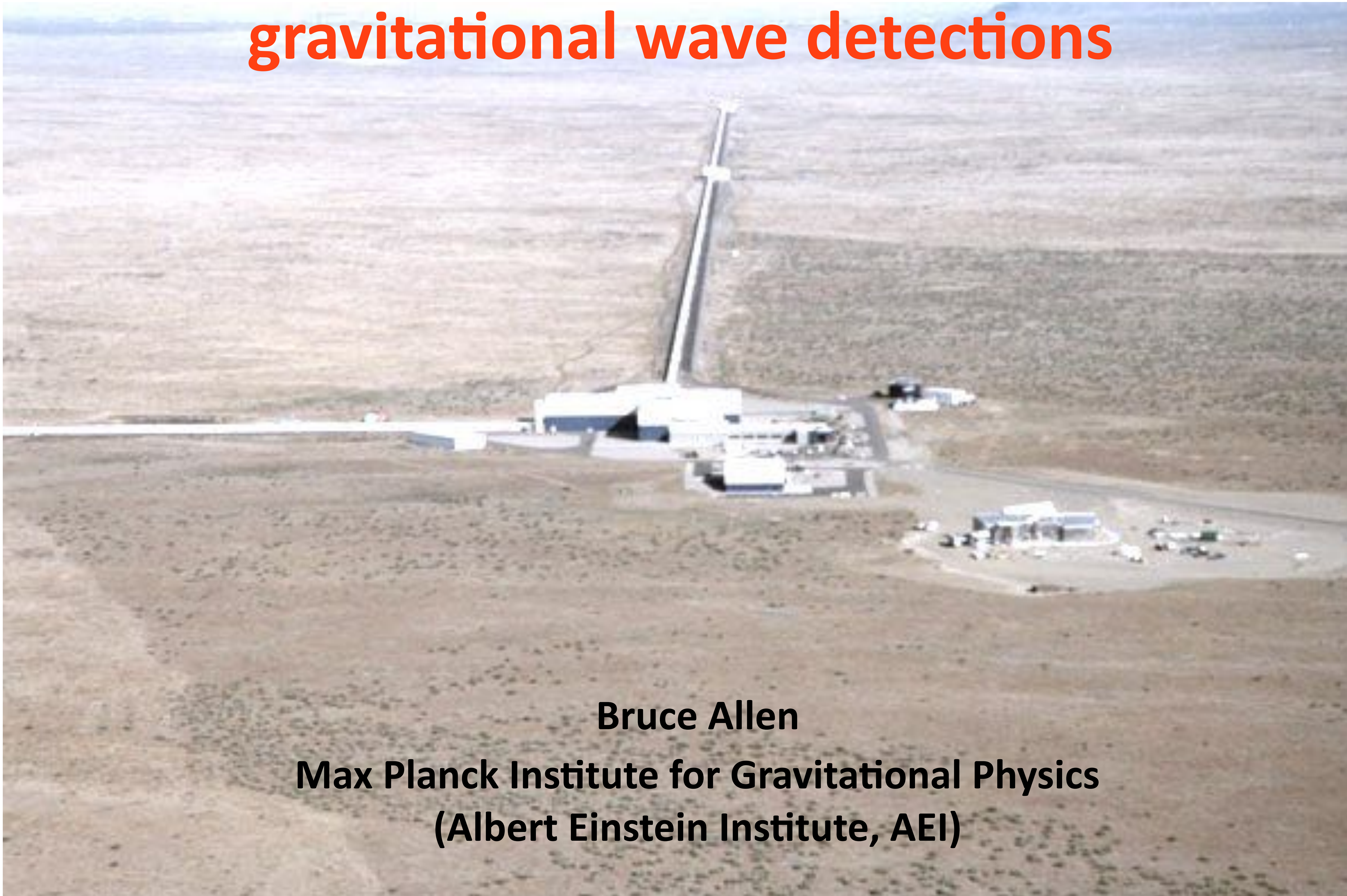
Stephen, Gary, and the first gravitational wave detections



Bruce Allen
Max Planck Institute for Gravitational Physics
(Albert Einstein Institute, AEI)

MIT Undergraduate 1976-80

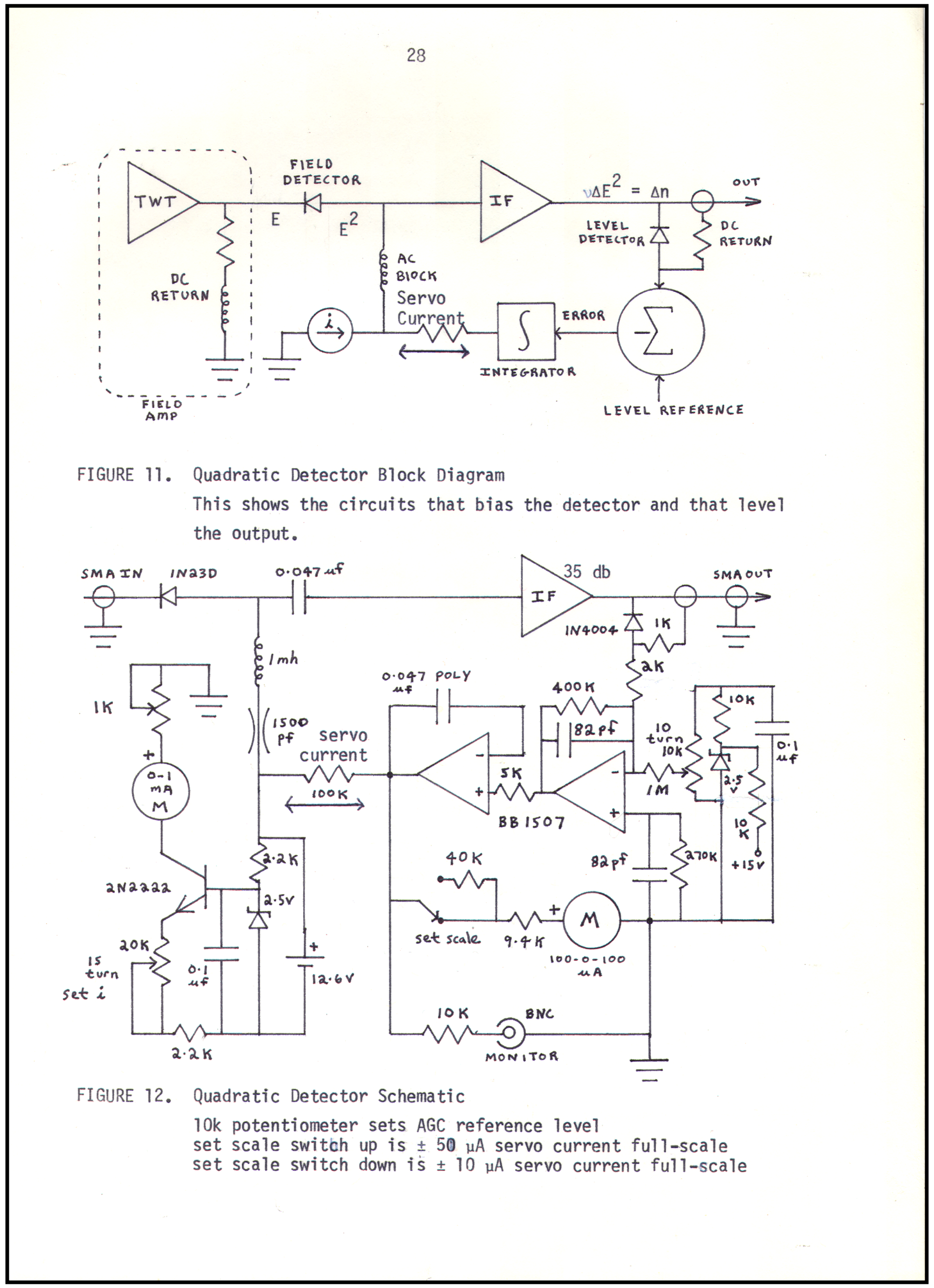


FIGURE 11. Quadratic Detector Block Diagram
This shows the circuits that bias the detector and that level the output.

FIGURE 12. Quadratic Detector Schematic
10k potentiometer sets AGC reference level
set scale switch up is ± 50 μA servo current full-scale
set scale switch down is ± 10 μA servo current full-scale

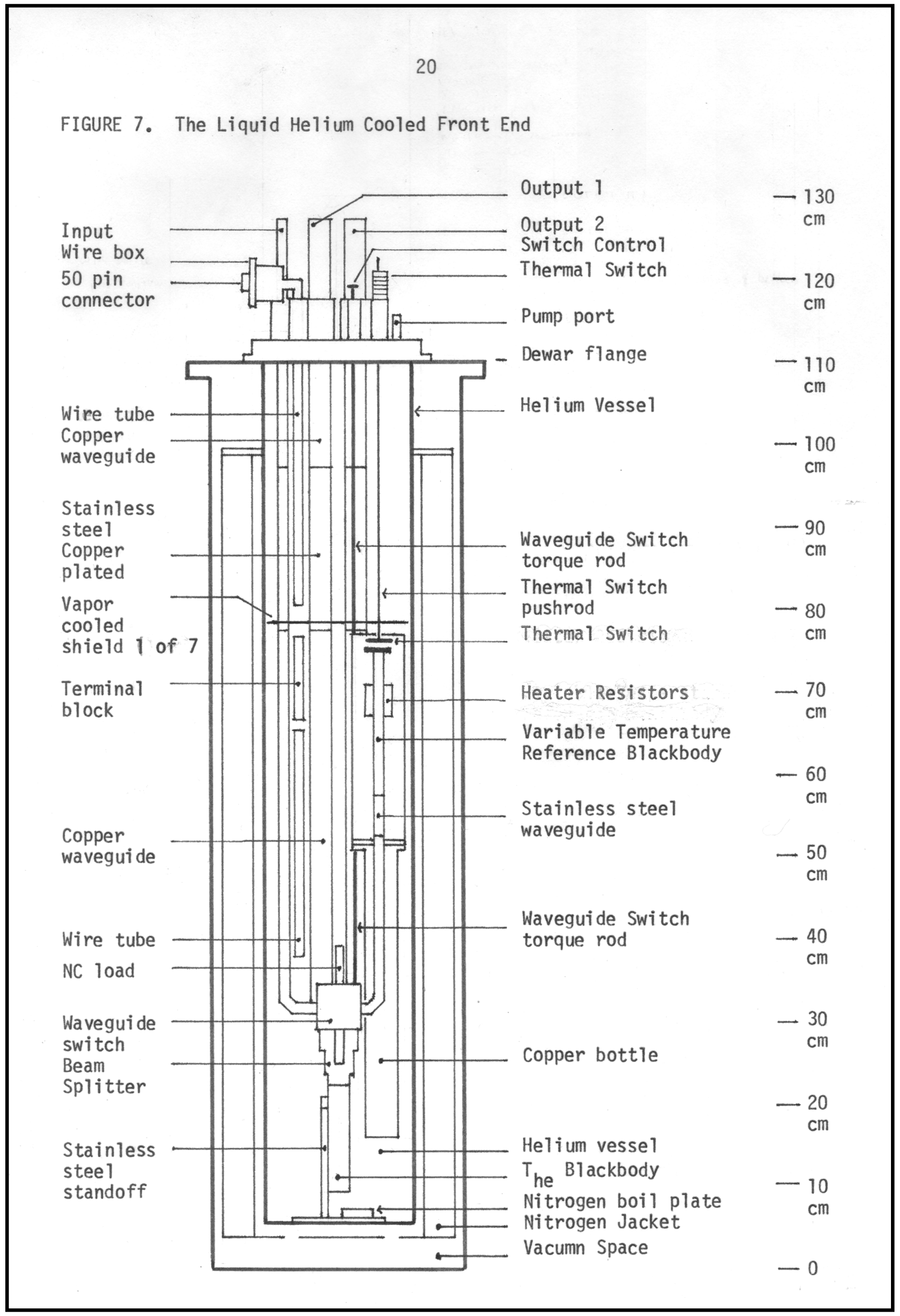
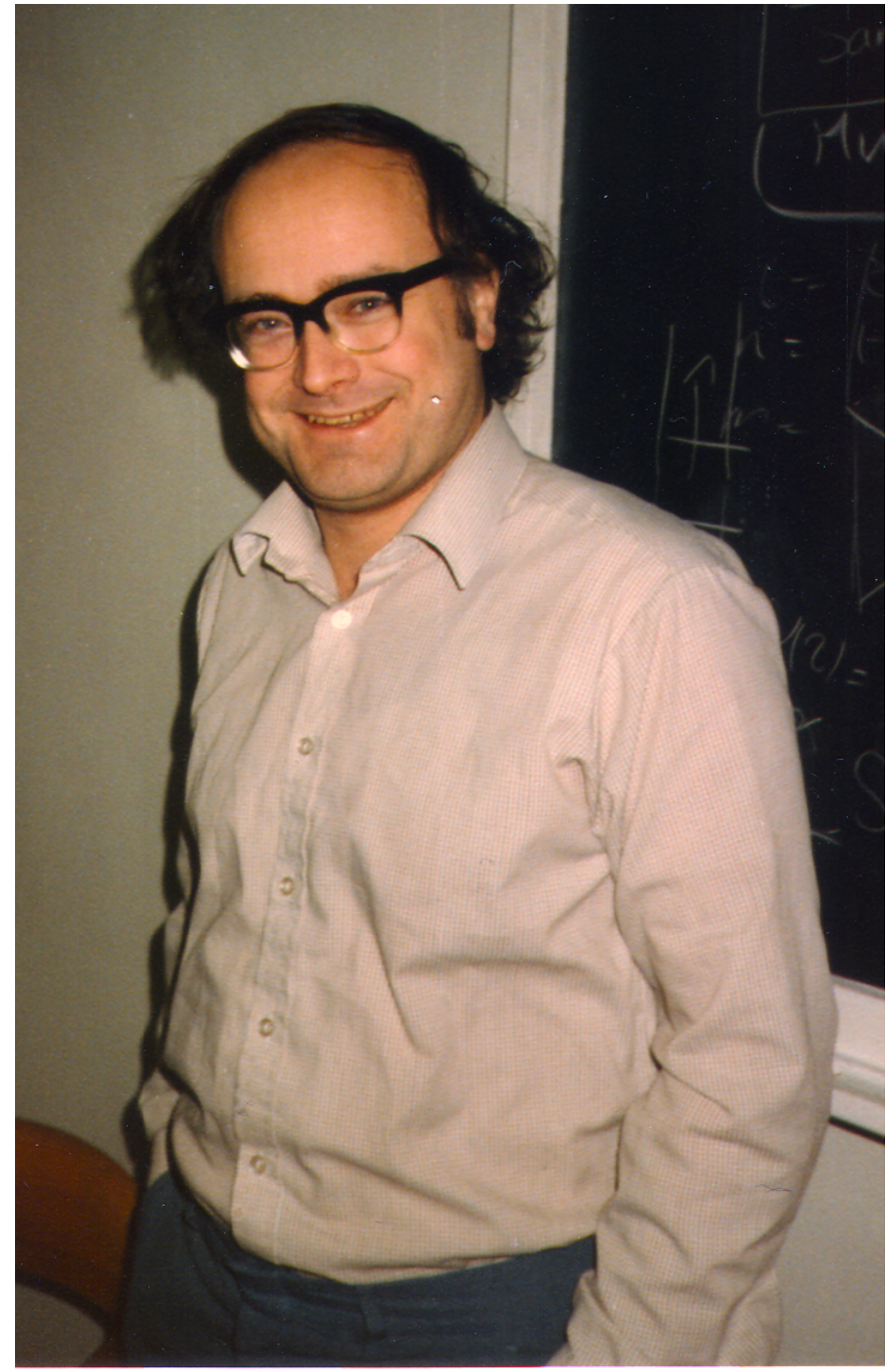


FIGURE 7. The Liquid Helium Cooled Front End

DAMTP
October 1980

Undergrad thesis, June 1980, CMB experiment

Cambridge 1980-83



Weber GW "detection"

EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

J. Weber
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 29 April 1969)

Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.

Some years ago an antenna for gravitational radiation was proposed.[1] This consists of an elastic body which may become deformed by the dynamic derivatives of the gravitational potentials, and its normal modes excited. Such an antenna measures, precisely, the Fourier transform of certain components of the Riemann curvature tensor, averaged over its volume. The theory has been developed rigorously, starting with Einstein's field equations to deduce[2] equations of motion. Neither the linear approximation nor the energy-flux relations are needed to describe these experiments, but their use enables discussion in terms of more familiar quantities. All aspects of the antenna response and signal-to-noise ratio can be written in terms of the curvature tensor. The theory was verified experimentally by developing a high-frequency source[3] and producing and detecting dynamic gravitational fields in the laboratory.

Several programs of research are being car-

array is a new set of windows for studying the universe.

Search for gravitational radiation in the vicinity of 1660 Hz.—A frequency in the vicinity of 1660 Hz was selected because the dimensions are convenient for a modest effort and because this frequency is swept through during emission in a supernova collapse. It was expected that once the technology was refined, detectors could be designed for search for radiation from sources with radio or optical emission, such as the pulsars. A knowledge of the expected frequency and Q of a source enormously increases the probability of successful search.

However, occasional signals were seen at 1660 Hz and small numbers of coincidences were observed on detectors[7,8] separated by a few kilometers. To explore these phenomena further, larger detectors were developed. One of these is now operating at Argonne National Laboratory.

Weber GW "detection"

GRAVITATIONAL RADIATION EXPERIMENTS*

J. Weber
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 8 September 1969)

A summary is given of the statistics and coincidences of the Argonne-Maryland gravitational-radiation-detector array. New experiments have been carried out. These include a parallel coincidence experiment in which one coincidence detector had a time delay in one channel and a second coincidence detector operated with no time delays. Other experiments involve observations to rule out the possibility that the detectors are being excited electromagnetically. These results are evidence supporting an earlier claim that gravitational radiation is being observed.

An earlier Letter[1] described an experiment involving coincidences of gravitational radiation detectors at Argonne National Laboratory and the University of Maryland. This is the first experiment which tests directly the dynamics of gravitational fields. The results may have significance for physics, astronomy, and cosmology. Further experiments were therefore carried out to verify claims that the coincidences were not all accidental and that neither seismic nor electromagnetic effects were causing them.

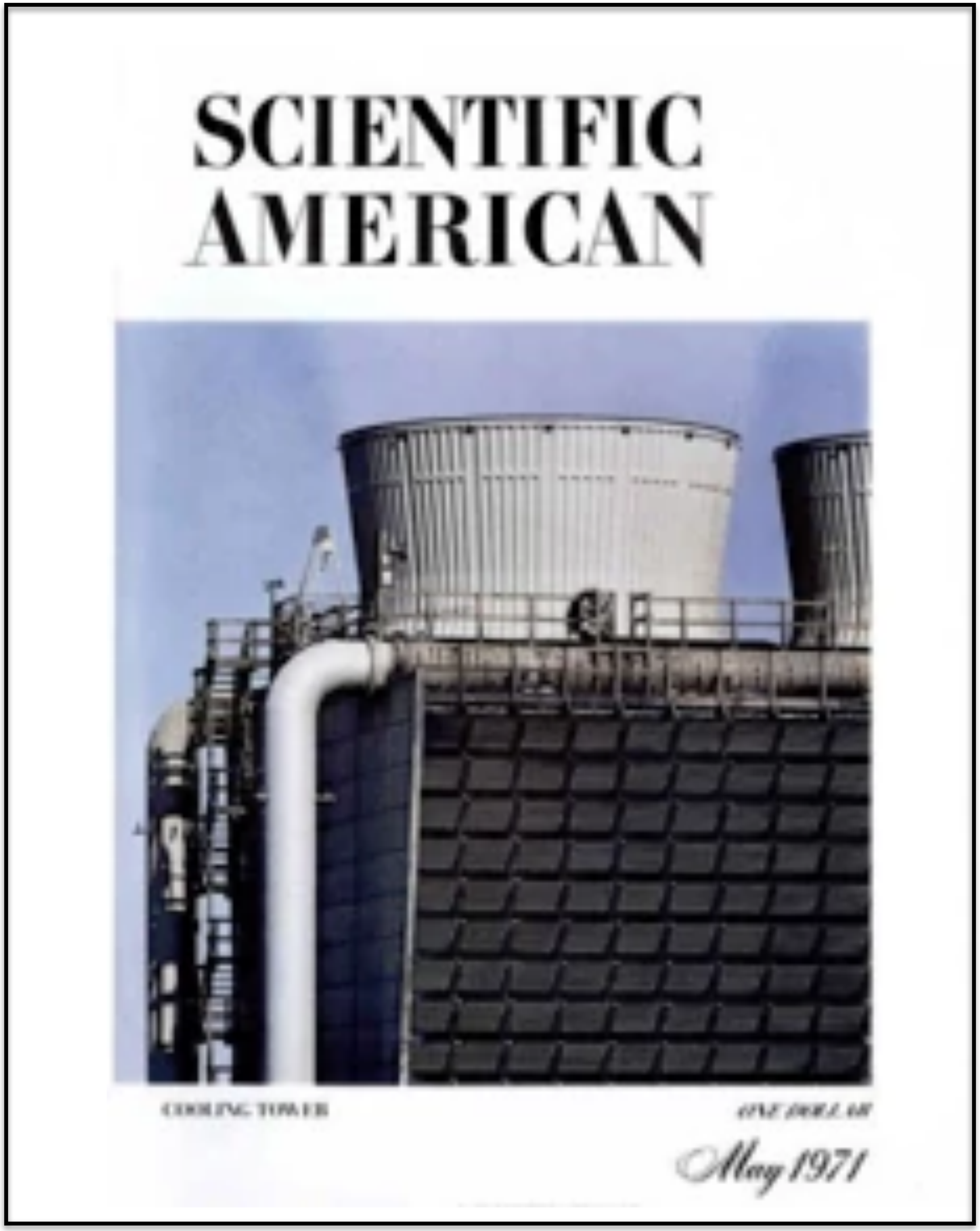
Statistics.—Each gravitational-radiation-detector voltage output consists primarily of the thermal fluctuations of the suspended cylinder's lowest frequency compressional mode. A coincidence is recorded if the output voltages of two detectors cross some arbitrarily set threshold in the positive direction within some small time interval Δt. A classification scheme is set up for the coincidences. For each class the number of observed coincidences is compared with the expected number of accidental ones. A signifi-

Phys. Rev. Lett. 24, 276, 1970

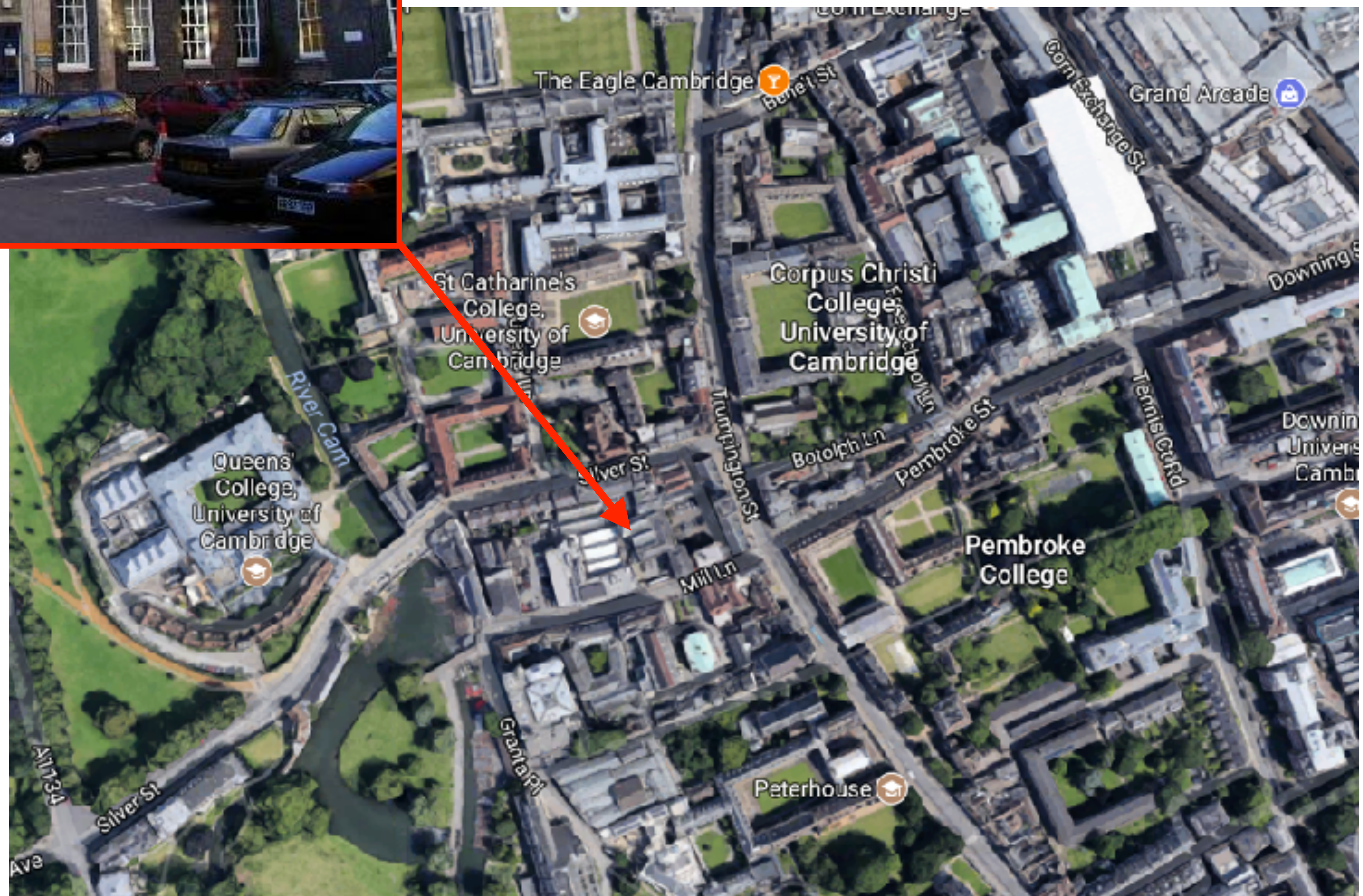
Weber GW “detection”

The Detection of Gravitational Waves, by Joseph Weber

The existence of such waves is predicted by the theory of relativity. Experiments designed to detect them have recorded evidence that they are being emitted in bursts from the direction of the galactic center”



DAMTP
Silver Street



PHYSICAL REVIEW D VOLUME 4, NUMBER 8 15 OCTOBER 1971

Theory of the Detection of Short Bursts of Gravitational Radiation

G. W. Gibbons
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, England

and

S. W. Hawking
Institute of Theoretical Astronomy, University of Cambridge, England
(Received 30 November 1970)

It is argued that the short bursts of gravitational radiation which Weber reports most probably arise from the gravitational collapse of a body of stellar mass or the capture of one collapsed object by another. In both cases the bulk of the energy would be emitted in a burst lasting about a millisecond, during which the Riemann tensor would change sign from one to three times. The signal-to-noise problem for the detection of such bursts is discussed, and it is shown that by observing fluctuations in the phase or amplitude of the Brownian oscillations of a quadrupole antenna one can detect bursts which impart to the system an energy of a small fraction of kT. Applied to Weber's antenna, this method could improve the sensitivity for reliable detection by a factor of about 12. However, by using an antenna of the same physical dimensions but with a much tighter electromechanical coupling, one could obtain an improvement by a factor of up to 250. The tighter coupling would also enable one to determine the time of arrival of the bursts to within a millisecond. Such time resolution would make it possible to verify that the radiation was propagating with the velocity of light and to determine the direction of the source.

I. INTRODUCTION

In this paper we discuss the problem of detecting short bursts of gravitational radiation. This is rather different from the detection of continuous

like Weber's, which has a very low electromechanical coupling, this method would improve the sensitivity for reliable detection by a factor of about 12. However, by using a detector consisting of two metal bars connected by a piezoelectric trans-

PRD 4, 2191–2197 (1971) cited 64 times: **Astone**, Billing, **Blair**, Caves, Dewey, **Drever**, **Hamilton**, **Hough**, Isaacson, Lobo, Michelson, Misner, Pizzella, Press, Ruffini, **Sathyaprakash**, **Saulson**, **Schutz**, **Thorne**, Trimble, **Vinet**, Weber, Winkler

- Merger of “collapsed objects” and “neutron stars”. Does not contain the words “black hole”
- Correct time-scales and energy estimates (msec per solar mass) when objects approach O(Schwarzschild radius)
- Concept of matched filtering (not with that name) to “dig into the noise”. x12 better sensitivity
- Precision of arrival-time determination, use of triangulation to determine direction to source
- Does not mention orbital behaviour (head-on collision?)
- Amusing typos (“Earth orbiting around the sun radiates 1kW at a frequency of 3 cycles/year.”)

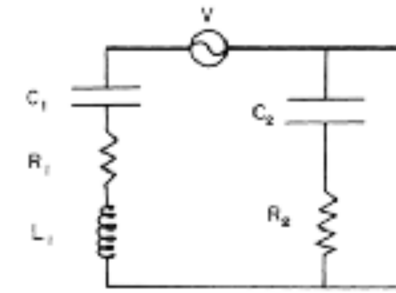


FIG. 4. The equivalent circuit of the detector and transducer.

$$L_1 = (\omega_0^2 \beta C_2)^{-1},$$
$$C_1 = \beta C_2,$$
$$R_1 = (Q\omega_0 \beta C_2)^{-1},$$

and

$$V = -c^2 l R_{1010}(2\omega_0)^{-1} m^{1/2} (\beta C_2)^{-1/2}.$$

The equivalent circuit of the detector and transducer is therefore given by Fig. 4.[13]

We shall assume that the output of this equivalent circuit is fed into a high-impedance amplifier with gain A and that the amplifier output is divided in the ratio Z_3/Z_4 (Fig. 5). In this figure Z_1 represents the impedance of the series $L_1C_1R_1$ and Z_2 represents the impedance of the series C_2R_2. The impedances Z_3 and Z_4 are chosen as $Z_3 = D(Z_1+Z_2)^{-1}$ and $Z_4 = HZ_2^{-1}$, where D and H are constant with $D >> H$. The Z_3, Z_4 circuit "undoes" the effect of the resonance of the detector and gives an output signal voltage

$$V_S = AHD^{-1} V$$
$$= -AHD^{-1} c^2 l R_{1010}(Z\omega_0)^{-1} m^{1/2} (\beta C_2)^{-1/2}.$$

It is not necessary to use such an inverse circuit but it is convenient for discussing the signal/noise ratio. The impedances Z_3 and Z_4 could be realized physically by a parallel LCR circuit and a parallel LR circuit respectively, though one might need to use a superconducting inductance in Z_3 to obtain a sufficiently high Q. It would probably be more convenient to simulate the Z_3, Z_4 circuit electronically.

Superimposed on the output signal voltage will be the Johnson noise produced by R_1 which represents the Brownian noise of the detector. This will produce at the output a flat noise spectrum with a mean-square voltage

$$V_B^2 = (AHD^{-1})^2 2kT\tau^{-1}(Q\omega_0\beta C_2)^{-1}$$

per unit bandwidth. The transducer noise produced by the resistance R_2 will give at the output a mean-

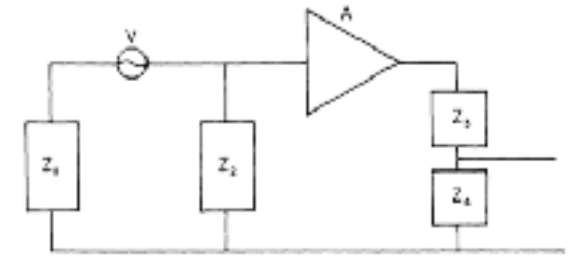


FIG. 5. The equivalent circuit for discussing the signal/noise ratio.

square voltage of

$$V_T^2 = (AHD^{-1})^2 2kT\tau^{-1} \tan\delta\, (\omega_0 C_2)^{-1} |Z_1|^2 |Z_2|^{-2}$$
$$= (AHD^{-1})^2 2kT\pi^{-1} \tan\delta\, (\omega_0 C_2)^{-1} (\beta\omega_0)^{-2}$$
$$\times [\omega^2 Q^{-2} + \omega_0^{-2}(\omega^2 - \omega_0^2)^2]$$

per unit bandwidth. This has a sharp minimum at the resonant frequency ω_0. The noise produced by the amplifier will have a rather similar spectrum at the output. Using modern techniques, it seems possible to reduce the amplifier noise below the transducer noise and it will be neglected.

Suppose now that the output of the circuit in Fig. 5 is fed into a filter of bandwidth $\Delta\omega$. If the signal is of the form suggested in Sec. 2, i.e., a burst of one to three cycles, its Fourier transform will have a maximum at a frequency ω_1 of the order of $2\pi\tau^{-1}$ and a half-width of the same order. Therefore, if the filter pass band is centered at ω_1, the *amplitude* of the transmitted signal will be $V_S \Delta\omega\omega_1^{-1}$ and the *power* will be $V_S^2(\Delta\omega)^2\omega_1^{-2}$ for $\Delta\omega \ll \omega_1$. This behavior distinguishes short bursts from continuous incoherent radiation where the power is proportional to $\Delta\omega$. It is the reason why it is desirable to use a fairly large value of $\Delta\omega$, i.e., good time resolution.

If the resonant frequency ω_0 is chosen to be equal to ω_1,[14] the filter will transmit a Brownian noise power approximately equal to

$$(AHD^{-1})^2 2kT\pi^{-1}(Q\omega_0\beta C_2)^{-1}\Delta\omega$$

and a transducer noise power

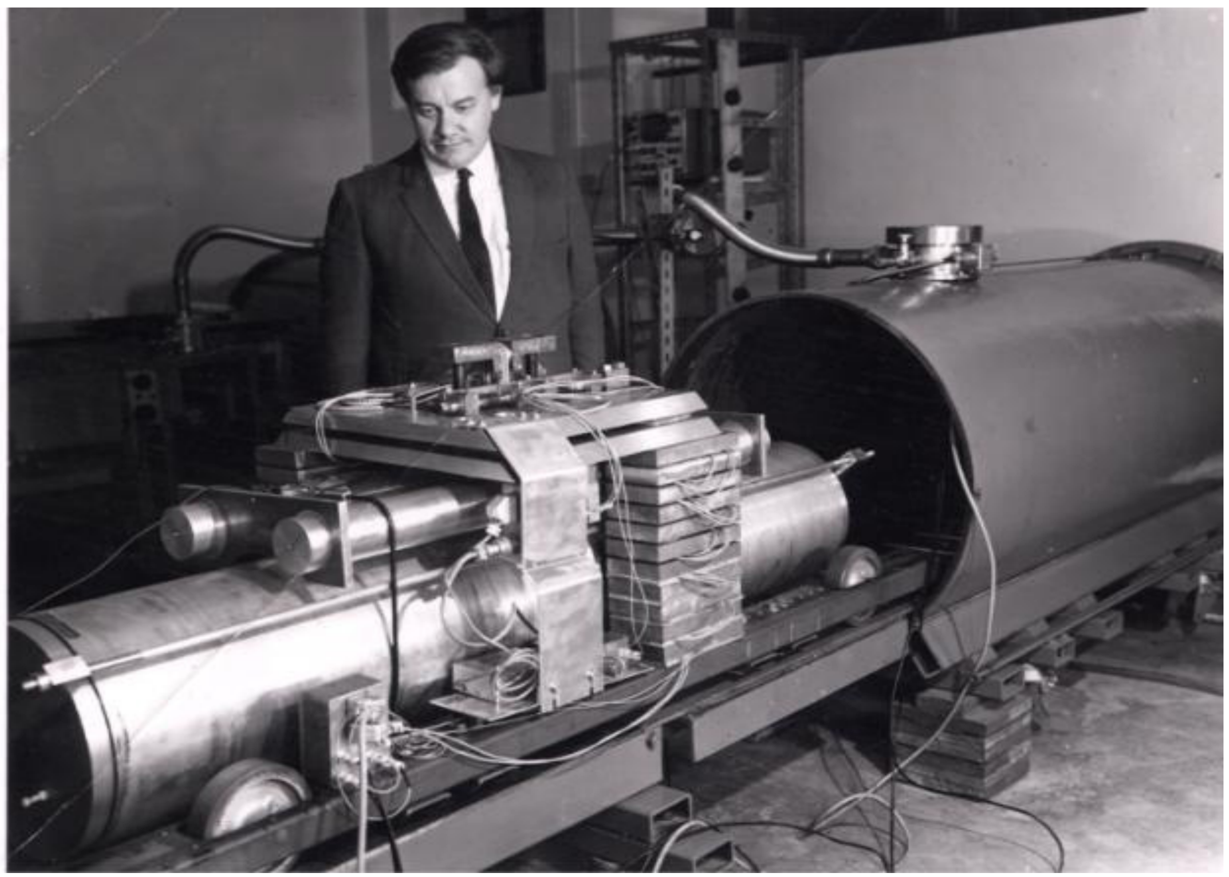
$$(AHD^{-1})^2 kT(3\pi)^{-1} \tan\delta\, \beta^{-2} C_2^{-1} (\Delta\omega)^3 \omega_0^{-2}.$$

The optimum value of $\Delta\omega$ will be the smaller of ω_0 and the value for which the transmitted noise power equals the transmitted transducer noise power. This gives

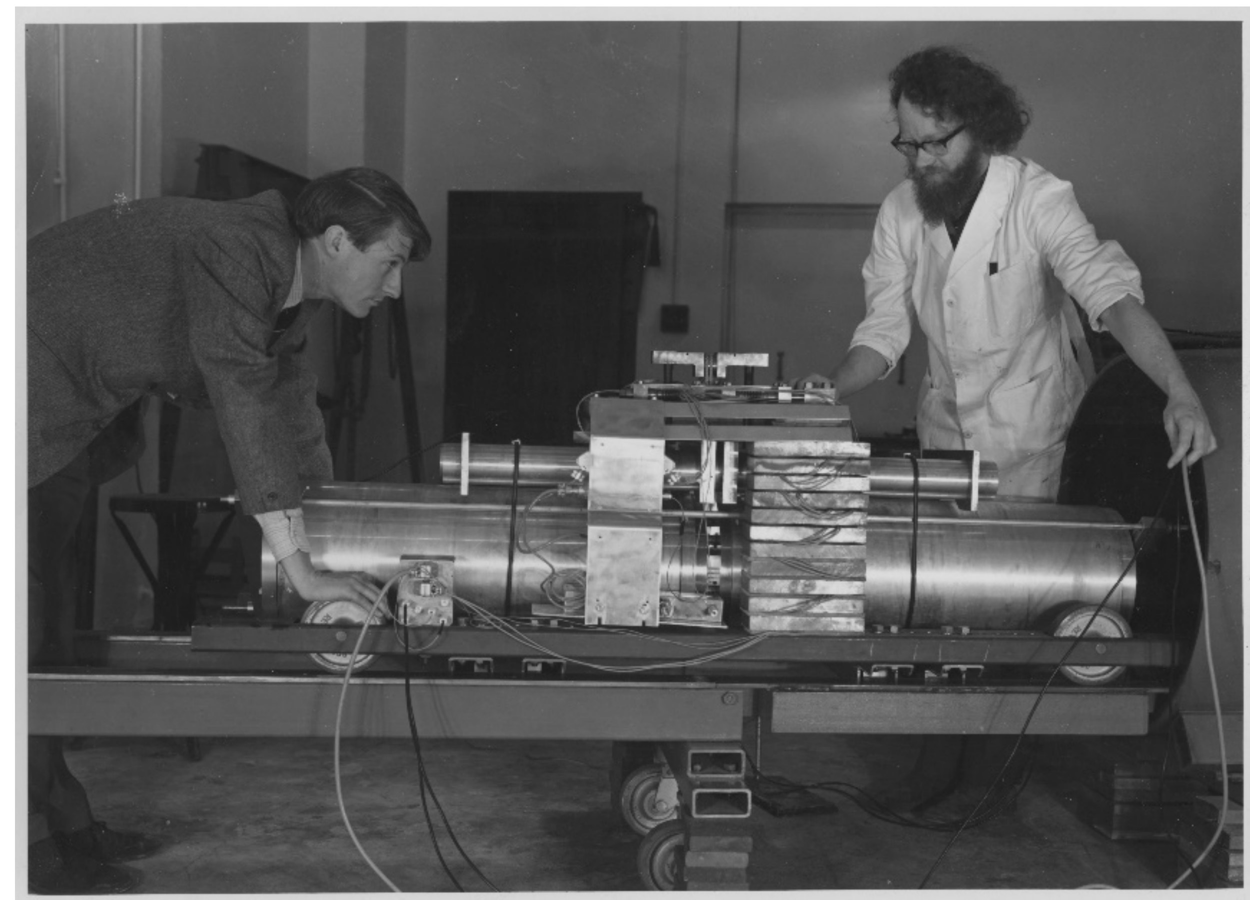
$$(\Delta\omega\omega_0^{-1})^2 = 6\beta(Q\tan\delta)^{-1}$$

which agrees almost exactly with Eq. (13). With this value of $\Delta\omega$ one could detect against the noise a short burst in which the amplitude of R_{1010} was

Glasgow, 1971



Ron Drever



Jim Hough (L) and Stuart Cherry (R)

Hawking's Area Theorem PRL 21, 1344 (1971)

Gravitational Radiation from Colliding Black Holes

S. W. Hawking
Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England
(Received 11 March 1971)

It is shown that there is an upper bound to the energy of the gravitational radiation emitted when one collapsed object captures another. In the case of two objects with equal masses m and zero intrinsic angular momenta, this upper bound is $(2-\sqrt{2})m$.

Weber[1-3] has recently reported coinciding measurements of short bursts of gravitational radiation at a frequency of 1660 Hz. These occur at a rate of about one per day and the bursts appear to be coming from the center of the galaxy. It seems likely[3,4] that the probability of a burst causing a coincidence between Weber's detectors is less than $\frac{1}{10}$. If one allows for this and assumes that the radiation is broadband, one finds that the energy flux in gravitational radiation must be at least 10^{10} erg/cm^2 day.[4] This would imply a mass loss from the center of the galaxy of about $20\,000 M_\odot$/yr. It is therefore possible that the mass of the galaxy might have been considerably higher in the past than it is now.[5] This makes it important to estimate the efficiency with which rest-mass energy can be converted into gravitational radiation. Clearly nuclear reactions are insufficient since they release only about 1% of the rest mass. The efficiency might be higher in either the nonspherical gravitational collapse of a star or the collision and coalescence of two collapsed objects. Up to now no limits on the efficiency of the processes have been known. The object of this Letter is to show that there is a limit for the second process. For the case of two colliding collapsed objects, each of mass m and zero angular momentum, the amount of energy that can be carried away by gravitational or any other form of radiation is less than $(2-\sqrt{2})m$.

I assume the validity of the Carter-Israel conjucture[6,7] that the metric outside a collapsed object settles down to that of one of the Kerr family of solutions[8] with positive mass m and angular momentum a per unit mass less than or equal to m. (I am using units in which $G=c=1$.) Each of these solutions contains a nonsingular *event horizon*, two-dimensional sections of which are topographically spheres with area[9]

$$8\pi m[m+(m^2-a^2)^{1/2}]. \tag{1}$$

The event horizon is the boundary of the region of space-time from which particles or photons can escape to infinity. I shall consider only

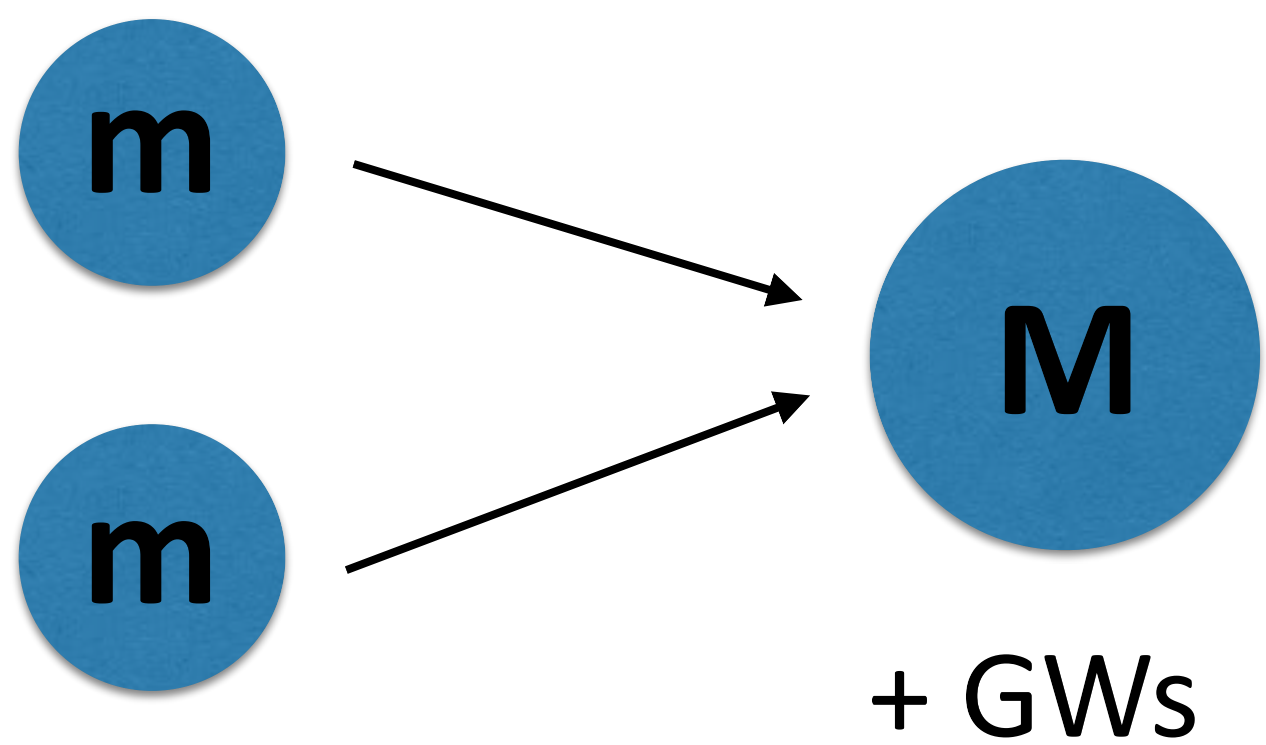
Hawking's Area Theorem PRL 21, 1344 (1971)

Non-spinning area $A = 4\pi r_S^2 = 16\pi m^2$

$A_1 + A_2 \leq A_3$

Saturate: $2m^2 = M^2$

Efficiency:
$(2m - M)/2m =$
$(2 - \sqrt{2})/2 =$
29.3 % of energy in GWs



New Scientist, 11.12.1975

No gravity waves to be found in Berkshire

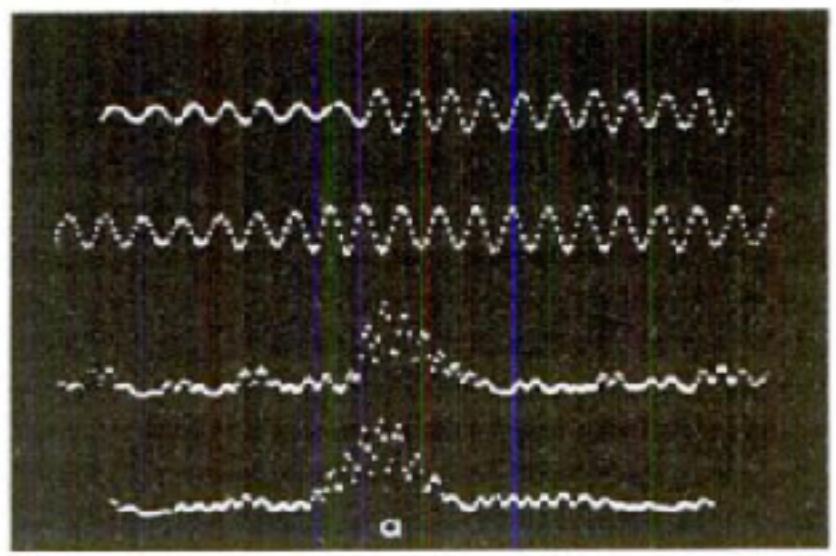
Jo Weber's claim, made first in 1969, that he has detected and continues to detect gravitational waves has suffered a further blow. For another experiment, this time by Dr C. Christodoulides and Professor W. D. Allen of Reading University and the Rutherford Laboratory, has failed to detect the radiation.

Published in a recent issue of *Journal of Physics A* (vol 8, p 1726) Allen and Christodoulides' results support those of the other groups that also began work after Weber's initial announcement—the groups at Bell Laboratories, Glasgow, Moscow, and Munich.

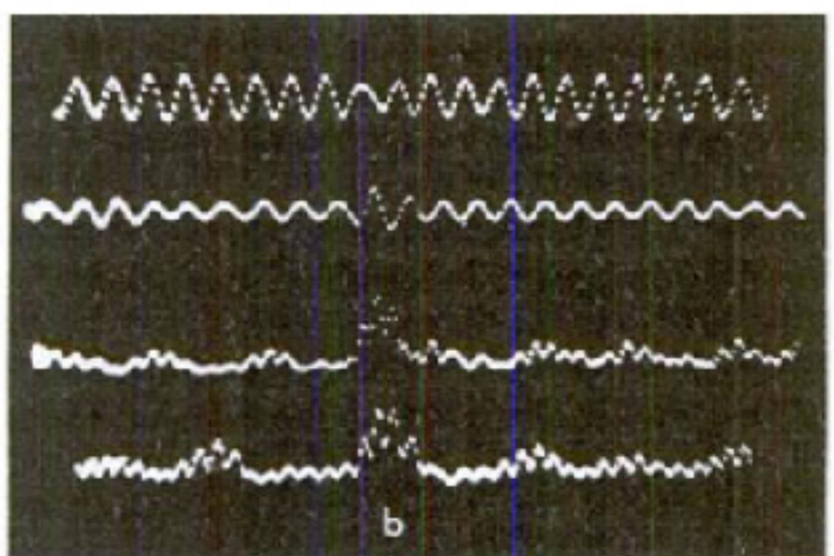
Each of Allen's and Christodoulides' two detectors, one of which was set up at the Rutherford Laboratory, the other at Reading, followed a design (also employed by the Glasgow group) due to Dr Peter Aplin of Bristol University. The detectors are made from 1·5 m long cylinders sliced in two across their axes, the two halves being joined by piezoelectric transducers bonded between them. In this position the transducers very efficiently transform ringing along the axes of the cylinders into electrical signals.

The electrical output from the ringing of the detectors is shown in the figures below. In each figure the output of the Rutherford pair of cylinders makes the first trace; the Reading pair makes the second. Each cycle of the ringing takes about 0·8 msec. The traces are of an amplitude corresponding to the thermal energy ½kT of each ringing mode; they represent movements of a small fraction of a nuclear diameter in the ends of the cylinders.

In figure a the two traces are disturbed simultaneously by a calibration pulse



a



b

given electrostatically to the ends of the bars. The output of the electronics (designed to filter out the resonance of the bars) is shown in the third track (for Rutherford) and the fourth (for Reading), and clearly shows the effect of the calibration pulse.

In figure b a similar pulse occurs in both detectors but this time the pulse is natural. The question arises whether this is an incident gravitational wave hitting both detectors simultaneously (simultaneously because gravitational waves should move at the speed of light; at that speed it takes only one tenth of a ringing cycle to get from Reading to the Rutherford) or whether it is a coincidence of "noise" caused by random thermal movements in each bar. Random coincidences like these should occur at the same rate whatever delay is applied between the signals from each pair of cylinders—whereas coincidences induced by gravitational waves would vanish for any delay other than zero. The rate of coincidence Allen and Christodoulides found was independent of delay and so due only to noise.

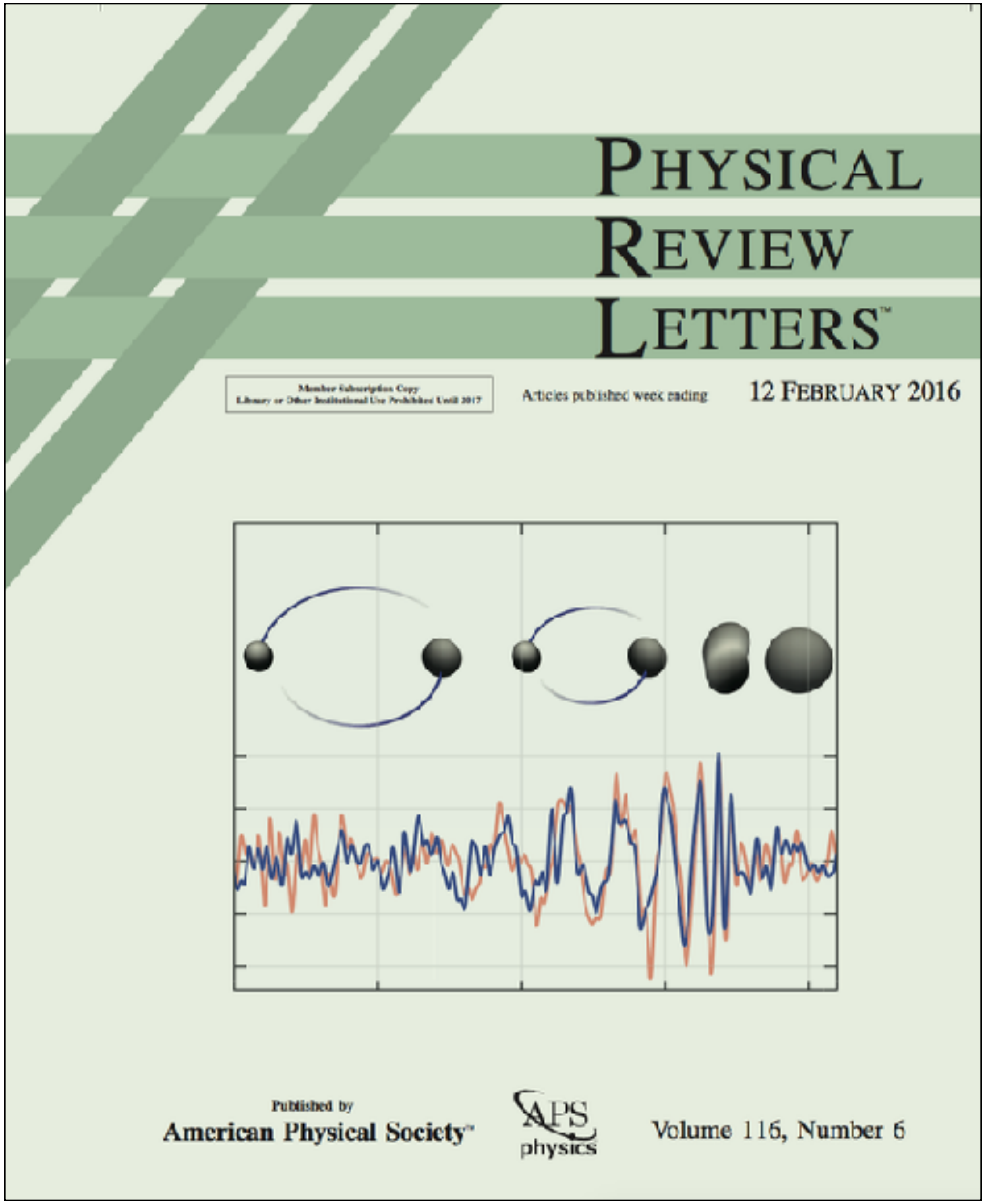
The remaining question is whether as (Professor Allen puts it) "God claps his hands" when He makes gravity waves. Only Weber's detector is sensitive to waves which come in long (tenth-second) bursts with a slow rise and fall in intensity; the newer detectors have been sensitive only to much shorter (millisecond) bursts.

Fast-forward 45 years,
from 1971 to 2016…

First Detection

14 September 2015:
Advanced LIGO records
merger of a 29 and 36 solar mass BH

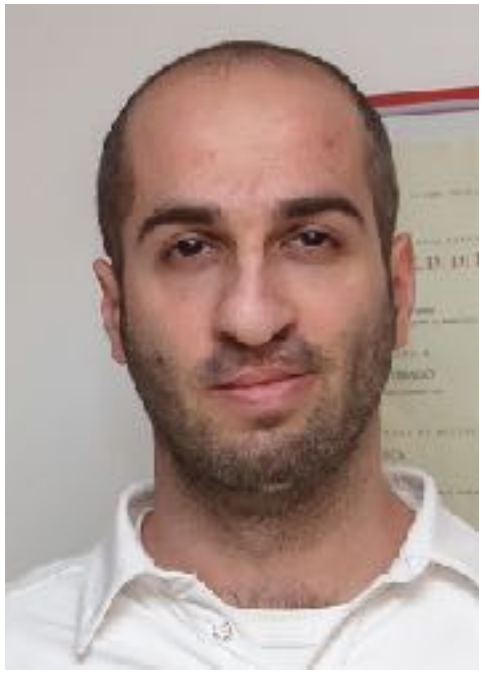
References: PRL 116, 061102 (2016); PRX 6, 041015 (2016); Ann. Phys. 529, 1600209 (2017); PRL 118, 221101 (2017)



GW150914

- First observing run (O1, science operations) start scheduled **18 September 2015**
- Event at **09:50 UTC on 14 September 2015, four days before O1 start**

AEI Hannover, September 14, 2015



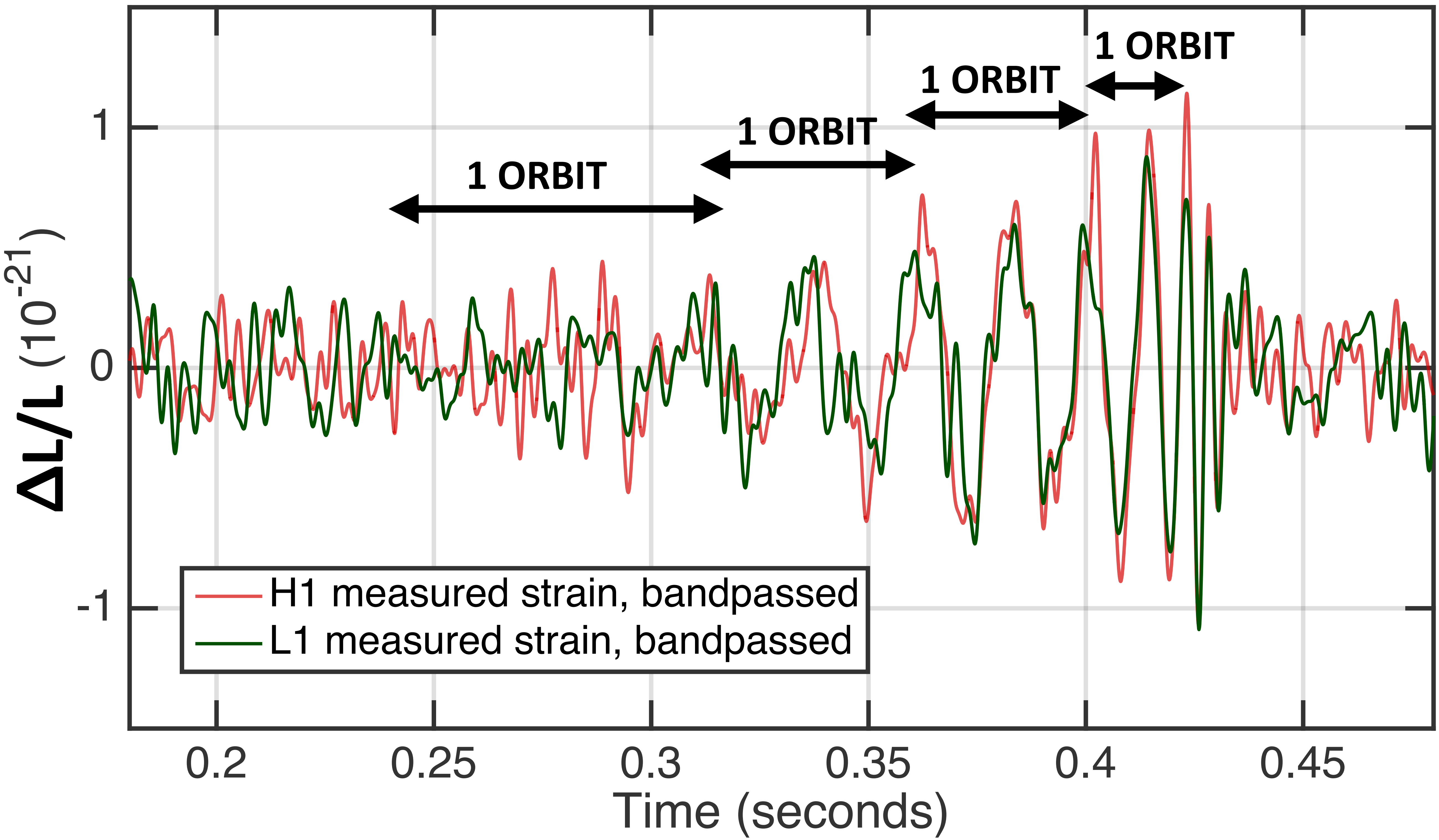
Marco Drago

Andrew Lundgren

- 11:50 Monday morning in Germany (02:50 in Hanford, 04:50 in Livingston)
- Event database had ~1000 entries
- Marco and Andy checked injection flags and logbooks, data quality, made Qscans of LHO/LLO data.
- Contacted LIGO operators: “everyone’s gone home”

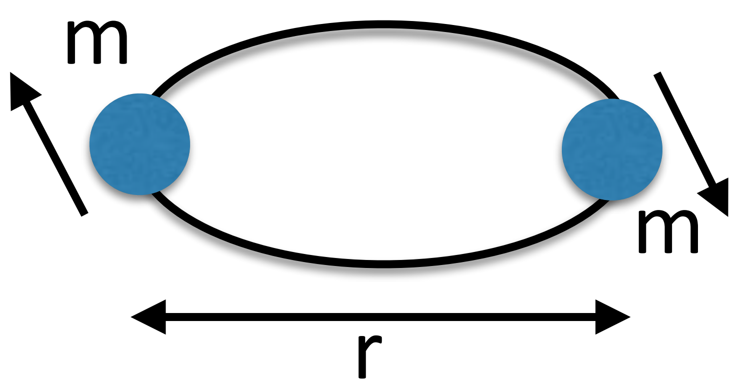
- At 12:54, Marco sent an email to the collaboration, asking for confirmation that it’s not a hidden test signal (hardware injection)
- Next hours: flurry of emails, decision to lock down sites, freeze instrument state

The Chirp



1 ORBIT
1 ORBIT
1 ORBIT
1 ORBIT
1
0
-1
ΔL/L (10^{-21})
H1 measured strain, bandpassed
L1 measured strain, bandpassed
0.2
0.25
0.3
0.35
0.4
0.45
Time (seconds)

Gravitational waves from orbiting masses



orbital angular frequency ω

$$\text{Newton}: \frac{Gm^2}{r^2} = m\omega^2\left(\frac{r}{2}\right) \Rightarrow r^3 = \frac{2Gm}{\omega^2}$$

$$\mathrm{E}_{\text{mechanical}} = \frac{1}{2}m\left(\frac{\omega r}{2}\right)^2 + \frac{1}{2}m\left(\frac{\omega r}{2}\right)^2 - \frac{Gm^2}{r} = -\frac{Gm^2}{2r} = -\frac{G^{2/3}m^{5/3}}{2^{4/3}}\omega^{2/3}$$

$$\text{GW Luminosity} = \frac{G}{5c^5}\left(\frac{d^3}{dt^3}Q_{ab}\right)\left(\frac{d^3}{dt^3}Q_{ab}\right) = \frac{8G}{5c^5}m^2r^4\omega^6 = \frac{2^{13/3}G^{7/3}m^{10/3}}{5c^5}\omega^{10/3}$$

$$\text{GW Luminosity} = -\frac{d}{dt}\mathrm{E}_{\text{mechanical}} = \frac{G^{2/3}m^{5/3}}{3\cdot 2^{1/3}}\omega^{-1/3}\frac{d\omega}{dt}$$

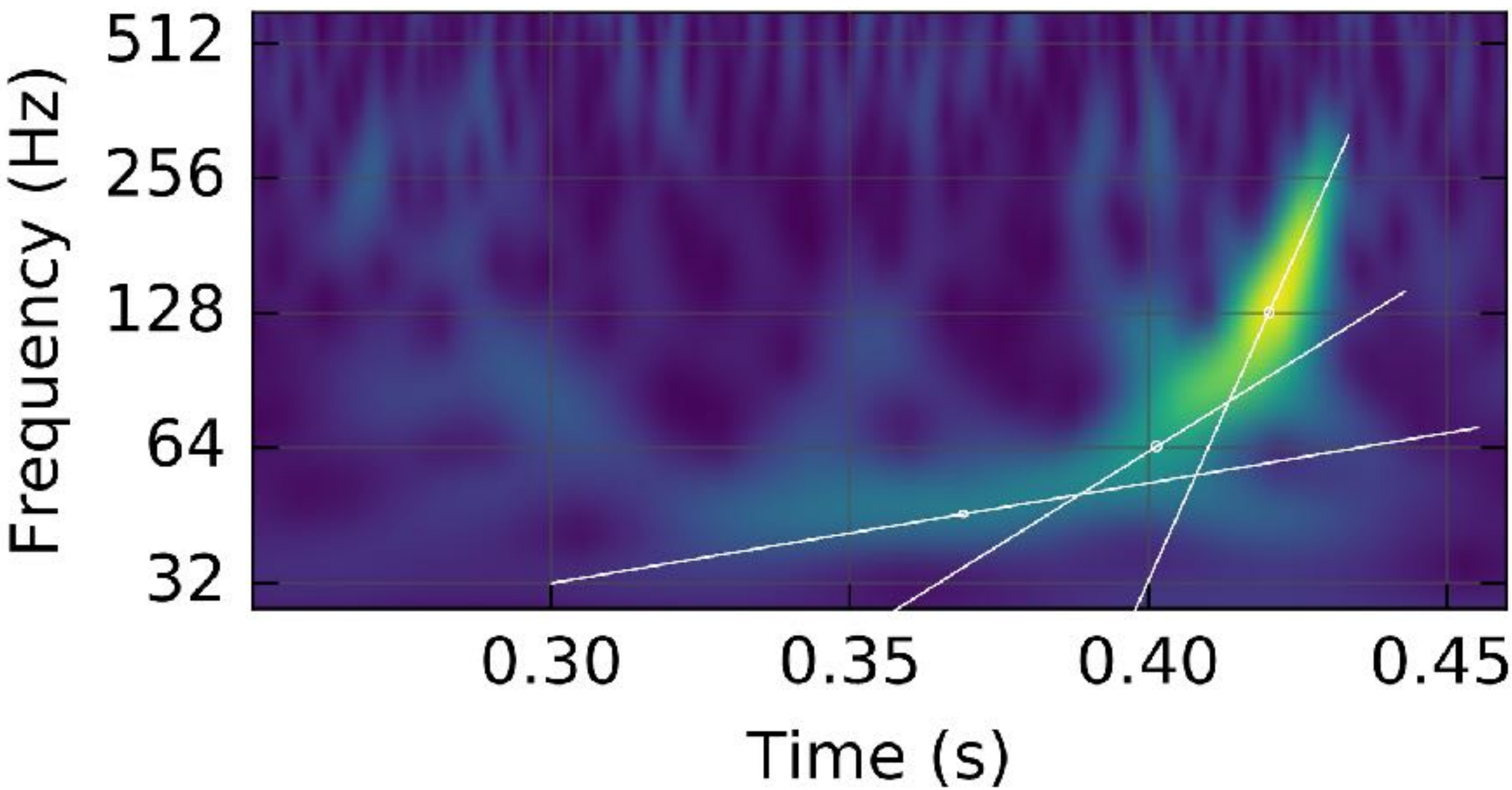
GW frequency
$f = 4\pi\omega$

$$\frac{d\omega}{dt} = \frac{3\cdot 2^{14/3}G^{5/3}m^{5/3}}{5c^5}\omega^{11/3}$$

get mass from frequency and its rate of change!

Masses from the rate of frequency increase

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3}\dot{f}\right]^{3/5} = 30\ M_\odot$$



Can only be two black holes!

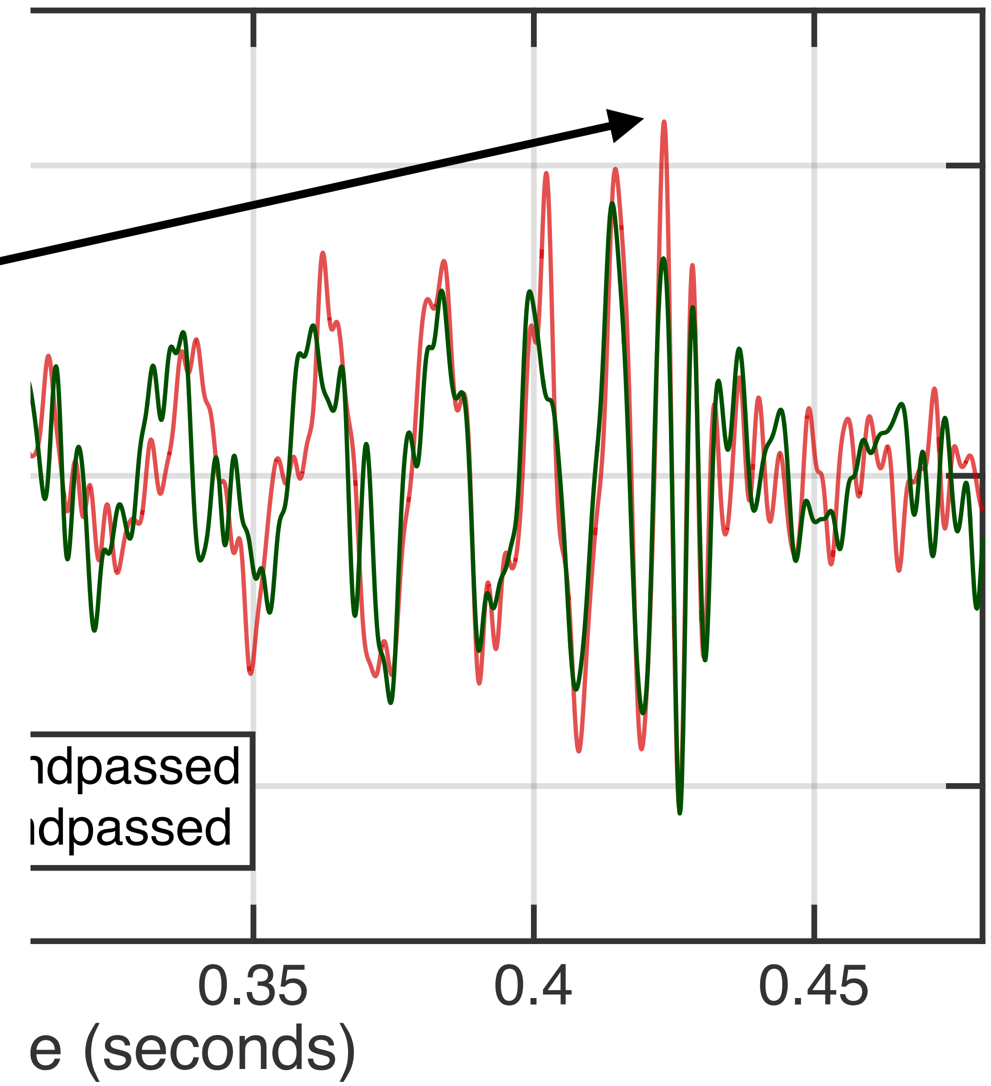
- Chirp mass $\mathcal{M}$ ~ 30 M⊙

  => m_1, m_2 ~ 35 M⊙ =>

  Sum of Schwarzschild radii ≥206km
- At peak f_{GW} = 150 Hz, orbital frequency = 75 Hz separation of Newtonian point masses 346 km
- **Ordinary stars** are 10^6 km in size (merge at mHz). **White dwarfs** are 10^4 km (merge at 1 Hz). They are too big to explain data!
- **Neutron stars** are also not possible:

  m_1 = 4 M⊙ => m_2=600 M⊙

  =>Schwarzschild radius 1800km => too big!

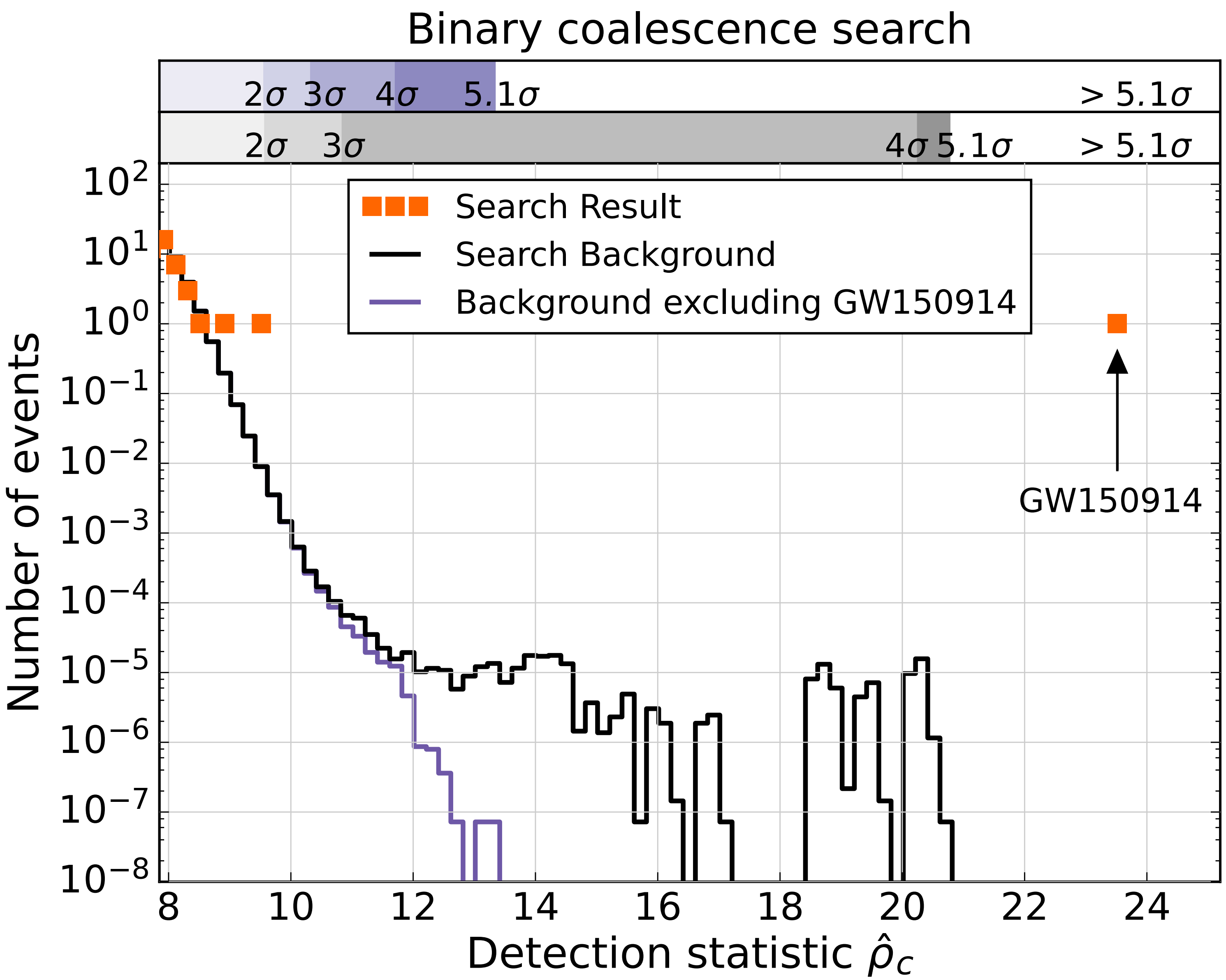


Among known objects, only black holes are heavy enough and small enough!

Random Noise?

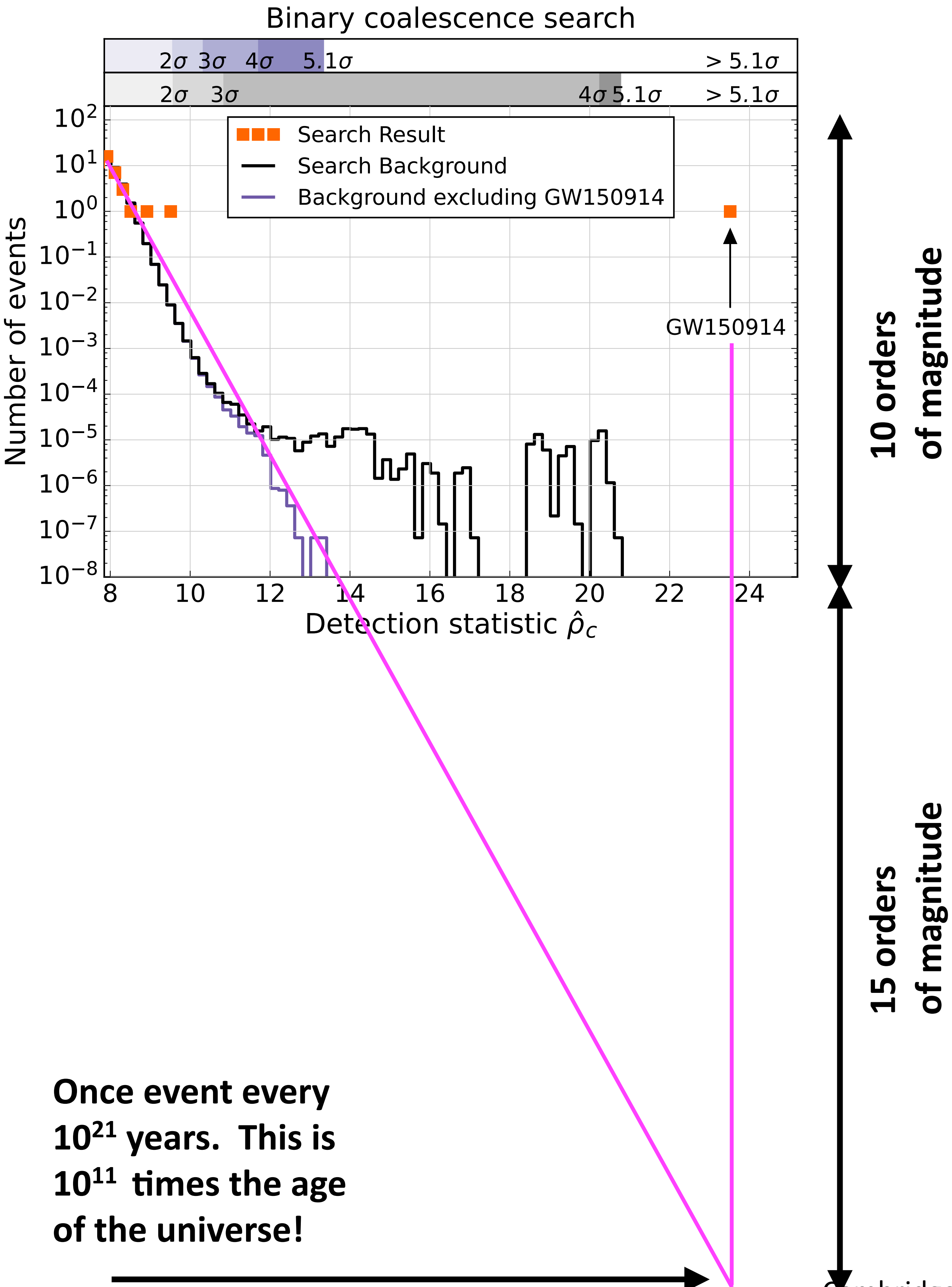
More than 200,000 years before noise in the detector would mimic this signal, or a similar signal of the types that we search for.

What is the false alarm probability?



- Orange squares: highest SNR events in the first 16 days of data collected (12 Sept - 20 Oct)
- Estimate background by shifting instrumental data in time at one site in 0.1 second increments (>> 10 msec light-travel time) approximately $2\text{x}10^6$ times.
- Generate 608,000 years of "artificial" data, search for events
- Including trials factor, false alarm rate < 1 in 203,000 years
- For a Gaussian process, this is > 5.1σ

What is the false alarm probability?



Once event every 10^{21} years. This is 10^{11} times the age of the universe!

- Orange squares: highest SNR events in the first 16 days of data collected (12 Sept - 20 Oct)
- Estimate background by shifting instrumental data in time at one site in 0.1 second increments (>> 10 msec light-travel time) approximately 2×10^6 times.
- Generate 608,000 years of “artificial” data, search for events
- Including trials factor, false alarm rate < 1 in 203,000 years
- For a Gaussian process, this is > 5.1σ
- Real false alarm rate much much less! **We got lucky, could have confidently detected it 70% farther away.**

Energy lost, power radiated

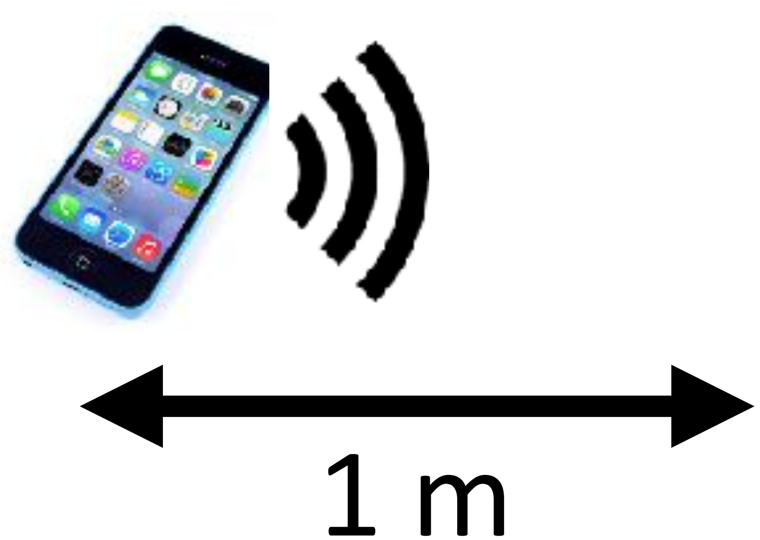
Primary black hole mass	$36^{+5}_{-4}\,M_\odot$
Secondary black hole mass	$29^{+4}_{-4}\,M_\odot$
Final black hole mass	$62^{+4}_{-4}\,M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift, z	$0.09^{+0.03}_{-0.04}$

- Radiated energy $(3\pm0.5)M_\odot$
- Peak luminosity $O(G/c^5)$

  $= 3.6 \times 10^{56}$ erg/s

  $= 200\ M_\odot/s$

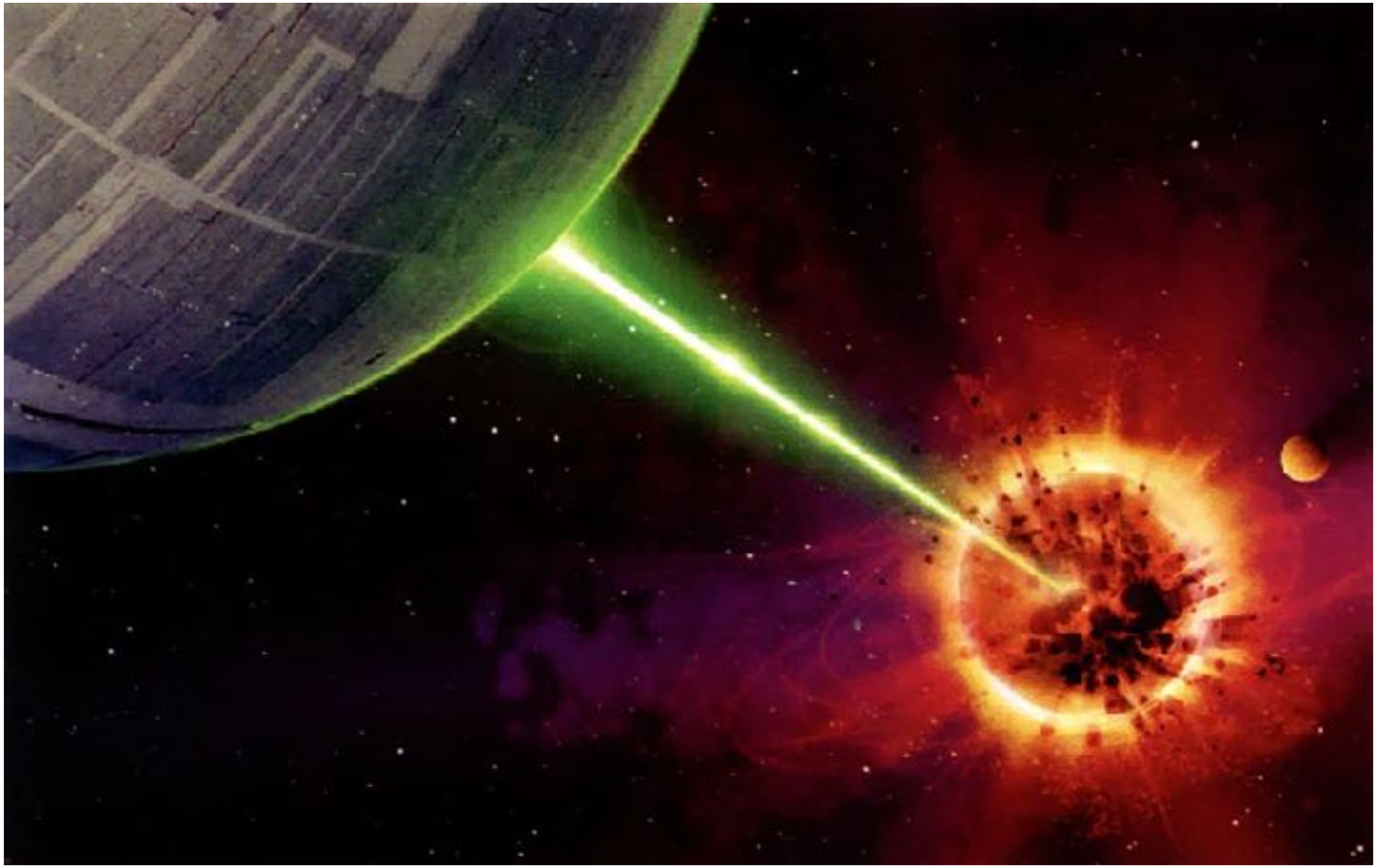
- Flux about $1\mu W/cm^2$ at detector, $\sim 10^{12}$ millicrab
- Cell phone at 1 meter!



Estimate of Radiated Energy in GWs

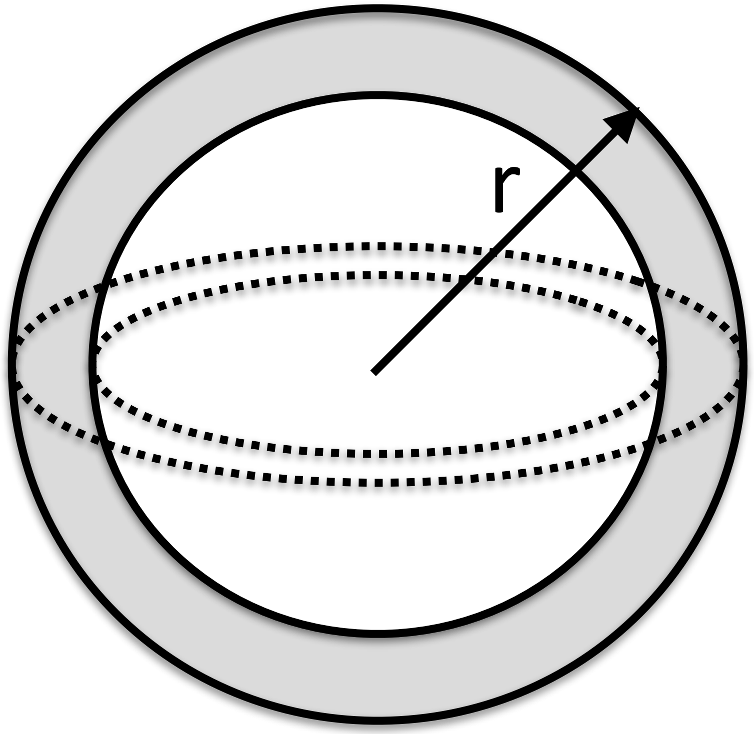
$$\mathrm{E}_{\mathrm{mechanical}} = -\frac{Gm^2}{2r}$$

Set m = 35 $M_\odot$ and r=346 km, obtain **$\mathbf{E_{mechanical}}$ ~ 3 $\mathbf{M_\odot c^2}$**



3 solar masses in gravitational waves

- Most energy emitted in ~40 msec
- 10 msec after merger, expanding shell of GW energy 15,000 km in radius. Energy density in GW: ~60 kg/cm^3
- 1 sec after merger, shell 300,000 km radius, energy density in shell ~ 100 g/cm^3.
  You could safely observe from this distance in a space-suit: strain would change your body length by ~1mm
- 10 s after merger, shell has 3,000,000 km radius. Energy density in GW: ~1 g/cm^3



$r \sim t$

$\rho \sim r^{-2} \sim t^{-2}$

Hawking's Area Theorem PRL 21, 1344 (1971)

Gravitational Radiation from Colliding Black Holes

S. W. Hawking
Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England
(Received 11 March 1971)

It is shown that there is an upper bound to the energy of the gravitational radiation emitted when one collapsed object captures another. In the case of two objects with equal masses m and zero intrinsic angular momenta, this upper bound is $(2-\sqrt{2})m$.

Weber[1-3] has recently reported coinciding measurements of short bursts of gravitational radiation at a frequency of 1660 Hz. These occur at a rate of about one per day and the bursts appear to be coming from the center of the galaxy. It seems likely[3,4] that the probability of a burst causing a coincidence between Weber's detectors is less than $\frac{1}{10}$. If one allows for this and assumes that the radiation is broadband, one finds that the energy flux in gravitational radiation must be at least 10^{10} erg/cm^2 day.[4] This would imply a mass loss from the center of the galaxy of about $20\,000 M_\odot$/yr. It is therefore possible that the mass of the galaxy might have been considerably higher in the past than it is now.[5] This makes it important to estimate the efficiency with which rest-mass energy can be converted into gravitational radiation. Clearly nuclear reactions are insufficient since they release only about 1% of the rest mass. The efficiency might be higher in either the nonspherical gravitational collapse of a star or the collision and coalescence of two collapsed objects. Up to now no limits on the efficiency of the processes have been known. The object of this Letter is to show that there is a limit for the second process. For the case of two colliding collapsed objects, each of mass m and zero angular momentum, the amount of energy that can be carried away by gravitational or any other form of radiation is less than $(2-\sqrt{2})m$.

I assume the validity of the Carter-Israel conjucture[6,7] that the metric outside a collapsed object settles down to that of one of the Kerr family of solutions[8] with positive mass m and angular momentum a per unit mass less than or equal to m. (I am using units in which $G=c=1$.) Each of these solutions contains a nonsingular *event horizon*, two-dimensional sections of which are topographically spheres with area[9]

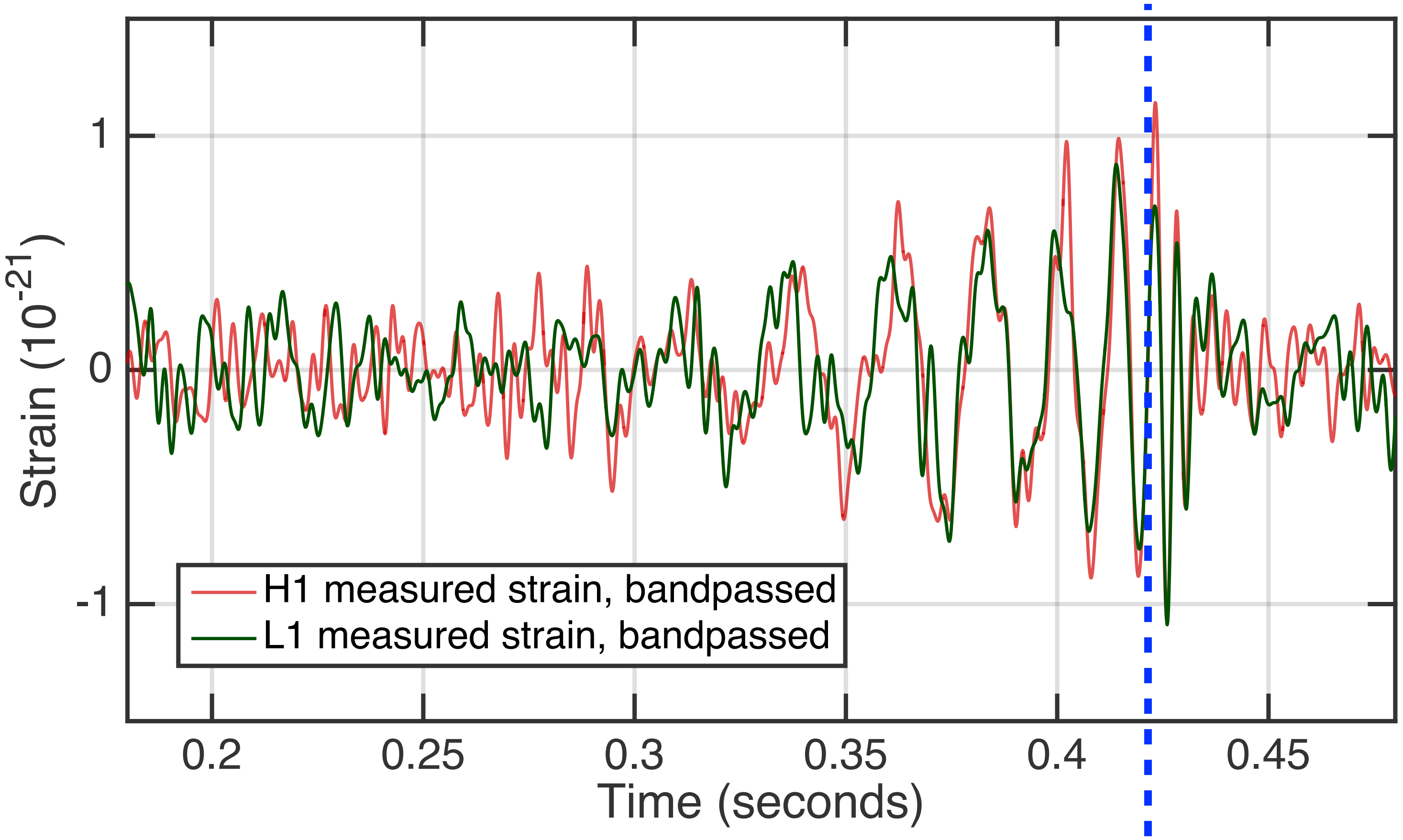
$$8\pi m[m+(m^2-a^2)^{1/2}]. \qquad (1)$$

The event horizon is the boundary of the region of space-time from which particles or photons can escape to infinity. I shall consider only

Primary black hole mass	$36^{+5}_{-4}\ \mathrm{M}_\odot$	
Secondary black hole mass	$29^{+4}_{-4}\ \mathrm{M}_\odot$	
Final black hole mass	$62^{+4}_{-4}\ \mathrm{M}_\odot$	
Final black hole spin	$0.67^{+0.05}_{-0.07}$	

$$m_f^2\left(1+\sqrt{1-s_f^2}\right) > m_1^2\left(1+\sqrt{1-s_1^2}\right) + m_2^2\left(1+\sqrt{1-s_2^2}\right)$$

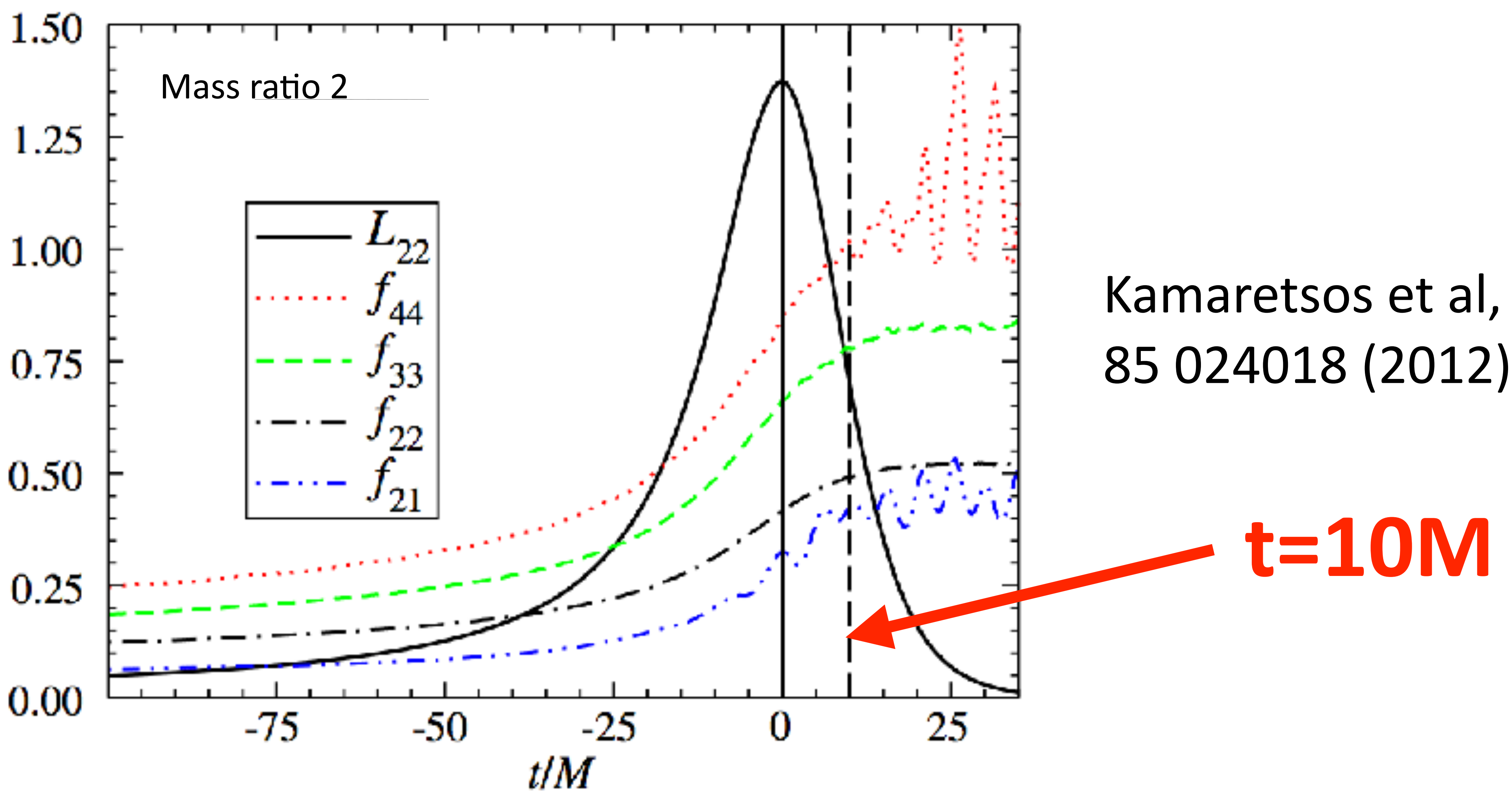
GW150914 test area theorem? No!



PRL 116 (22), 221101

- Most SNR before merger: only values of m_1, m_2 are determined independently.
- m_f and s_f determined by numerical/analytic evolution
- If area theorem were NOT satisfied, then the numerical/ analytic solution of Einstein equations are faulty

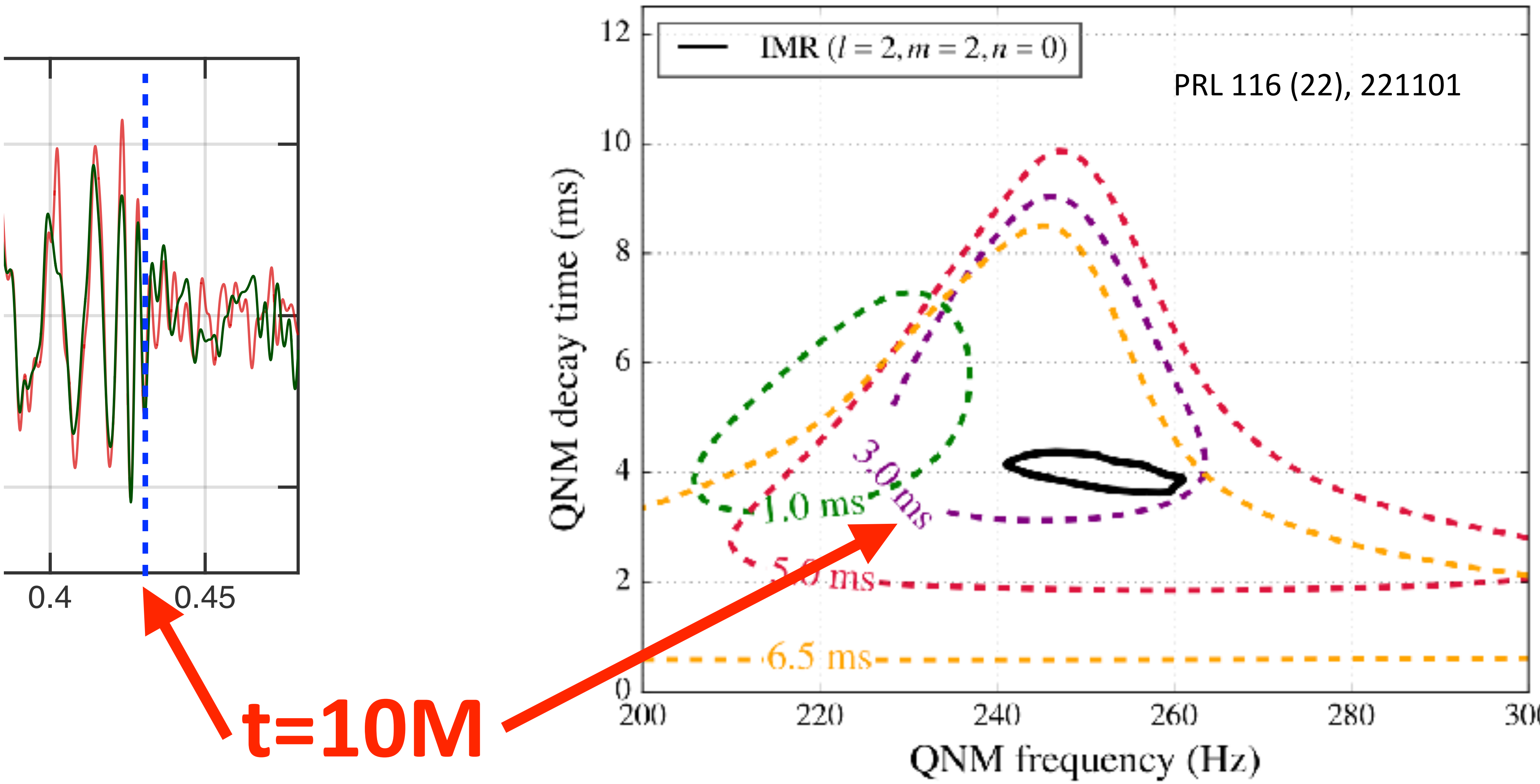
GW150914 test area theorem? No!



Kamaretsos et al, PRD 85 024018 (2012)

- Quasi-normal modes characterised by frequency and decay time: function of mass and spin of final black hole
- Modes "stabilise" about 10M after merger
- In detector frame: M=340 µsec, so 10M = 3.4 msec

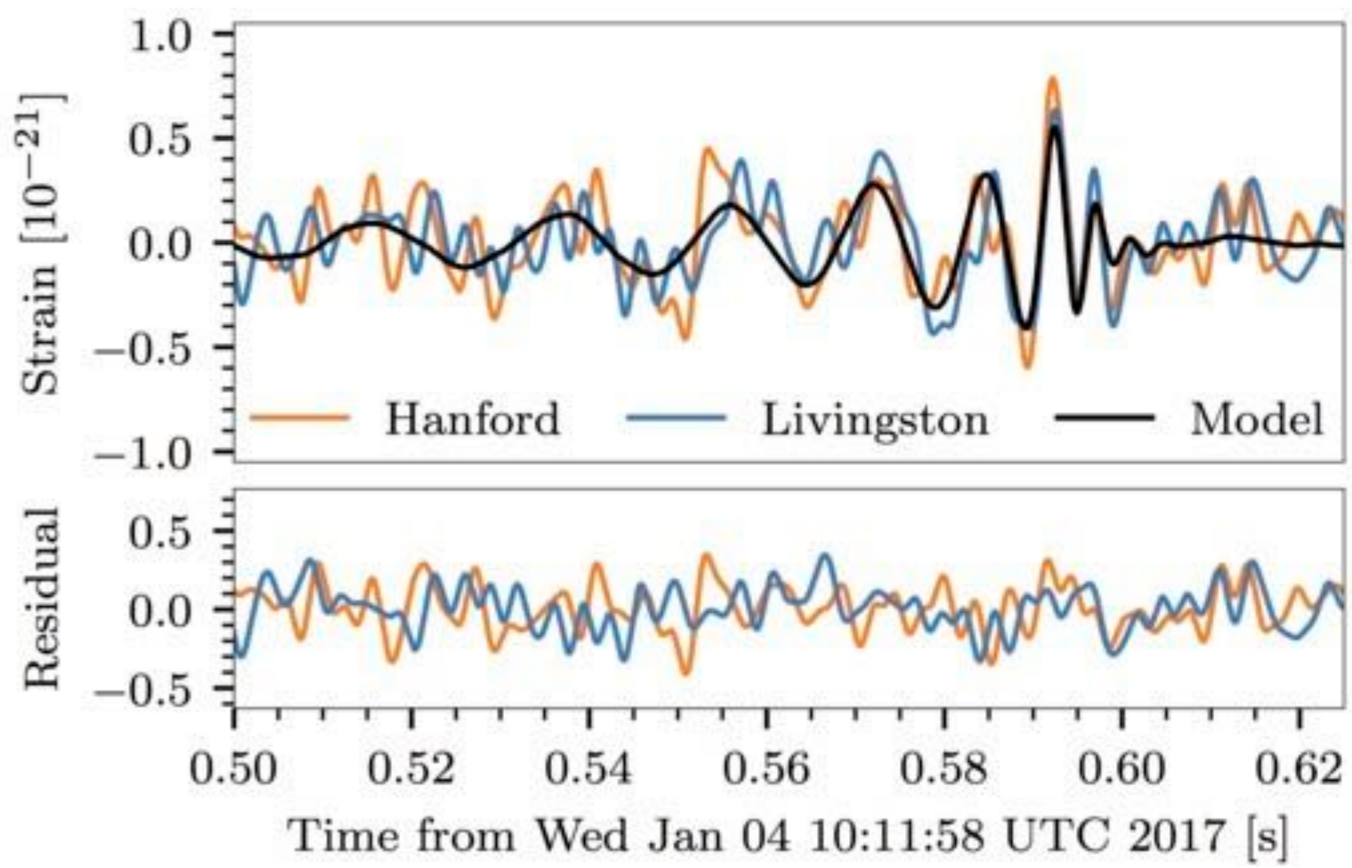
GW150914 test area theorem? No!



- Signal remaining 3.4 msec after peak (merger) constrains frequency/damping time to purple region
- Does not tightly constrain final mass and spin
- Need a stronger signal: ideally get frequency/damping time of TWO quasi-normal mode

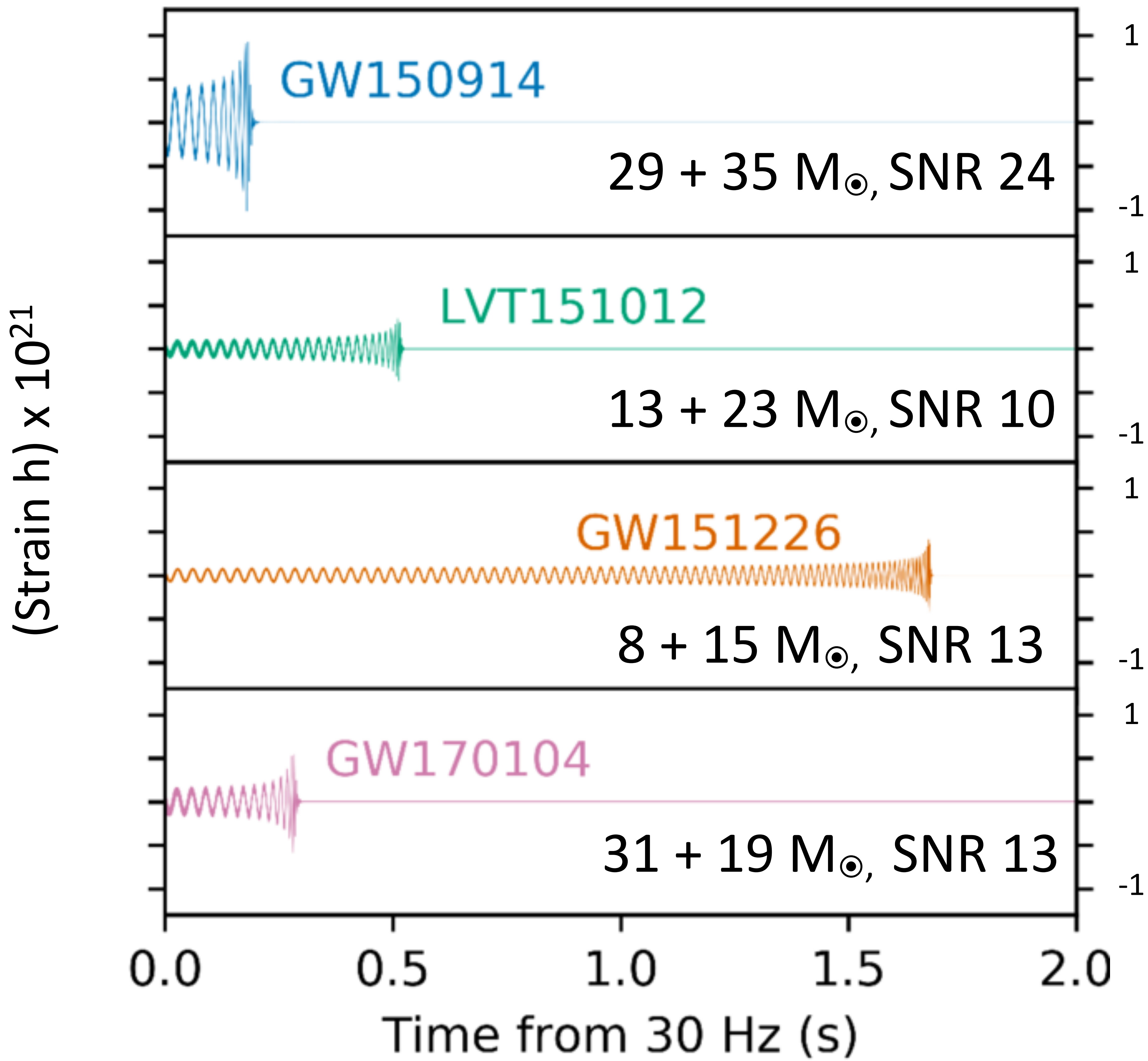
GW170104: first Detection in O2

- Merger of 31 and 19 $M_\odot$ black holes
- 2 $M_\odot$ lost in GWs
- Distance: redshift 0.18 corresponding to 880 Mpc
- Like first detection GW150914, only at twice the distance!
- *“GW170104 was first identified by inspection of low latency triggers from Livingston data. An automated notification was not generated as the Hanford detector’s calibration state was temporarily set incorrectly in the low-latency system.”*

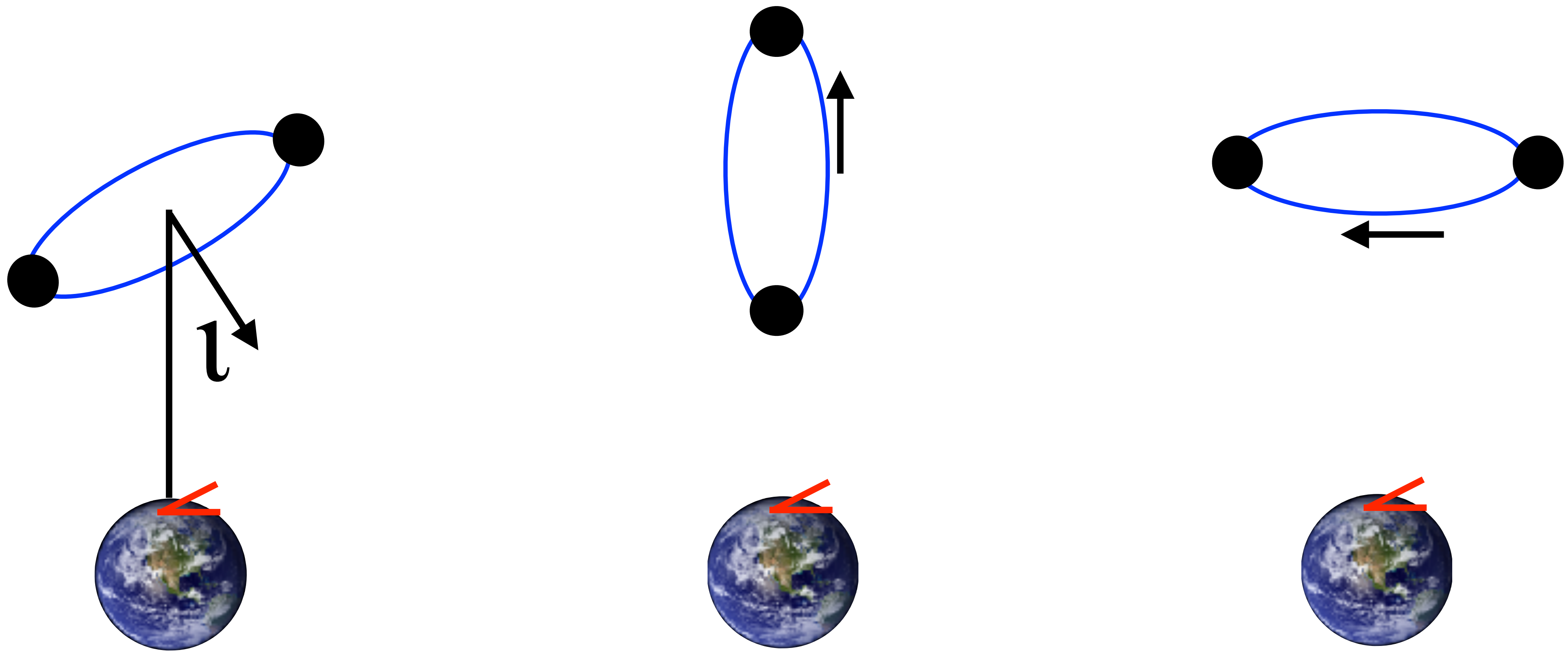


Phys. Rev. Lett. 118, 221101 (2017)

Binary Black Holes in O1/O2



Why is spin hard to observe?

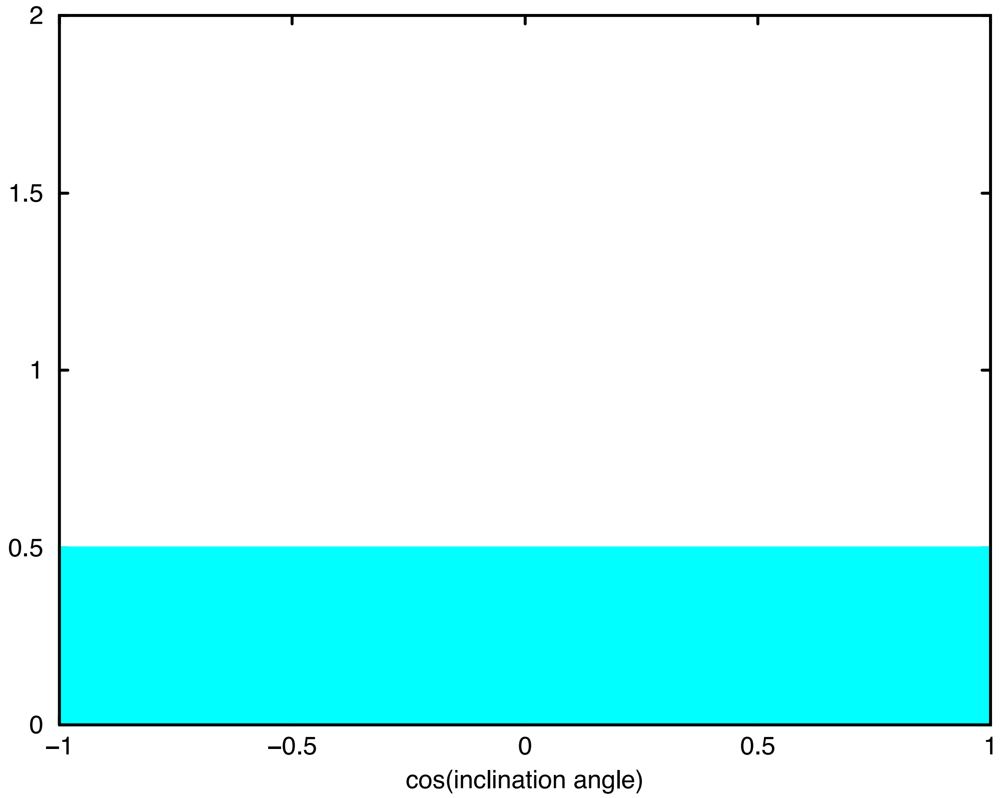


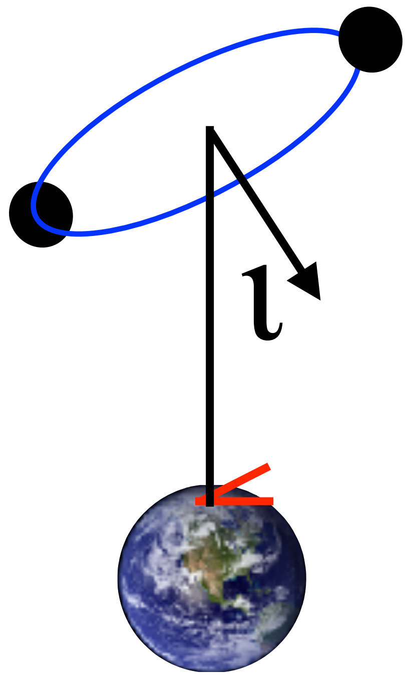
Orbital plane edge-on
GWs have linear polarisation

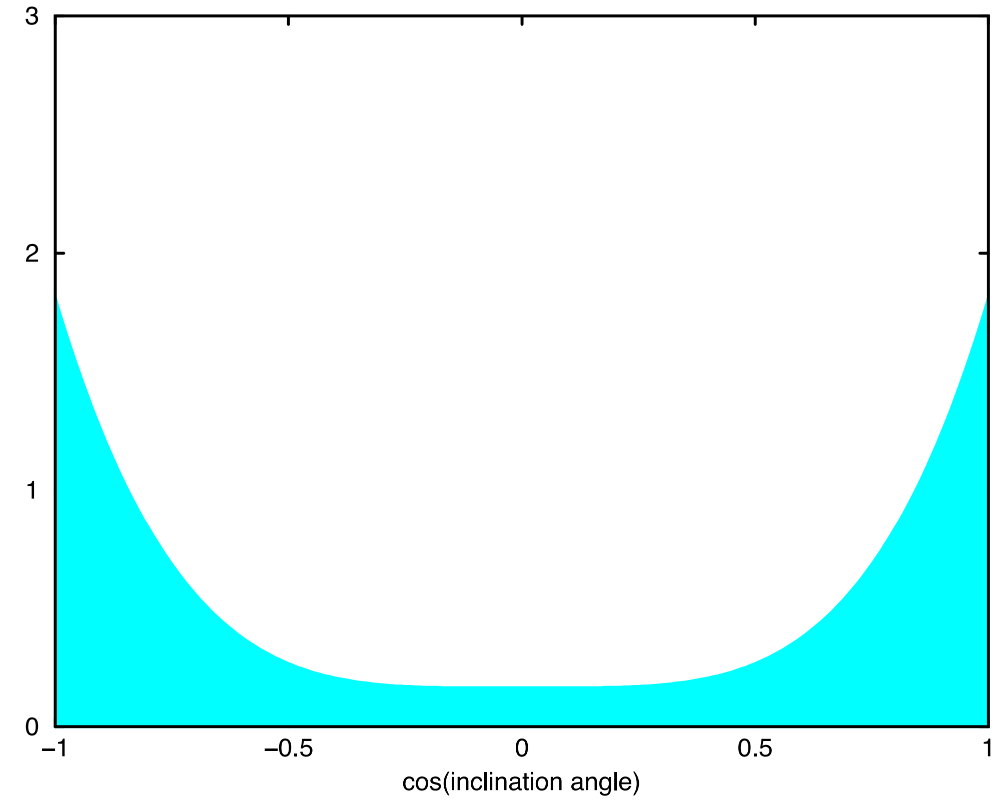
Orbital plane face-on
GWs have circular polarisation

- Face-on: produces strong signal, regardless of detector orientation
- Edge-on: cosine factor. Amplitude in detector depends upon its orientation

Why is spin hard to observe?





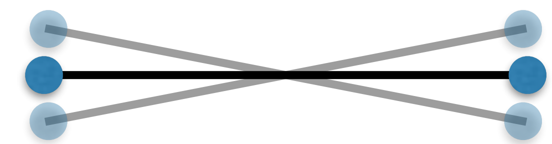


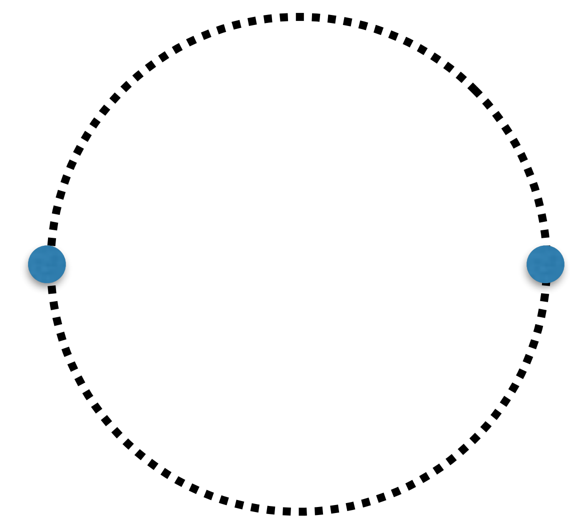
Prior (astrophysical) probability distribution for cos(ι)

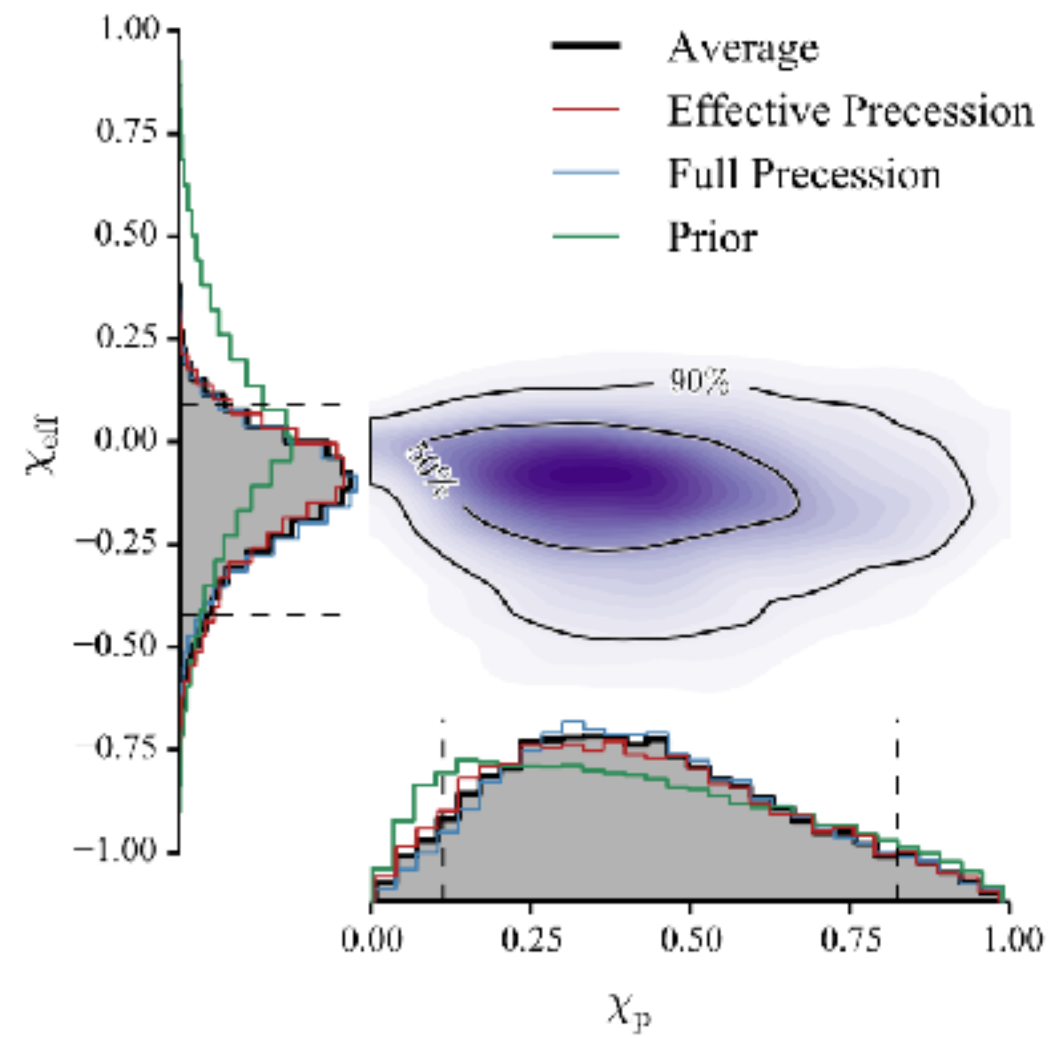
Posterior distribution after detecting a signal

Why is spin hard to observe

- "Smoking gun": precession of the orbital plane
- Hard to detect: effects on waveform strongest when orbital plane viewed edge-on; hidden when viewed face-on/off.
- A network of detectors with different orientations will make us more likely to detect systems that are not face-on/off
- Compare priors and posteriors







GW170104

Closing

- aLIGO observations a wonderful way to observe dynamic strong-field gravity. Fully consistent with GR
- 1970s work by Hawking and Gibbons was a significant factor leading to first detections 45 years later
- Coming years: find mass and spin distributions of these BH systems. Clues about their origins.
- To test area theorem directly, need a source x2 closer with detectors that are x3 more sensitive. Hopefully by end of the decade. Perhaps might also "average" many weaker events.
- Data analysis: very compute intensive, but the human element is still important
- Solar masses radiated in tens of milliseconds is dramatic, but nevertheless ineffectual
- When looking at posterior probability distributions for parameters, be sure to compare this with priors. What comes from the data itself?