

# *Gravitational Wave Observation of Dynamical, Strong-field Gravity*

*Frans Pretorius  
Princeton University*

*Gravity and Black Holes,  
Stephen Hawking 75<sup>th</sup> Birthday Conference  
Cambridge, July 4, 2017*

# Outline I

- General Relativity in the wake of GW150194, GW151226 and GW170104
  - Entering the era of observational dynamical, strong-field gravity
  - Can now start to test the most non-linear aspects of Einstein gravity, and begin to constrain modifications to GR that predict deviations here
    - GR150914 could *eventually* be a game-changer constraining modifications to GR, and exotic alternatives to black holes within GR, but not until alternatives can provide concrete predictions for merger events
    - For now, can focus on showing self-consistency within GR

# Outline II

- Looking ahead
  - Once LIGO reaches design sensitivity, can within a few years expect a very rich data base of events
  - Can use *signal-stacking* to enhance the science that can be gleaned from a population of similar GW events
  - First case study : going after the sub-leading quasi-normal mode (QNM) of black hole ringdown to test the “no-hair” property of Kerr black holes.
    - To set the stage, show that GW150914 is *already* providing a zeroth-order consistency test of the no-hair properties (or more correctly with the “final state conjecture”)
    - Use similar arguments to show how we can enhance the measurement of a chosen, “collective” higher order harmonic of a set of merger events
- Conclusions

# Strong Field Gravity

- This is the regime of general relativity (GR) where typical curvature scales are comparable to, or larger than other relevant scales in the problem
  - GR has no intrinsic length scale, so the scale where gravity becomes strong is always relative to some other physical scale in the problem
    - for compact objects (black holes and neutron stars) the radius of the object sets the scale
    - for the universe as a whole, the Hubble radius is the relevant scale

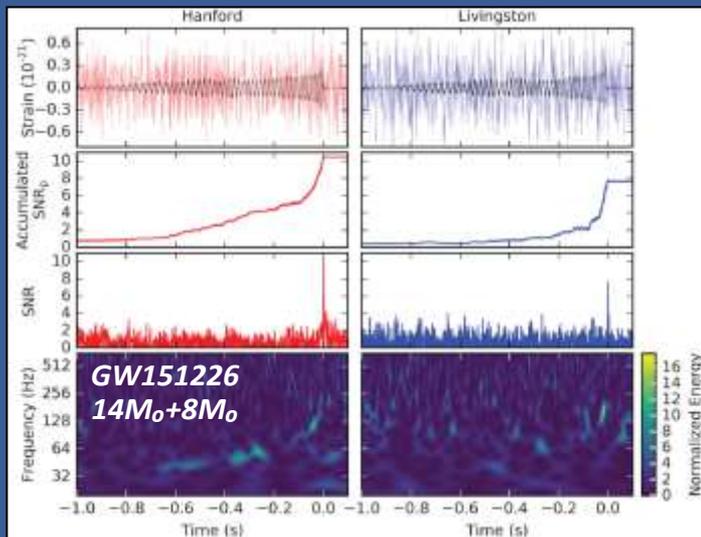
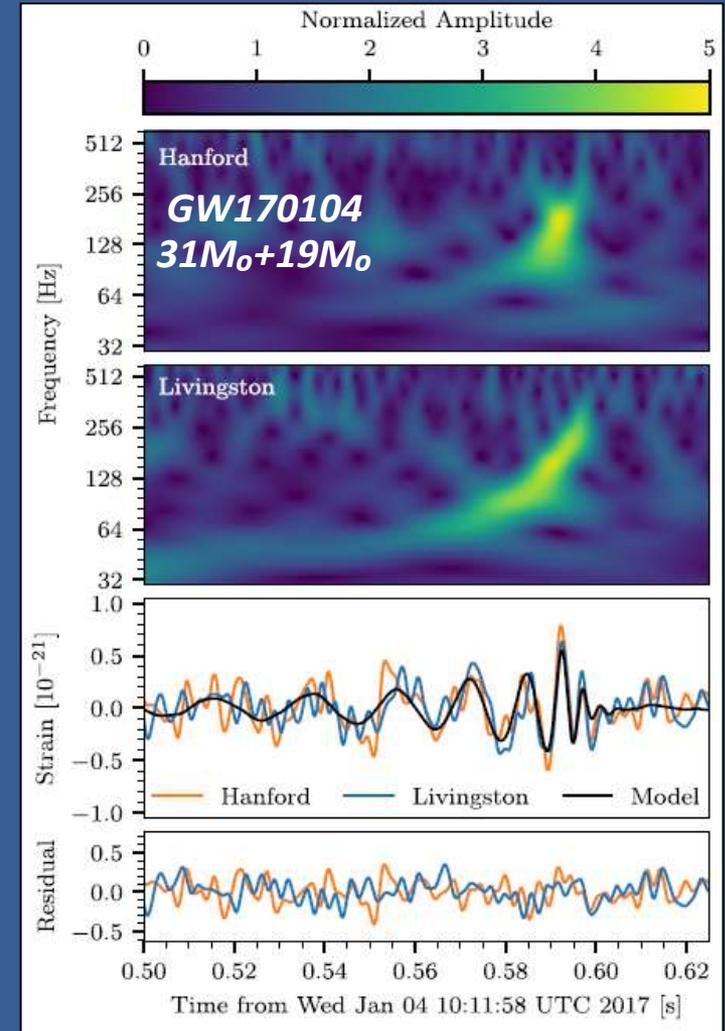
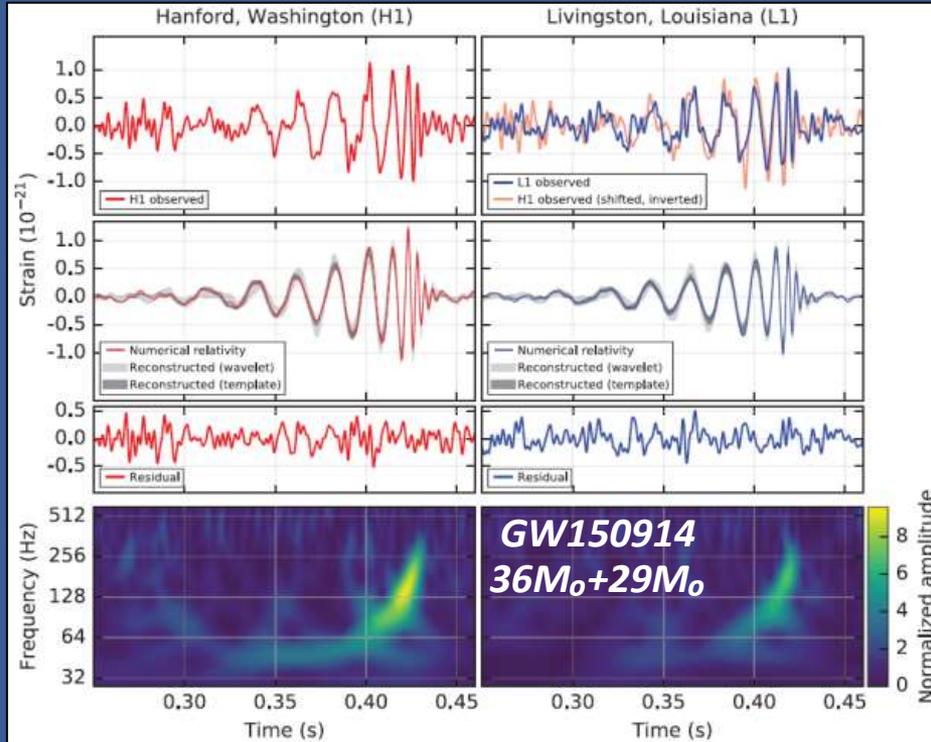
# Strong Field Gravity

- The most extreme manifestation of strong field gravity is the presence of a *horizon*
  - general relativity then mandates that some form of singularity in the geometry is present somewhere in the spacetime
  - in a cosmological setting on scales of the Hubble radius there is not a horizon in the same sense as a black hole, nevertheless here the structure of spacetime is likewise markedly different from that of weak-field gravity (i.e. Minkowski spacetime)
- In dynamical situations the gravitational wave luminosity can approach a decent fraction of the Planck luminosity
  - the Planck luminosity  $L_p = c^5/G$  does not depend on  $\hbar$ , but in some sense is a limiting luminosity even in classical GR

# Why gather evidence for the GR description of strong-field gravity?

- GR itself has no intrinsic scale, and so one could argue the numerous existing confirmations of its weak-field properties should give confidence in all its predictions
- However, aside from basic scientific inquiry, there are reasons to be more cautious about blindly accepting GR's extreme gravity predictions
  - *the fundamental inconsistency with quantum mechanics*
    - ostensibly tensions should only manifest near the Planck scale, but some “firewall” proponents argue otherwise
  - *the existence of dark energy and dark matter*
    - the evidence for the latter does not rely on strong field gravity, but some have suggested the two phenomena are connected, e.g. Verlinde's emergent gravity proposal

# The era of observational, dynamical strong-field gravity has arrived



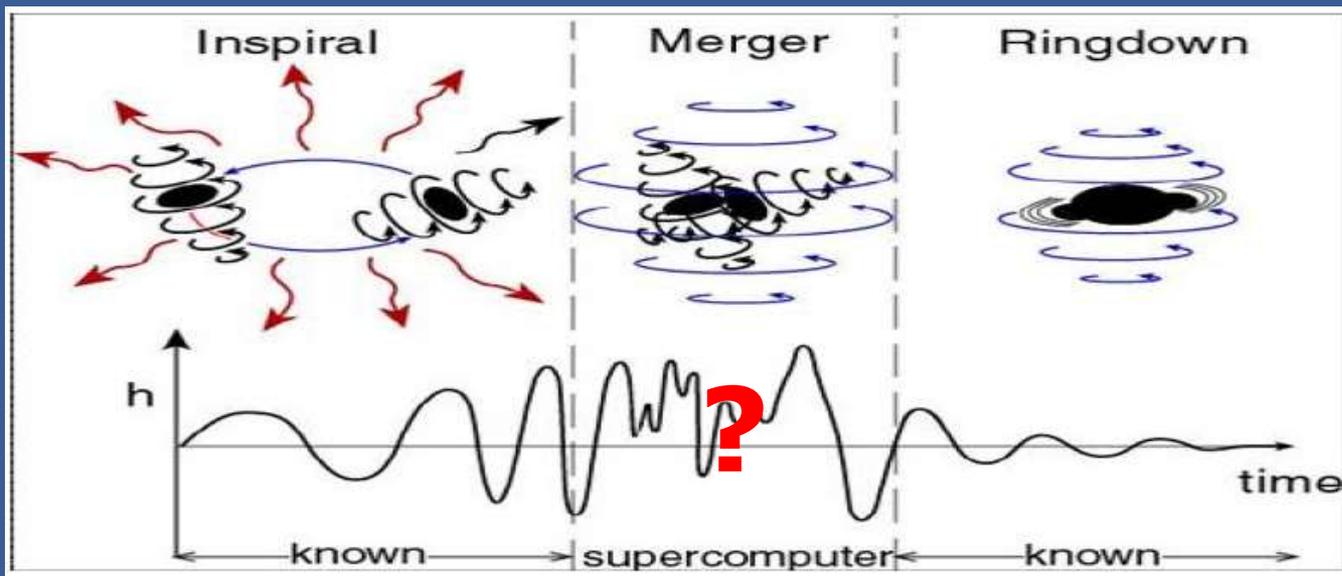
*PRL 116, 061102 (2016), PRL 116, 241103 (2016)  
PRL 118, 221101 (2017); LIGO & Virgo Collaboration*

# The physics of GW150914/GW170104

- The residuals subtracting the best-fit numerical relativity templates for binary black hole mergers is consistent with noise [*PRL 116, 221101 (2016); PRL 118, 221101 (2017); LIGO/Virgo Collab.*]
  - For GW150914, fractional deviations of  $> 4\%$  in the waveform from the GR prediction not supported by the data (other than those that can be absorbed in a re-definition of the parameters of the binary)
- This folds in all the rich physics of black hole collisions within general relativity
  - Runaway inspiral due to GW emission
  - No naked singularities in the collision, the horizons merge, and the collective area increases
  - Astonishingly simple (as characterized by the waveform) transition from inspiral to merger-ringdown
  - Very rapid ringdown to a unique, quiescent Kerr black hole remnant

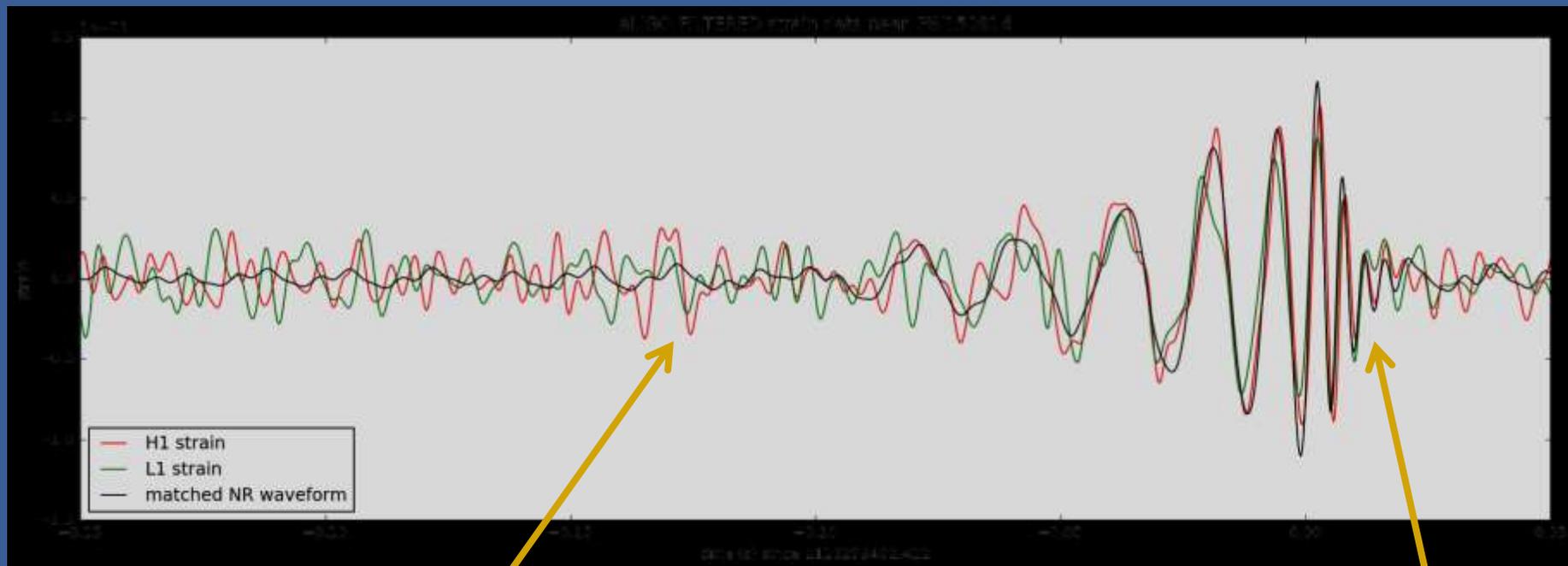
# Beyond GR

- There is no anomaly in GW150914/170104 that defies a conventional explanation, so the main significance of these event is to constrain/rule-out alternatives
- The problem with doing so now, is pretty much all alternative theories, or “exotica” (boson stars, gravastars, traversable wormholes, etc.) are in the following, or worse situation:



*Illustration by Kip Thorne*

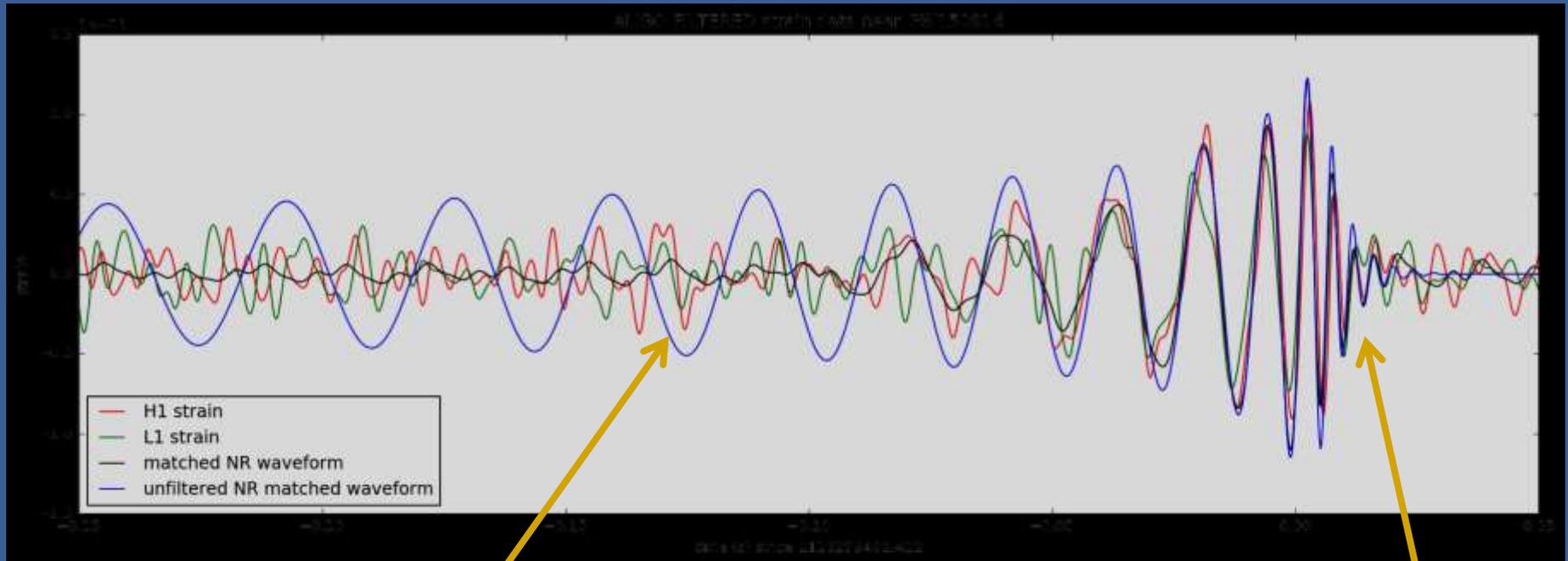
# GW150914, Filtered Signal plus Best-Fit Template



*Inspiral*

*Ringdown*

# Filtered Signal plus Filtered & Unfiltered Best-Fit Template

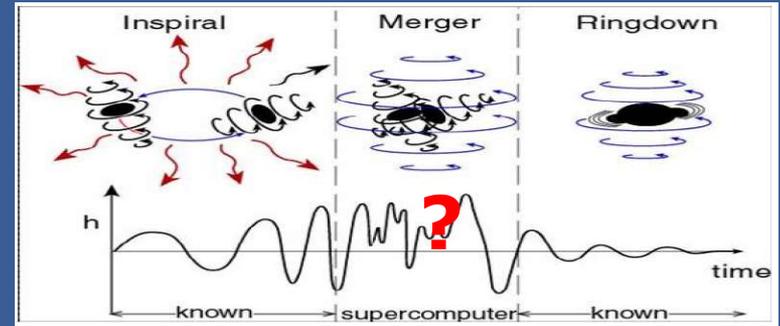


*Inspiral*

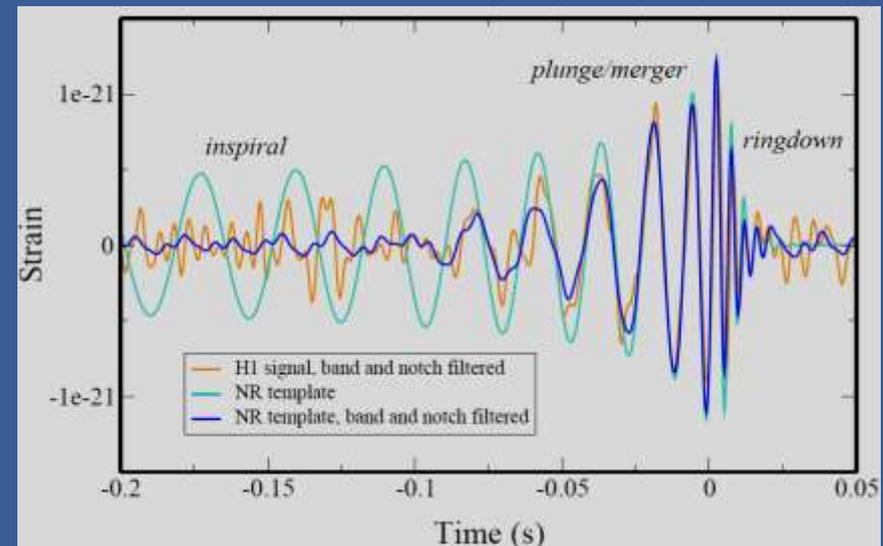
*Ringdown*

# Beyond GR

- Because of the “?” in non-conventional GR, essentially all methods people have devised to constrain GR or to search for deviations are based on



- **The early inspiral**, where post Newtonian-like expansions are available, and reasonably well-motivated generic deformations of these, such as the parameterized post Einsteinian (ppE) approach have been developed
- **Stationary isolated solutions**, where ringdown modes can be computed, or images of accretion disks about these solutions can be studied to be confronted with anticipated data from the event horizon telescope
- After GW150914 this no longer suffices; the bar has been raised for any alternative to claim viability in light of all experimental and observational data
  - Some limited constraints possible using only the inspiral, or constraints on qualitative properties exotica must have to merge and ringdown as rapidly as Kerr black holes  
See e.g. *Yunes, Yagi and FP, PRD 94 (2016)*



# Testing General Relativity using GW150914

- That the residual of the full event is consistent with noise is the most powerful, agnostic test
- General relativity does not break the event apart into distinct regimes, phases or concepts, however doing so is essential for a deeper understanding of black holes and their dynamics
  - One of the cherished properties of vacuum black holes in GR that we can go after in this way stems from the “final state conjecture” (FSC) :

*The exterior spacetime of any sufficiently isolated, vacuum black hole asymptotes to a member of the 2-parameter  $(a, M)$  Kerr family of solutions*

# Testing General Relativity using GW150914

- This property is often colloquially referred to as the “no-hair” property, but it implies much more than the no-hair theorems
  - all single, asymptotically flat, stationary black holes in 4D, vacuum GR (with no exterior naked singularities) are uniquely described by a member of the 2-parameter  $(a, M)$  Kerr family of solutions

*[Israel '67 for static blackholes, later Carter, Robinson, Hawking, ... for the stationary case]*

- taken by itself, this would suggest either
  - (a) black hole solutions are sets of measure zero and not of astrophysical relevance at all
  - (b) the Kerr family are “dynamical attractors” reached once gravitational collapse occurs

# Testing General Relativity using GW150914

- Many profound consequences of the FSC; most relevant for testing GR with binary mergers is:
  - The full structure of spacetime exterior to the horizons of all vacuum binary black hole spacetimes allowed in GR, prepared in relative isolation sufficiently far to the past of coalescence, are essentially *uniquely* characterized by a *small, finite* set of numbers  $N$
  - A merger waveform observed with large signal-to-noise ratio (SNR) will, from an information-theoretic perspective, require a correspondingly large set of numbers  $M$  to describe
  - For  $M \gg N$ , multiple independent subsets of  $M$  can be used to reconstruct consistent representations of  $N$  (to within degeneracies and noise-uncertainty)

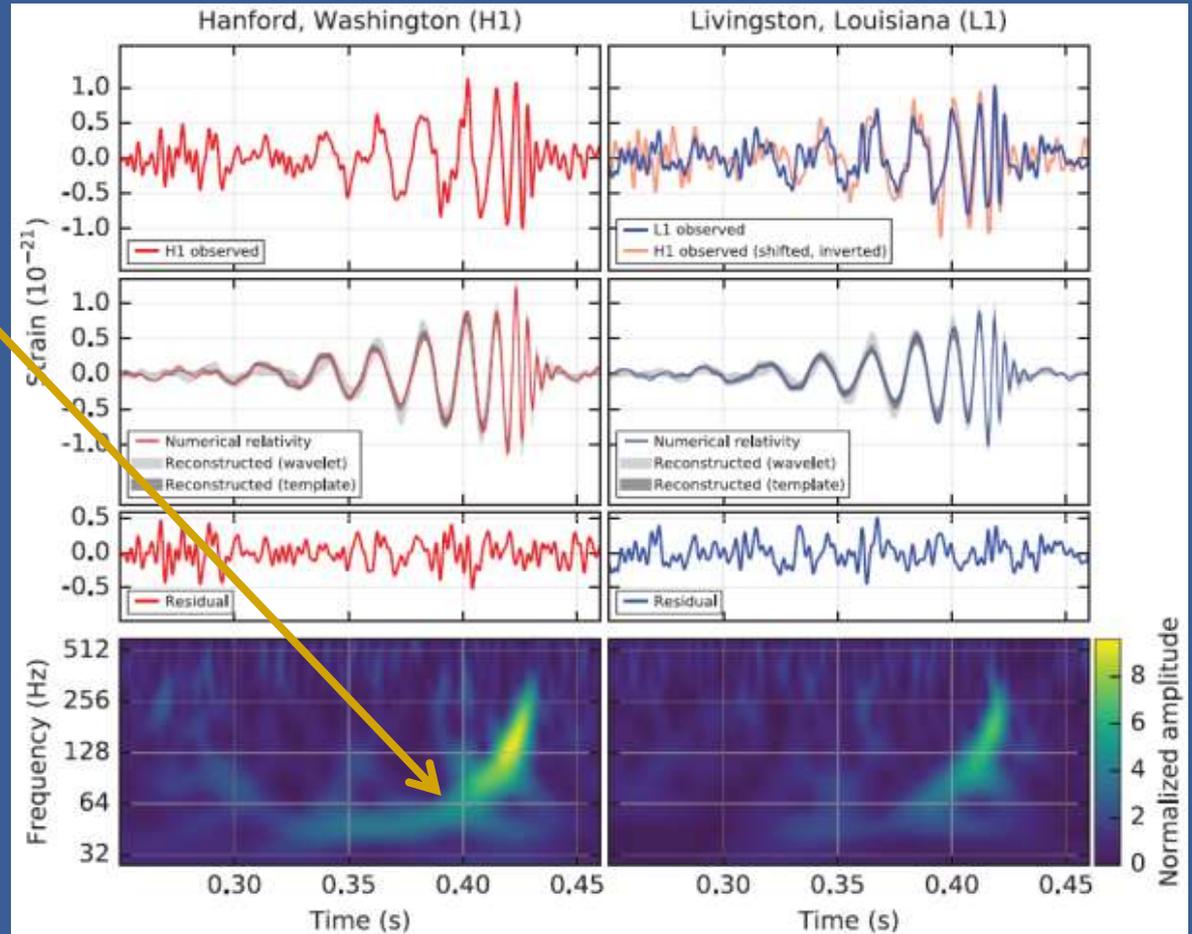
# Testing General Relativity using GW150914

- Note : this goes beyond what has traditionally been referred to as *black hole spectroscopy* [Detweiler, Dreyer et al., 2004, Berti et al., 2006]
  - all infinitely many quasi-normal mode (QNM) *frequencies* of a perturbed Kerr BH are uniquely characterized by  $(a, M)$ ; hence, measurement of multiple QNM frequencies in a ringdown waveform can be inverted to give multiple, independent estimates of  $(a, M)$
- In mergers, the entire waveform, including *the full spectrum* (amplitudes, phases) of all QNMs excited in the merger, plus non-linear effects, are uniquely determined by the small set of parameters describing the initial binary
  - here, use independent parts of the signal of GW150914 to reconstruct the mass and spin of the remnant, and check for consistency

# GW150914: The Zeroth-order Test

During the inspiral, how rapidly the signal sweeps up in frequency in time-frequency space can be used to compute the *chirp mass* of the binary :

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

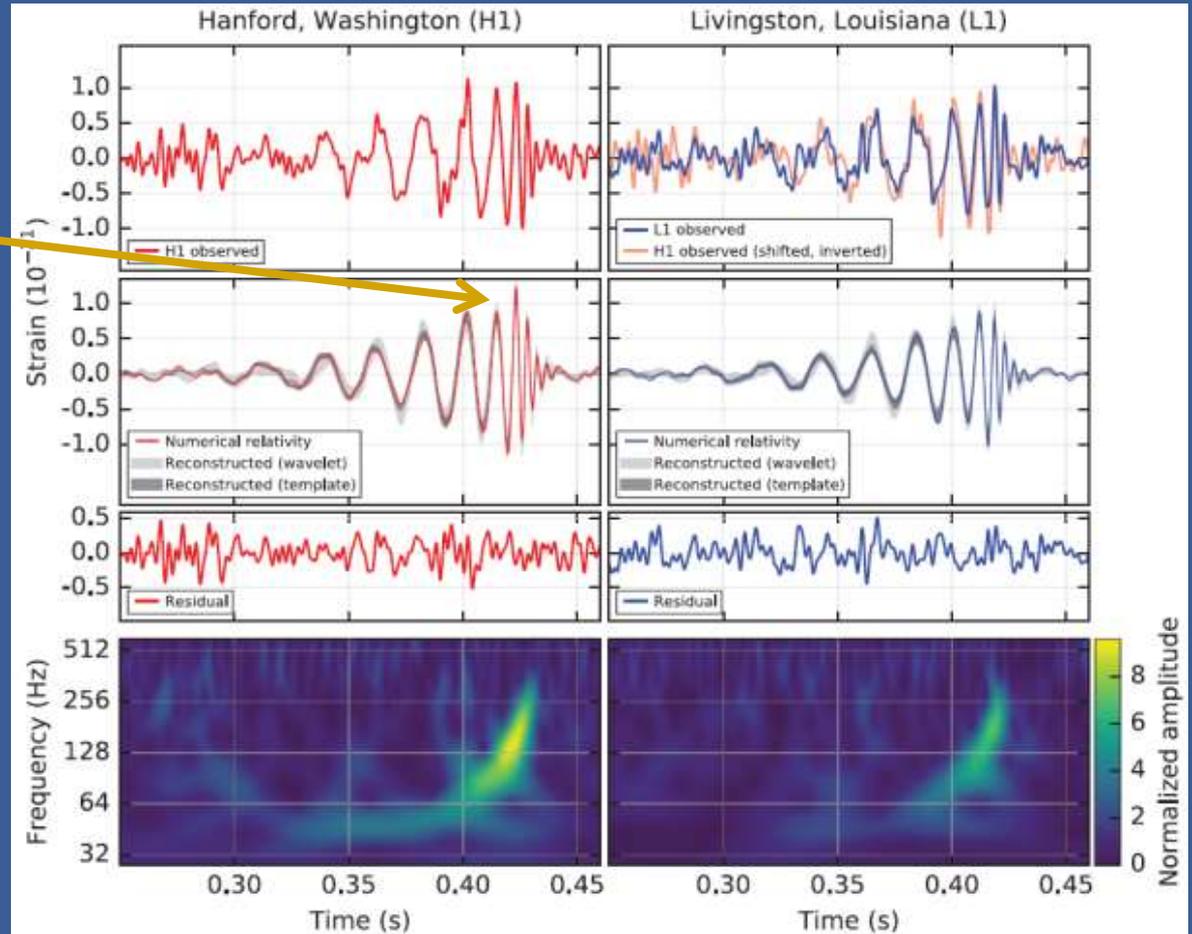


# GW150914: The Zeroth-order Test

The frequency roughly a cycle before peak amplitude sets the scale of the binary just before the black holes merge, and is a function of the total mass

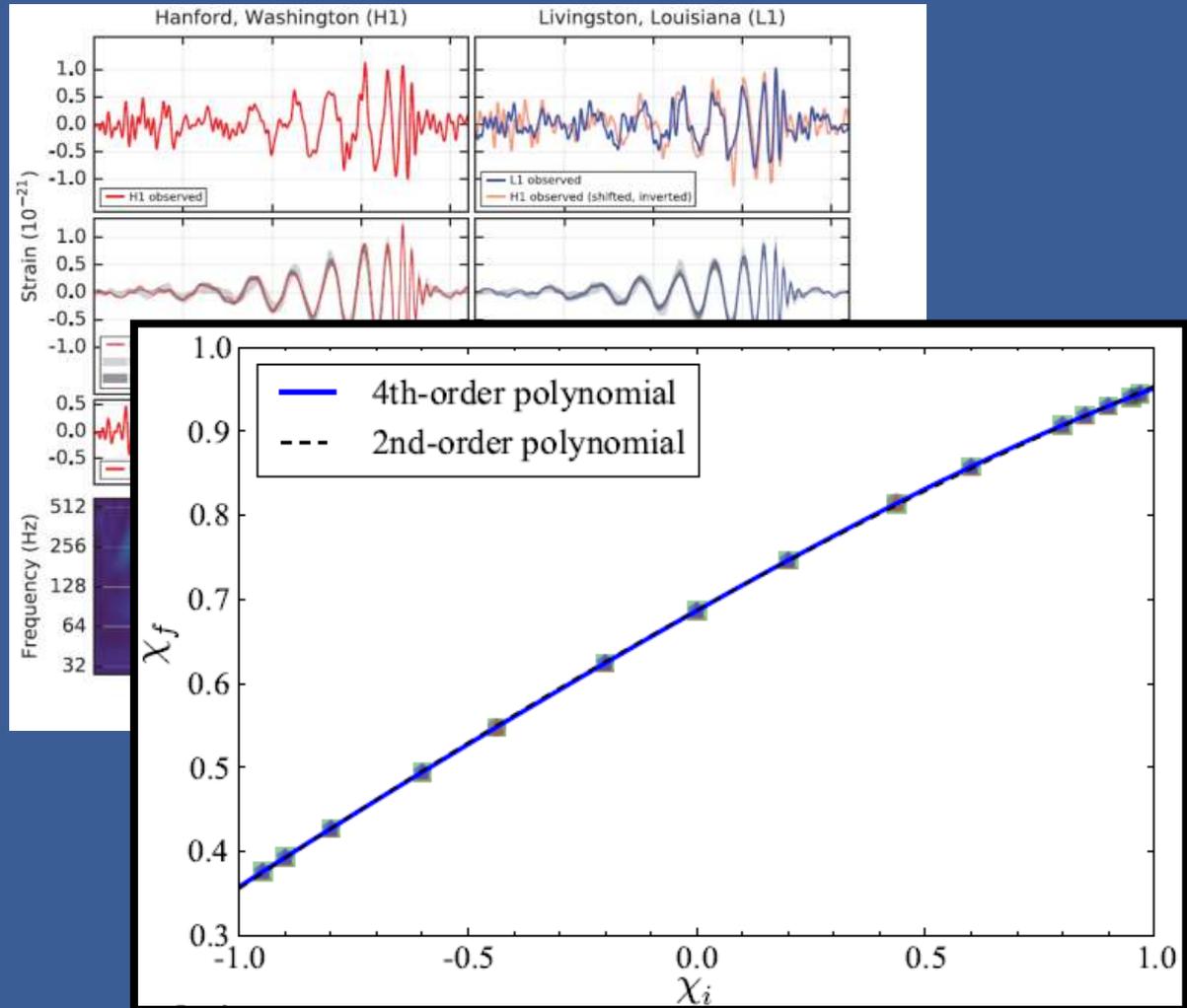
$$M = (m_1 + m_2)$$

Numerical solutions show a few % of the energy of the system is radiated after this point, so a mass derived from this frequency gives a good estimate of the remnant mass



# GW150914: The Zeroth-order Test

Not enough cycles above the noise floor prior to merger to get a good handle on the individual spins (through various spin-spin and spin-orbit interactions), though for comparable mass systems as these are, the orbital angular momentum prior to plunge offers the leading order contribution to the final spin of the remnant



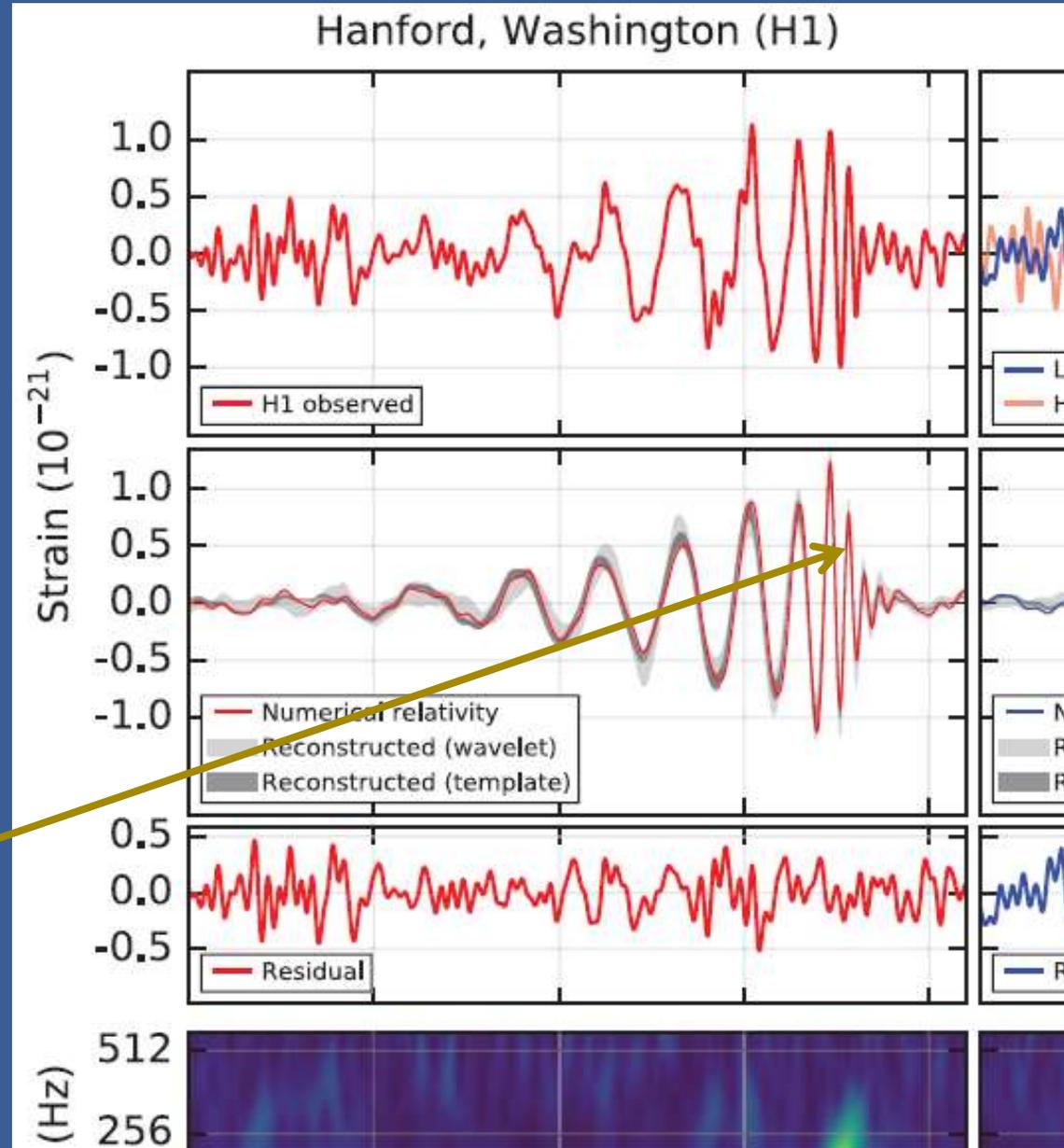
*Equal mass, non-precessing mergers,  
from D. Hemberger et al., PRD88 (2013)*

# GW150914: The Zeroth-order Test

The “late-time” GW emission of the merger remnant is dominated by quasi-normal ringdown modes

NR solutions show one is, to good approximation, within this regime almost immediately after peak amplitude is reached, and the signal is dominated by the least damped quadrupole ( $l=m=2$ ) mode

The observed frequency and decay time of the full signal gives a good proxy to this, and can be inverted to yield an independent estimate of the mass and spin of the remnant

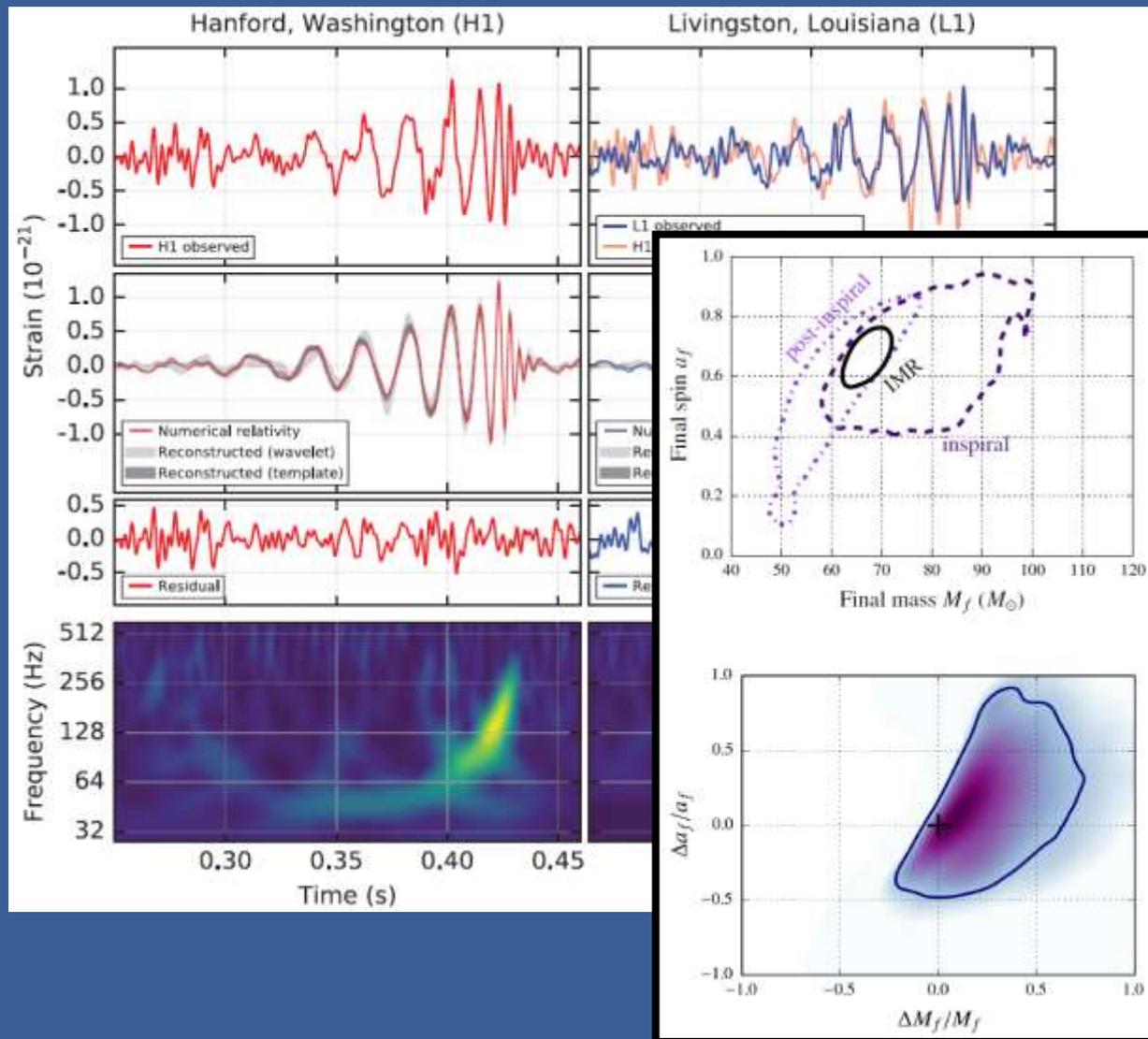


# GW150914: The Zeroth-order Test

We thus have estimates of the mass and the spin of the remnant from two distinct regimes of the event:

- The two-body inspiral
- The ringdown of the remnant to Kerr

Extracted  $(a, M)$  pairs are consistent with each other, albeit with large uncertainties



# Beyond Zeroth Order

- Obtaining additional constraints requires measuring sub-leading QNMs
  - higher  $(l,m)$  modes (and overtones) also probe shorter characteristic scales
  - unfortunately, the initial amplitudes excited in a comparable mass merger drop with  $(l,m)$ , as do the quality factors
  - With GW150914, the next sub-leading modes have  $\text{SNR} < 1$ ; expect to need an event similar to GW150914 with  $\sim 10$  times the SNR to directly measure a second QNM

# Beyond Zeroth Order

- How to go after subtle differences?
  - be patient : with time (next generation detectors and luck of a nearby event) we will get ever stronger signals, and ever stronger constraints
  - less patient : dig as deeply into the data as we can with novel analysis strategies
    - might not be as robust or give results with as high-confidence as traditional techniques, but would give earlier signs of new physics
    - **example** : *coherently stack multiple ringdown signals to search for higher order QNMs.*

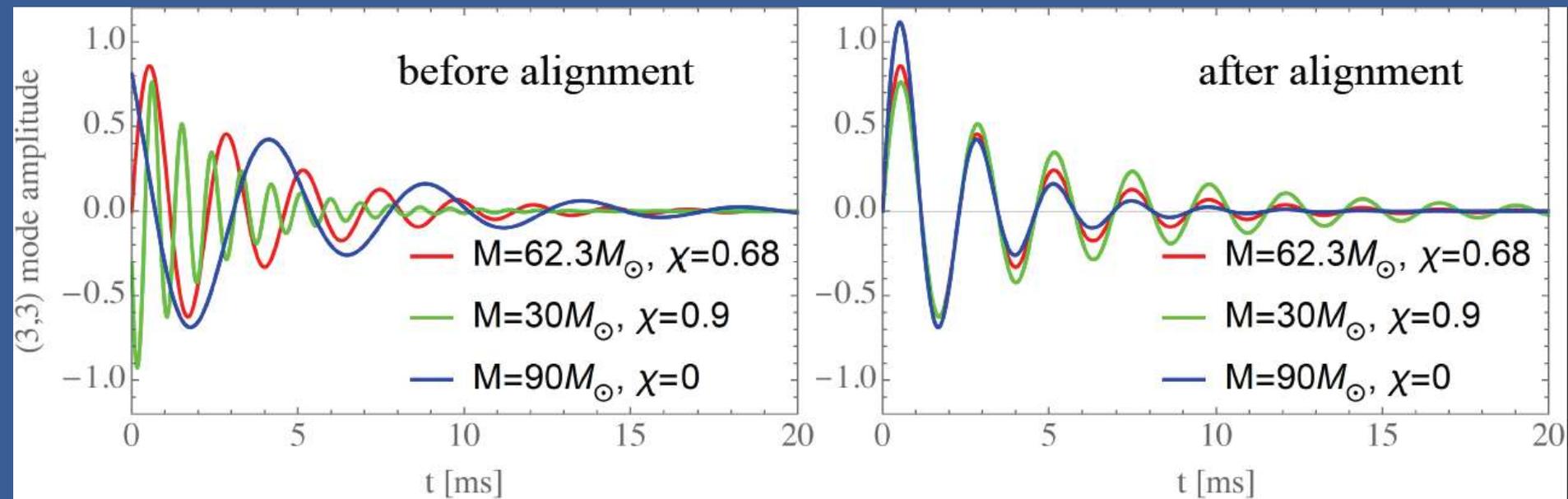
# Coherent mode stacking

*Work with H. Yang, K. Yagi, L. Lehner, V. Paschalidis,  
N. Yunes and J. Blackman, PRL 118 (2017)*

- The main issues preventing a “naïve” stacking of the ringdown signal from a population of mergers with different parameters (masses, spins, distances, source orientation) are :
  - Different remnants have different masses/spins, so the same  $(l,m)$  multiple modes will have different frequencies/decay times
  - Without additional information we do not know the phases of sub-leading modes, especially as we are targeting events where we do not expect these modes to have a large enough SNR to be detectable in isolation
    - without phase information one could implement incoherent (power) stacking, but that only achieves a theoretical maximum of  $[N]^{1/4}$  scaling improvement

# Coherent mode stacking

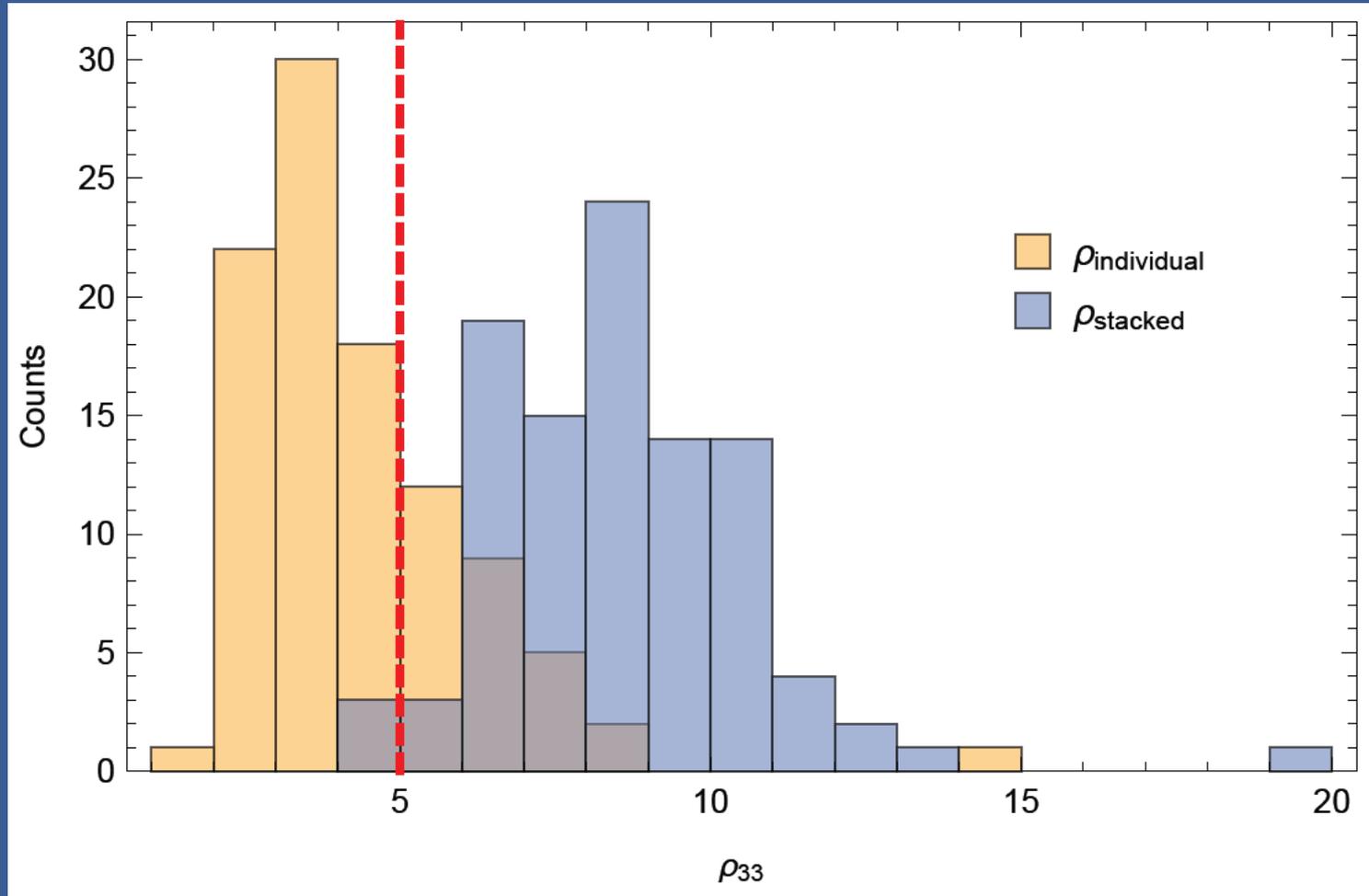
- To attempt to solve these issues, we do the following
  - Crucially, we restrict to the set of mergers where an inspiral *and* leading order ( $l=2, m=2$ ) mode are measurable
    - this allows us to measure the parameters of the binary with sufficient accuracy to allow a calculation (via numerical simulations, reduced basis models, etc. ) of the amplitudes and phases of all sub-leading modes
  - In the search we target one sub-leading mode per event, here the fundamental harmonic of the ( $l=3, m=3$ ) mode. We then scale/shift each signal by appropriate constants to phase and frequency align the target modes amongst all events



# Coherent mode stacking

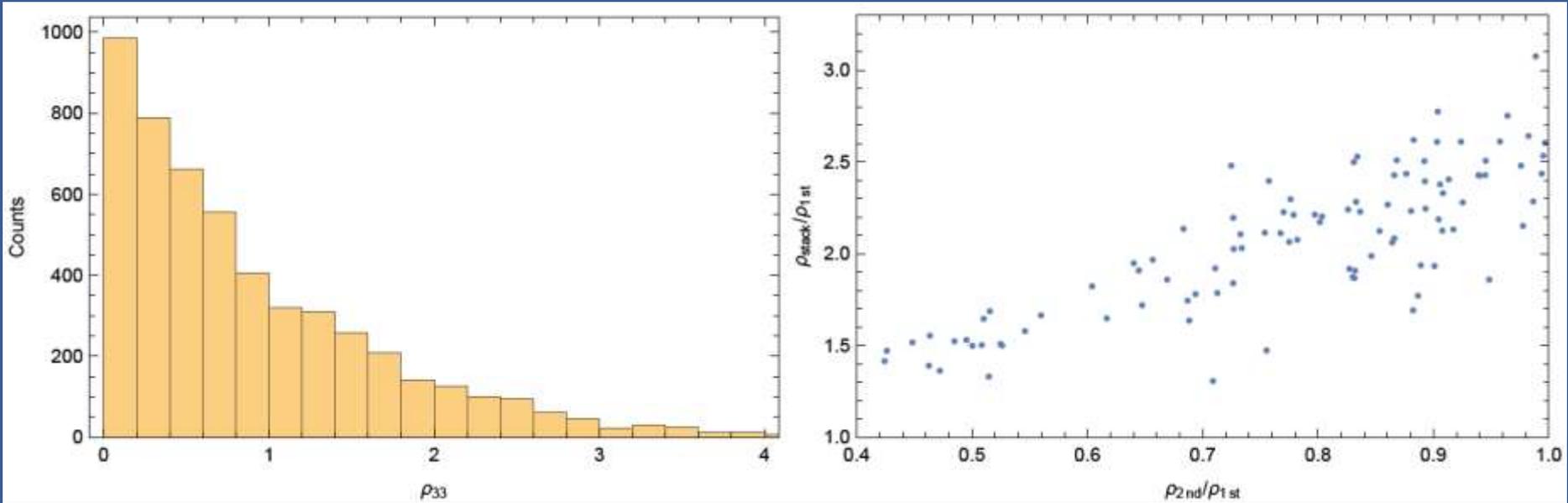
- This introduces a few additional complications, most notably
  - A parameter estimation “noise” coming from uncertainties extracting the parameters of each binary
  - We are adding *scaled* detector noise in the stacking
  - How to properly weight the different events in the sum as the population will not be homogeneous, in particular in SNR
- For this first “proof of principle” result, we do the following
  - Restrict to initially non-spinning black holes
  - Assume a uniform distribution of black hole masses from 10-50  $M_{\odot}$ , and the optimistic end of the merger rate of 40/Gpc<sup>3</sup>/yr
  - Only select events where the (2,2) mode by itself is detectable with SNR > 8 (in our 100 Monte Carlo runs there were 40-65 such events per year); and for now only stacking the 15 loudest
  - Assume parameter estimation noise that scales like 1/SNR, calibrated (for all) by that of GW150914
  - Use the “downhill simplex optimization” method to choose stacking weights to maximize the SNR

# Results



- Counts from 100 Monte Carlo simulations of 1 year of detections at AdLIGO design sensitivity : 30% chance for detection of  $(3,3)$  mode from single loudest event, 97% chance from stacked signals

# Results



- Left : Histogram of  $(3,3)$  mode SNR for all events in all year-simulations
- Right : Ratio of the stacked  $(3,3)$  mode SNR to the single loudest in each year of simulated data, plotted vs the ratio of the SNR of the second loudest to loudest events.
  - Demonstrates we are not achieving the theoretical maximum of  $\sqrt{N} \sim 4$ , due largely to the inhomogeneous distribution of events, and hints that we get the most enhancement if we have a few loud outliers
  - Still, even in the worst cases get some improvement

# Testing General Relativity with Coherent Stacking

- Once we have a set of merger events where GR predicts that the stacked signal should be detectable, failure to see it will show something is wrong in the assumptions leading to the predicted SNR
  - here, most crucially is the assumption that the full spectrum of QNMs excited in a merger is uniquely governed by the small set of parameters describing the binary, which holds in GR precisely because of the final state conjecture
  - Of course, this would not “discover” the source of the problem, but only point to a problem

# Conclusions

- We are all eagerly anticipating more events from ground based GW detectors, including those with EM counterparts; Can anticipate data in 3 broad categories
  - **Statistical :  $O(100)$  binary merger events**
    - start to search for small, systematic deviations from GR from the collection of inspirals
    - gain evidence for/against speculative scenarios, such as the existence of ultra-light scalars that spin-down stellar mass black holes [the “axiverse”], observably bright EM counterparts to binary BH mergers, etc.
    - Use novel approaches like stacking to get more information from a population of events; preliminary results for targeting sub-leading QNM from BH mergers promising
  - **Loud : an SNR  $O(100)$  event**
    - higher precision tests of GR/discovery of strong-field deviations, e.g. certain resolutions of the black hole information paradox/fire-wall problem propose macroscopic near-horizons deviations from classical physics [see e.g. Giddings *arXiv:1602.03622*], though because these proposals do not yet make concrete predictions will need a signal loud enough to give a measurable residual from the purely classical prediction
  - **Rare :**
    - low probability events [eccentric mergers, large mass ratios, near extremal spins, etc.] that may be more sensitive to certain kinds of strong-field deviations