Exotic Compact Objects (ECOs) [Cardoso-Pani, 1904.05363]

↳ any compact object that is NOT a BH nor a NS

(1)

1°) MOTIVATION

I) Novel dark objects? New GW sources? (New "species") (DM candidates)
We'll never find them if we don't model
(Lesson from particle physics!)

II) Quantifying the evidence for BHs

↳ given observation X, can we place a bound on the "BH-ness"?

↳ Horizons are unavoidable in GR → utmost importance to test!

III) Singularity problem, Weak CC

↳ Horizons are NECESSARY for the self-consistency of GR!

IV) INFORMATION LOSS PARADOX [eg firewalls, fuzzballs]

↳ something must happen at the horizon scale, no matter the CURVATURE - maybe ensamble of regular, horizonless microstates?]

V) DARK MATTER CONNECTION

↳ Can ECOs be (part of) the DM?

1°) OUTLINE

↳ A toy model of ECOs in GR

↳ Geodesics, light ring, energy balance

↳ GW Signatures

↳ Spin-induced multiple moments

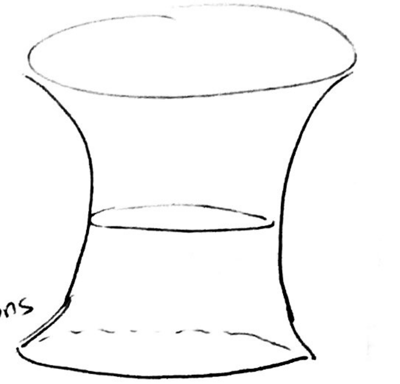
↳ Tidal Heating

↳ TLNs

↳ QNMs / echoes

10) TOY MODEL FOR ECOs

[Damour-Solodukhin, 2007 ; Visser 1995]



$$ds^2 = -\left(1 - \frac{2M}{r} + \lambda^2\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

$$\lambda \sim e^{-M^2/\ell_p^2}$$

↳ $r=2M$ is null-like — but separates two timelike regions → $\boxed{r_0 = 2M \text{ THROAT}}$

$n_\mu n^\mu = g^{\mu\nu} n_\mu n_\nu = g^{rr} = \left(1 - \frac{2M}{r}\right)$

↳ Solution of GR ⊕ exotic matter $[\rho = 0 \,;\, p_r < 0 \,;\, p_t > 0 \,;\, p_t \gg |p_r|] \Rightarrow$ NEC $\neq 0$ ($T_{\mu\nu} k^\mu k^\nu \geq 0$ $\forall$ null k^μ $\Rightarrow$ WEC, DEC, SEC)

$$R = -\frac{1}{8\lambda^2 M^2} \;;\; R_{abcd} R^{abcd} \sim \frac{1 + 24\lambda^4}{\lambda^4} \rightarrow \underline{\text{HUGE CURVATURE!}}$$

(only at the throat)

If instead $\lambda = 0 \Rightarrow$ Two copies glued at $\boxed{r = r_0 = 2M(1+\varepsilon)}$

↳ <u>Thin shell</u> $\rightarrow S_{\mu\nu} = -\frac{1}{8\pi}(\underbrace{K_{\mu\nu}}_{\text{extrinsic curvature}} - K \underbrace{h_{\mu\nu}}_{\text{induced metric}})$ [Israel]

$S_{\mu\nu} = \mathrm{diag}(-\sigma, P, P)$

$\boxed{\sigma < 0 \,,\, P \rightarrow \infty \text{ as } \varepsilon \rightarrow 0}$

(Violates WEC and DEC)

WEC: $T_{\mu\nu} V^\mu V^\nu \geq 0$ $\forall$ timelike V^μ

DEC = WEC ⊕ $T_{\mu\nu} V^\nu \neq$ spacelike

SEC = $(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) V^\mu V^\nu \geq 0$ $\forall$ timelike V^μ

<u>Note</u>: gravastars [Mazur-Mottola 2001] have similar properties

↳ We can use this metric – supplemented with $\boxed{\text{BCs}}$ at $r = r_0$ – as a Toy model.

• Buchdal's theorem:

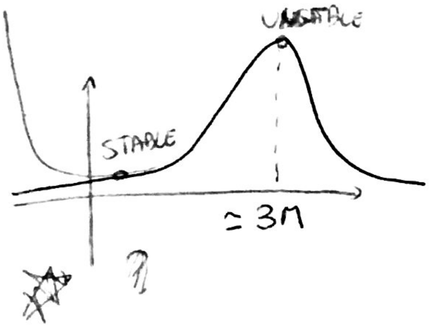
↳ GR is correct
↳ sph. symm
↳ single perfect fluid
↳ $P_r \geq P_t$ (quasi-isotropic)
↳ $P_r \geq 0$, $\rho \geq 0$
↳ $\rho'(r) < 0$

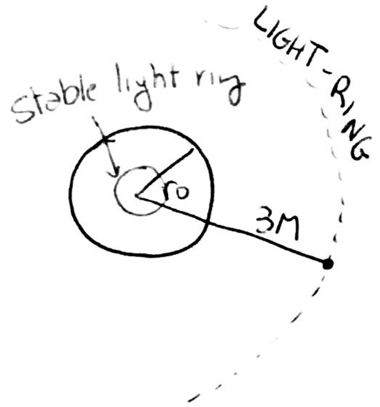
$\Rightarrow \varepsilon > 1/8$ $\left(R_0 > \frac{9}{4} M\right)$

↳ Many ways to evade it!

10 | GEODESICS and EM TESTS

At $r > r_0$ gedelesics are $\simeq$ those of Schwareschild





$$\tau_{instab} \simeq 3\sqrt{3}\, M \qquad \tau \sim \int_{r_0}^{3M} dr \sqrt{\frac{g_{rr}}{g_{tt}}} \sim \sqrt{15} M$$

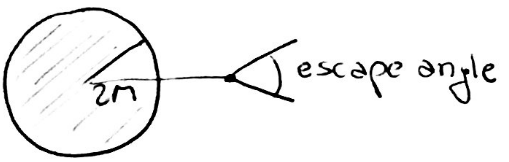
$\tau \gg \tau_{instab}$

$\Downarrow$

$$\boxed{\epsilon \lesssim 0.01}$$

Clean Photon-Sphere

→ Classification: CO → ISCO
UCO → Photon-sphere
ClePhOs → Clean photon-sphere



$\Delta\Omega \sim O(\epsilon)$ [And $\tau_{roundtrip} \sim O(M)$

↳ Geodesics and EM radiation are SIMILAR to BH as $\epsilon \to 0$

→ Strongest constraint: EQUILIBRIUM with ENVIRONMENT [Broderick-Narayan '06]

$$\Delta E \sim \left(1 - (1-\epsilon)^N\right) \delta M \qquad N = \frac{T_a}{\tau_{roundtrip}} \qquad T_a \sim \tau_s \sim 4.5 \cdot 10^7 \text{yr}$$

$$\simeq \epsilon \frac{T_a}{\tau_{roun}} \delta M \qquad \dot{M} \simeq f_{edd} \underbrace{\dot{M}_{Edd}}_{\simeq 10^{33} \frac{M}{M_\odot} \text{erg/s}}$$

$$\Rightarrow \boxed{\dot{E} \sim 10^{-25} \left(\frac{\epsilon}{10^{-15}}\right) \left(\frac{f_{edd}}{10^{-9}}\right)} \qquad \boxed{\epsilon \lesssim 10^{-15} ; 10^{-14}}$$

↳ there might be enegy in other channels.

→ NOTE: above constraint assume PERFECT REFLECTION ($|R|^2 = 1$). If $|R|^2 < 1$ ⇒ weak Bounds and "dirty" [Di Filippo+, 20..]

: Bottom line: EM tests are weakly constraining

→ UCOs will have similar shadows than BHs

↳ Accretion at the center? $\frac{\Delta M}{M} \sim \frac{\dot{M}}{M} \Delta t \sim f_{edd} \frac{\dot{M}_{Edd}}{M} \tau_{Hubble} \sim 3 \cdot 10^{-4} \left(\frac{f_{edd}}{10^{-4}}\right)$

GW1: (SPIN-INDUCED) MULTIPLE MOMENTS

For a Kerr BH: [Hansen, 1970s]

$$\boxed{M_\ell^{BH} + i S_\ell^{BH} = M^{\ell+1} (i\chi)^\ell}$$ $\ell = 0, 1, 2 \ldots$

$\chi = \frac{S_1}{M^2}$; $M = M_0$

$\Rightarrow \begin{cases} M_\ell \propto \chi^\ell & ; \quad M_\ell = 0 \quad \ell \text{ odd} \\ S_\ell \propto \chi^\ell & \quad\; S_\ell = 0 \quad \ell \text{ even} \end{cases}$

For an ECO:

$$M_\ell^{ECO} = M_\ell^{BH} + \delta M_\ell \quad ; \quad S_\ell^{ECO} = S_\ell^{BH} + \delta S_\ell$$

↳ symmetries might be BROKEN (e.g. star with a quadrupole even if $\chi = 0$)

↳ δM_ℓ and δS_ℓ are MODEL-DEPENDENT!

↳ How to estimate them?

→ SMALL-MULTIPOLE EXPANSION:

$r > r_0 \to$ vacuum $\to R_{\mu\nu} = 0 \Rightarrow \underbrace{R^{(0)}_{\mu\nu}}_{\text{Schwarzschild}} + \delta R_{\mu\nu} = 0$ $\quad g_{\mu\nu} = g^{(0)}_{\mu\nu} + \underbrace{h_{\mu\nu}}_{\text{Stationary}}$, $|h_{\mu\nu}| \ll 1$

$h_{\mu\nu} = h^{polar}_{\mu\nu} + h^{axial}_{\mu\nu}$ [Regge-Wheeler '60s]

↳ Expanded in SPHERICAL HARMONICS

$$h^{polar}_{\mu\nu} = \begin{pmatrix} H_0^\ell P_\ell(\cos\theta) & & & 0 \\ & H_2^\ell P_\ell & & \\ & & r^2 K_\ell P_\ell & \\ 0 & & & r^2 K_\ell P_\ell \sin^2\theta \end{pmatrix} ; \quad h^{axial}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0 P_\ell' \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ h_0 P_\ell' & 0 & 0 & 0 \end{pmatrix}$$

↳ Parity and angular-mom. sum rules

Es: SPIN-INDUCED

$$H_0^\ell = \chi^2 \Big\{ \overbrace{\left(\frac{1}{r^3} + \frac{1}{r^4}\right)}^{\text{regular}} + \overbrace{\delta M_2 \log\left(1 - \frac{2M}{r}\right)}^{\text{irregular}} \Big\}$$

↳ ASYMP. FLATNESS

$O(\varepsilon) \to \ell = 1$, axial $\Rightarrow S_1$

$O(\varepsilon^2) \to \ell=1 \oplus \ell=1 \to \ell=0 \to \delta M_0$ (rescaled)

$\to \ell=2 \to \delta M_2 \to$ QUADRUPOLE MOMENT

↳ Dipole: $\boxed{D_2 h_0^\ell = 0} \to h_0^\ell \propto \frac{x}{r}$

↳ Quadrupole eq: $\boxed{D_1 H_0^\ell = T_1^\ell} \longrightarrow$

$$D_1 \equiv \frac{d^2}{dx^2} + \frac{2f}{r}\frac{d}{dx} - \left(f\frac{\ell(\ell+1)}{r^2} + \frac{4M^2}{r^4}\right) \; ; \; D_2 = \frac{d^2}{dx^2} + \frac{4M - \ell(\ell+1)}{r^3 F}$$

☆ ↳ Requiring $\underbrace{K}_{\text{curvature}} M^4 \lesssim O(1)$ as $\varepsilon \to 0$ ("soft ECOs") $\Rightarrow \boxed{\frac{\delta M_2}{M^3} \to \frac{\chi^2}{\log\varepsilon}}$ or faster

In general, for SOFT ECOs [Raposo, Emparan, PP, 2018]

$$\boxed{\frac{\delta M_\ell}{M^{\ell+1}} \to \underbrace{a_\ell \frac{\chi^\ell}{\log \varepsilon}}_{\text{spin-induced}} + \underbrace{b_\ell\, \varepsilon}_{\text{Non-spin induced}} + \cdots} \text{ (or faster)}$$

Non-spin induced [eg. $\ell=2$ at $O(\varepsilon)$]

↳ deviations must die off suff. fast for curvature to be small

↳ $\log(\varepsilon)$ scaling VERY COMMON and helpful!!

→ How to measure δM_2?

→ How to measure the GW signal of a binary:

↳ Quadrupole moments enters the

☆ $$\boxed{\mathcal{L} = \frac{\mu v^2}{2} + \frac{\mu M}{r} - \frac{Q_{ab}Q^{ab}}{4\lambda} + \frac{1}{2} G_{ab} Q^{ab} + \cdots}$$ $\cancel{\frac{1}{2} Q_{ab} \mathcal{E}^{ab}}$ [Bernard lecture]

(STF tensors)

[Poisson-Will]

↳ EQ. of Motion for $\boxed{r = r(\varepsilon)}$ (orbital distance)

$$E_{\text{binding}} = -\frac{m_1 m_2}{2r} \Longrightarrow \dot{E}_{GW} = \frac{32}{5}\left(\frac{M^3 \mu^2}{r^5}\right) = -\dot{E}_{\text{binding}} \Rightarrow \boxed{r = r(\varepsilon)}$$

$$\Rightarrow \Omega = \sqrt{\frac{M}{r^3}} \to 2\Omega = \frac{d\phi_{GW}}{dt} \Rightarrow \phi_{GW} \to \text{GW phase} \; \boxed{\frac{\partial^2 \Psi}{\partial \omega^2} = \frac{2}{\dot{E}}\frac{\partial E}{\partial \omega}}$$

$$\phi_{GW} = \left(\frac{3}{128 \nu x^{5/2}}\right)\left[1 + \gamma \underbrace{\frac{m_2 Q_1 + m_1 Q_2}{m_1^2 m_2^2}}\, v^2\right] \longrightarrow \boxed{2PN} \text{ CORRECTION}$$

$x = (M\Omega)^{2/3}$ $\quad \nu = \frac{m_1 m_2}{M^2}$

1°] EFFECT OF SPIN : ERGOREGION INSTABILITY [Friedman 1970s]
('let's do it at the end)

1°] GW II : TIDAL ~~HEATING~~ LOVE NUMBERS $D_1 H_0^\ell = 0 \quad \ell = 2$

↳ Quadrupolar pert. of spherical bkg due to tidal field

$$\star \quad g_{tt} = -\left(1 - \frac{2M}{r}\right) + \frac{3Q_{ij}}{r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \mathcal{E}_{ij} x^i x^j$$

↳ BH: regularity at the horizon implies $\boxed{K_2 = \frac{M_2}{E_2} = 0}$

$\boxed{\text{TLNs of a BH} = 0}$ (so far!) $\boxed{H_0 = \alpha \, \mathrm{Reg}(r-2M) + \beta(\mathrm{Irreg})}$

↳ does NOT mean BHs don't get tidally deformed!

↳ For ECOs: generic "Robin" BCs $\boxed{a\Psi + b\frac{d\Psi}{dx} = c}$ (on Zerilli)
at $r = r_0$

$$\boxed{K_2 \sim \frac{4a - 3c}{15a \log \mathcal{E}}} \to \text{again } \log \mathcal{E}$$

↳ How to measure it?

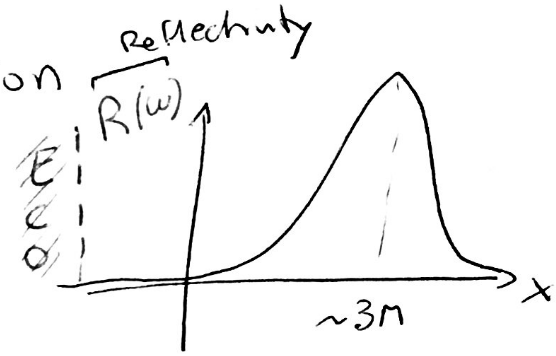
$$\mathcal{L} \supset \frac{1}{\lambda} Q_{ab} Q^{ab} \Rightarrow Q_{ab} \sim \lambda \mathcal{E}_{ab} \sim \lambda \left(\frac{m_2}{r^3}\right)$$
↳ 3PN

$$\Rightarrow \phi_{\text{Tidal}} \sim (2PN) \times \mathcal{E}_{ab} = \underline{\underline{5PN}} \qquad \boxed{\frac{\lambda}{M^5} = K_2 \left(\frac{R}{M}\right)^5}$$

↳ same eff as for NSs
$\star$

$\boxed{\text{ECO PERTURBATIONS}}$

$r > r_0 \to$ Zerilli's and RW equation



$\frac{dr}{dx} = 1 - \frac{2M}{r}$

$$\psi''(x) + (\omega^2 - V)\psi = 0$$

$$V_{RW} = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right)$$

BH BCs:

↳ $\psi \sim e^{\pm i\omega x}$ $\quad x \to \pm\infty$ $\quad \cancel{\dot{E}_\pm \propto \omega^2 |\psi|^2}$

↳ define QNMs [Cardoso's lecture]

In presence of source:

$\psi'' + (\omega^2 - V)\psi = S \implies$ Green's function:

$$\psi(x) = \frac{\psi_+}{W}\int_{-\infty}^{r} dx\, \psi_- S + \frac{\psi_-}{W}\int_{r}^{+\infty} dx\, \psi_+ S$$

$\underbrace{\psi_+ \to e^{i\omega x}\ \ x \to \infty \ ; \ \psi_- \to e^{-i\omega x}\ \ x \to -\infty}_{\text{homog. sol.}}$

$$\implies \boxed{\dot{E}_\pm \propto \omega^2 |\psi|^2}$$

↳ Energy fluxes

$$\boxed{\dot{E}_H = \dot{E}_- = \dot{M} \propto \frac{\Omega_K^6}{M^2}}$$ → For a particle in circular orbit, low-freq.

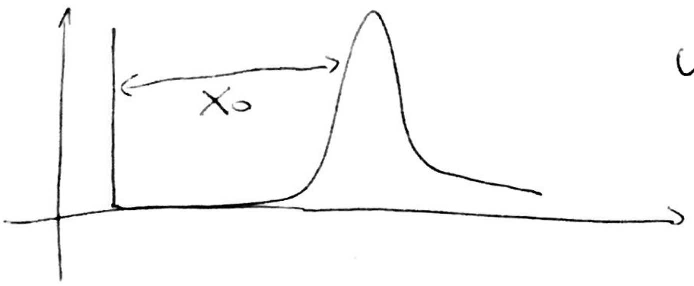
↳ $\dot{E}_{bind} = -\dot{E}_H \implies$ TIDAL HEATING!

↳ For an ECO $\boxed{\dot{E}_{TH} \sim (1 - |R|^2)\dot{E}_{BH}}$ $\quad (\simeq 0$ if $|R|^2 \simeq 1)$

↳ ($2.5^{PN} \times \log v$) effect if spinning $\quad \left(\dot{E}_H \propto \frac{\Omega_K^5}{M^2}\underbrace{(\Omega_K - \Omega)}_{\text{superradiance}}\right)$

↳ $4PN \times \log v$ " " nonspinning

☐ QNMs : change BC at $r = r_0$



$$\omega_{QNM} \sim \frac{2\pi n}{X_0} \sim \frac{1}{\log \varepsilon}$$ $\rightarrow$ LOW FREQUENCY !!

$\omega_I \rightarrow$ tunneling probability [Cardoso's lecture] $\quad \varphi \sim \varphi_0 e^{i\omega_I t}$

$$|A|^2 \sim (M\omega_R)^{2\ell+1} \ll 1$$

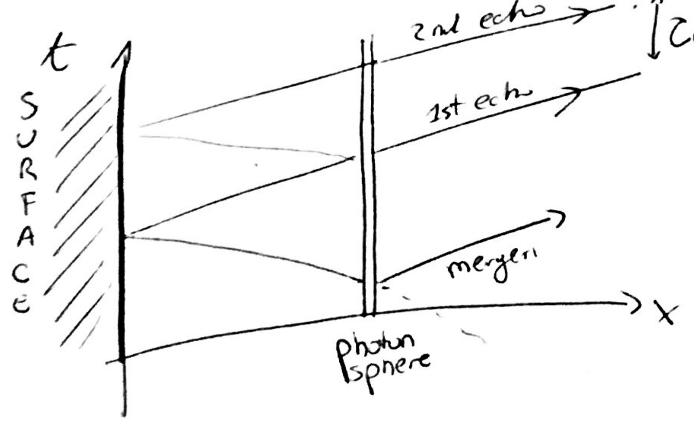
$$N \sim \frac{t}{X_0} \text{ reflections} \Rightarrow \varphi = \varphi_0 (1 - |A|^2)^N \simeq \varphi_0 \left(1 - \frac{t}{X_0} |A|^2\right)$$

ω_I

$$\Rightarrow \boxed{\omega_R \sim \frac{1}{X_0} \; ; \; \omega_I \sim \frac{|A|^2}{X_0} \sim \omega_R^{2\ell+3}}$$

Narrow Resonances

→ Does the prompt ringdown change? → No! → ECHOES



$$\tau_{echo} \sim \int_{3M}^{r_0} dr \sqrt{\frac{g_{rr}}{g_{tt}}} \sim \log \varepsilon$$

$$\psi_- = \begin{cases} e^{-i\omega x} & x \to -\infty \\ A_{in} e^{-i\omega x} + A_{out} e^{i\omega x} & x \to +\infty \end{cases}$$

$$\psi_+ = \begin{cases} B_{in} e^{-i\omega x} + B_{out} e^{i\omega x} & x \to -\infty \\ e^{i\omega x} & x \to +\infty \end{cases}$$

$$R_{BH} = \frac{B_{in}}{B_{out}} \; ; \; T_{BH} = \frac{1}{B_{out}}$$

(1) $\psi_{ECO} = K_0 \left[e^{-i\omega x} + R e^{i\omega x} \right] \quad x \to x_0$

↳ Different integration constants in the Green's function

(2) $\psi_{ECO} = \psi_{BH} + K \psi_+ \int_{-\infty}^{+\infty} dx \frac{S \psi_+}{W}$

$$(1) = (2) \Rightarrow \boxed{K = \frac{T_{BH} R}{1 - R_{BH} R}}$$

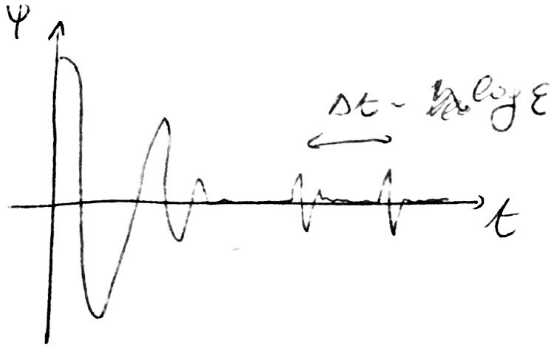
TRANSFER FUNCTION [Mark+, 2017]

Near $+\infty$

$$\Psi_{ECO} = \Psi_{BH}(r\to+\infty) + \underbrace{K\, e^{2i\omega x}\, \Psi_{BH}(r\sim r_+)}_{\text{TRASF FUNCT}}$$

TRASF FUNCT REPROCESSES BH RESPONSE AT HORIZON.

$$\boxed{K = T_{BH} R \sum_{n=1}^{\infty} (R R_{BH})^{n-1}}$$ geom. series



- Echo signal VERY RICH :
  - ↳ Transient effect (QNMs only at $t\to+\infty$)
  - ↳ Analytical modelling complicated
  - ↳ Source dependence ?
  - ↳ Spin effects important [Maggio+ 2019]
    - → Mixing of polarizations
    - → Frequency Modulation ($R_{BH} = R_{BH}(\omega,\chi)$)

↳ progress in modelling but still open problems

⊙ BONUS: ERGOREGION INSTABILITY

$$\omega_I \propto \omega_R^{2\ell+3} \xrightarrow{\text{spin}} \omega_R^{2\ell+2}\underbrace{(\omega_R - m\Omega)}_{\text{superradiance}}$$