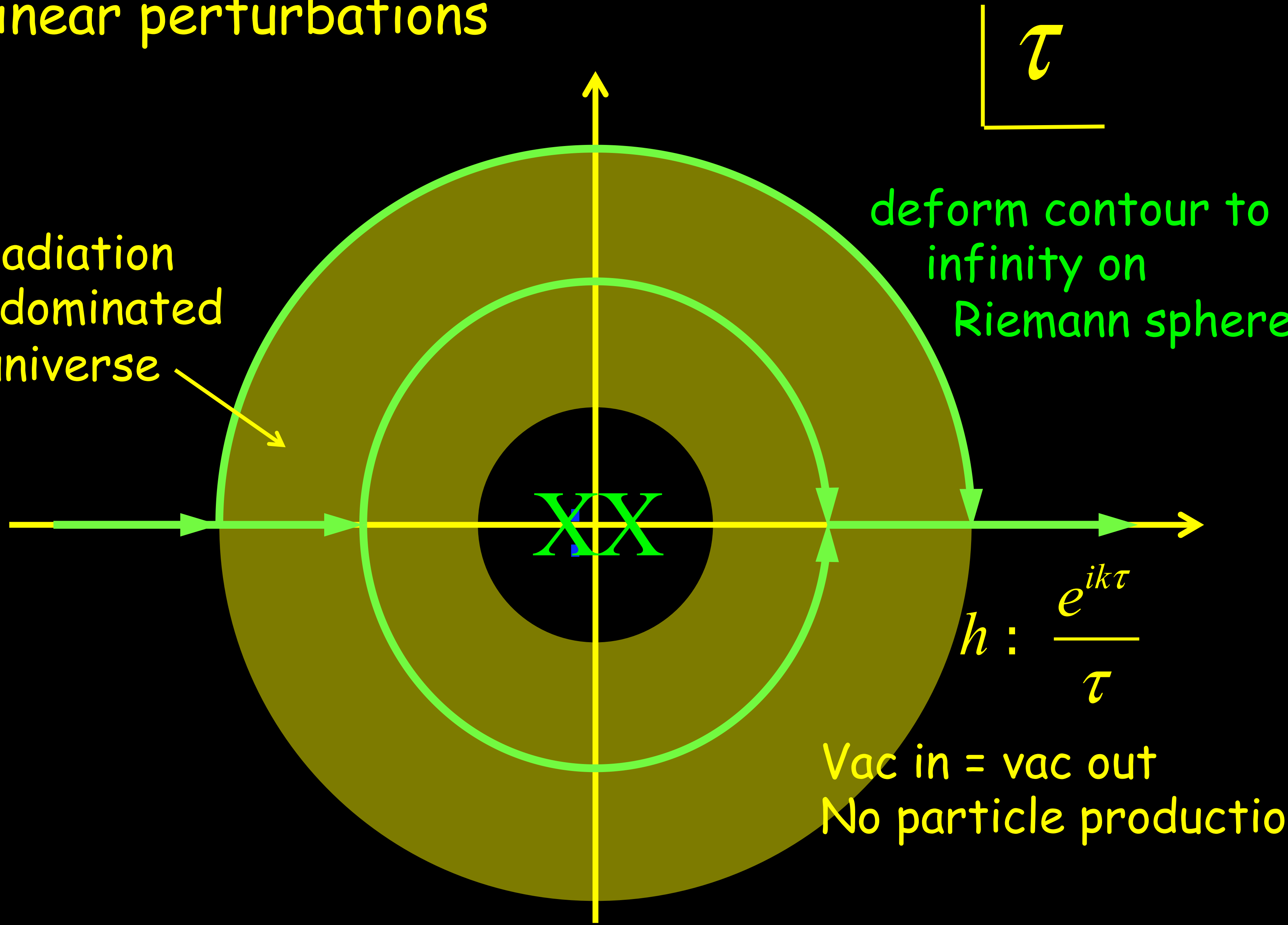
, sim^y $a_{1,2}$

Uniqueness of solution

unique extension of s, $a_{1,2}$ to complex t-plane

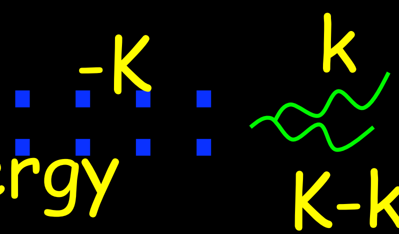


Also, as approach singularities, action has asymptotic special conformal O(4,2) symmetry: matching its (conserved) generators across singularities yields this as the unique solution linear perturbations



Vac in = vac out
No particle production

Antigravity region -> unstable?

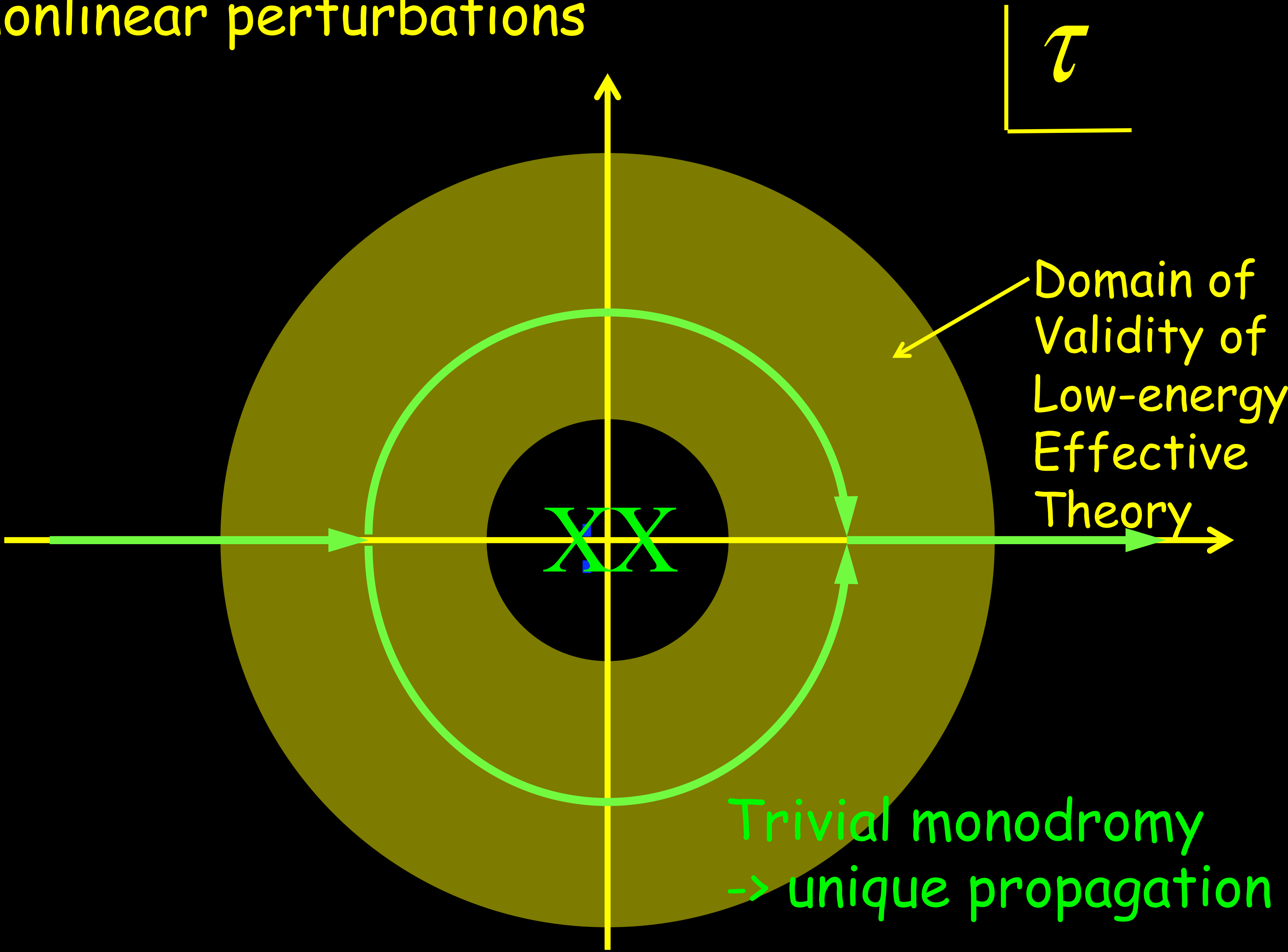


(only consider k-modes for which low-energy effective action is valid)

No: asymptotic states are positive energy,

$$\text{amplitude}: \int_{-\infty}^{\infty} \frac{d\tau}{\tau}\, e^{i(K+k+|K-k|)\tau} = 0 \quad \text{(Jordan)}$$

nonlinear perturbations



If: 1) low-energy effective theory valid along solution contour

2) matter consists of radiation, $T_{\lambda}^{\lambda} = 0$ which can be treated as perfect fluid

Double series in pert amplitude, τ^{-1}

-> trivial monodromy around singularity!

-> propagation of long wavelength modes across singularity is actually independent of microphysics at the singularity!

Conclusions

* Unique continuation of classical 4d GR-scalar effective theory 'around' cosmic singularities

* Surprisingly, it involves a brief antigravity phase

* Particle production in modes for which effective theory is valid is under control
  (massive fields -> finite particle production)

* Can be used to study cosmological ensembles defined by S_∞ in the past, to compare inflation, ekpyrotic/cyclic theories (in progress)

Happy 70

Stephen !