N=8 Supergravity

Renata Kallosh

Based on RK, 1103.4115, 1104.5480,
Carrasco, RK, Roiban, 1108.4390,
Chemissany, RK, Ortin, 1112.0332,
Broedel, Carrasco, Ferrara, RK, Roiban, work in progress,
RK, Ortin, work in progress.

S. W. Hawking

“Is the End in Sight for Theoretical Physics: An Inaugural Lecture” 1980

- “At the moment the N=8 supergravity theory is the only candidate in sight. There are likely to be a number of crucial calculations within the next few years which have the possibility of showing that the theory is no good.”

  1981: 3-loop counterterms are found, with unknown coefficient

- “ If the theory survives these tests, it will probably be some years more before we develop computational methods that will enable us to make predictions.”

  2007: Calculations show that the coefficient in front of the 3-loop counterterm vanishes!!!

- “These will be the outstanding problems for theoretical physicists in the next twenty years or so.”

  Is N= 8 supergravity all-loop finite? If so, what does it mean?

Old Wisdom, and why did we quit in 1981

Using the existence of the covariant on-shell superspace Brink, Howe, 1979 and the background field method in QFT one can use the tensor calculus and construct the invariant candidate counterterms RK; Howe, Lindstrom, 1981.

Such geometric counterterms have all known symmetries of the theory, including $E_{7(7)}$. They start at the 8-loop level.
Linearized ones start at the 3-loop level, RK; Howe, Stelle, Townsend, 1981

Clarification of the 1/8 BPS 7-loop candidate,
Bossard, Howe, Stelle, Vanhove, 2011.

For example one can use a superspace torsion

$$S^8 \sim \kappa^{14} \int d^4x \, d^{32}\theta \, \mathrm{BerE} \, \mathrm{T}_{\mathrm{ijk}\alpha}(\mathrm{x},\theta)\overline{\mathrm{T}}^{\mathrm{ijk}\dot\alpha}(\mathrm{x},\theta)\mathrm{T}_{\mathrm{mnl}}{}^{\alpha}(\mathrm{x},\theta)\overline{\mathrm{T}}^{\mathrm{mnl}}{}_{\dot\alpha}(\mathrm{x},\theta) \ .$$

Tensor calculus = infinite proliferation of candidate counterterms

New era, new people, new computers, new rules: never use N=8 supergravity rules, build it from N=4 Super-Yang-Mills

Explicit calculations: no UV divergences at 3- and 4 loops

Bern, Carrasco, Dixon, Johansson, Kosower, Roiban, 2007-2009

Five-loop progress is continuing but no new results yet (as of January 3).

Explanation?

- Light-cone superspace counterterms are not available at any loop order (prediction of UV finiteness). RK 2008, 2009. This is still the case in 2012:

- 3-loop finiteness follows from $E_{7(7)}$

  Broedel, Dixon
  Beisert, Elvang, Friedman, Kiermaier, Morales, Stieberger
  Bossard, Howe, Stelle, 2009-2010

- String theory -> field theory, M. Greene's talk

The $E_{7(7)}$ symmetry of N=8 supergravity was discovered by Cremmer and Julia in 1979

Cremmer, Julia 1979 ; Cremmer, Julia, Sherk, 1978

De Wit, Nicolai 1982; de Wit, Freedman, 1977

If we trust continuous global $E_{7(7)}$ at the 3-loop quantum level what is the prediction at higher loops?

$E_{7(7)}$ revisited: RK, 2011

Noether-Gaillard-Zumino current conservation is inconsistent with the $E_{7(7)}$ invariance of the candidate counterterms

$E_{7(7)}$ revisited: Bossard-Nicolai, 2011

Yes, NGZ current conservation is inconsistent with the $E_{7(7)}$ invariance of the candidate counterterms. However, there is a procedure of deformation of the linear twisted self-duality constraint, which should be able to fix the problem. Examples of U(1) duality.

$E_{7(7)}$ revisited: Carrasco, RK, Roiban, 2011

Bossard-Nicolai deformation procedure needs a significant modification to explain the simplest case of Born-Infeld deformation of the Maxwell theory which conserves the NGZ current

A duality doublet depends on fields in action,

F=dA and on $G = 2\frac{\delta\mathcal{L}}{\delta F}$

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} \qquad \delta\mathcal{L} = \frac{1}{4}\left(\tilde{F}\, C\, F + \tilde{G}\, B\, G\right)$$

Noether-Gaillard-Zumino current conservation requires that the action transforms in a particular way, INSTEAD OF BEING INVARIANT

$$\partial^\mu \tilde{F}_{\mu\nu} = 0$$
$$\partial^\mu \tilde{G}_{\mu\nu} = 0$$

Duality symmetry rotates vector field equations and Bianchi identities

U(1) case is simple $\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}$ $\qquad F\tilde{F} + G\tilde{G} = 0$

Canonical example: Maxwell vs. Dirac-Born-Infeld

$$\mathcal{L}_{\rm Max} = -\frac{1}{4}F^2 \;\rightarrow\; G = \tilde{F},\; \tilde{G} = -F \;\rightarrow\; \tilde{F}F + \tilde{G}G = 0$$

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F^2 + \frac{1}{32}g^2((F^2)^2 + (\tilde{F}F)^2) \;\rightarrow\; G_{\rm eff} = F\left(1 - \tfrac{1}{4}g^2F^2\right) + \tfrac{1}{4}g^2F(\tilde{F}F)$$

$$F\tilde{F} + \tilde{G}_{\rm eff}G_{\rm eff} = \mathcal{O}(g^4) \neq 0$$

$$\mathcal{L}_{\rm BI} = \frac{1}{g^2}(1 - \sqrt{1 + 2g^2(F^2/4) - g^4(F\tilde{F}/4)^2}) = -\frac{1}{4}F^2 + \frac{1}{32}g^2\left((F^2)^2 + (F\tilde{F})^2\right) + \cdots$$

$$G^{BI}_{\rm eff} = \frac{\tilde{F} + \frac{1}{4}g^4(\tilde{F}F)F}{\sqrt{1 + 2g^2(F^2/4) - g^4(\tilde{F}F/4)}}$$

$$F\tilde{F} + \tilde{G}^{BI}_{\rm eff}G^{BI}_{\rm eff} = 0$$

To preserve the U(1) NGZ current conservation one needs all powers of F, once F^4 was added, one needs F^n!

X

Given some quantum generated duality-invariant term in the effective action of some duality-preserving theory, is it possible to add higher-order terms to restore the invariance of the field equations?

Carrasco, Kallosh, RR

focus on nonlinear E&M-type duality

we presented an algorithmic construction of the BI action by requiring duality

$$T = F - iG \qquad T^* = F + iG \qquad T^{\pm} = \tfrac{1}{2}(T \pm i\tilde{T})$$

$$T^+ = 0 \;\Leftrightarrow\; \tilde{G} = -F$$ -- Twisted linear self-duality

Structure of the deformation; nonlinear twisted self-duality

$$T^+ = \frac{\delta \mathcal{I}(T^-, T^{*+}, g)}{\delta T^{*+}}$$

$$\mathcal{I}(T, T^*)$$

Must be manifestly duality invariant, according to Bossard, Nicolai

But what is it? For example for BI?

The construction

Carrasco, RK, Roiban

Start with $\mathcal{I}$ and construct nonlinear twisted self-duality equation

$$T^{+}_{\mu\nu} = \frac{g^2}{16} T^{*}{}^{+}_{\mu\nu} (T^{-})^2 \Big[1 + \sum_{n=0} d_n \Big(\tfrac{1}{4} g^4 (T^{*+})^2 (T^{-})^2 \Big)^n \Big]$$

Make an ansatz for the action

$$\mathcal{L} = \Big(g^{-2} \sum_{m=0,p=0} g^{2(p+2m)} c_{(p,2m)} t^p z^{2m} \Big) - c_{(0,0)} g^{-2} \qquad z = F\tilde{F}/4 \qquad t = F^2/4$$

Easy to recover any desired/known action exhibiting duality

There is an infinite supply

Gibbons, Rasheed, 1995; Gaillard, Zumino,1997, …

Courant-Hilbert diff eq

For the Born-Infeld action one needs:

$$\mathcal{I}(T^-, T^{*+}, g) = \frac{6}{g^2}\left(1 - {}_3F_2\left(-\tfrac{1}{2}, -\tfrac{1}{4}, \tfrac{1}{4}; \tfrac{1}{3}, \tfrac{2}{3}; -\tfrac{1}{27}\, g^4\, (\overline{T}^+)^2\, (T^-)^2\right)\right)$$

$$T^+_{\mu\nu} = \tfrac{1}{16}\, g^2\, T^{*+}_{\mu\nu}\, (T^-)^2\ {}_3\mathrm{F}_2\left(\tfrac{1}{2}, \tfrac{3}{4}, \tfrac{5}{4}; \tfrac{4}{3}, \tfrac{5}{3}; -\tfrac{1}{27}\, g^4\, (T^{*+})^2 (T^-)^2\right)$$

$$\mathcal{L}_{\mathrm{BI}} = \frac{1}{g^2}\left(1 - \sqrt{1 + 2g^2(F^2/4) - g^4(F\tilde{F}/4)^2}\right) = -\frac{1}{4}F^2 + \frac{1}{32}g^2\left((F^2)^2 + (F\tilde{F})^2\right) + \cdots$$

Remarkably complicated source of deformation $\mathcal{I}$, is there a reason for it?

Here we knew what we wanted to obtain and using that information we constructed $\mathcal{I}$ to all orders

But if we do not know the action, how do we find $\mathcal{I}$?

New U(1) duality invariant models, unknown before

Chemissany, RK, Ortin, ``Born-Infeld with Higher Derivatives,'' 1112.0332

The first quartic correction to Maxwell was known from open string theory and D3-brane quantum corrections: $(dF)^4$
Andreev, Tseytlin 1988,
Shmakova; De Giovanni, Santambrogio, Zanon , 1999; Green, Gutperle, 2000

$$(\alpha')^4(s^2+t^2+u^2)t^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}F_{\mu_1\nu_1}(p_1)F_{\mu_2\nu_2}(p_2)F_{\mu_3\nu_3}(p_3)F_{\mu_4\nu_4}(p_4)$$

$$t^{(8)} \equiv t^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}$$

Green-Schwarz
8-tensor, 1982

Chemissany, de Jong , de Roo, 2006: U(1) NGZ current is conserved at $(dF)^4$ level, but broken at higher levels. How to add $d^{4n} F^{2n+2}$ to restore the NGZ current conservation?

Using Carrasco, RK, Roiban algorithm we found a recursive answer

Born-Infeld model with higher derivatives and NGZ current conservation

$$\mathcal{I}_A^{(1)}(T) \;\equiv\; \frac{\lambda}{2^3} t^{(8)}{}_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \partial_\alpha T^{*+\,\mu_1\nu_1} \partial^\alpha T^{-\,\mu_2\nu_2} \partial_\beta T^{*+\,\mu_3\nu_3} \partial^\beta T^{-\,\mu_4\nu_4}$$

Recursion relation generates n-order from the previous ones: $[\mu\nu] \Rightarrow a$

$$\sum_{n=1}^{\infty} \lambda^n T_a^{(n)+} = t^{(8)}{}_{abcd} \sum_{p,q,r=0} \lambda^{p+q+r+1} \partial_\alpha (\partial^\alpha T^{(p)-b} \partial_\beta T^{(q)*+c} \partial^\beta T^{(r)-d})$$

Algorithm for an action at any order in $d^{4n}\, F^{2n+2}$ For example

$$\frac{1}{2} \int d^4x\, t^{(8)}{}_{abcd} \partial_\alpha F^{+\,a} \partial^\alpha F^{-\,b} \partial_\beta F^{+\,c} \partial^\beta F^{-\,d} \qquad d^4\, F^4$$

$$-\frac{1}{2} \int d^4x \left\{ t^{(8)}{}_{abcd} t^{(8)}{}_{defg} \partial_\alpha \left(\partial^\alpha F^{-\,b} \partial_\beta F^{+\,c} \partial^\beta F^{-\,d}\right) \partial_\gamma \left(\partial^\gamma F^{-\,e} \partial_\delta F^{+\,f} \partial^\delta F^{-\,g}\right) + \text{c.c.} \right\}$$

$d^8\, F^6$

etc

New form of the reconstructive identity for U(1) duality

$$S(F) = \frac{1}{4\lambda} \int d^4x \, d\lambda \, F\tilde{G}(\lambda)$$

If we know $\lambda \ \mathcal{I}(T, T^*)$

the manifestly invariant source of deformation,

we know $\tilde{G}(\lambda)$

and we know the action, which preserves the NGZ current conservation in all orders in λ

Case of open string (dF)^4 corrections

This corresponds to the following 4-point amplitude

$$S^{(1)} = \frac{\lambda}{2^4} \int d^4x \, t^{(8)}_{abcd} \partial_\mu F^a \partial^\mu F^b \partial^\nu F^c \partial_\nu F^d$$

Once we have identified a manifestly U(1) invariant source of deformation,

$$\mathcal{I}_{\rm string} = \lambda \, t^{(8)}_{abcd} [\partial_\mu T^{*+\,a} \partial^\mu T^{-\,b} \partial_\nu T^{*+\,c} \partial^\nu T^{-\,d} + \frac{1}{2} \partial_\mu T^{*+\,a} \partial^\mu T^{*+\,b} \partial_\nu T^{-\,c} \partial^\nu T^{-\,d}]$$

the complete action with all higher derivative terms follows from the algorithm

U(1) duality and N=2 supersymmetry

Broedel, Carrasco, Ferrara, RK, Roiban

Combinations of superfields	Chirality	Charge
$T^+ = \mathcal{W} - i\mathcal{M}$	+	+
$T^{*+} = \mathcal{W} + i\mathcal{M}$	+	-
$T^- = \overline{\mathcal{W}} - i\overline{\mathcal{M}}$	-	+
$T^{*-} = \overline{\mathcal{W}} + i\overline{\mathcal{M}}$	-	-

Superfield action reconstructive identity

$$S = \frac{i}{8\lambda}\int d\lambda \Big[\int \mathrm{d}^8\mathcal{Z}\, \mathcal{W}\mathcal{M} - \int \mathrm{d}^8\overline{\mathcal{Z}}\, \overline{\mathcal{W}\mathcal{M}}\Big]$$

New class of models! NGZ current conservation: all powers of W's in the action.
We recovered all models discovered before and an infinite class of new ones.

We learned how to get higher order in F terms to restore the NGZ current conservation for U(1) duality

- Born-Infeld type models, known before but now we follow a new procedure
- N=1 supersymmetric BI type models, known before but now we follow a new procedure which is supposed to work for the Born-Infeld N=8 supergravity

- Born-Infeld type models with higher derivatives, NEW!
- Generic N=2 supersymmetric BI type models, Broedel, Carrasco, Ferrara, RK, Roiban, NEW!

Lesson for N=8 supergravity: to be able to construct all higher order in F terms with derivatives, once we add the 4-vector counterterms, we need to produce N=8 Born-Infeld type supergravity: tall order!

Can we say more today?

New class of $E_{7(7)}$ invariants and UV properties of N=8

Work in progress with Tomas Ortin

- We construct new manifest $E_{7(7)}$ invariants using the octonionic nature of the $E_{7(7)}$ and the structure of the BI U(1) duality invariant model with higher derivatives.

- Our new $E_{7(7)}$ invariants, based on a Jordan triple system, generalize the Cartan-Cremmer-Julia quartic invariant. They may be useful for amplitudes instead of the black hole horizon area. They do not depend on scalars.

- We show that the corresponding $E_{7(7)}$ invariants, required for the Bossard-Nicolai deformation procedure, are inconsistent with the UV counterterms.

- The class of $E_{7(7)}$ invariants constructed from the SU(8) covariant tensors may, or may not provide the source of deformation, depending on vectors as well as scalars. They may, or may not lead to Born-Infeld N=8 supergravity. This remains to be seen!

Manifest $E_{7(7)}$ and Local SU(8)?

- In the candidate counterterms we used the building blocks which are local SU(8) tensors, "blind" under $E_{7(7)}$, scalars in $\frac{E_{7(7)}}{SU(8)}$

- A possibility to have local SU(8) symmetry simultaneously with the manifestly $E_{7(7)}$ symmetric abelian U(1) gauge symmetry relies heavily on the linear twisted self-duality for graviphotons

$$T^+_{AB} = 0$$

- To construct $\mathcal{I}(\mathcal{T}, \bar{\mathcal{T}})$ $\qquad T^+_{AB} \neq 0$

- with local SU(8) graviphotons, manifest $E_{7(7)}$ covariant **56** U(1)'s is the challenge!

The manifest $E_{7(7)}$ invariants are rare!

The fundamental **56** of $E_{7(7)}$ is spanned by the two antisymmetric real tensors (F, G)

$$\delta F^{ij} \;=\; \Lambda^i{}_k F^{kj} - \Lambda^j{}_k F^{ki} + \Sigma^{ijkl} G_{kl}$$

$$\delta G_{ij} \;=\; \Lambda^k{}_i G_{jk} - \Lambda^k{}_j G_{ik} + \Sigma_{ijkl} F^{kl}$$

$$\Sigma_{ijkl} \;=\; \frac{1}{24} \epsilon_{ijklmnpq} \Sigma^{mnpq}$$

Cartan Quartic $E_{7(7)}$ Invariant

$$
\begin{aligned}
J &= F^{ij}G_{jk}F^{kl}G_{li} - \tfrac{1}{4}F^{ij}G_{ij}F^{kl}G_{kl} \\
&+ \tfrac{1}{96}\,\epsilon^{ijklmnpq}G_{ij}G_{kl}G_{mn}G_{pq} + \tfrac{1}{96}\,\epsilon_{ijklmnpq}F^{ij}F^{kl}F^{mn}F^{pq}
\end{aligned}
$$

Cremmer-Julia Quartic $E_{7(7)}$ Invariant

$$
\diamondsuit(X,\bar{X}) = Tr(X\bar{X})^2 - \frac{1}{4}(TrX\bar{X})^2 + 8\mathrm{Re}Pf\,X
$$

$$
X^{AB} = (X_{AB})^* = \frac{1}{4\sqrt{2}}(F^{ij} - \mathrm{i}\,G_{ij})\Gamma^{ij}_{AB}
$$

$$
\diamondsuit = -J
$$

Groups of type E7, 1969

R. Brown, Journal fur die reine und angewandte Mathematik, Vol 236 , 79

- Groups of type E7 are groups of linear transformations leaving invariant two multilinear forms: one is skew-symmetric bilinear

$$\{x, y\}$$

- and the other is a symmetric four-linear form

$$q(x, y, z, w) = \{T(x, y, z), w\}$$

- Here the $T(x, y, z)$ is the triple product defined by the cubic Jordan algebra
- An explicit expression in Gunaydin, Koepsell, Nicolai, 2000
- Review in Duff et al, 0809.4685

$$J \sim q(x, x, x, x)$$

was used for the black holes and quantum information

New $E_{7(7)}$ invariants

RK, Ortin

$$J_{(8)} \equiv J_{[\mu_1\nu_1][\mu_2\nu_2][\mu_3\nu_3][\mu_4\nu_4]} = q(x_{\mu_1\nu_1}, y_{\mu_2\nu_2}, z_{\mu_3\nu_3}, w_{\mu_4\nu_4})$$

$$t^{(8)[\mu_1\nu_1][\mu_2\nu_2][\mu_3\nu_3][\mu_4\nu_4]} J_{[\mu_1\nu_1][\mu_2\nu_2][\mu_3\nu_3][\mu_4\nu_4]} \equiv t^{(8)} \cdot J_{(8)}$$

Relevant for the amplitudes!

$$f(s,t,u)\, t^{(8)} \cdot J_{(8)}$$

Instead of electric and magnetic charges we have helicity amplitudes

$$(p,q) \to (F_{\mu\nu}, G_{\mu\nu})$$

When $E_{7(7)}$ is reduced to U(1), our new invariants describe the open string corrections

$$(\alpha')^4(s^2+t^2+u^2)t^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}F_{\mu_1\nu_1}(p_1)F_{\mu_2\nu_2}(p_2)F_{\mu_3\nu_3}(p_3)F_{\mu_4\nu_4}(p_4)$$

N=4 YM → N=8

Twistors → Octonions

BI $\mathcal{N}$=8???

- The classical $\mathcal{N}$=8 theory has one gravitational coupling and it is strictly quadratic in vector fields F
$$S_{N=8}(\kappa^2) = \frac{1}{2\kappa^2}(R - F\mathcal{N}(\phi)F + ...)$$
- The first L-loop UV divergence has necessarily a quartic term of the form $F^4 f(s,t,u)$. If we add such term to the action, it will break the NGZ current conservation.
- To restore the $E_{7(7)}$ current conservation we have to add to the action not just a counterterm but all higher orders of F^n with derivatives. One can think of two possibilities here.
- We will find that there is no consistent N=8 Born-Infeld-type supergravity. In such case an unbroken $E_{7(7)}$ would predict UV finiteness.
- We will be able to construct N=8 Born-Infeld-type supergravity.

$$S^{BI}_{N=8}(\kappa^2, g^2) = \frac{1}{2\kappa^2}(R - F\mathcal{N}(\phi)F + ...) + g^2 F^4 f(s,t,u) + ...g^{2m} F^n f(s,t,u,...) + ...$$

In such case it is not even clear how the existence of such $\mathcal{N}$=8 BI supergravity affects the predictions for the UV properties of the original $\mathcal{N}$=8 supergravity theory. We will deal with this issue if we find BI $\mathcal{N}$=8

S. W. Hawking, an optimist!

On N=8 supergravity in 1980

"These will be the outstanding problems for theoretical physicists in the next twenty years or so."

Stephen