The State of the No-Boundary Wave Function

James Hartle, UCSB, SantaBarbara

based on work with

Stephen Hawking, DAMTP, Cambridge
Thomas Hertog, Universiteit Leuven

Hawking70, Cambridge, January 7, 2012

A Quantum Universe

If the universe is a quantum mechanical system it has a quantum state.
What is it?

$$\Psi$$

A theory of the quantum state is the objective of Quantum Cosmology.

No State --- No Predictions

- The probability p at time t of an alternative represented by a projection P(t) (e.g a range of position) in a state $|\Psi\rangle$ is:

$$p = ||P(t)|\Psi\rangle||^2$$

$$P(t) = e^{iHt/\hbar} P(0) e^{-iHt/\hbar}$$

- If we don't have the operator P and both H and the state $|\Psi\rangle$ there are no probabilities and no predictions.

Contemporary Final Theories Have Two Parts

$$H \qquad \Psi$$

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Which regularities of the universe come mostly from H and which from ψ ?

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Which regularities of the universe come mostly from H and which from ψ ?

An unfinished task of unification?

H

- classical dynamics
- laboratory experiment eg CERN.

Ψ

- classical spacetime
- early homo/iso +inflation
- fluctuations in ground state
- arrows of time
- CMB, large scale structure
- isolated systems
- topology of spacetime
- num. of large and small dims.
- num. of time dimensions
- coupling consts. eff. theories

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How successful is the NBWF in explaining these features?

What is the meaning of `Success'

Hawking and Hertog

- A final theory (H, ψ) predicts probabilities p(hist) for alternative histories of the universe --- alternative spacetimes, field configurations observers, etc. (Bottom-up probabilities)
- A theory (H,ψ) is successful when it predicts high probabilities for the history of the universe that we observe. These are conditioned on the observational situation p(hist|us). (Top-down probabilities)
- The most probable history from p(hist) may be very different from that from p(hist|us).

Rules of Thumb for TD Probabilities

$$p(hist|us) = (\mathrm{TDfactor}) \times p(hist)$$

- If the universes in the classical ensemble are small enough that we are rare in each:

$$(\mathrm{TD\ factor}) \propto (\mathrm{volume\ of\ reheating\ surface})$$

(volume weighting --- favors inflation) (Page, Hawking)

- If the universes in the classical ensemble are so big that we are common in each:

$$(\mathrm{TD\ factor}) = 1$$

Two Themes

- A Quantum Universe: Treat the universe and everything in it quantum mechanically.

  fluctuations and backgrounds
  observers as physical systems within the universe.

- Focus on probabilities for observations in our Hubble volume and coarse-grain over everything outside our past light cone. (Quantum mechanics makes that feasible.)

Limitations of This Talk

- Assume spacetime geometry is a quantum mechanical variable (no dual description as in Hertog talk)
- Assume usual quantum mechanics (the generalized decoherent histories quantum mechanics of spacetime geometry.) (no emergent complex structure Gibbons).
- Assume usual quantum mechanics. The state supplies the measure for prediction (no further postulated measure needed.)
- Concentrate on the predictions for classcal histories in the semiclassical approximation. (No resolution of Page-Susskind challenges.)
- No Supersymmetry (D'Eath)

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This is neither a review talk,
nor a critical talk!

Wave Functions of the Universe

The state is not an initial condition

$$\Psi$$

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The state is not an initial condition

It predicts probabilities for all possible alternative 4-d histories of the universe ---

what went on then

what goes on now

what will go on in the future.

$$\Psi$$

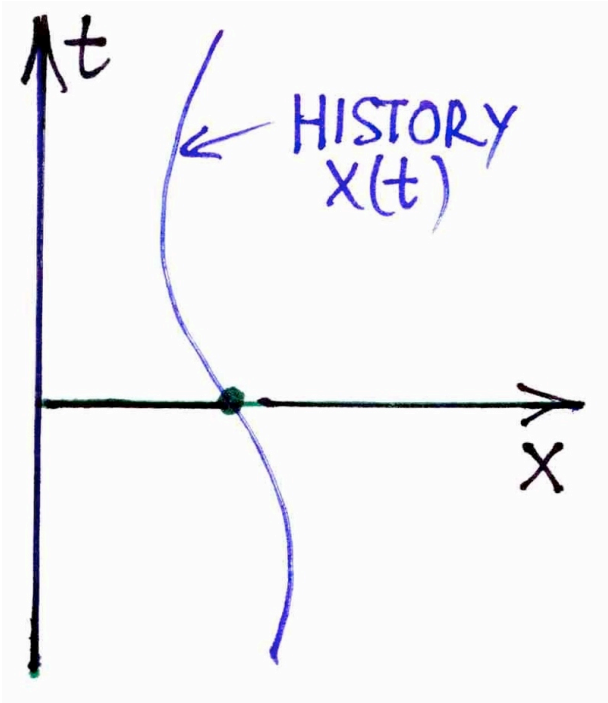
Wave Functions for the Universe

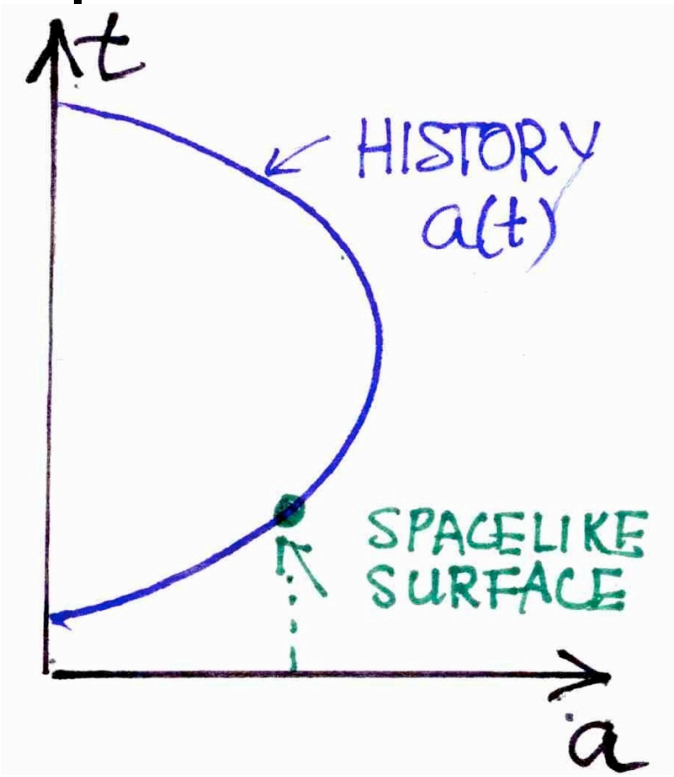
(minisuperspace models)

Geometry: Homogeneous, isotropic, closed.

$$ds^2 = (3/\Lambda)\left[N^2(\lambda)d\lambda^2 + a^2(\lambda)d\Omega_3^2\right]$$

Matter: cosmological constant plus scalar field





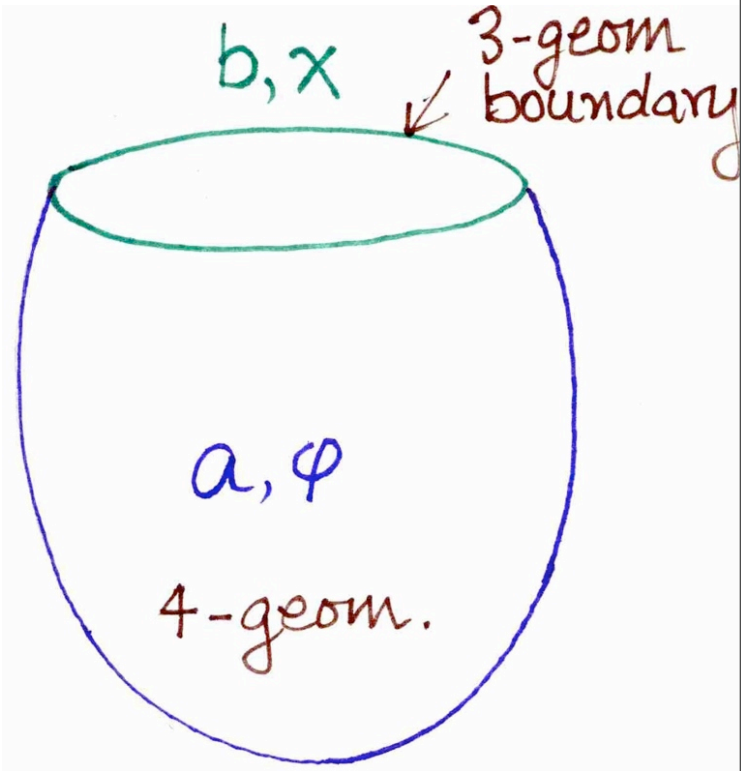
$$\psi = \psi(x, t)$$

$$\Psi = \Psi(b, \chi)$$

Hawking's No-Boundary Wave Function

Cosmological analog of ground state

No H to be a lowest eigenvalue of,
for a closed universe $H = 0$



$$\Psi(b,\chi) \equiv \int_{\mathcal{C}} \delta N \delta a \delta \phi \exp(-I[N(\lambda), a(\lambda), \phi(\lambda)]/\hbar).$$

The integral is over all $(a(\lambda), \phi(\lambda))$ which are regular on a disk and match the (b, χ) on its boundary.

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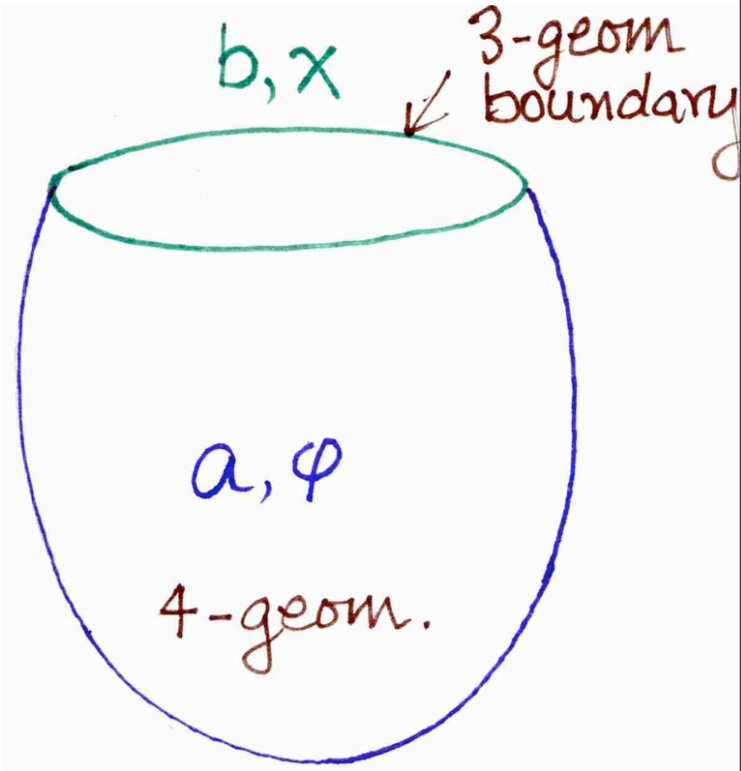

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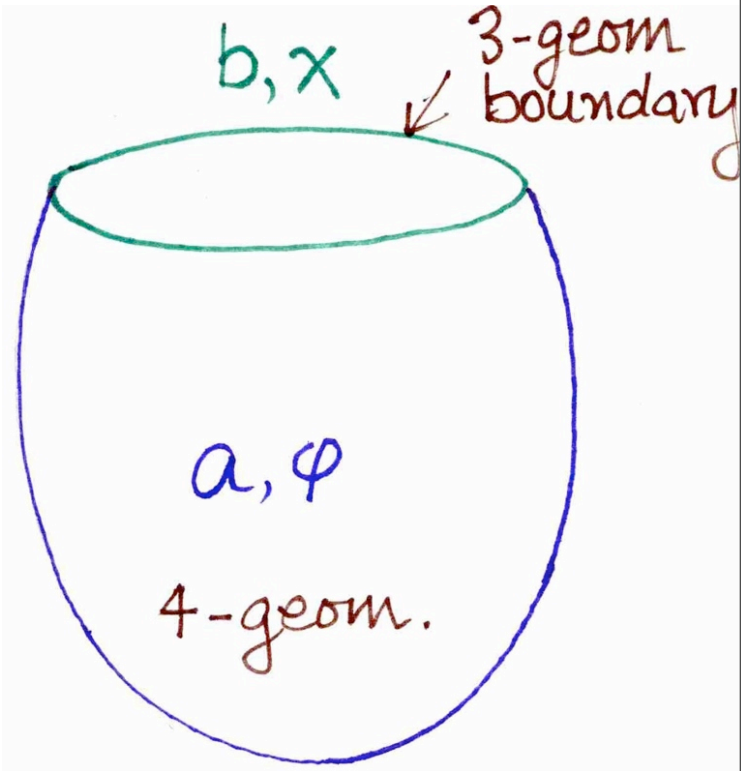

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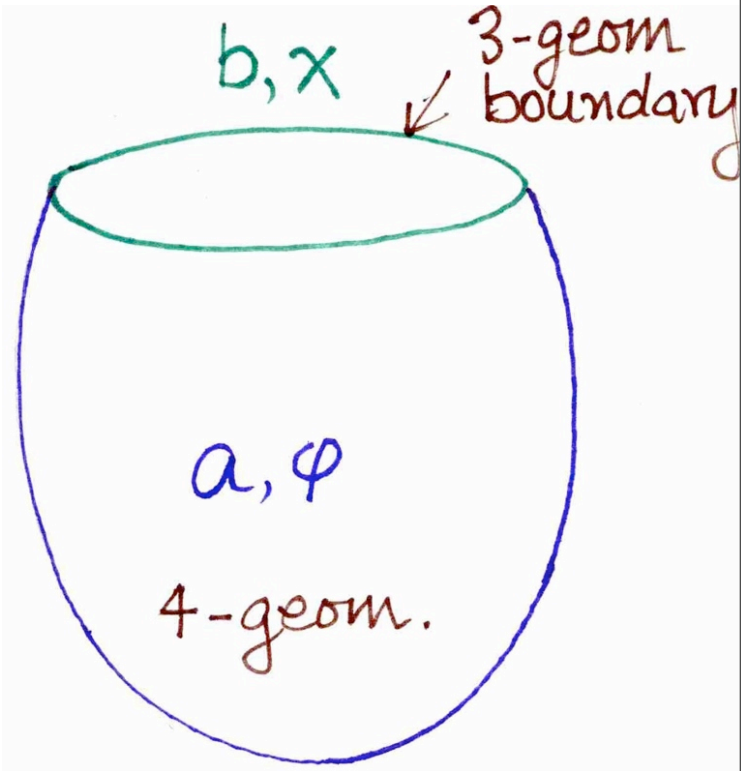

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A more precise definition of the NBWF may be possible using AdS/CFT dualities.
(Horowitz, Maldacena, Hertog talk.)

Departures from Symmetry Cost Action and Decrease Bottom-Up Probabilities

anisotropies,
inhomogeneities
bubbles
inflation
observers

Departures from Symmetry that Increase Volume Increase Top-Down Probabilities

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Probabilities for observation are a balance between these.

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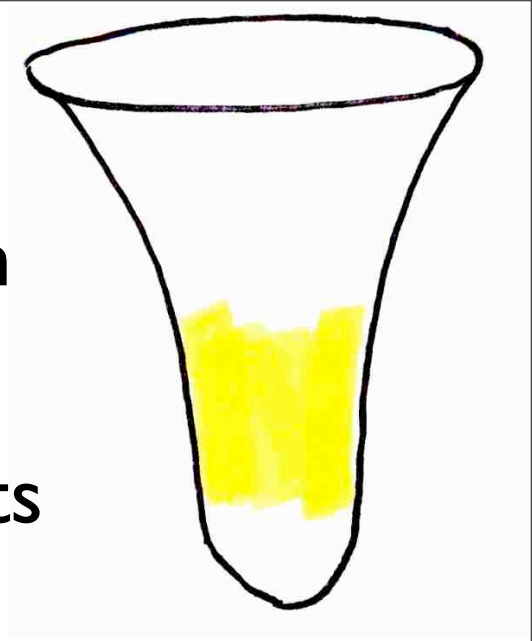
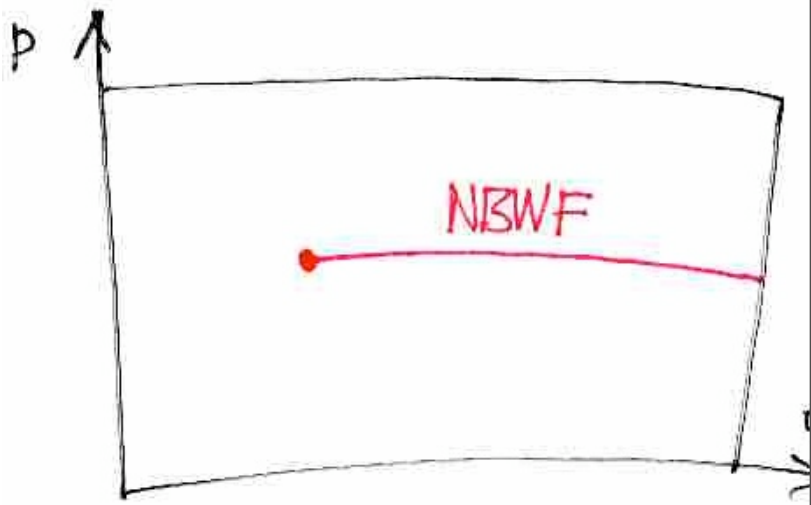
Classical Spacetime



The NBWF in the semiclassical approximation

$$\Psi(b,\chi) \approx \exp\{[-I_R(b,\chi) + iS(b,\chi)]/\hbar\}$$

where I_R and S are the real and imaginary parts of the action at a saddle point of the NBWF defining integral.



Predicted ensemble of classical histories (WKB):

$$p_A = \nabla_A S \quad \text{(integral curves of S)}$$

$$\text{prob(class hist)} \propto \exp(-2I_R/\hbar)$$

Provided! $|\nabla_A I_R| \ll |\nabla_A S|$ (classicality consdition)

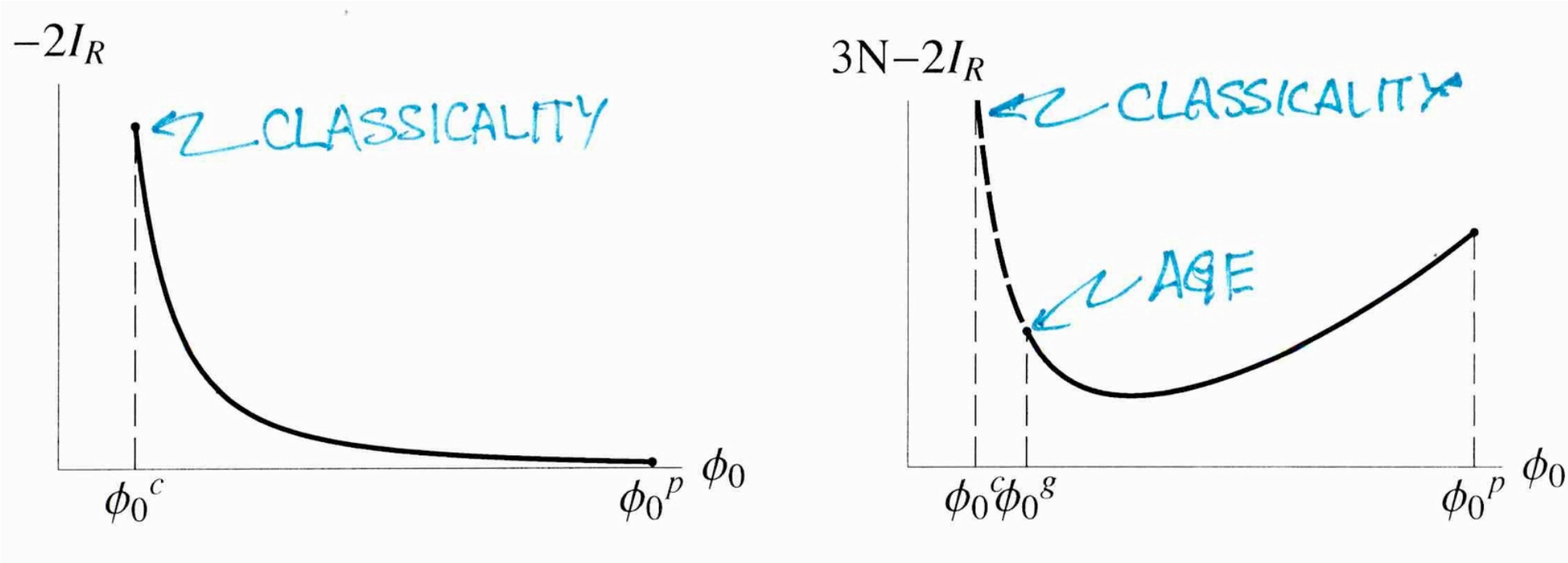
Not all classical spacetimes predicted

TD Weighting favors Inflation

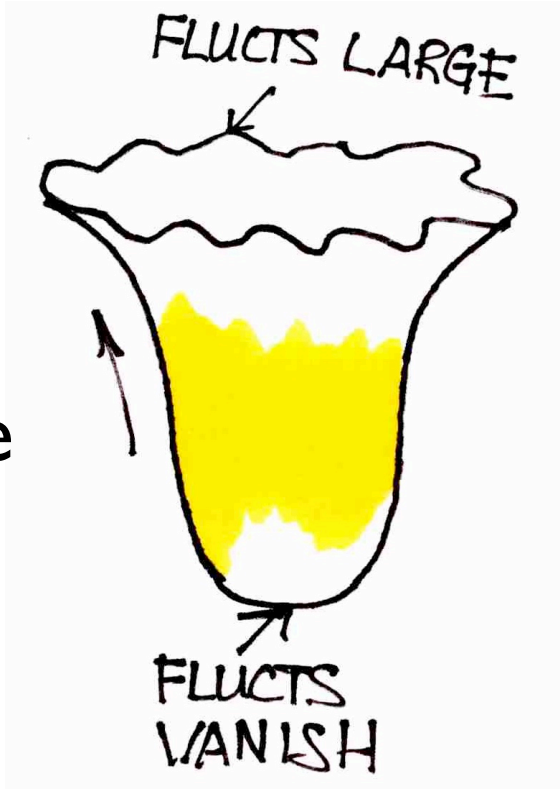
summary from Hertog talk

By itself, the NBWF + classicality favor low inflation, but we are are more likely to live in a universe that has undergone more inflation, because there are more places for us to be.

$$p(\phi_0|H_0,\rho) \propto \exp(3N)p(\phi_0) \propto \exp(3N - 2I_R)$$



Fluctuation Probabilities



- NBWF fluctuations start in their ground state (Halliwell & Hawking). Essentially the Bunch-Davies vacuum.

$$p(z_{(n)}|\phi_0) \approx \sqrt{\frac{\epsilon_* n^3}{2\pi H_*^2}} \exp\left[-\frac{\epsilon_*}{2H_*^2} n^3 z_{(n)}^2\right]$$

where ϵ_* and H_* are the slow roll and expansion parameters when the mode leaves the horizon.

- Fluctuations are large when

$$\frac{H_*^2}{\epsilon_*} \geq 1 \qquad \text{or} \qquad \frac{V^3}{V'^2} \geq 1$$

- That is eternal inflation.

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Arrows of Time

Our universe exhibits a number of arrows of time:

- Fluctuation Arrow ---- growth of fluctuations
- Thermodynamic arrow --- growth of entropy
- Radiation arrow --- retardation of E&M radiation
- Psychological arrow --- past, present, and future

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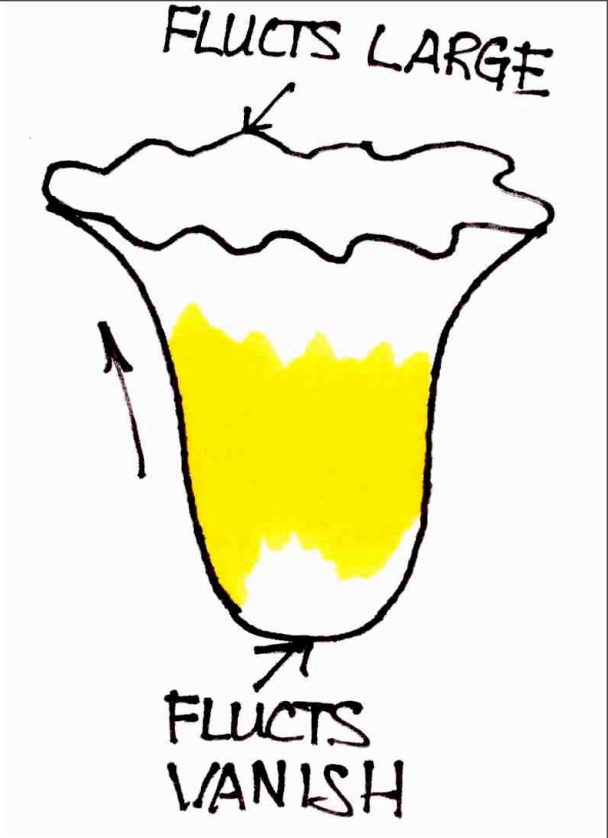
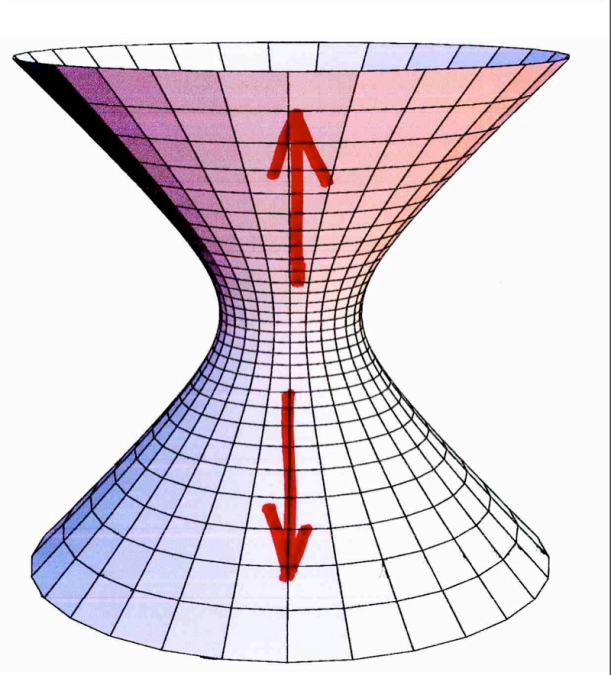
These arrows can be explained by a particular quantum state like the no-boundary wave function.

NBWF Arrows of Time

Hawking, Page, Laflamme, Lyons, Hertog, a.o.

- NBWF fluctuations vanish at only one place on the fuzzy instanton --- the South Pole.
- This means that fluctuations are small at only one of the places where the universe is small.
- For example, for bouncing universe fluctuations increase away from the bounce on both sides.
- Time's arrow points in opposite directions on the opposite sides of the bounce.

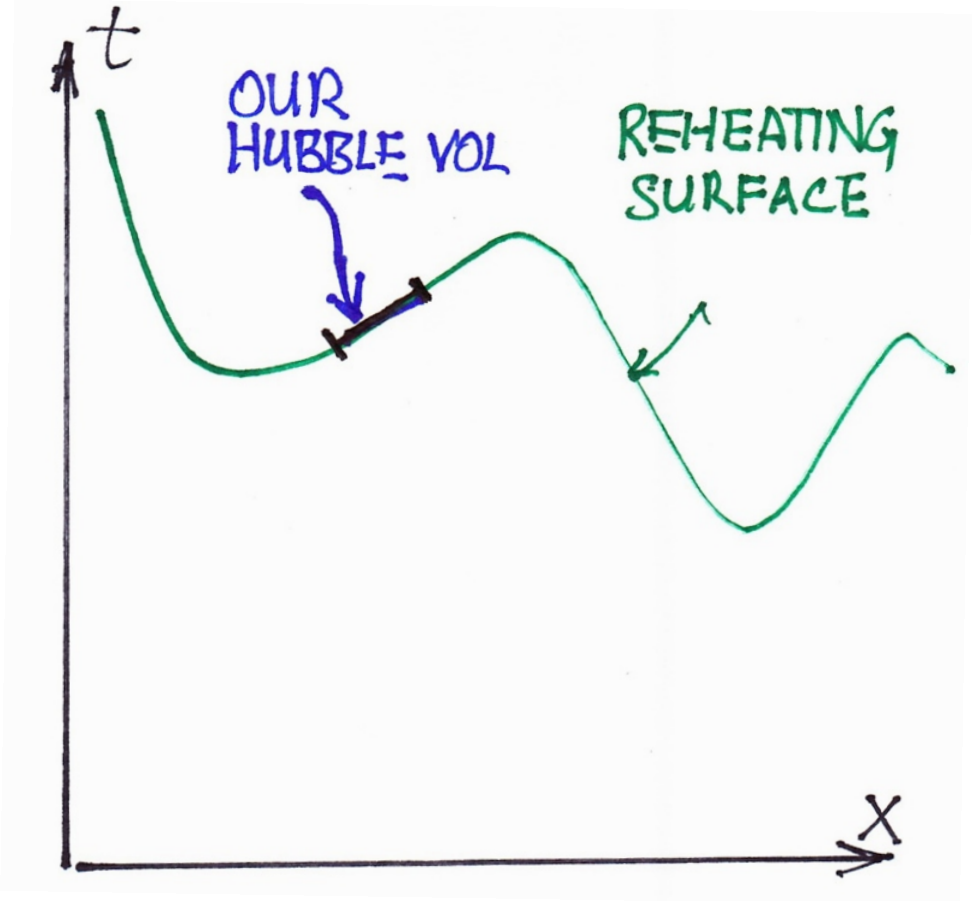




CMB, Large Scale Structure

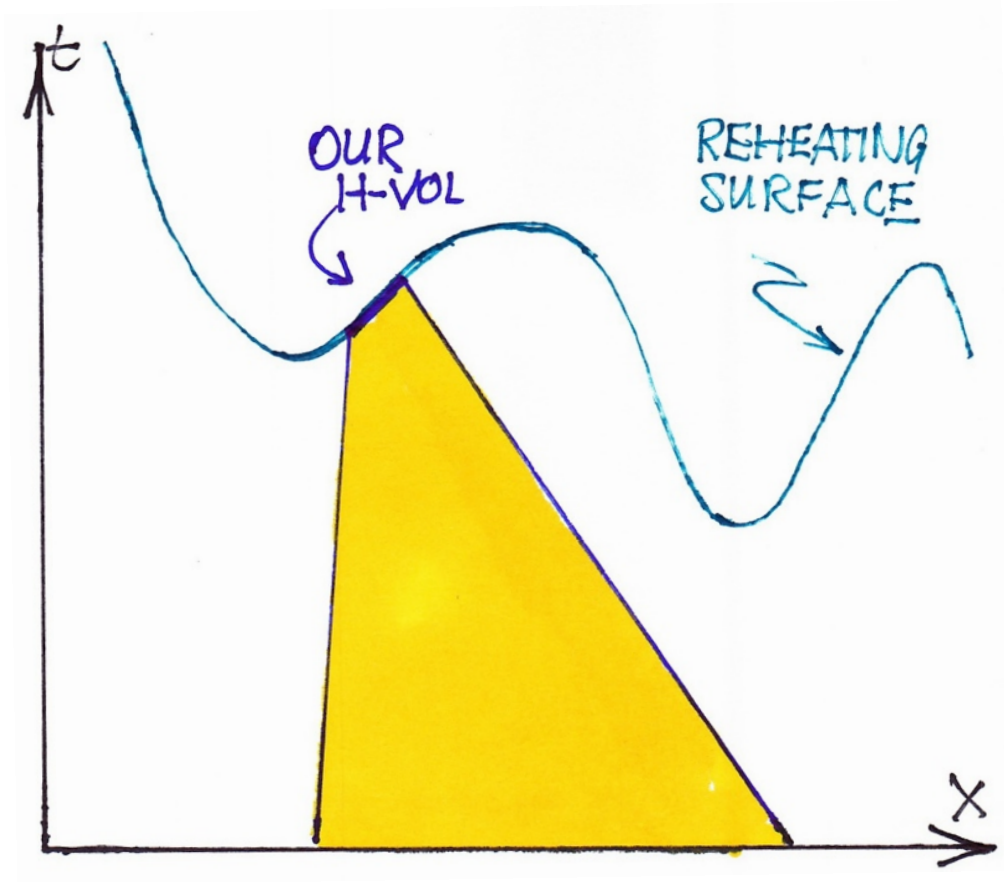
The NBWF predicts that typical histories become highly inhomogeneous on superhorizon scales due to fluctuations that leave the horizon during a regime of eternal inflation. (e.g. Creminelli, etal.)

Our Hubble volume is just a tiny, nearly uniform region on a vast reheating surface that is inhomogeneous on superhorizon scales.



EI,CMB, Large Scale Structure

- Reheating surface volume is so large that top-down=bottom up
- Our observations are influenced only by events in our past light cone.
- Coarse-grain (sum) over everything outside our past light cone and to the future.
- Probabilities for the $C_\ell^{\rm obs}$ are same as for a universe with no big large scale fluctuations.



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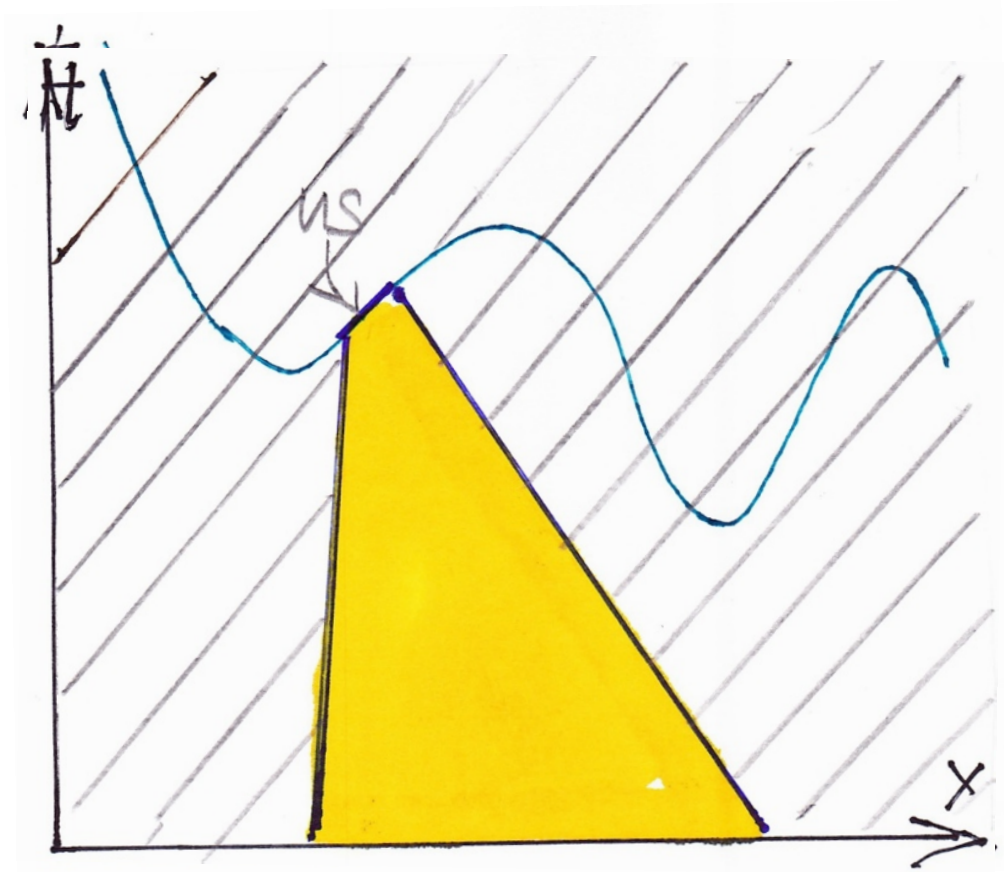

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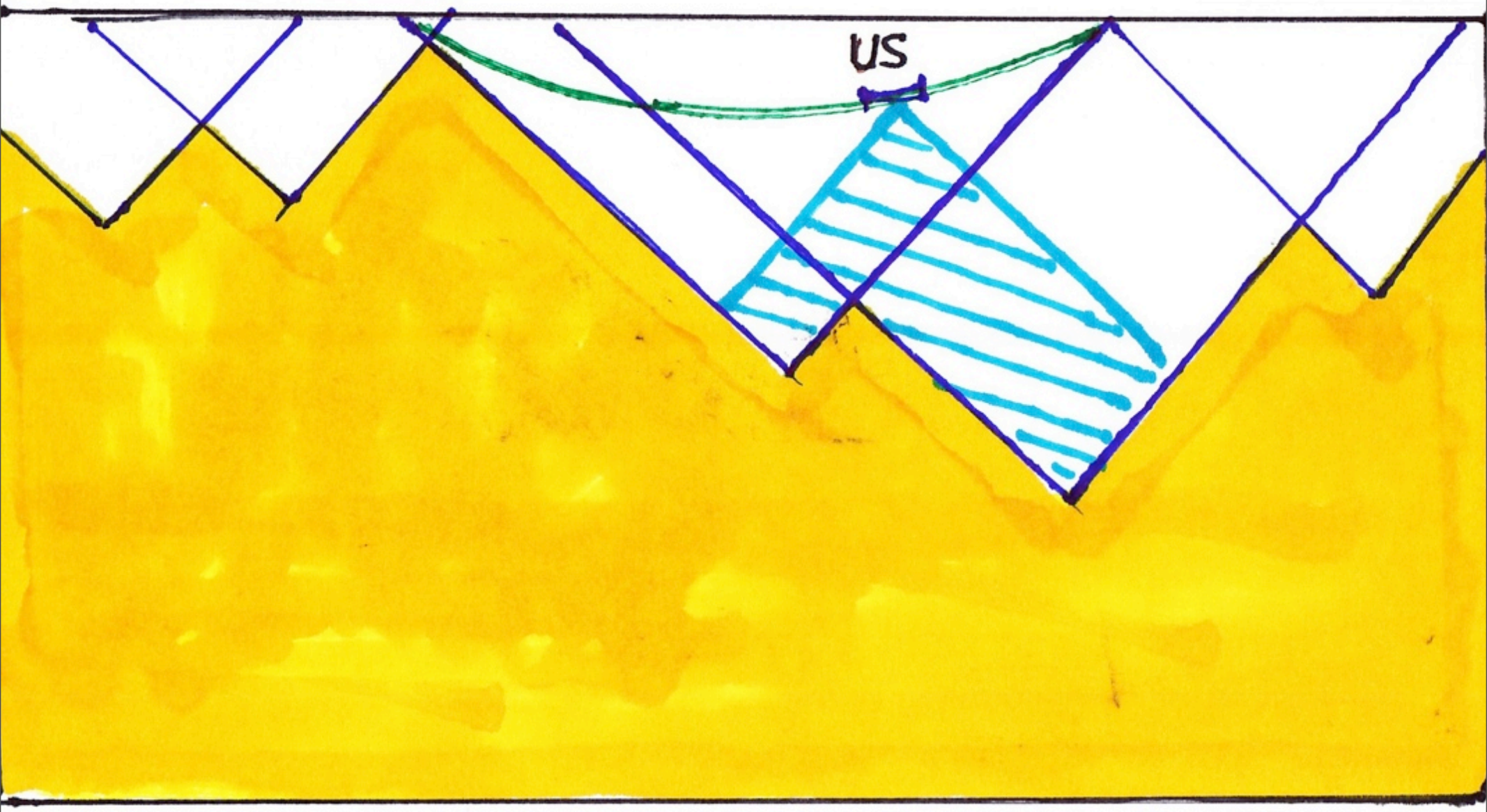
One Universe of Bubbles



Isolated Systems and Separation from the Planck Scale

- The behavior of subhorizon fluctuations is independent of specific superhorizon structure.
- Fluctuations grow and collapse gravitationally to produce approximately isolated subsystems (galaxies, stars, planets, ...)
- The expansion of the universe separates high energy scales from local ones leaving evaporating black holes as the only subsystems for which Planck scale physics is important.

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Expansion + Nuclear physics + cosmic censorship save us from local Planck scale physics.

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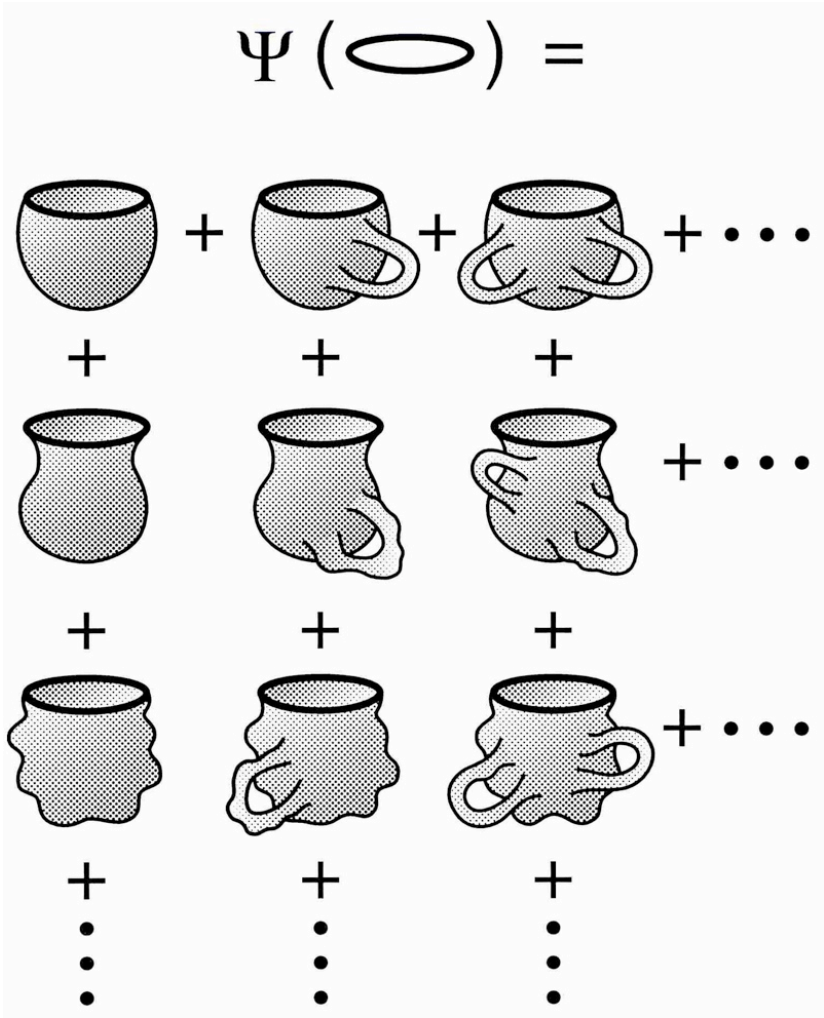
Topology of Spacetime



- A spacetime is a manifold with a metric. What is our manifold?
- Both the manifold and the metric may be quantum variables that are summed over.
- Some sum over topology is suggested by dualities (Maldacena, Hawking).
- Among known real saddle points the simplest topology has the least action. $S^2 \times S^2$, Page metric, Gibbons, a.o. Topology costs action.
- But we have little information about the complex saddle points on topologically complex manifolds.

Top

Sp

No evidence for interesting topology from the CMB. (Spergel talk)

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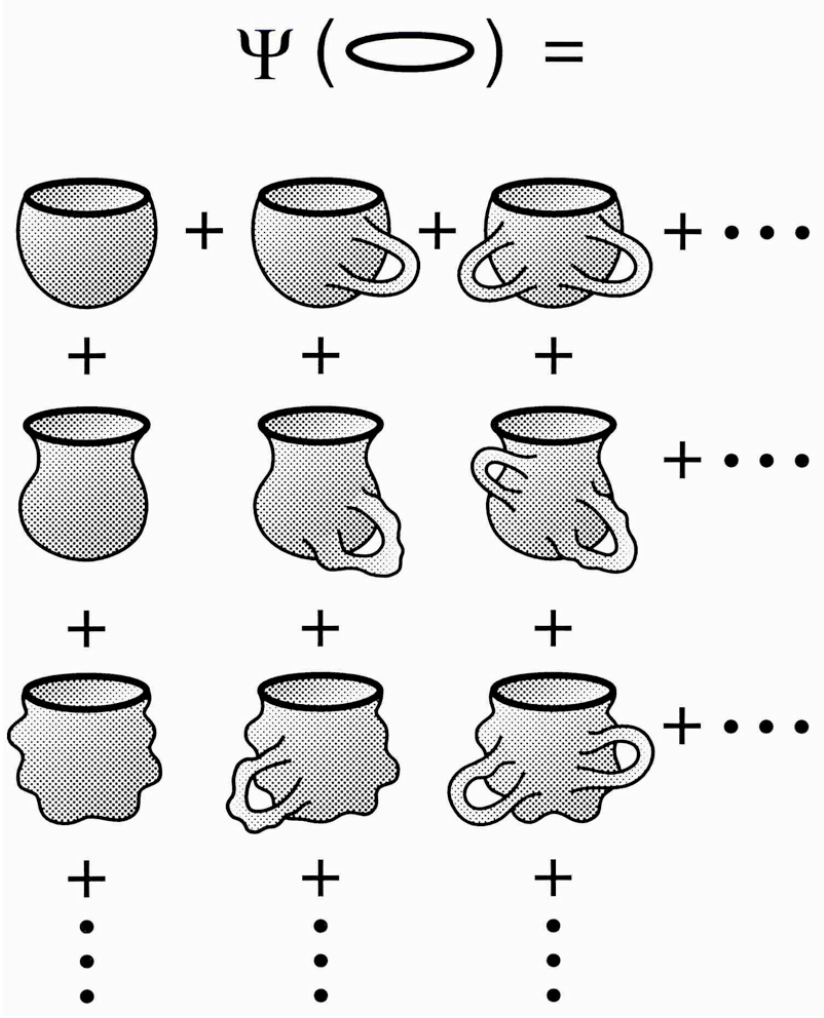

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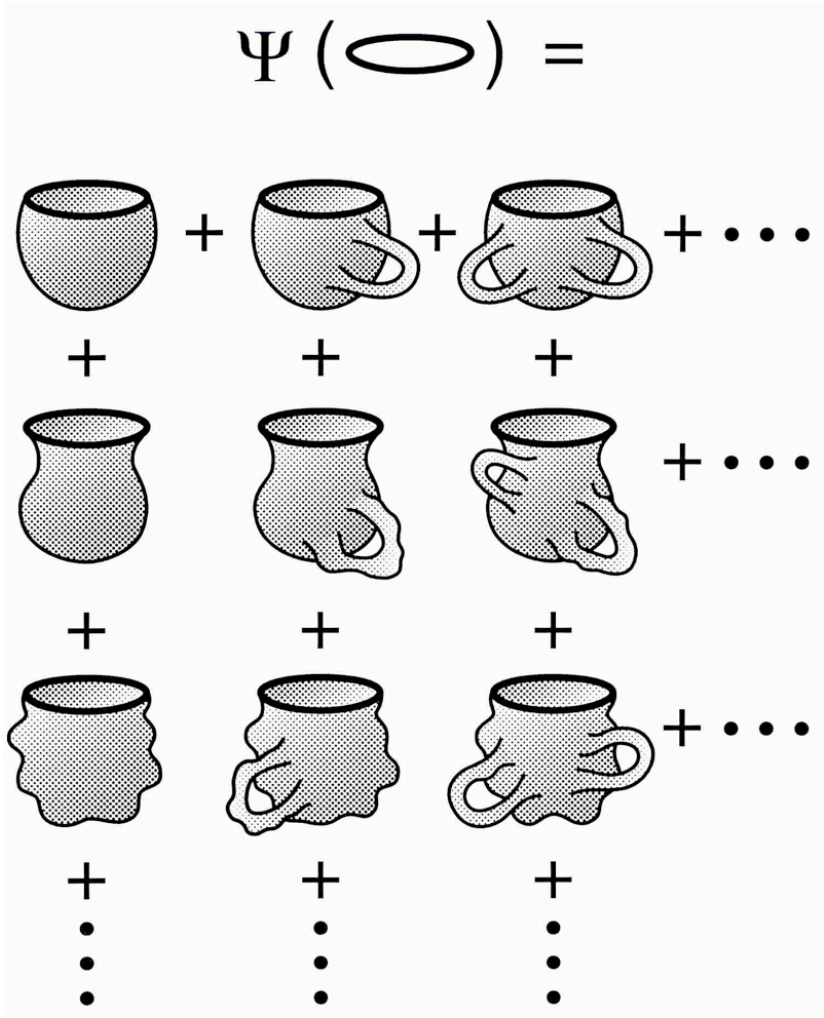

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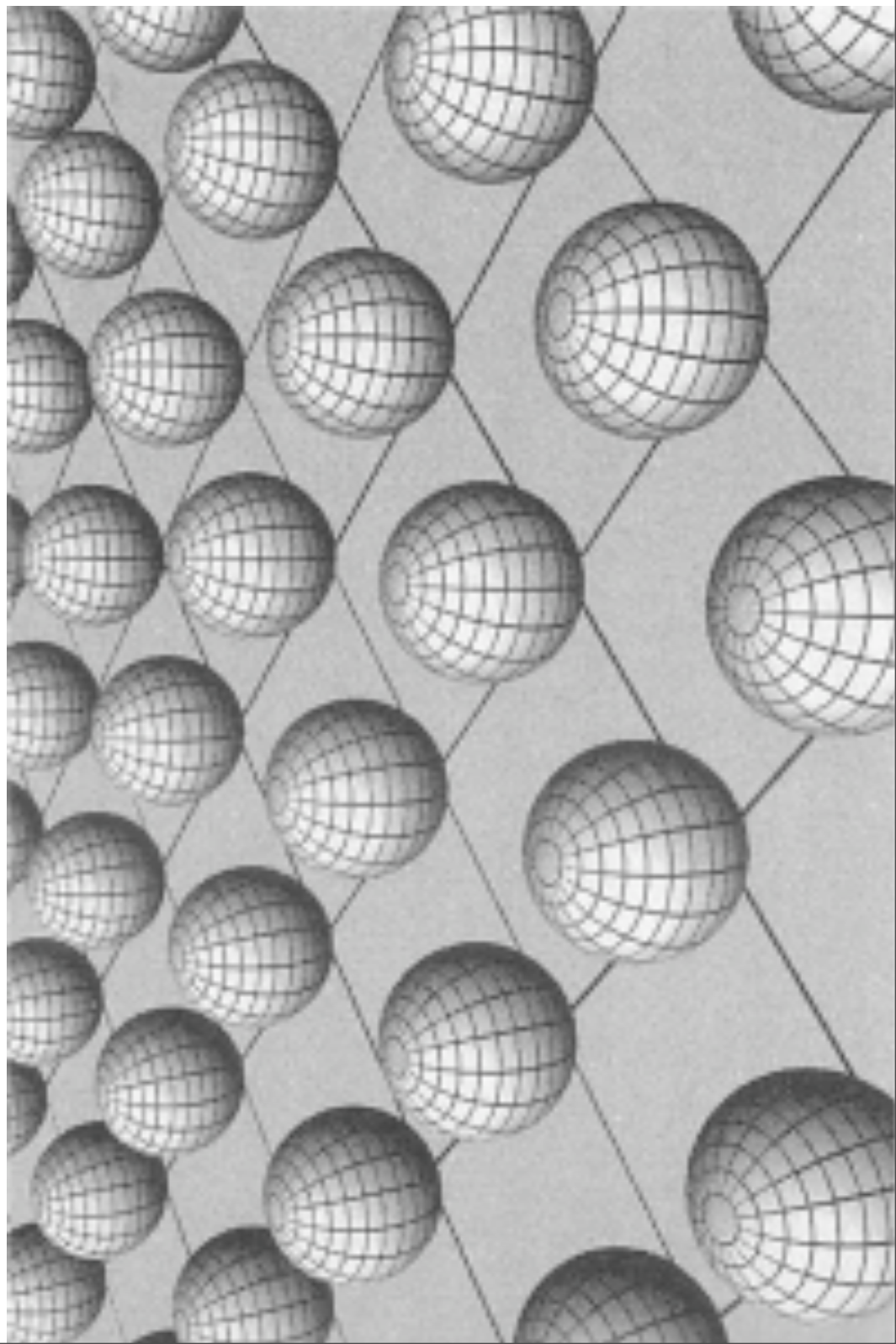
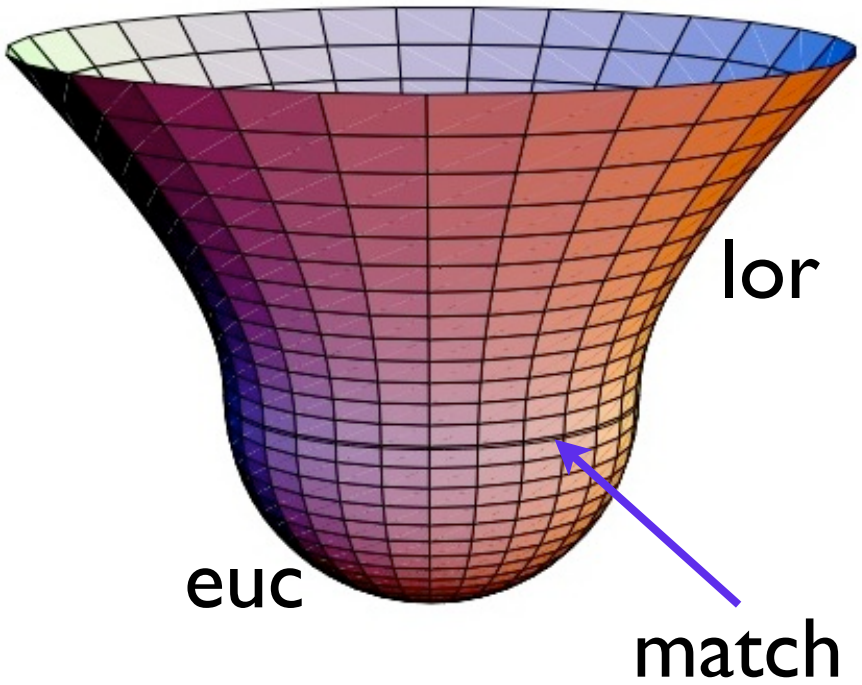
Large and Small Dimensions

Given that we observe four large dimensions, what is the probability that the others are small?



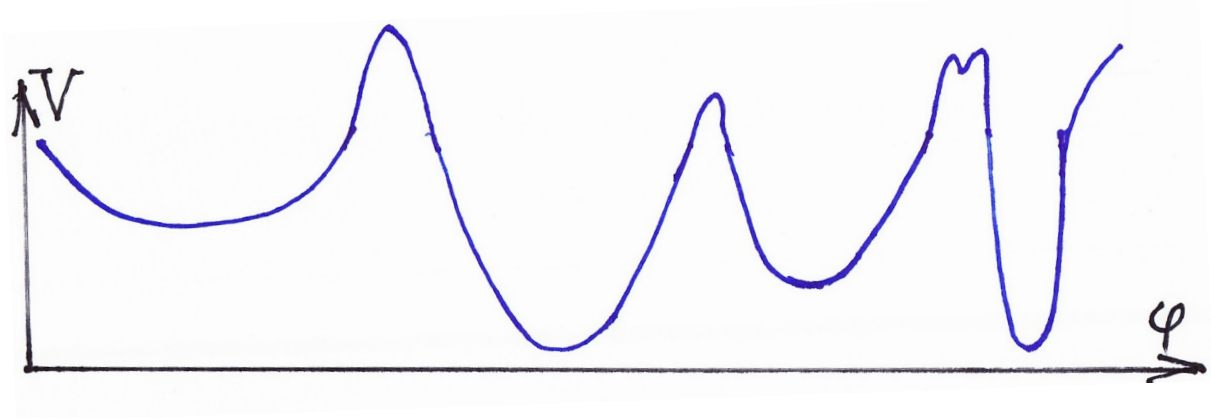
Number of Time Directions in Classical Spacetimes

Four-dimensional saddle-point geometries can be constructed by matching a real Euclidean geometry to a real Lorentzian geometry across a three-dimensional spacelike surface.



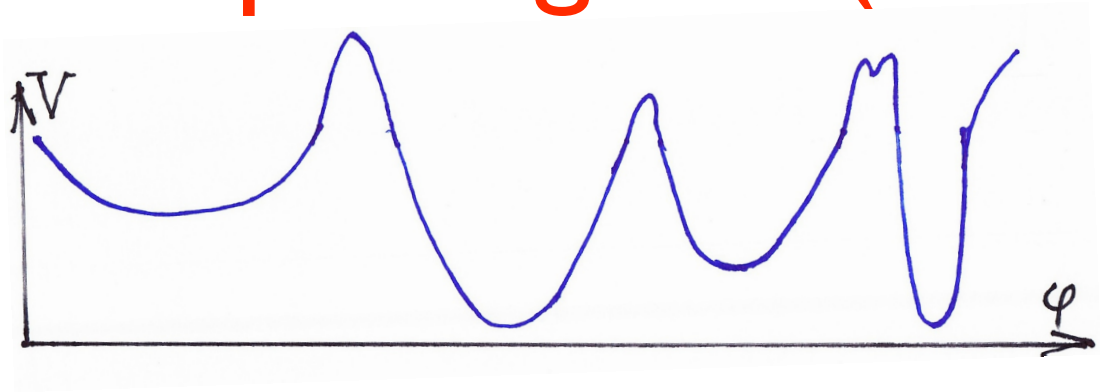
But this is possible only if the Lorentzian geomety has just one time dimension.

A Model Landscape



- Different minima K with $V_K(\phi) \approx \Lambda_K + (1/2)m_K^2\phi^2$ and big potential barriers between them (no tunneling in leading order semiclassical.)
- Which minimum we are in is a question of history. What is the probability we rolled down one hill or another.?
- Objective: The probability $p(\Lambda, m|D)$ for the parameters of our minimum given our data D.

Mechanisms for the Selection of Landscape Regions (`Potentials')



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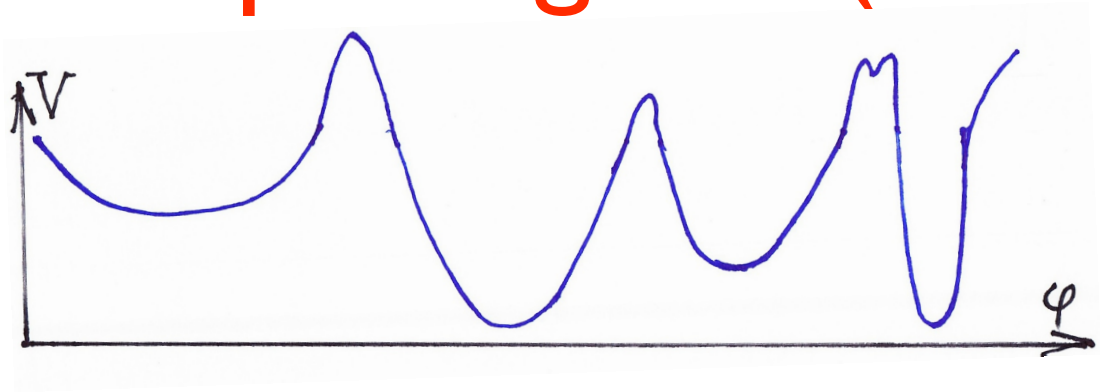

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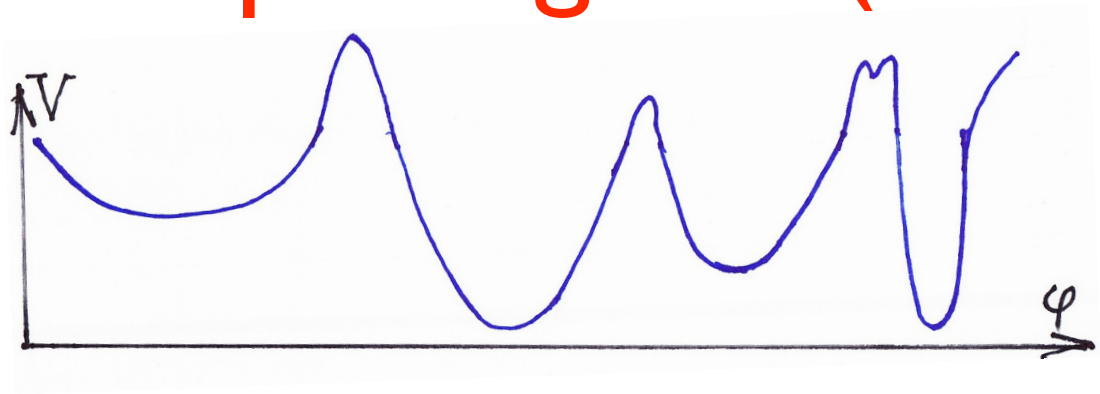

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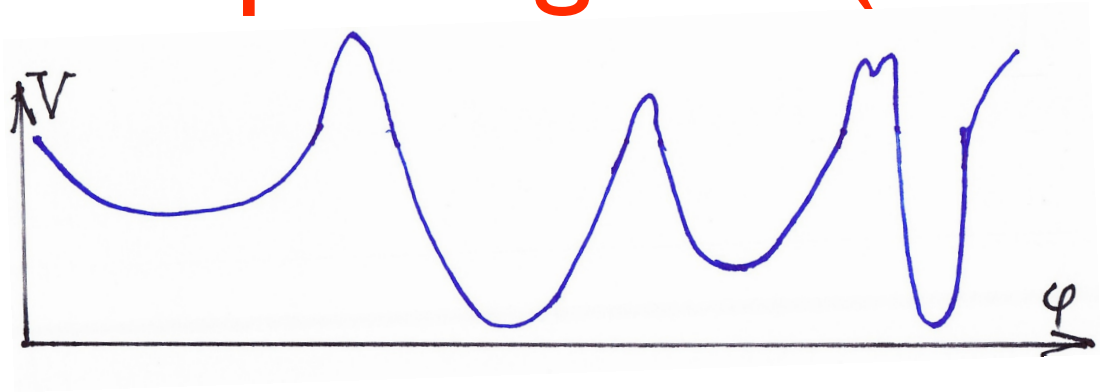

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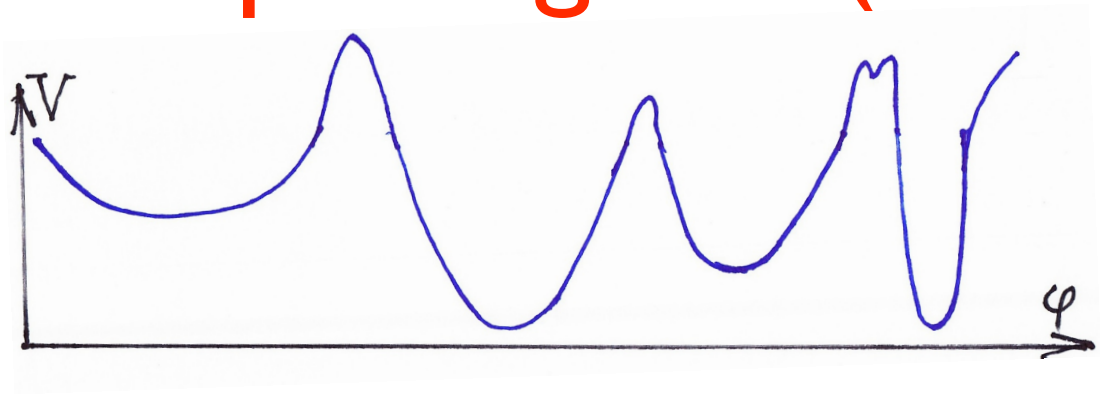

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- Selection for potentials that allow eternal inflation.
- Selection for histories around a given minimum that have the lowest exit from eternal inflation.
- Selection `anthropically' for parameters consistent with our local data.

Anthropic Reasoning is Automatic in Quantum Cosmology

If a constant Λ can vary (eg a landscape) then different histories can have different values.

$$p(\Lambda|us) = \frac{p(us|\Lambda)p(\Lambda)}{\sum_\Lambda p(us|\Lambda)p(\Lambda)}$$

If $p(us|\Lambda) = 0$ then $p(\Lambda|us) = 0$

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We won't see what is where we can't exist.

But the priors p(Λ) are supplied by the state, not flat priors representing ignorance.

Predicting Λ m from NBWF

$$V = \Lambda + \frac{1}{2}m^2\phi^2$$

dropped sub K's

We are interested in the probability $p(\Lambda, m|D)$ for the parameters given some part of our data D.

$$p(\Lambda, m|D) = \frac{p(D|\Lambda, m)p(\Lambda, m)}{\sum_{\Lambda,m} p(D|\Lambda, m)p(\Lambda, m)}$$

Take D to include the fact that we live in a Hubble volume of galaxies 13.7Gyr after the big bang.

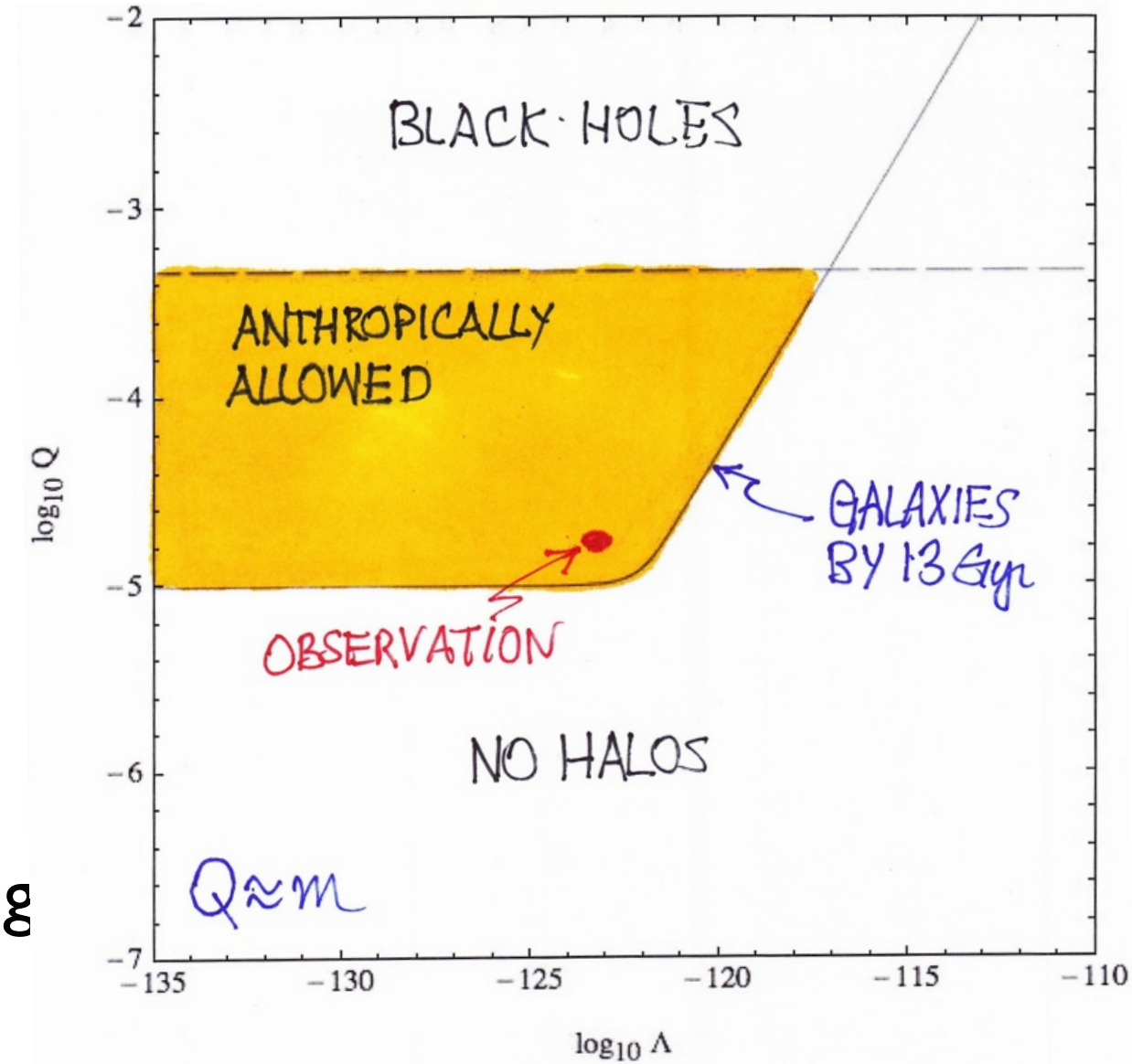
$$p(D|\Lambda, m) \propto p_g \langle N_g(\Lambda, m)\rangle$$

$\langle N_g(\Lambda, m)\rangle$ is the expected number of galaxies in a Hubble volume in the state of the fluctuations. p_g is the probability that we evolved in one galaxy --- small, unknown, but cancels out.

Anthropic Selection

$p(D|\Lambda, m)$ is the basis for traditional anthropic selection. Non-zero p is anthropically allowed.

Weinberg got good results by putting in the observed m and assuming a uniform prior for Λ.



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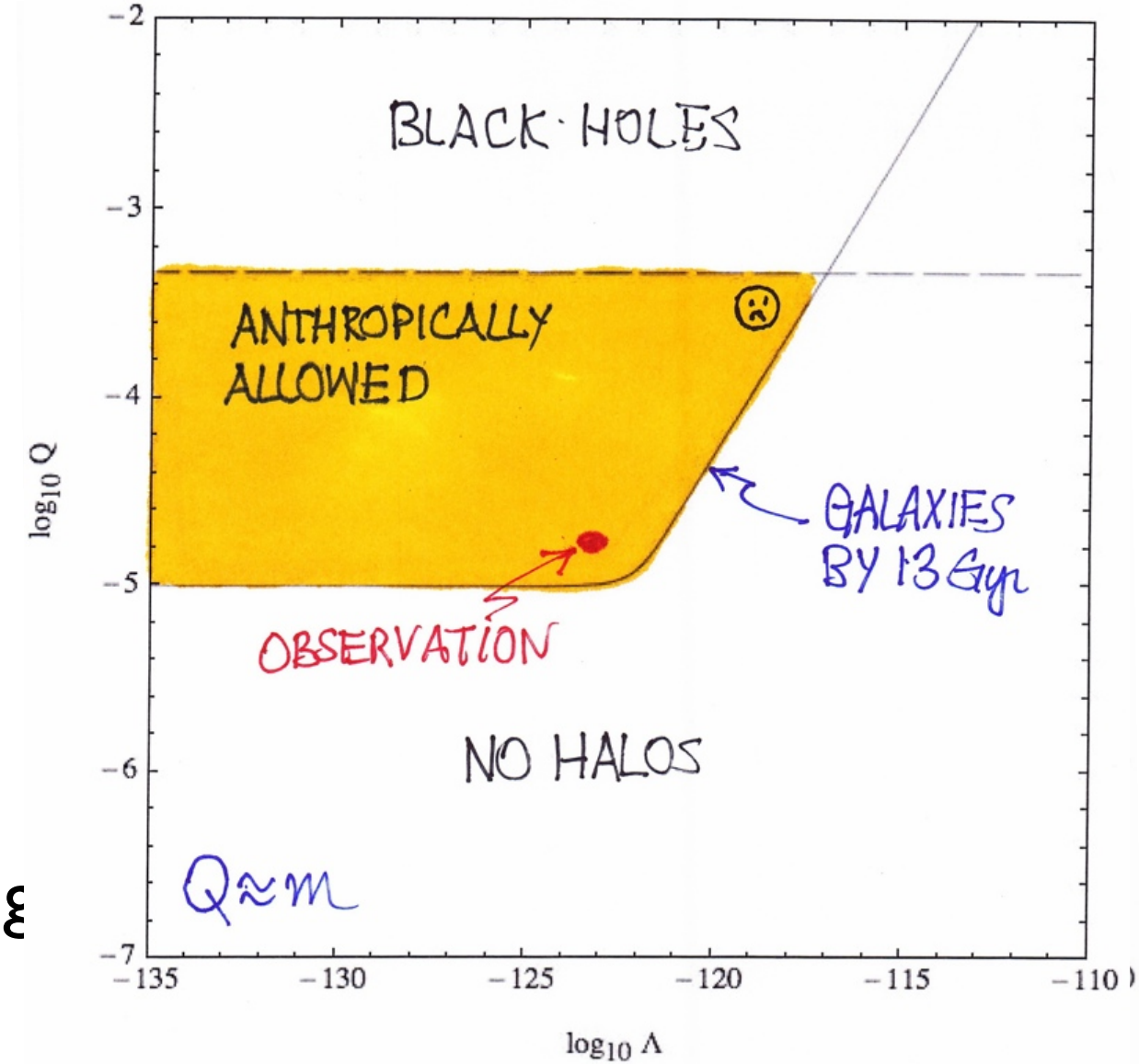

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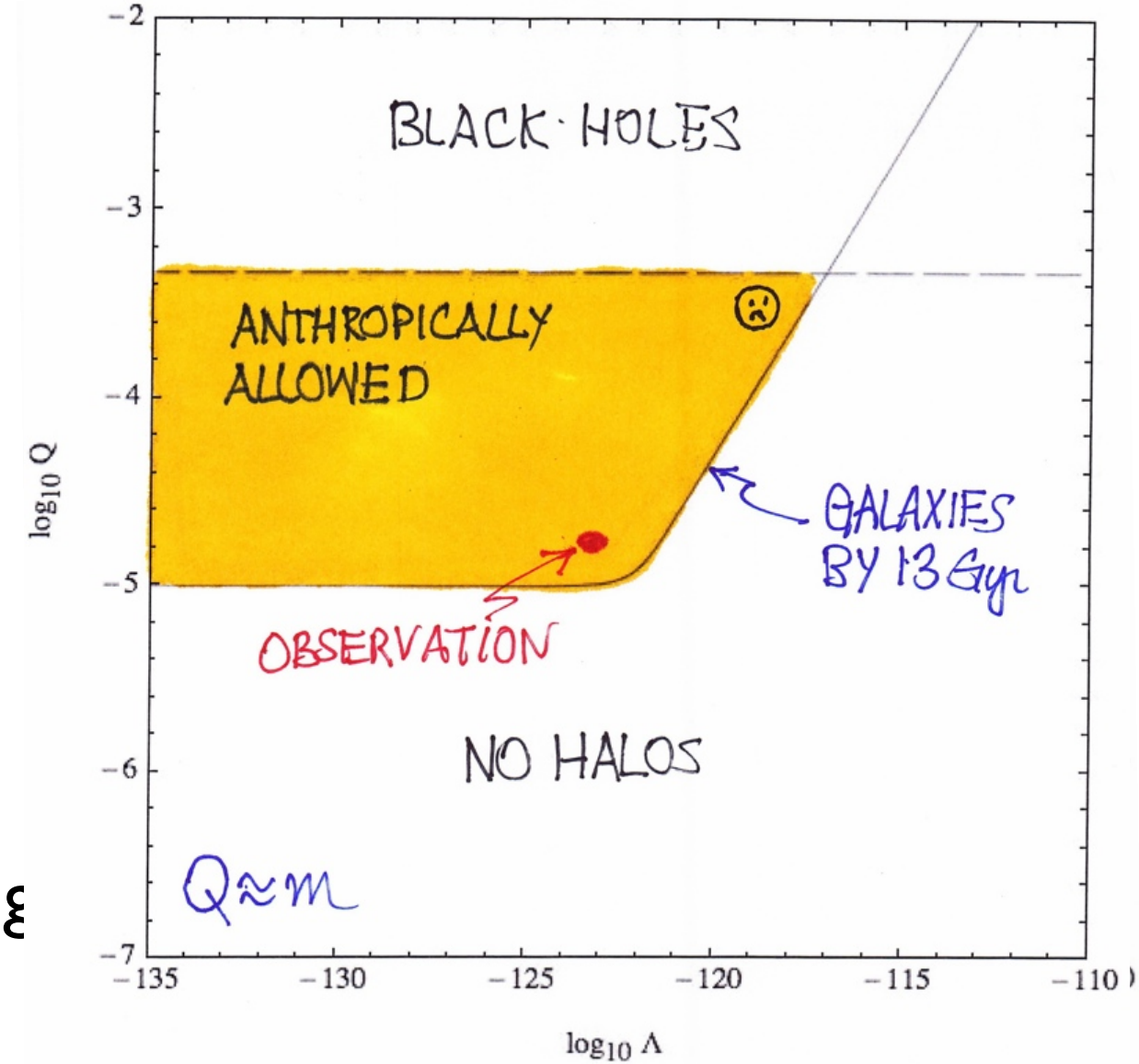

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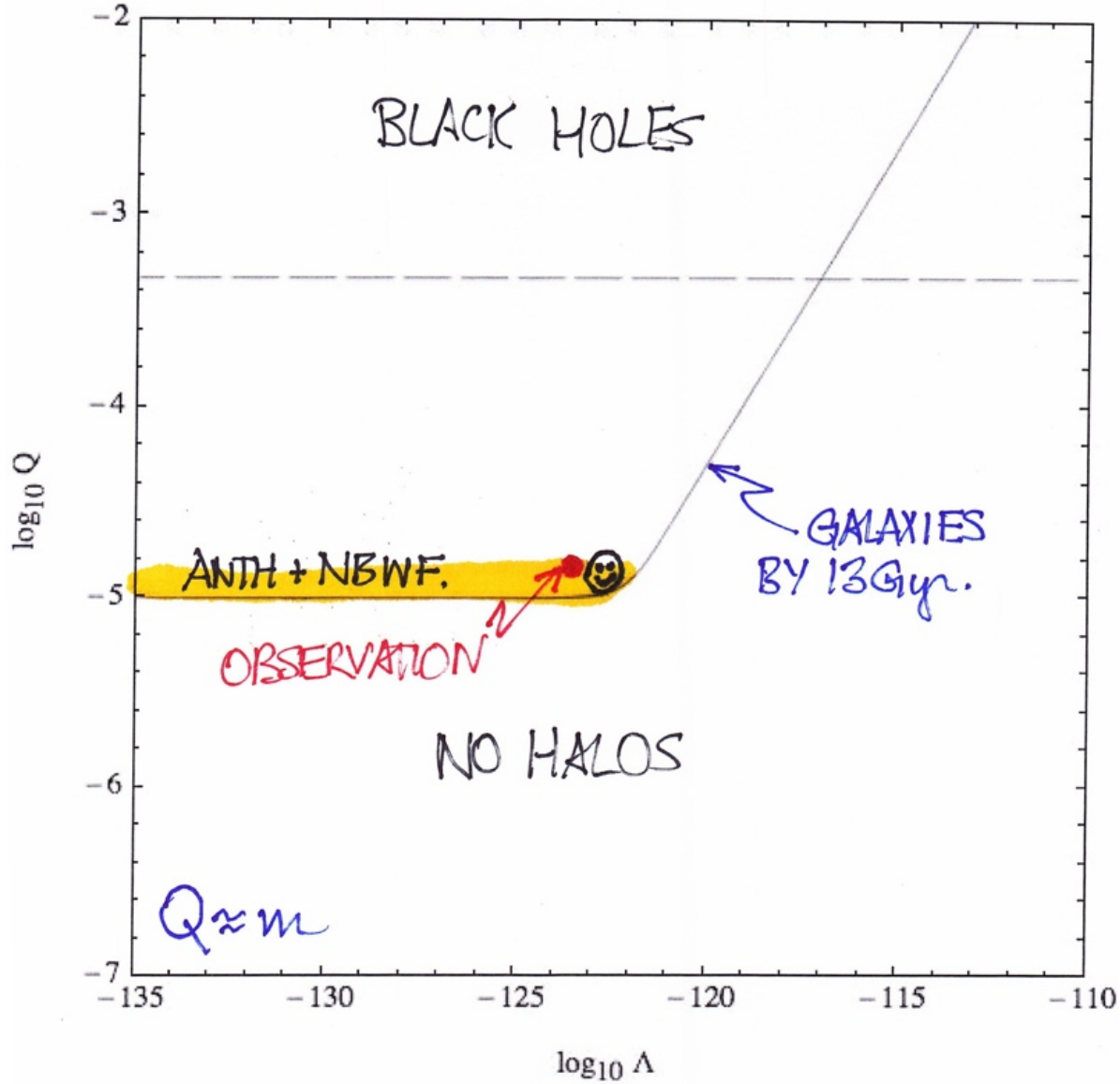
But the NBWF supplies the prior!

NBWF Aided Anthropics

$$p(\Lambda, m|D) \propto p(D|\Lambda, m)p(\Lambda, m)$$

$$\begin{aligned} p(\Lambda, m) &\approx \exp(\pi/V(\phi_{ei})) \\ &\approx \exp[\pi/(\Lambda + m/2)] \\ &\approx \exp(2\pi/Q) \end{aligned}$$

NBWF favors the lowest value of Q in the anthrop. allowed range.

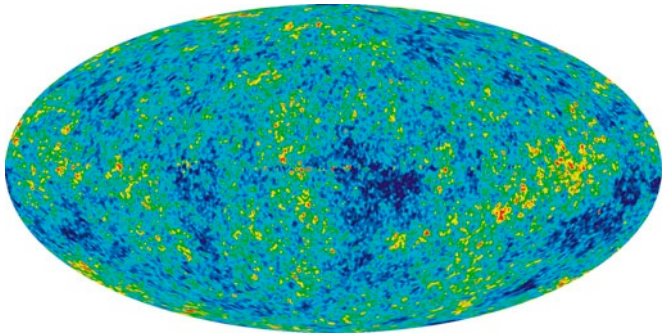


This restores Weinberg's anthropic argument for Λ.

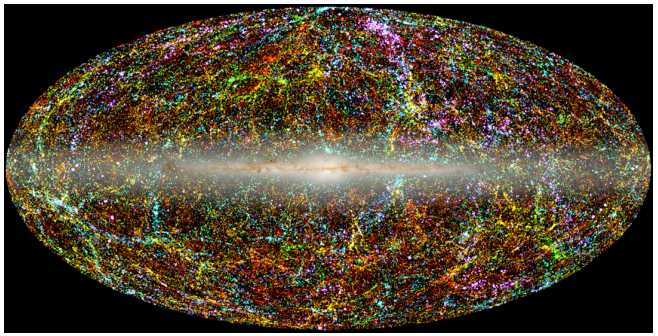
$$Q \sim 10^{-5}, \quad \Lambda \sim 10^{-123}.$$

Simplicity, Complexity, Simplicity

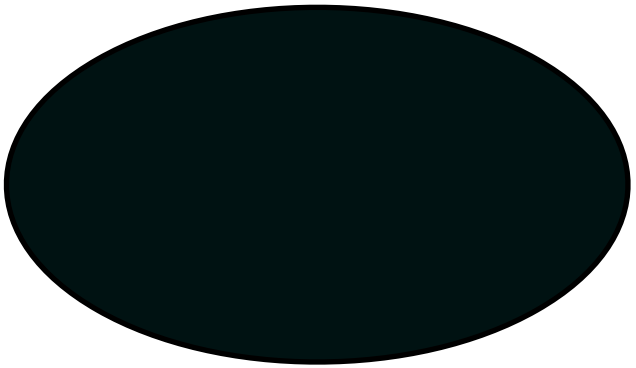
- The early universe is simple -- homogeneous, isotropic, matter nearly in thermal equilibrium...



- The middle universe is complex-- varied inhomogeneous structures (galaxies, stars, planets, biota, ...



- The late universe will be simple -- no protons, no stars, no black holes, no light....



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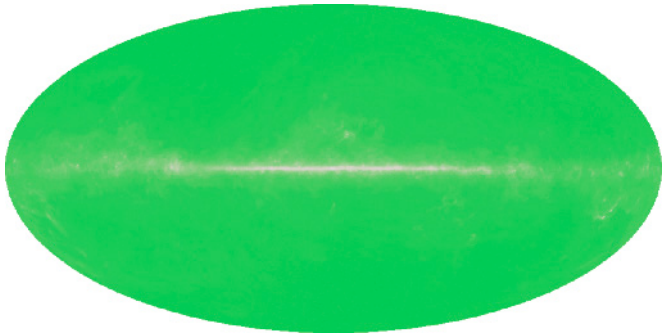

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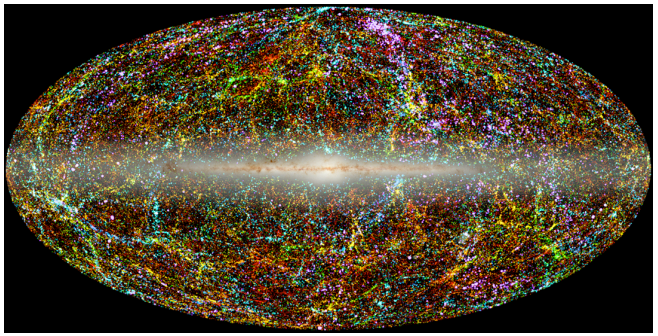

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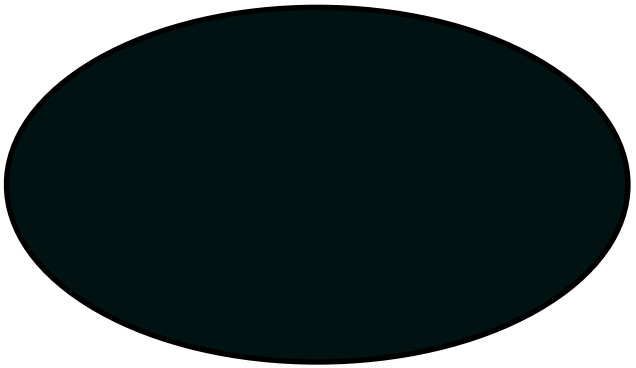

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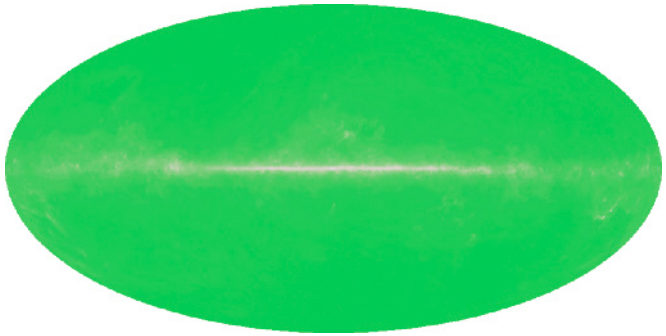

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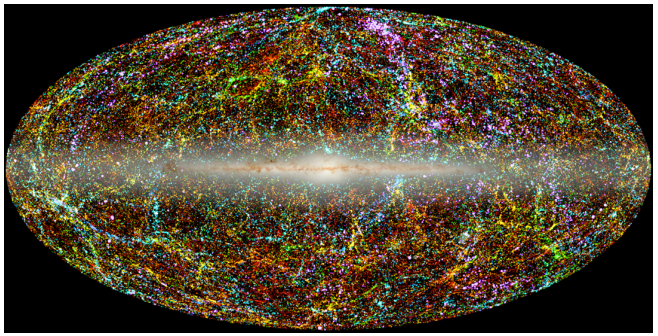
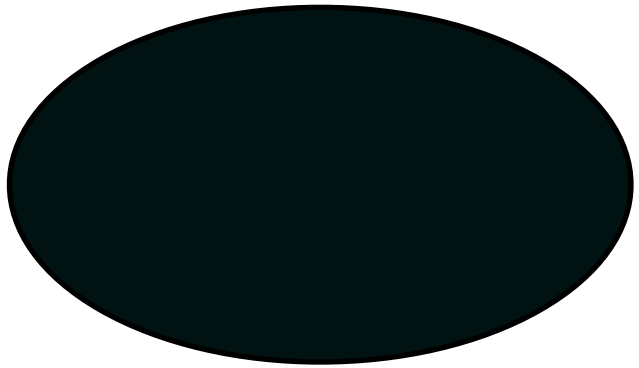


Is there a connection between mathematical simplicity of the theory and physical simplicity of its predictions?



Scorecard for the No-Boundary Wave Function

classical lorentzian spacetime	yes
early homo/iso + inflation	yes
fluctuations start in ground state	yes
arrows of time	yes
CMB, large scale structure	yes
isolated systems	yes
topology of spacetime	hints
num. of large & small dimensions	
number of time dimensions	hints
coupling consts. of effect. theories	some
complexity from simplicity	
connected to fundamental theory	recent progress


HAPPY BIRTHDAY STEPHEN
A. Zytkov


WE'RE NOT FINISHED YET!
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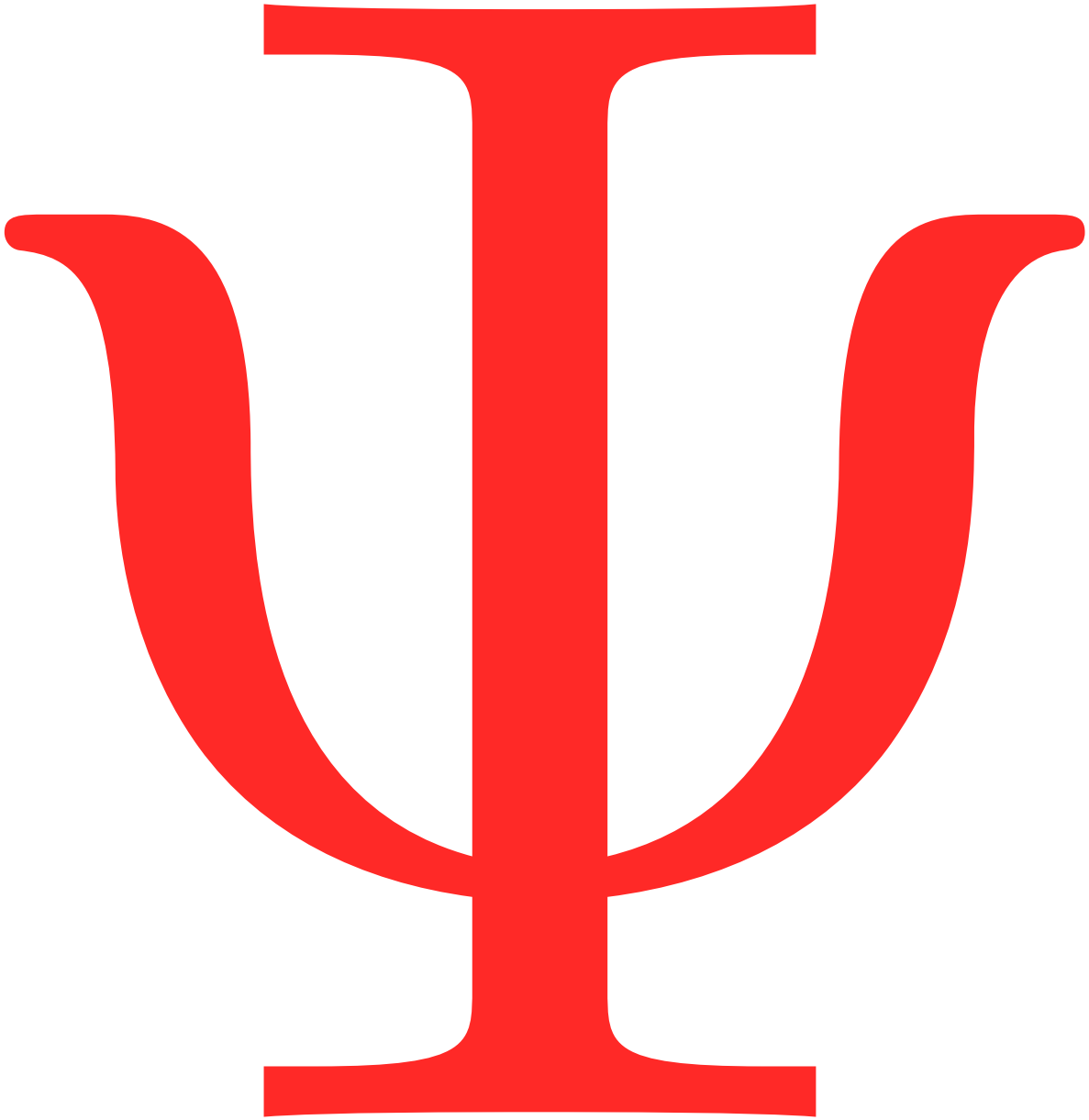

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