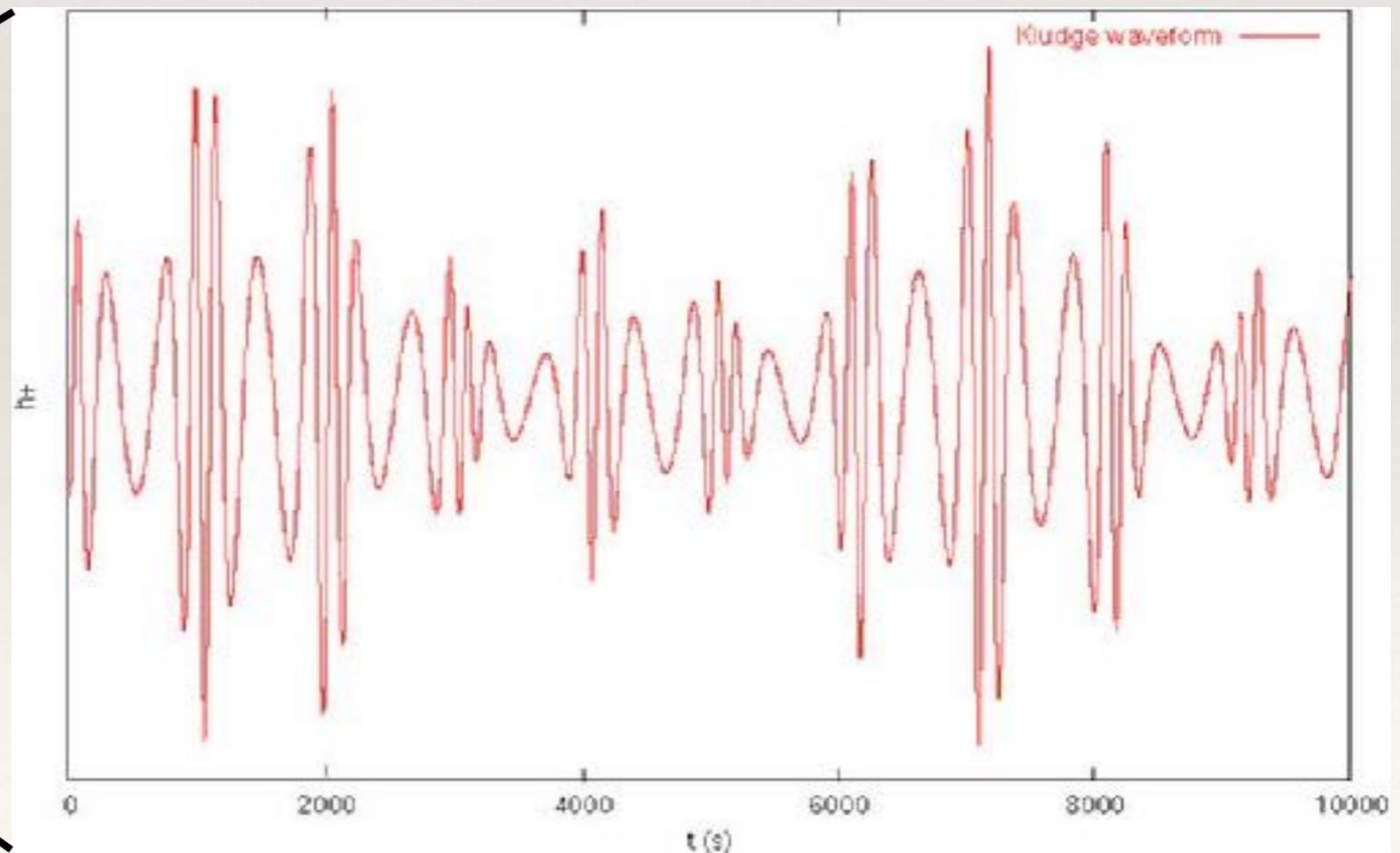
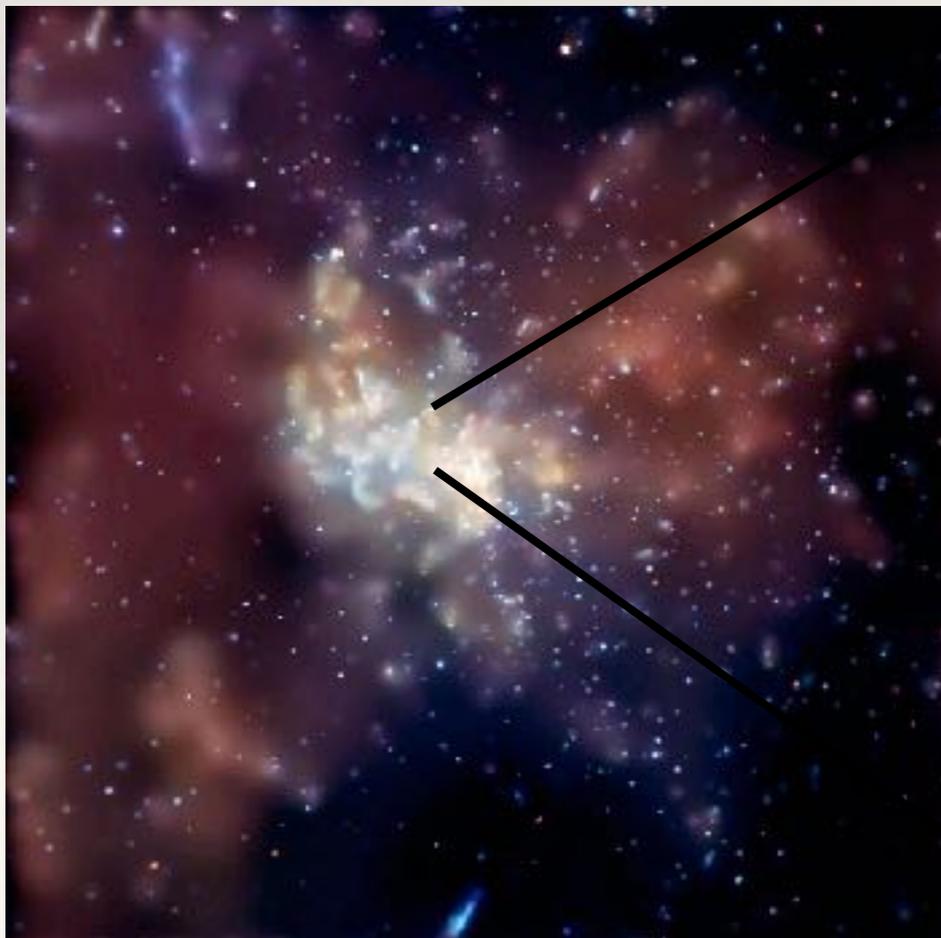


# Gravitational wave physics with LISA and pulsar timing arrays

Jonathan Gair, Albert-Einstein-Institute, Potsdam, Germany

Kavli RISE Summer School on Gravitational Waves, September 26th 2019



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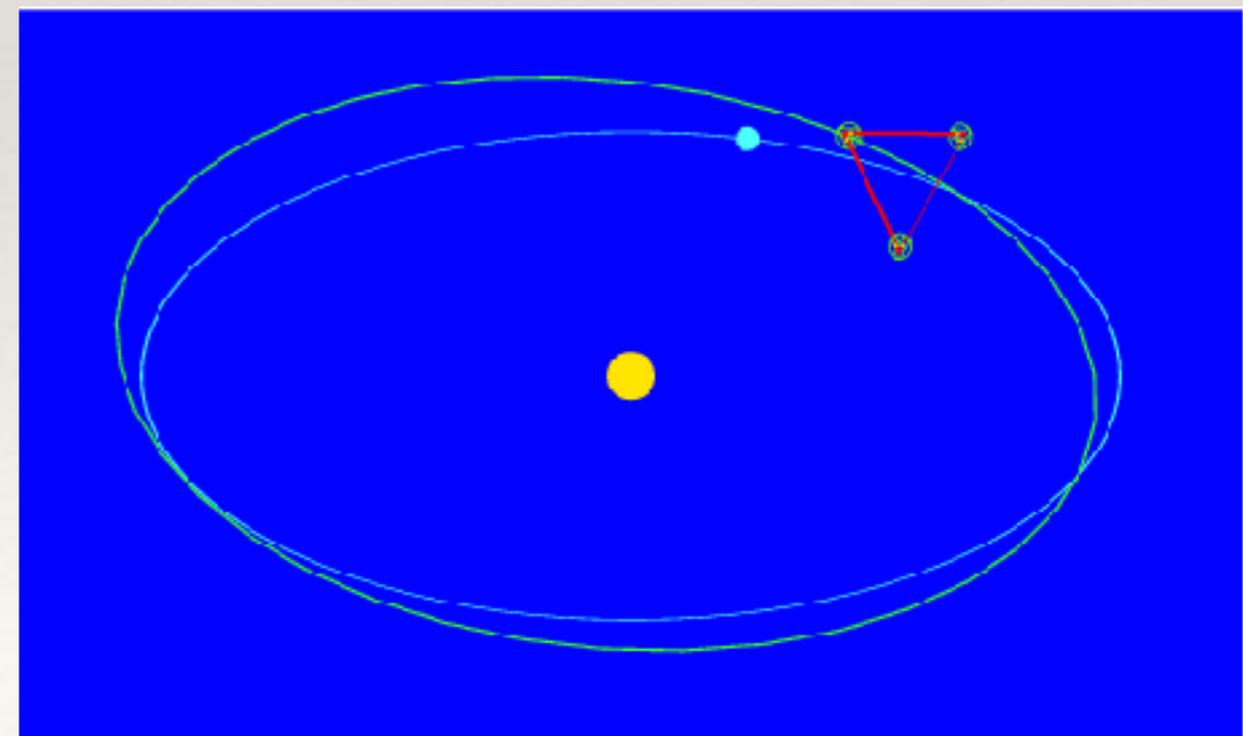
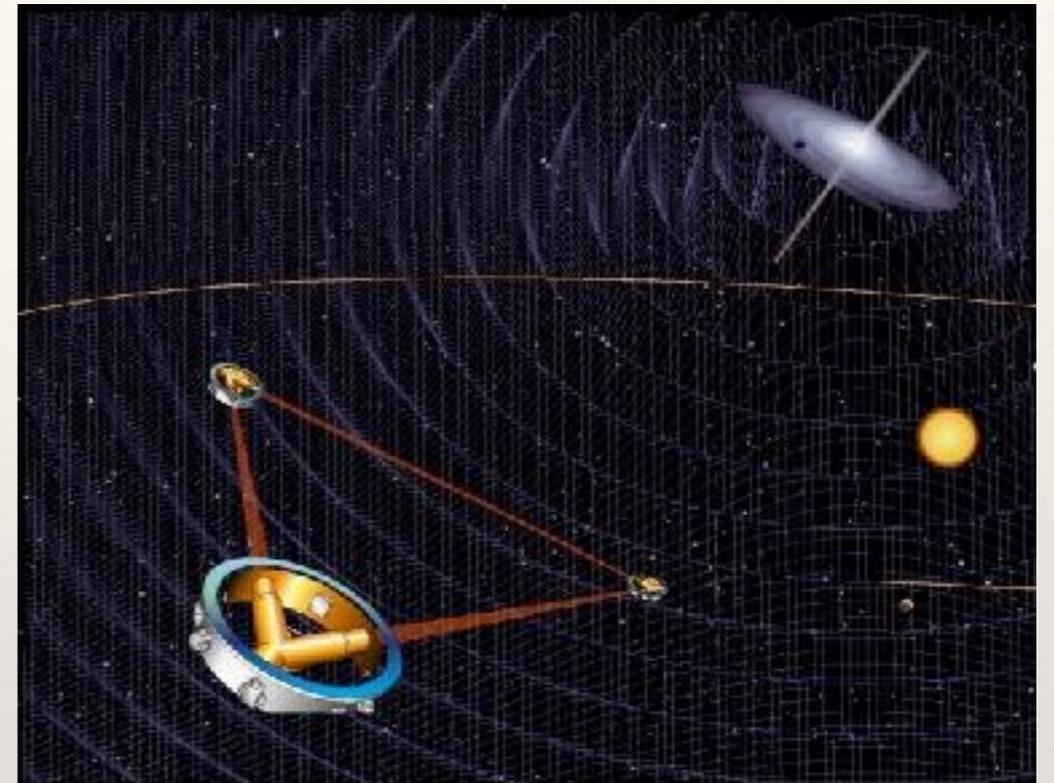
# Talk Outline

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- ❖ The Laser Interferometer Space Antenna (LISA)
- ❖ Pulsar-timing detection of gravitational waves
- ❖ Sources for LISA and pulsar timing arrays (PTAs)
- ❖ Tests of gravitational physics with LISA observations
- ❖ Cosmography with LISA observations
- ❖ Fundamental physics with PTA observations

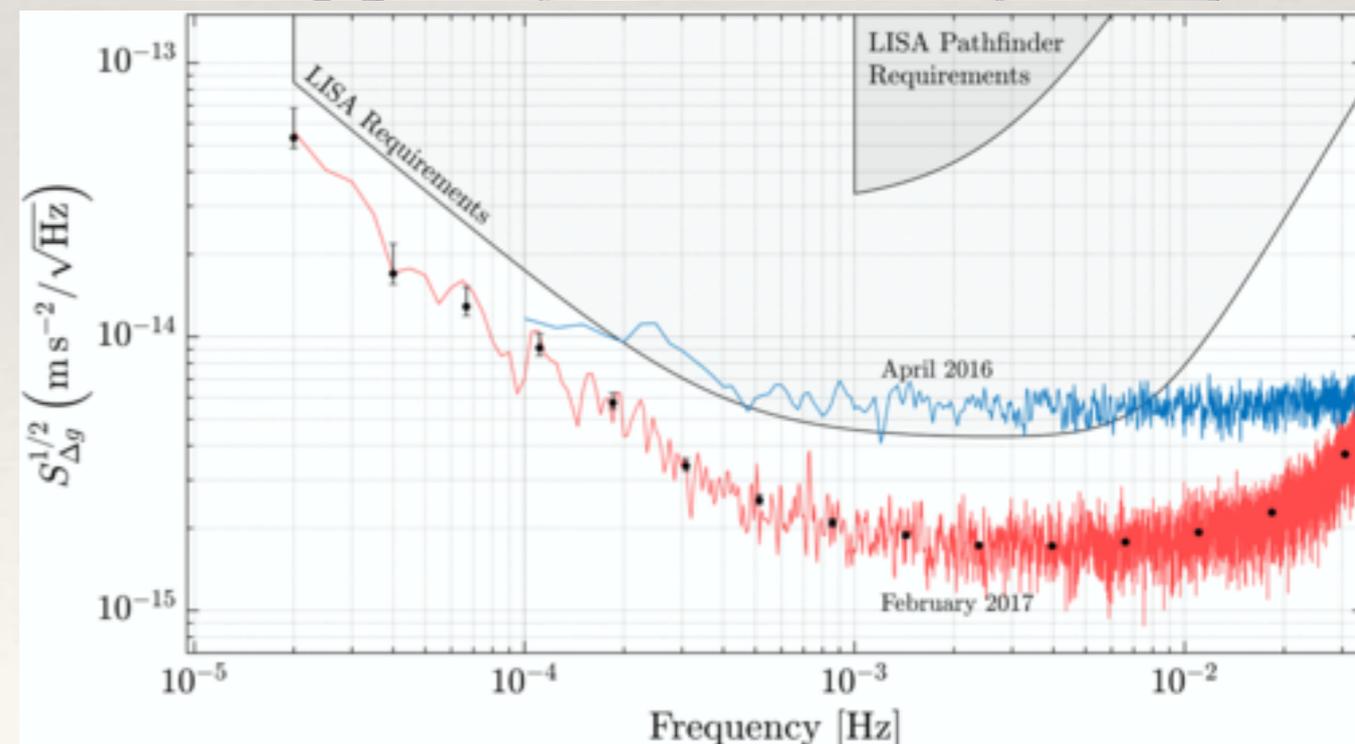
# The Laser Interferometer Space Antenna

- ❖ Long history. Original design (1998)
  - Gravitational wave detector operating in millihertz band.
  - Three satellites, 5 million km apart, in heliocentric, Earth-trailing orbit. 6 laser links.
  - Joint NASA / ESA project.
  - Technology demonstrator mission, LISA Pathfinder, approved. Launched 2015.
- ❖ NASA dropped out in 2011. New ESA-only mission eventually selected for L3 (2034).



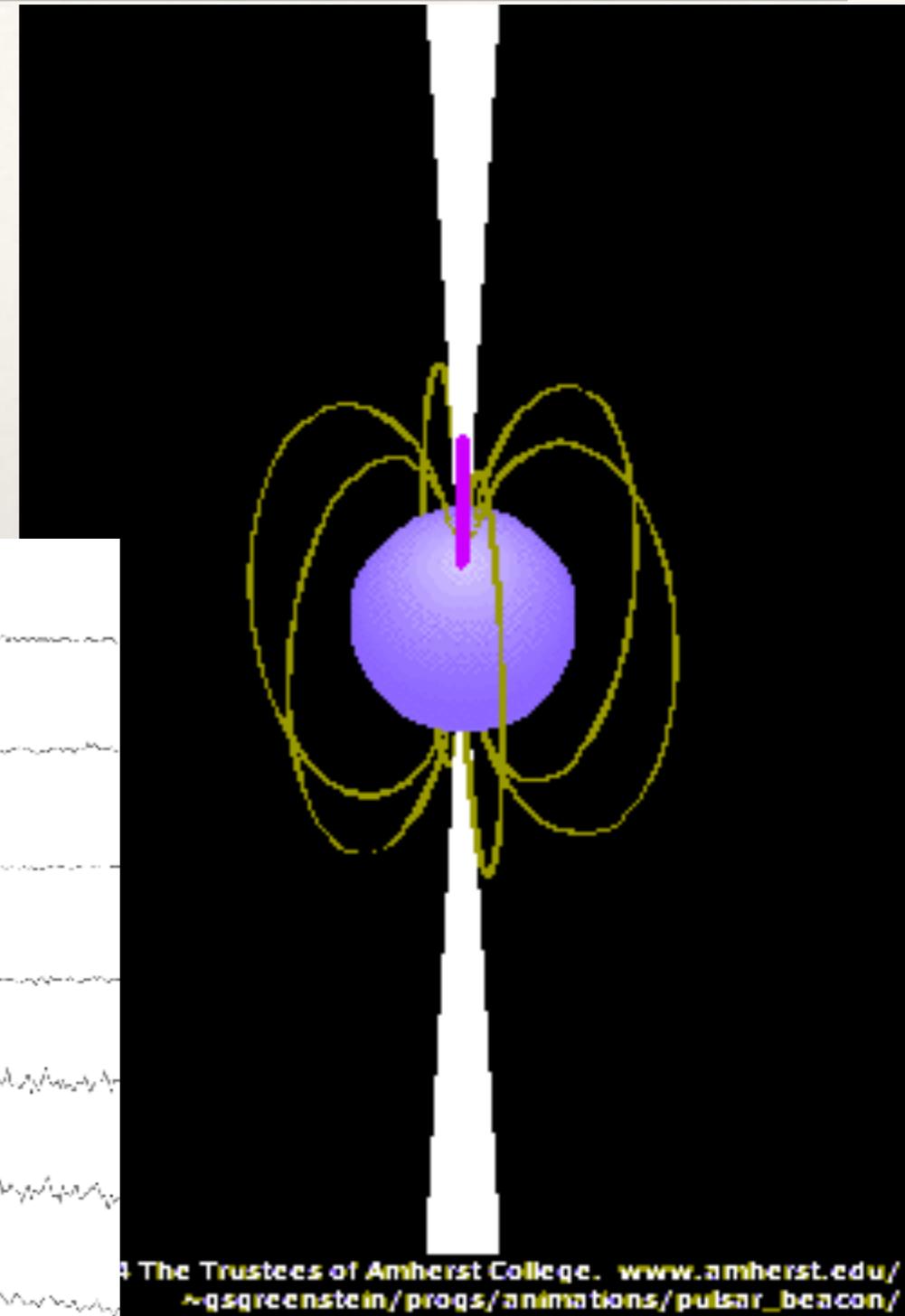
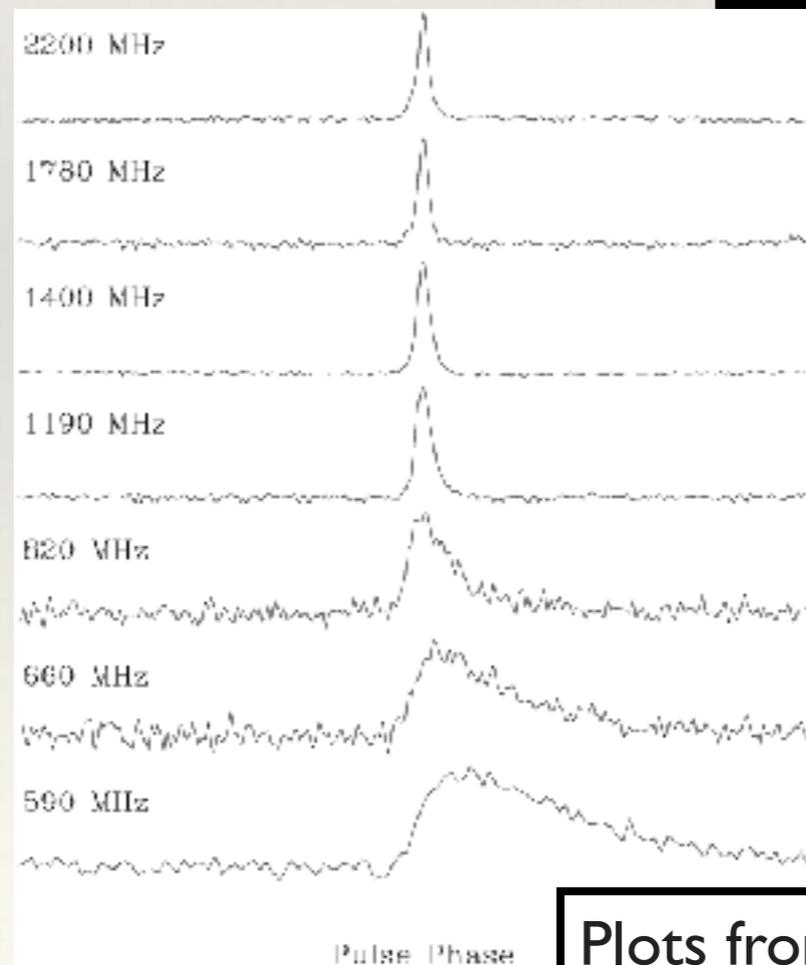
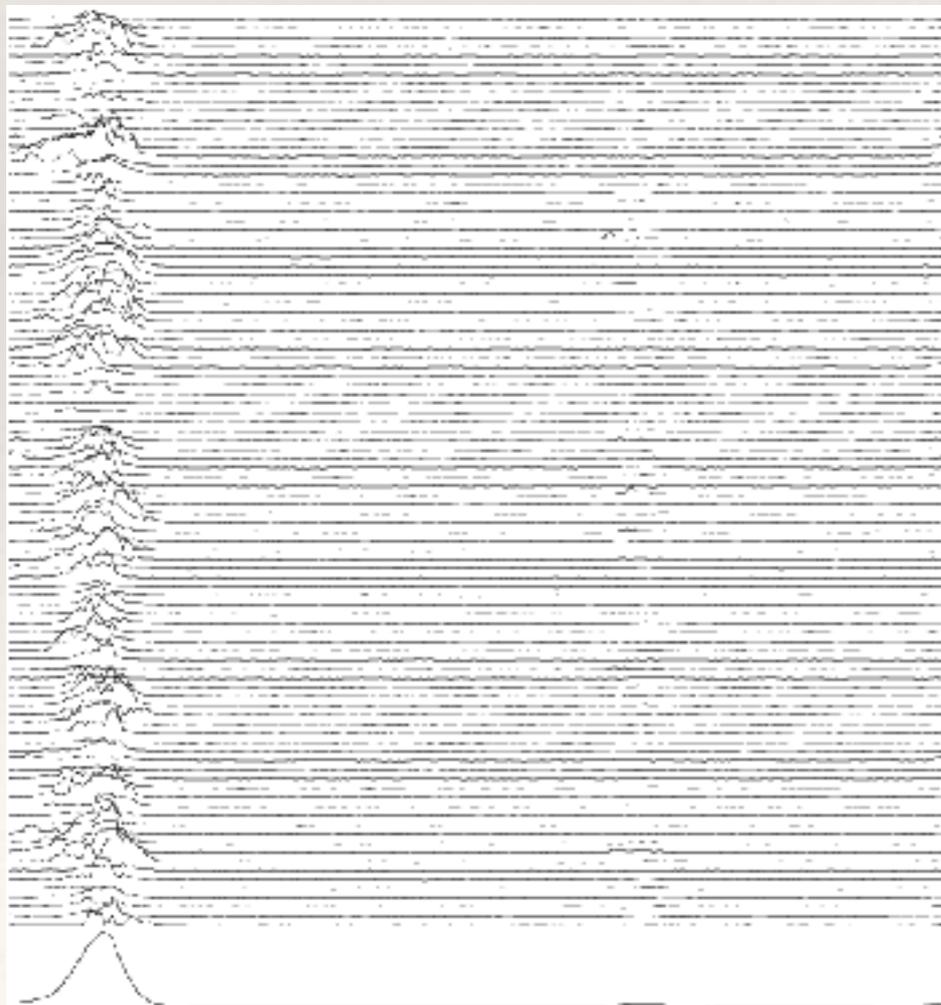
# LISA Status

- ❖ LISA now reinvigorated and timetable accelerated
  - LISA Pathfinder spectacularly demonstrated the technology.
  - Detection of GW150914+ renewed interest in gravitational waves.
  - mission now in phase A, adoption in 2022-2024;
  - mission launch: 2034.
- ❖ Mid-decadal review expressed strong support for NASA re-involvement, at probe-class level (~\$400m).
- ❖ Design: 2.5Gm arms, 6-link geometry.



# Pulsar timing

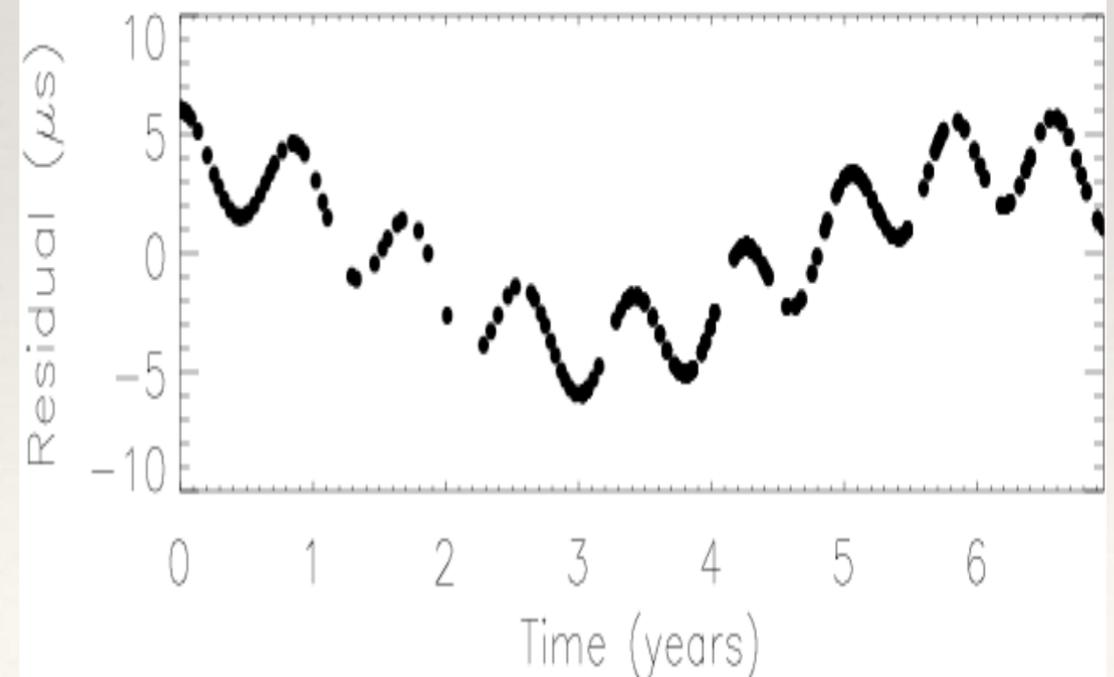
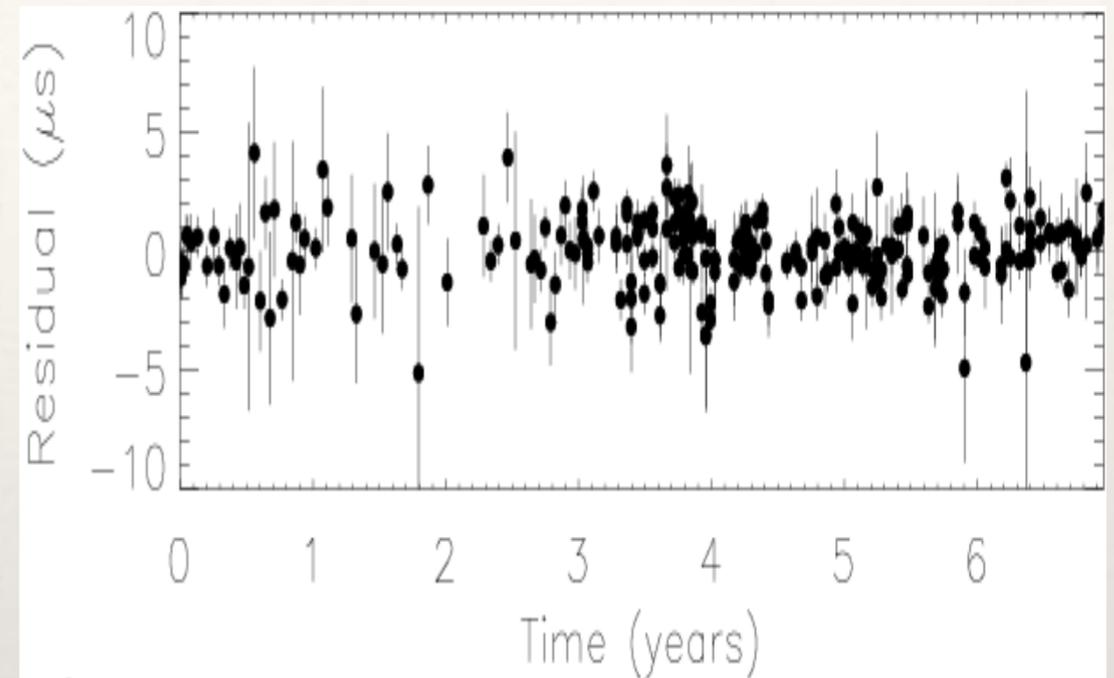
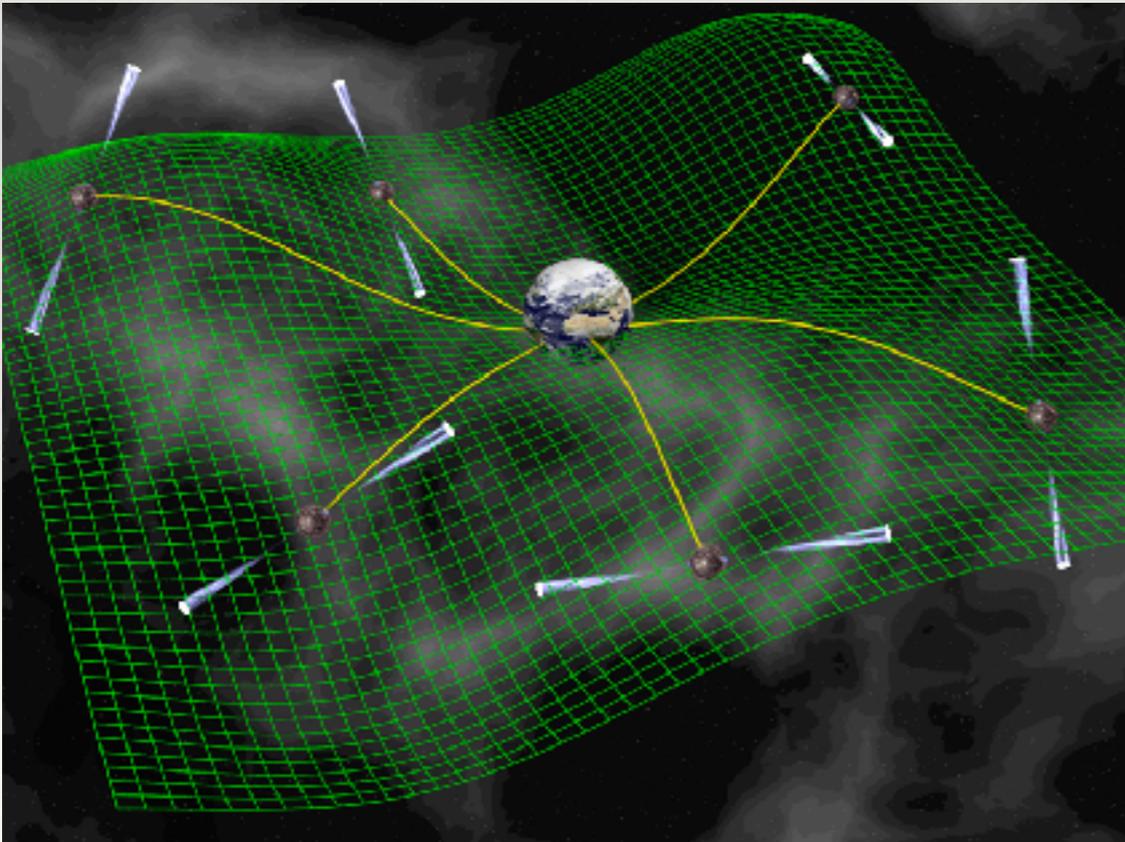
- ❖ Pulsars are rapidly rotating Neutron Stars. Observations indicate great homogeneity in pulse profile, and little variation in frequency.
- ❖ Pulsars are very accurate clocks.



Plots from I H Stairs (2003)

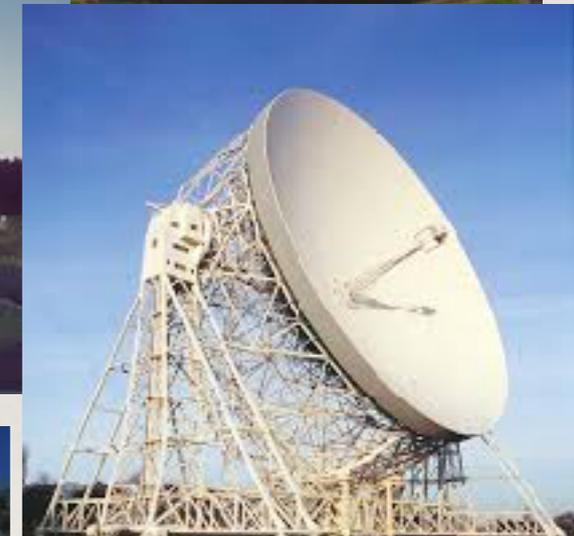
# Pulsar timing arrays

- ❖ GW passing between source and observer induces periodic change in pulse time of arrival.
- ❖ Use a network (array) of pulsars to increase signal to noise.



# Pulsar timing arrays

- ❖ There are three major pulsar timing efforts
  - **EPTA** - the European Pulsar Timing Array. Data collected from six telescopes in UK, Netherlands, France, Germany and Italy.



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- **NANOGrav** - US/Canada PTA. Data collected using Arecibo and the Green Bank Telescope.

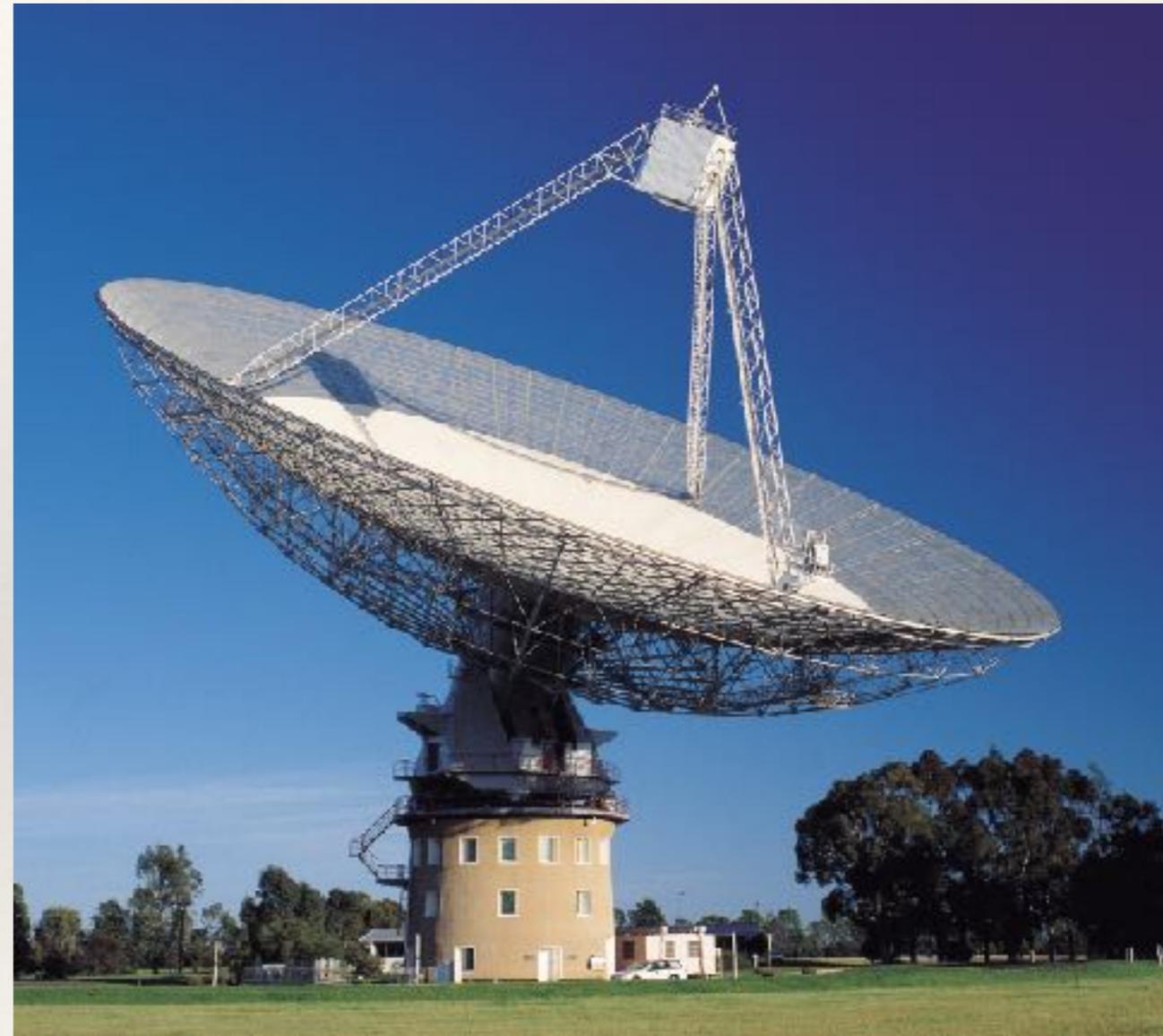


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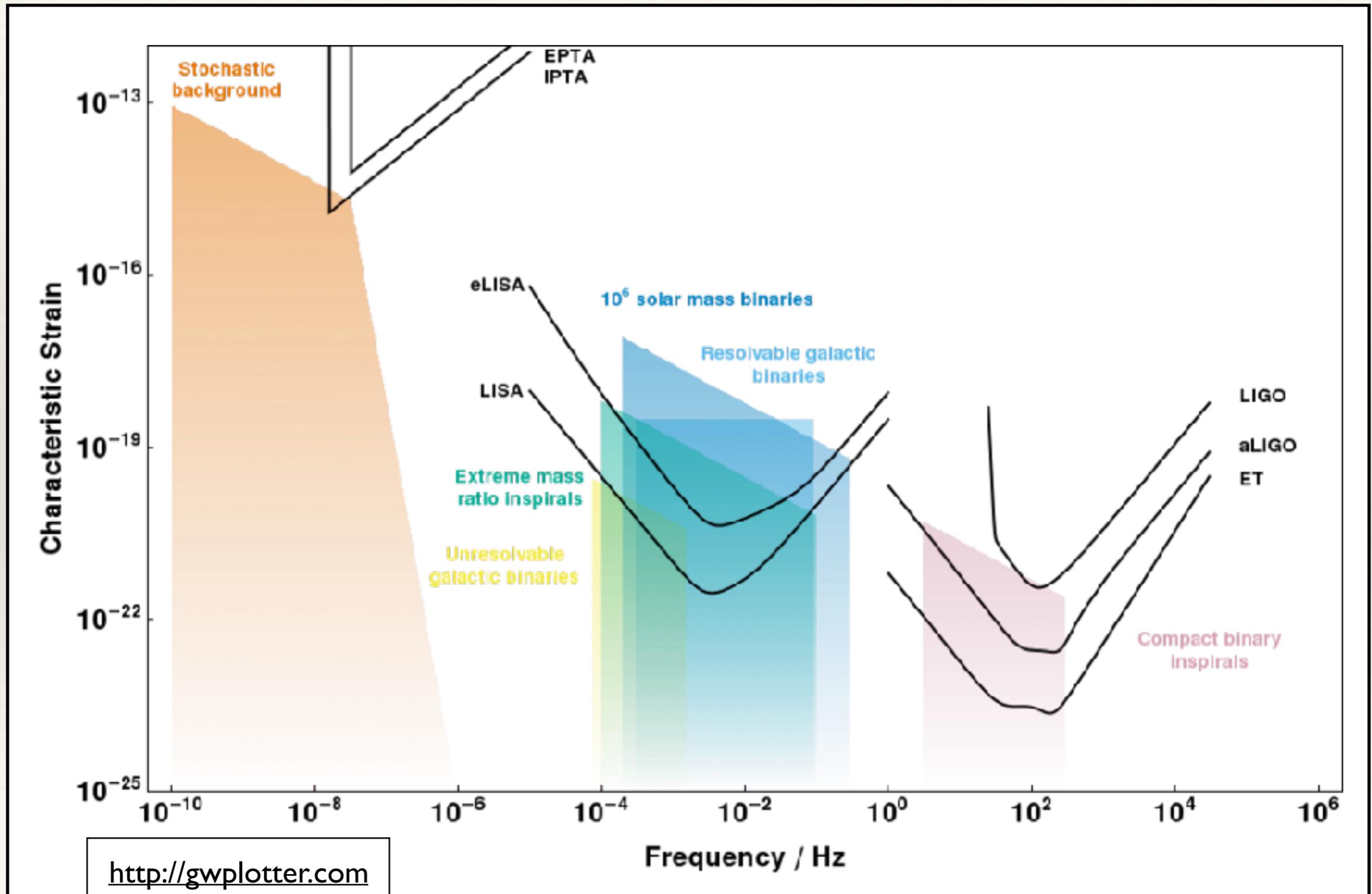
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  - **NANOGrav** - US/Canada PTA. Data collected using Arecibo and the Green Bank Telescope.
  - **PPTA** - the Parkes Pulsar Timing Array. Australian collaboration.
- ❖ The three PTAs combine data as the International Pulsar Timing Array (IPTA).



# Detector sensitivities



# Gravitational wave sources for LISA

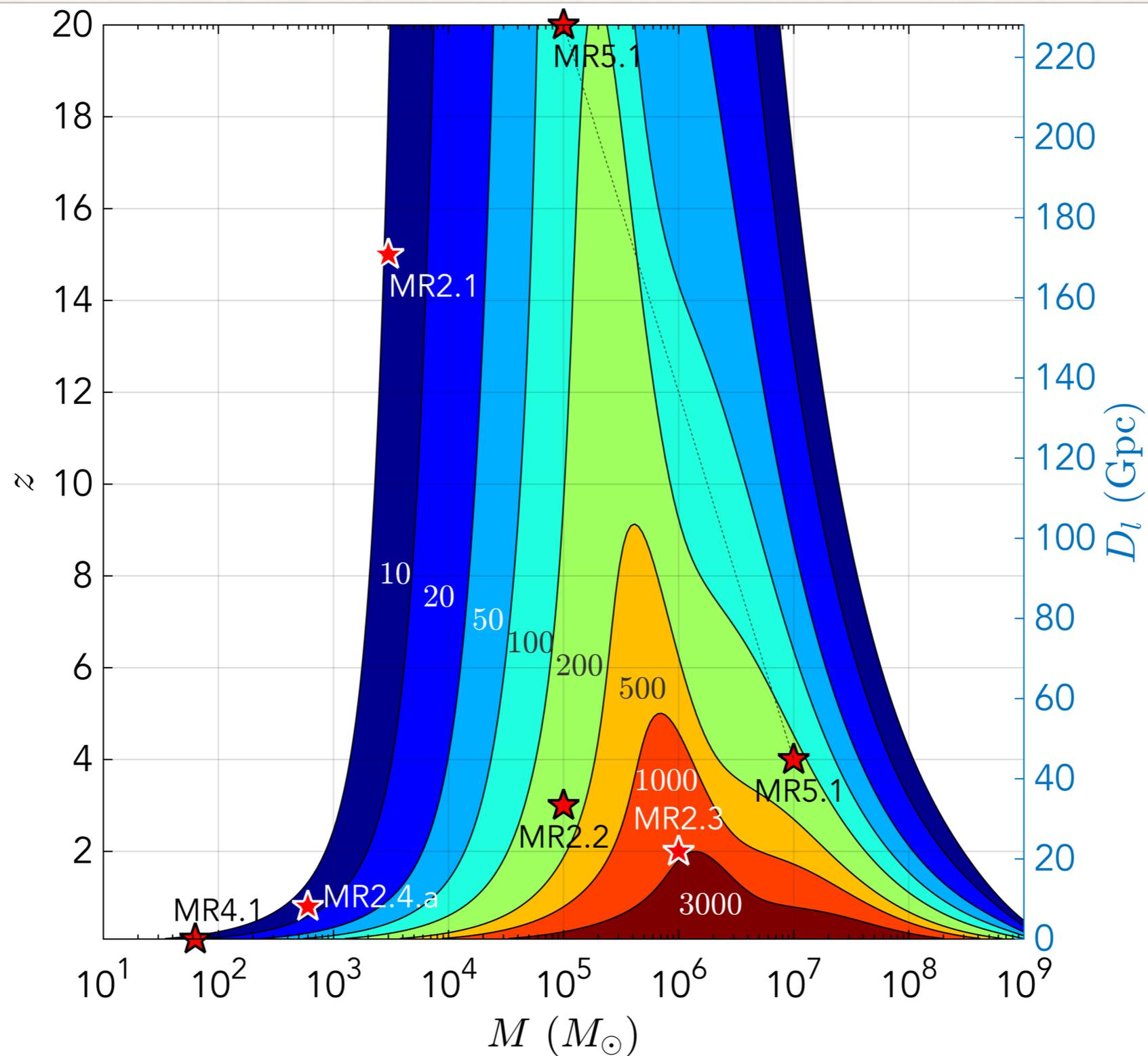
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# LISA sources: massive black hole mergers

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- ❖ Expected to occur following mergers of the host galaxies. LISA can observe gravitational waves from these with very high signal-to-noise ratio.

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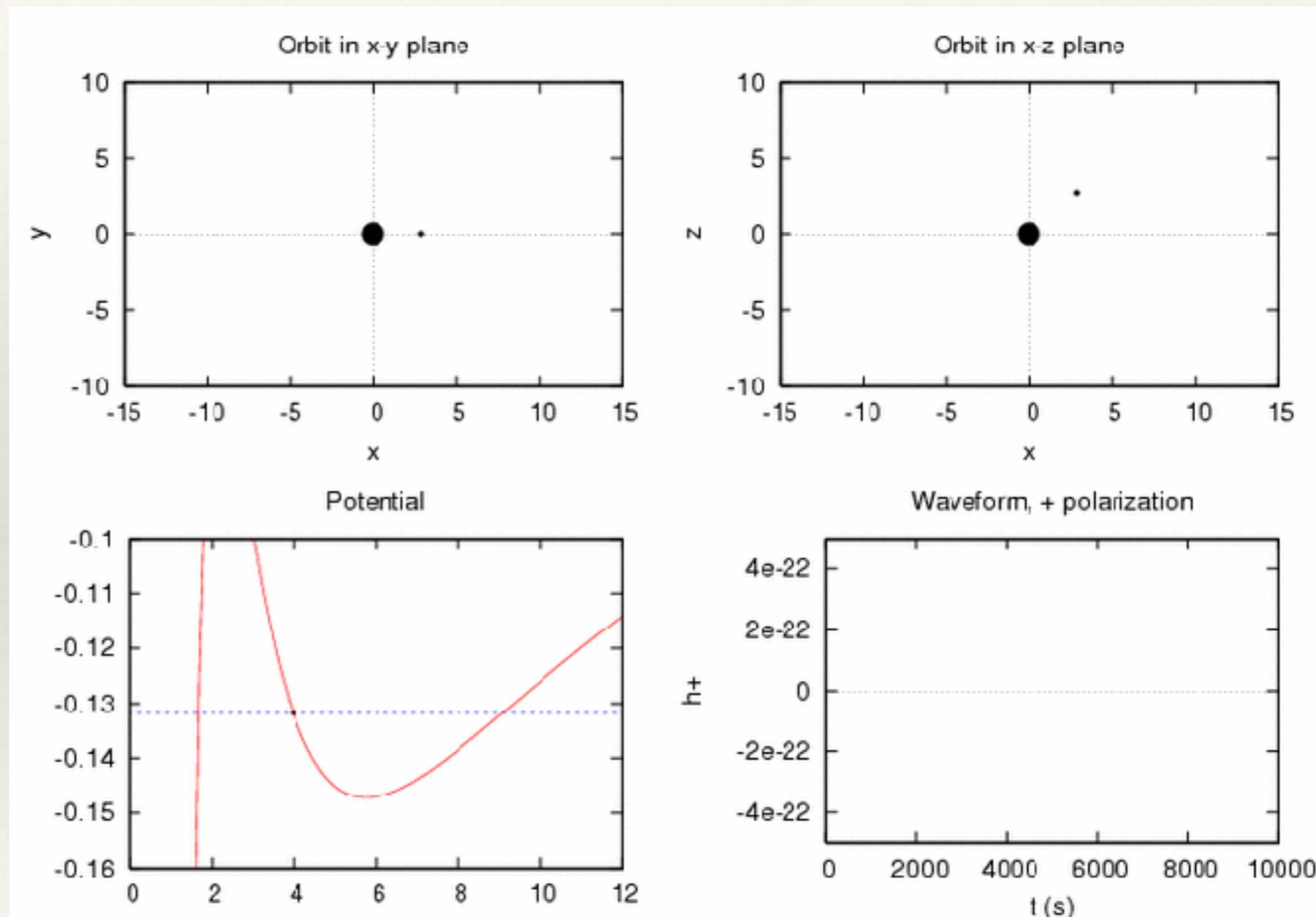
# LISA sources: massive black hole mergers

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- ❖ Expected to occur following mergers of the host galaxies. LISA can observe gravitational waves from these with very high signal-to-noise ratio.
- ❖ Expected event rate depends on assumptions about black hole population (Klein+, 2016)
  - Light pop-III seed model: baseline configuration expected to see ~350 events.
  - Heavy seed model, no delay in binary formation: ~550 events.
  - Heavy seed model, with delays: ~50 events.
- ❖ Baseline configuration would see 150 / 300 / 4 events at  $z > 7$  under the different models.

# LISA sources: extreme-mass-ratio inspirals

- ❖ The inspiral of a compact object into a massive black hole in the centre of a galaxy.
- ❖ Form as a result of scattering in dense galacto-centric stellar clusters.
- ❖ Orbits are expected to be both eccentric and inclined - rich waveform structure.



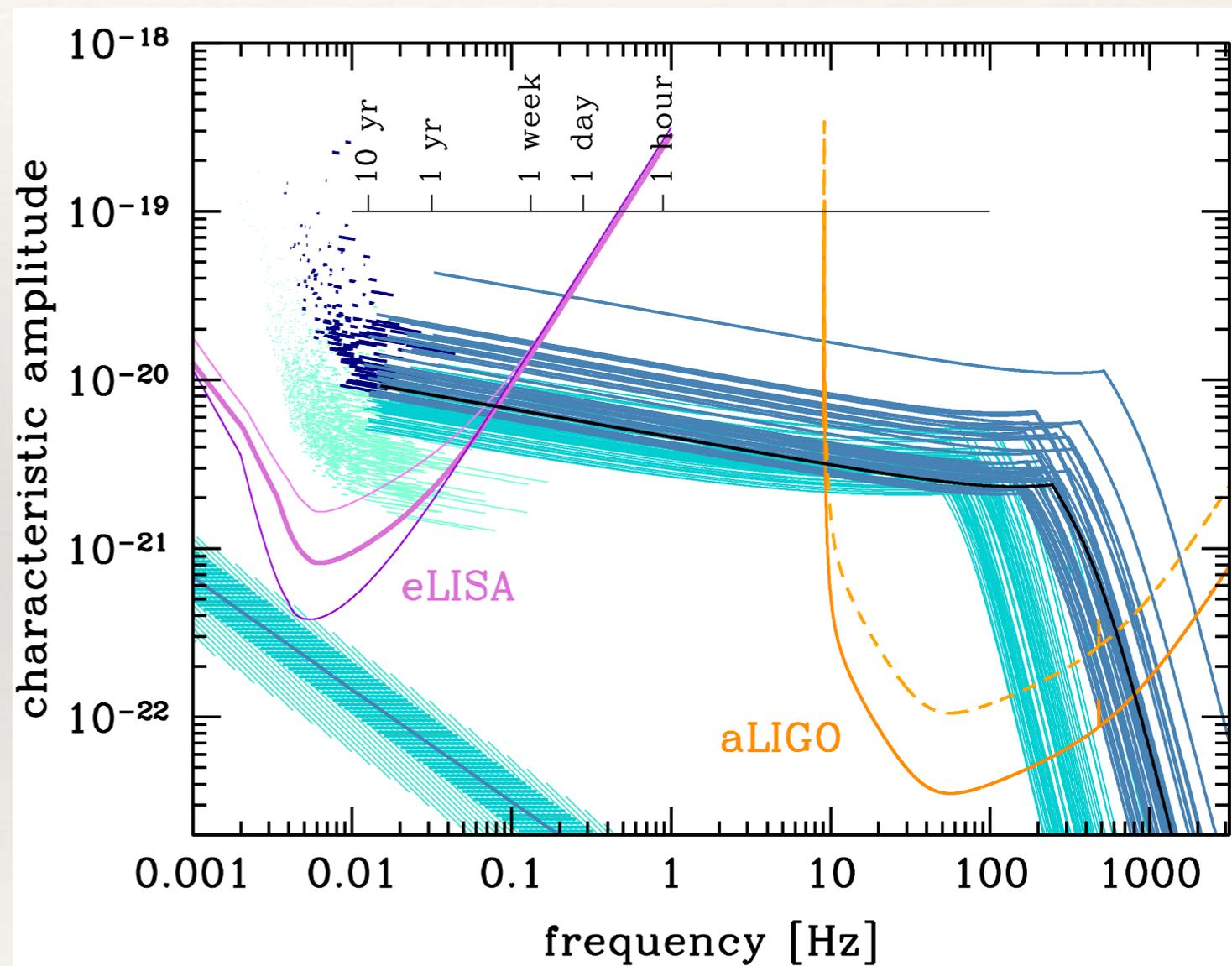
# LISA sources: extreme-mass-ratio inspirals

- ❖ There are large astrophysical uncertainties, but expect to see between a few tens and a few hundreds of events.

Model	Mass function	MBH spin	Cusp erosion	$M-\sigma$ relation	$N_p$	CO mass [ $M_\odot$ ]	Total	EMRI rate [ $\text{yr}^{-1}$ ] Detected (AKK)	Detected (AKS)
M1	Barausse12	a98	yes	Gultekin09	10	10	1600	294	189
M2	Barausse12	a98	yes	KormendyHo13	10	10	1400	220	146
M3	Barausse12	a98	yes	GrahamScott13	10	10	2770	809	440
M4	Barausse12	a98	yes	Gultekin09	10	30	520 (620)	260	221
M5	Gair10	a98	no	Gultekin09	10	10	140	47	15
M6	Barausse12	a98	no	Gultekin09	10	10	2080	479	261
M7	Barausse12	a98	yes	Gultekin09	0	10	15800	2712	1765
M8	Barausse12	a98	yes	Gultekin09	100	10	180	35	24
M9	Barausse12	aflat	yes	Gultekin09	10	10	1530	217	177
M10	Barausse12	a0	yes	Gultekin09	10	10	1520	188	188
M11	Gair10	a0	no	Gultekin09	100	10	13	1	1
M12	Barausse12	a98	no	Gultekin09	0	10	20000	4219	2279

# Stellar-origin black hole binaries

- ❖ GW150914 would have been observable by LISA ~5 years before being observed by LIGO, with  $S/N \sim 10$  in a 5yr observation. (Sesana 2016)
- ❖ LISA provides sky location to ~0.few square degrees and time of coalescence to ~few s.
- ❖ Number of events could be high (several hundred) but there are significant uncertainties.



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Mass distribution	$R/(\text{Gpc}^{-3}\text{yr}^{-1})$		
	PyCBC	GstLAL	Combined
	Event based		
GW150914	$3.2^{+8.3}_{-2.7}$	$3.6^{+9.1}_{-3.0}$	$3.4^{+8.6}_{-2.8}$
LVT151012	$9.2^{+30.3}_{-8.5}$	$9.2^{+31.4}_{-8.5}$	$9.4^{+30.4}_{-8.7}$
GW151226	$35^{+92}_{-29}$	$37^{+94}_{-31}$	$37^{+92}_{-31}$
All	$53^{+100}_{-40}$	$56^{+105}_{-42}$	$55^{+99}_{-41}$
	Astrophysical		
Flat in log mass	$31^{+43}_{-21}$	$30^{+43}_{-21}$	$30^{+43}_{-21}$
Power Law (-2.35)	$100^{+136}_{-69}$	$95^{+138}_{-67}$	$99^{+138}_{-70}$

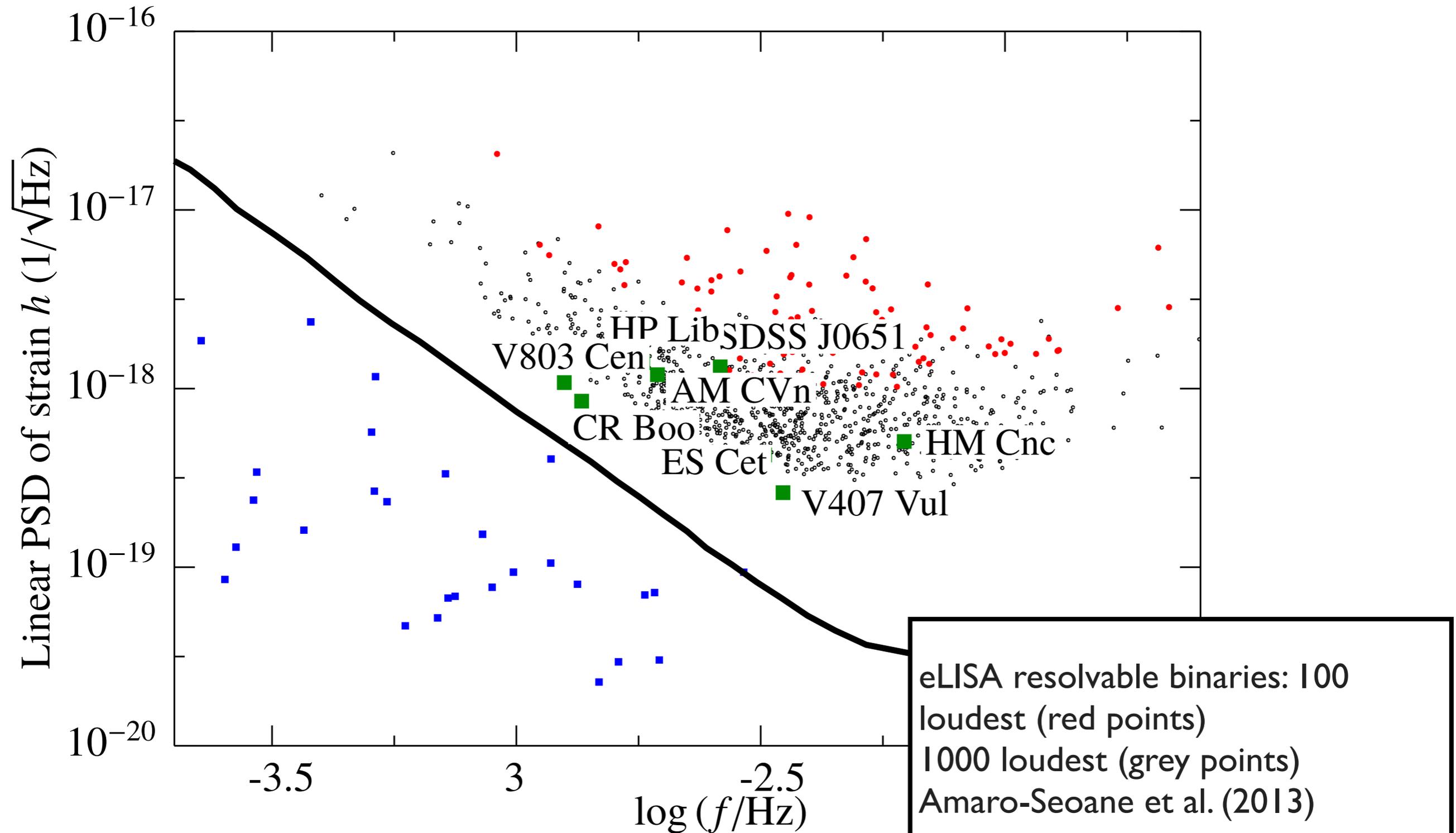
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# Other sources for LISA

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- ❖ Compact binaries in the Milky Way
  - Binaries of stellar remnants (white dwarfs or neutron stars) with orbital periods of  $\sim 1$  hour.
  - Known (verification) and unknown sources.
  - Signals almost monochromatic.

# Other sources for LISA



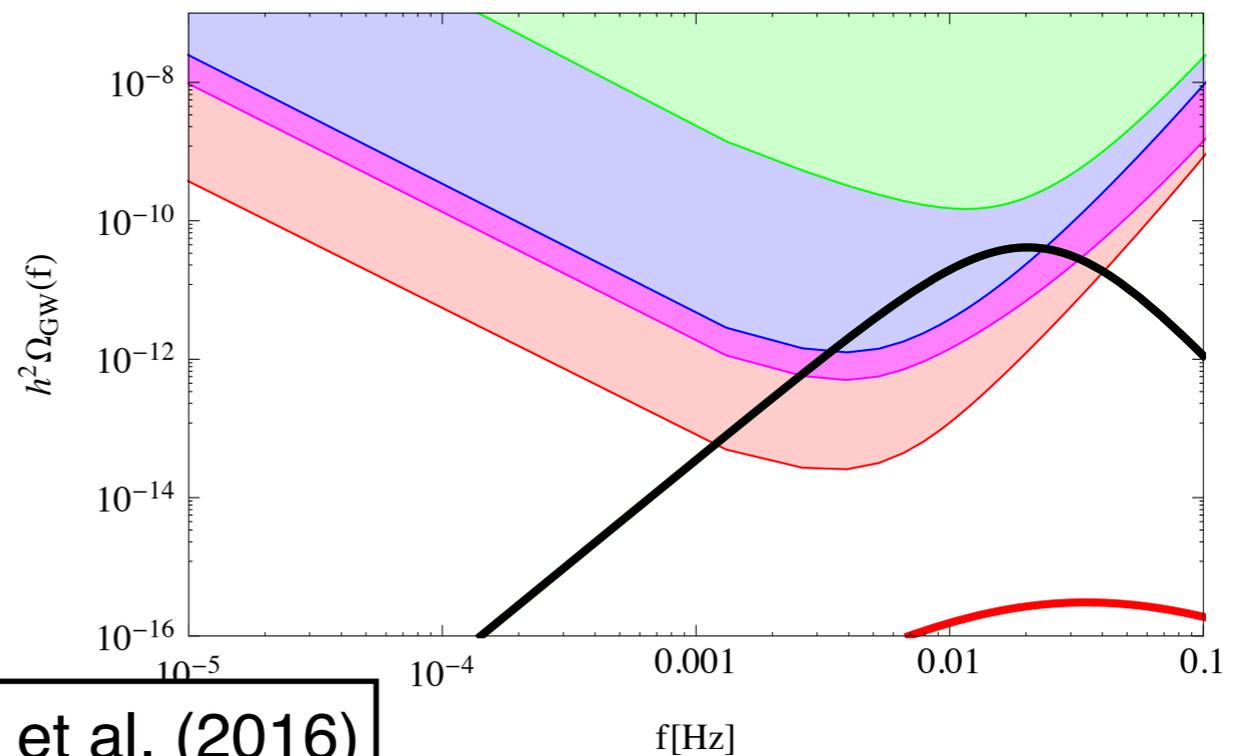
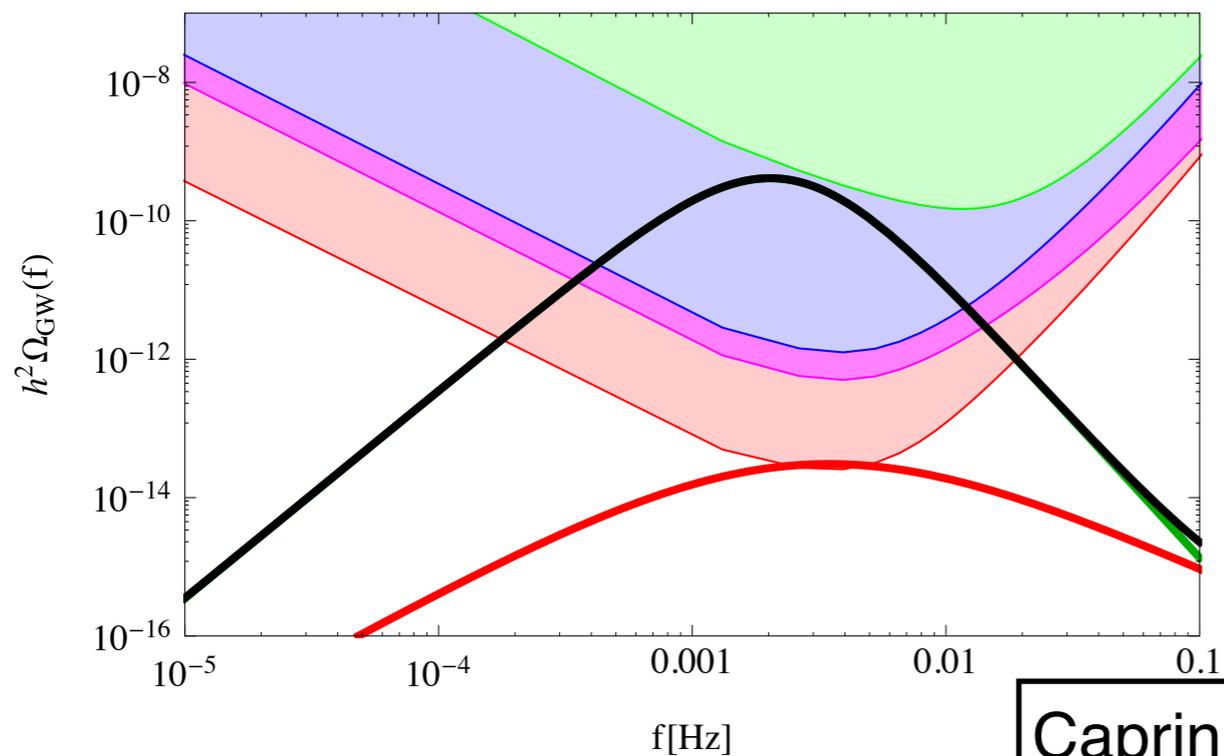
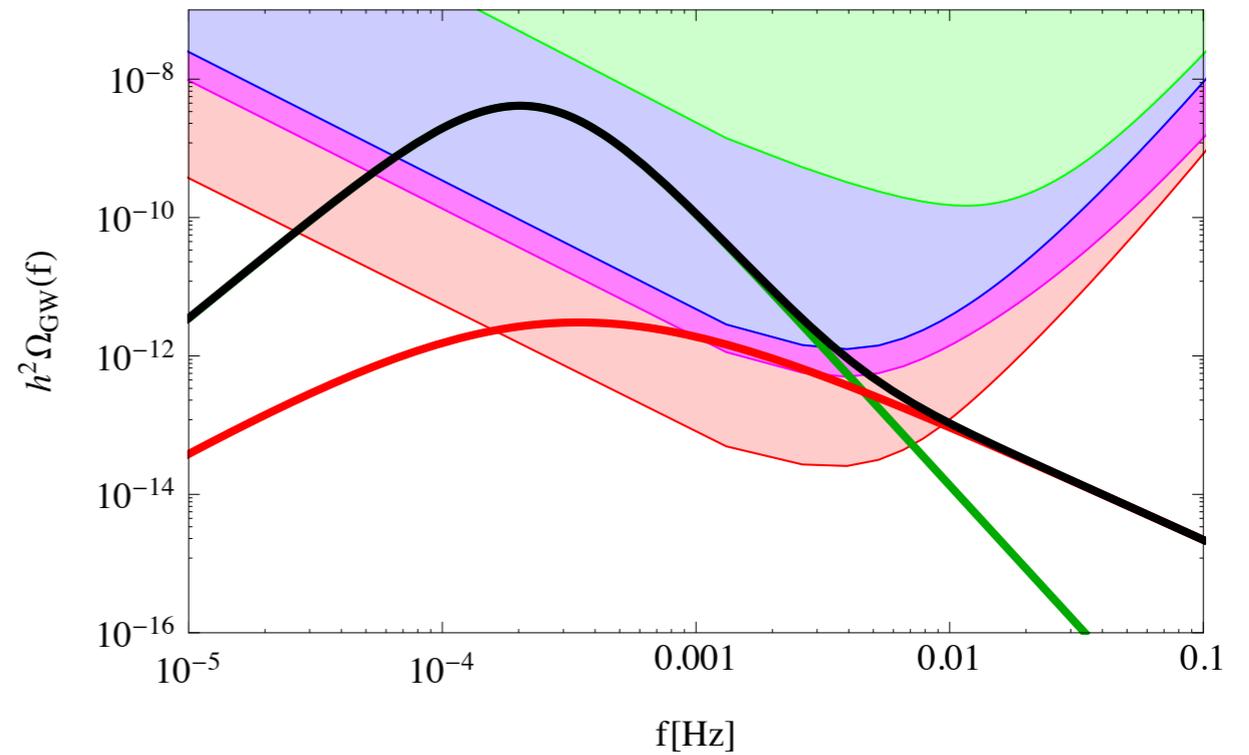
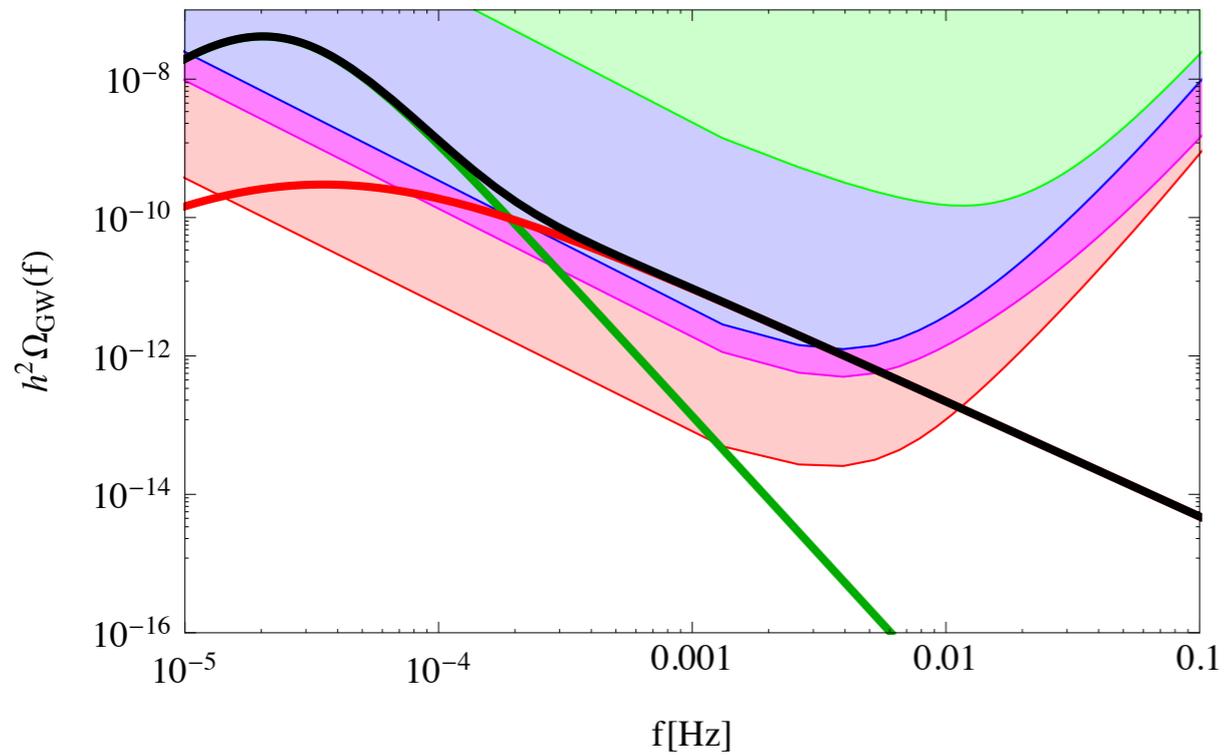
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  - Known (verification) and unknown sources.
  - Signals almost monochromatic.
  - LISA expected to detect  $\sim 15000$  binaries with  $S/N > 7$ .
  - LISA should determine 2D/3D location for 4500/1250 sources, measure  $df/dt$  for 3000 and  $d^2f/dt^2$  for  $\sim 3$ .
- ❖ Cosmological sources
  - Processes occurring at the TeV scale in the early Universe could generate a mHz stochastic gravitational wave background.
  - Cosmic string networks could produce both individual burst events and a stochastic background.

# Other sources for LISA



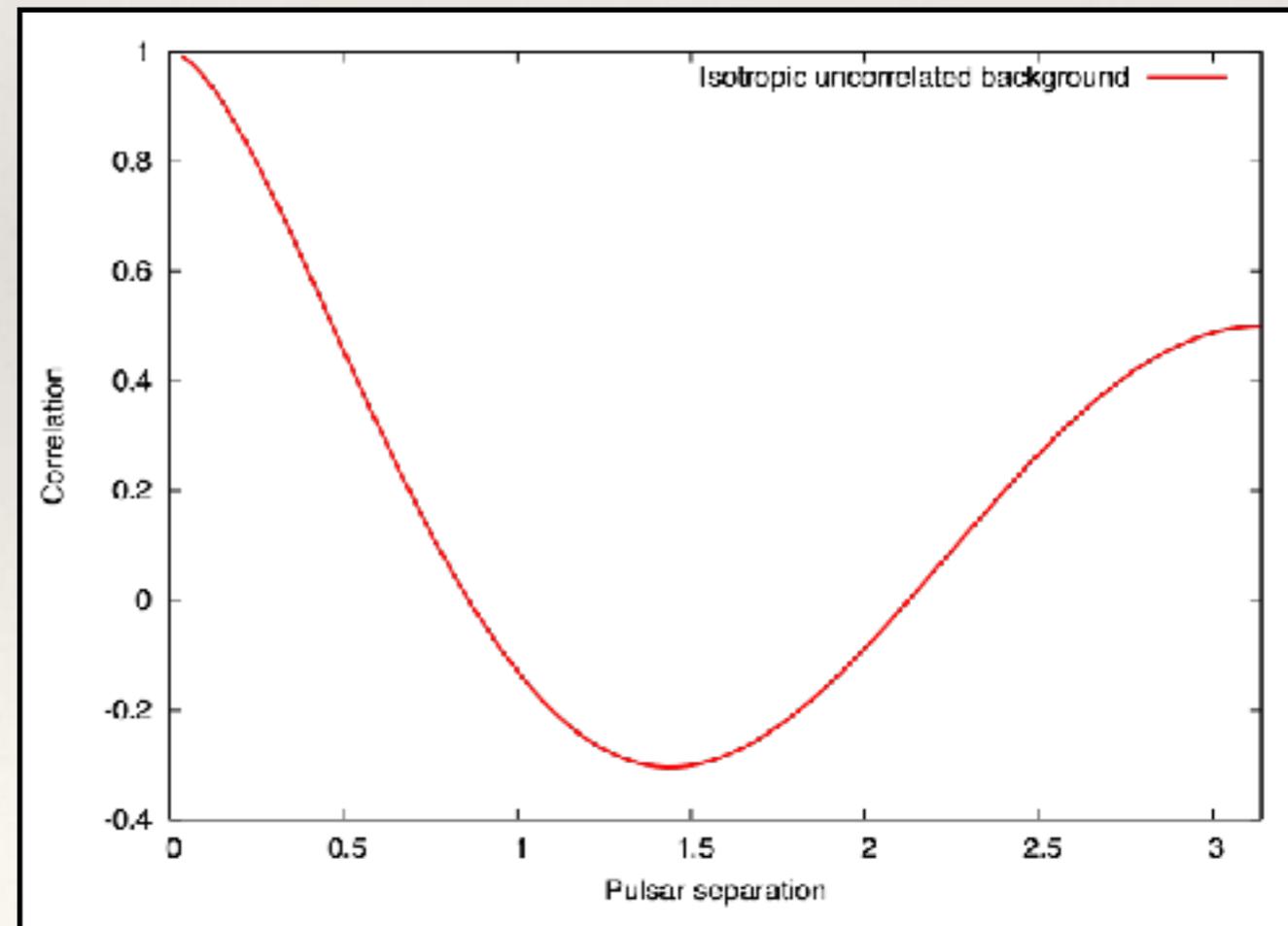
Caprini et al. (2016)

# Gravitational wave sources for pulsar timing arrays

# Sources for pulsar timing arrays

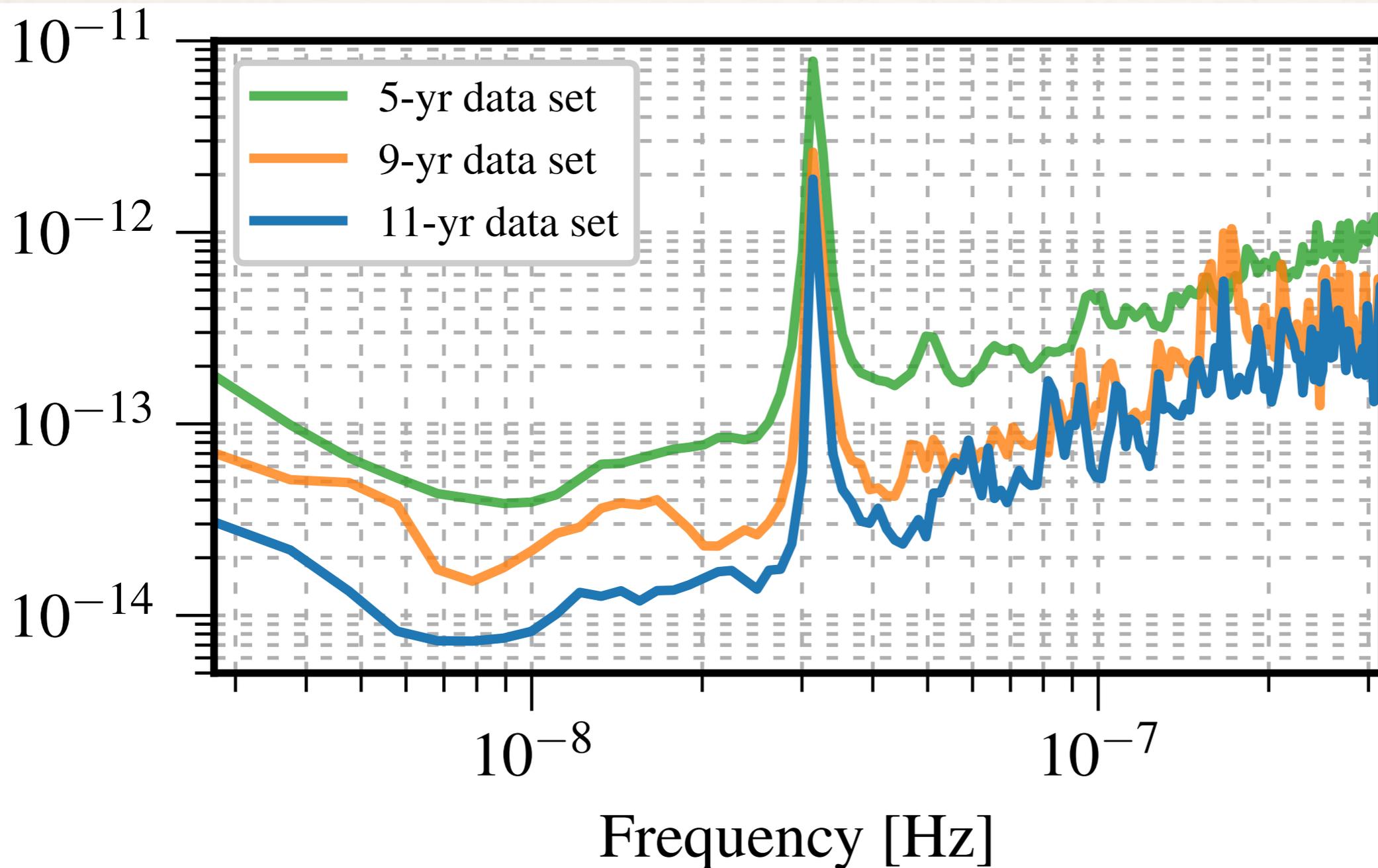
- ❖ Primary source for pulsar timing: pre-merger supermassive black hole binaries. Signal almost monochromatic.
- ❖ Expect to observe stochastic background. An stationary, isotropic, uncorrelated background produces a characteristic correlation signature

$$\langle s_a(t)^* s_b(t') \rangle = \Gamma_{ab} C(|t - t'|)$$



# Current PTA limits

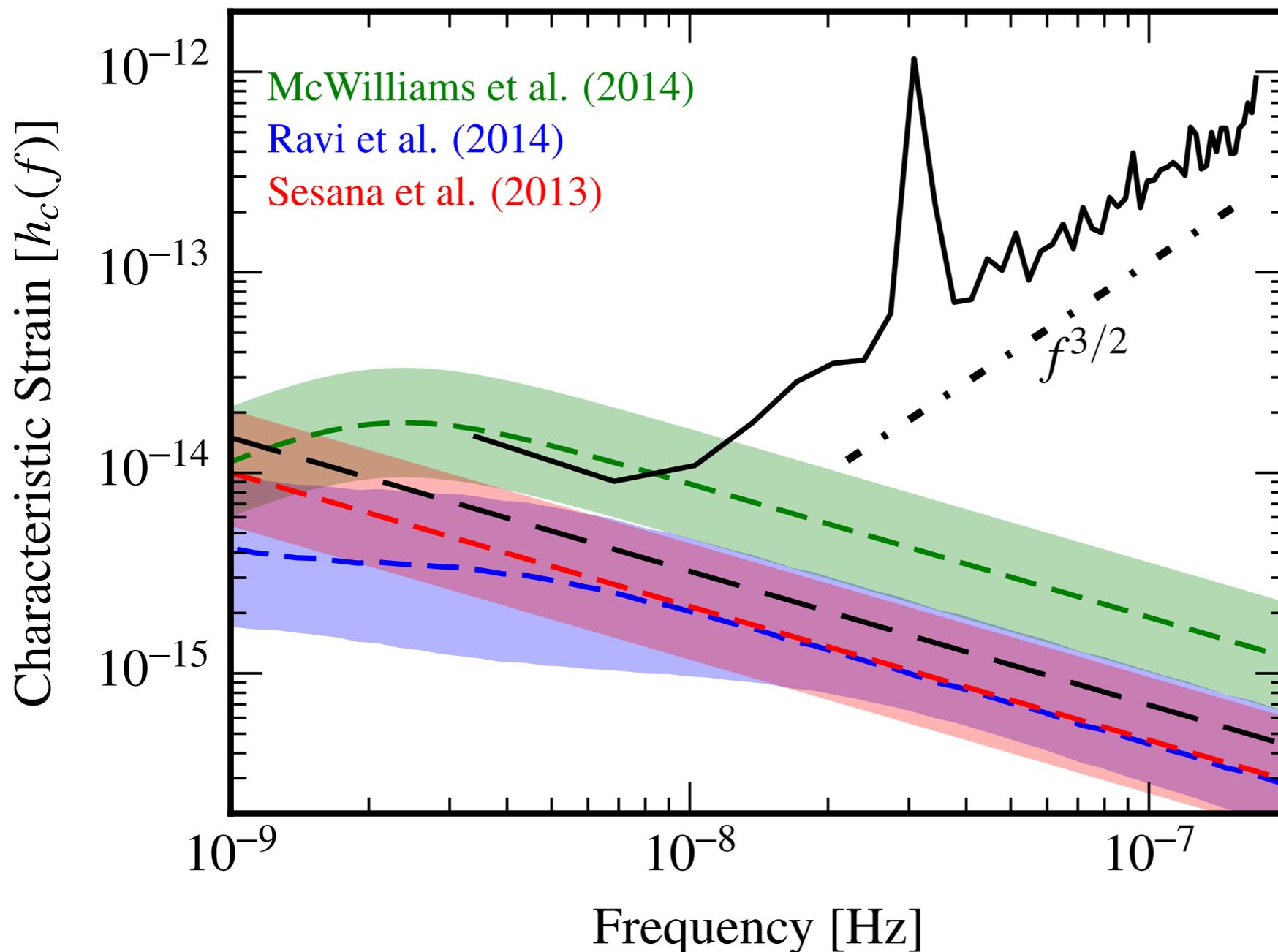
- ❖ No detection yet, but recent limits are starting to become astrophysically interesting.



NANOGrav  
11-year  
results  
[Aggarwal et  
al. (2019)]

# Current PTA limits

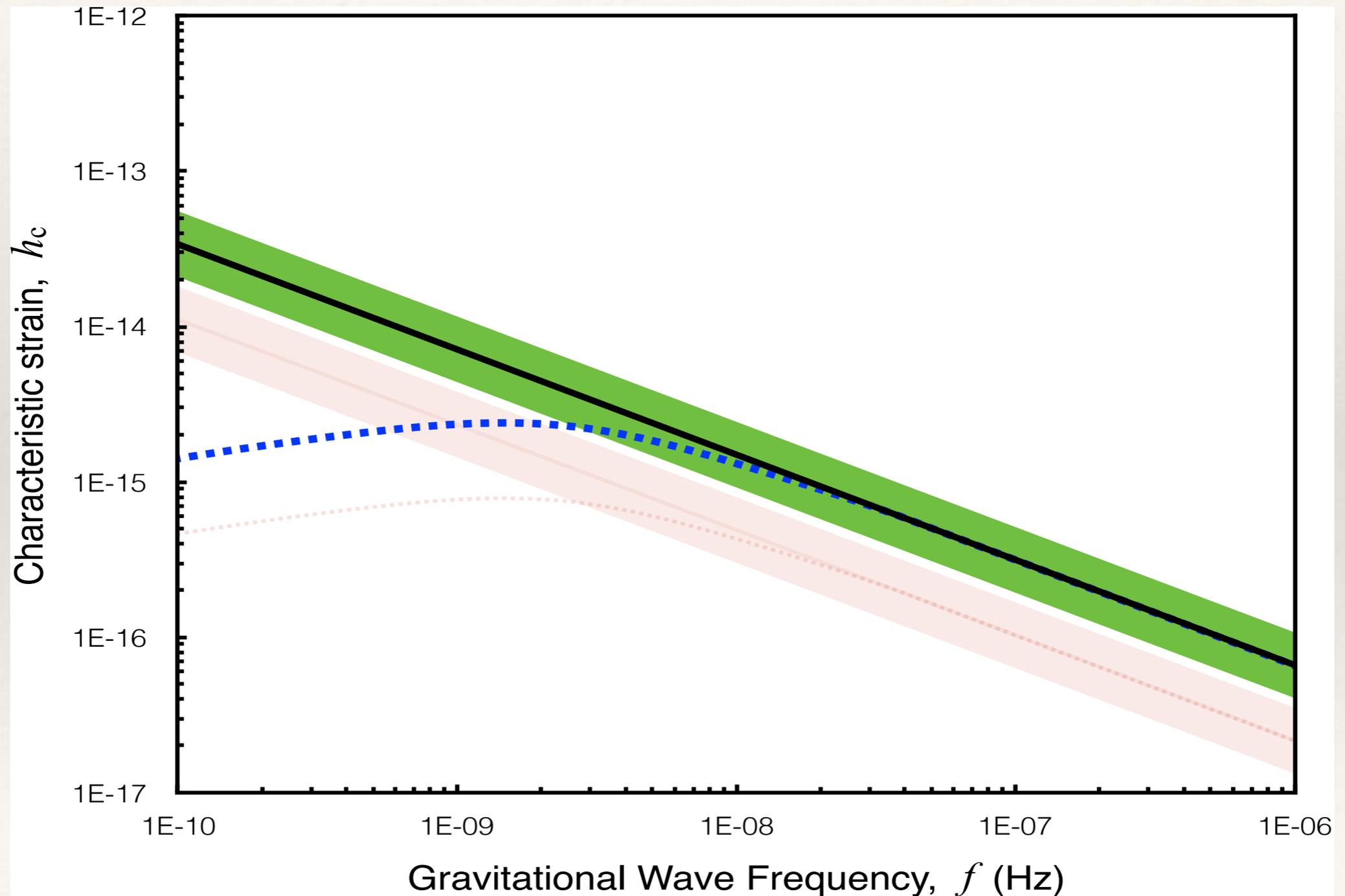
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NANOGrav  
9-year results  
[Arzoumanian  
et al. (2015)]

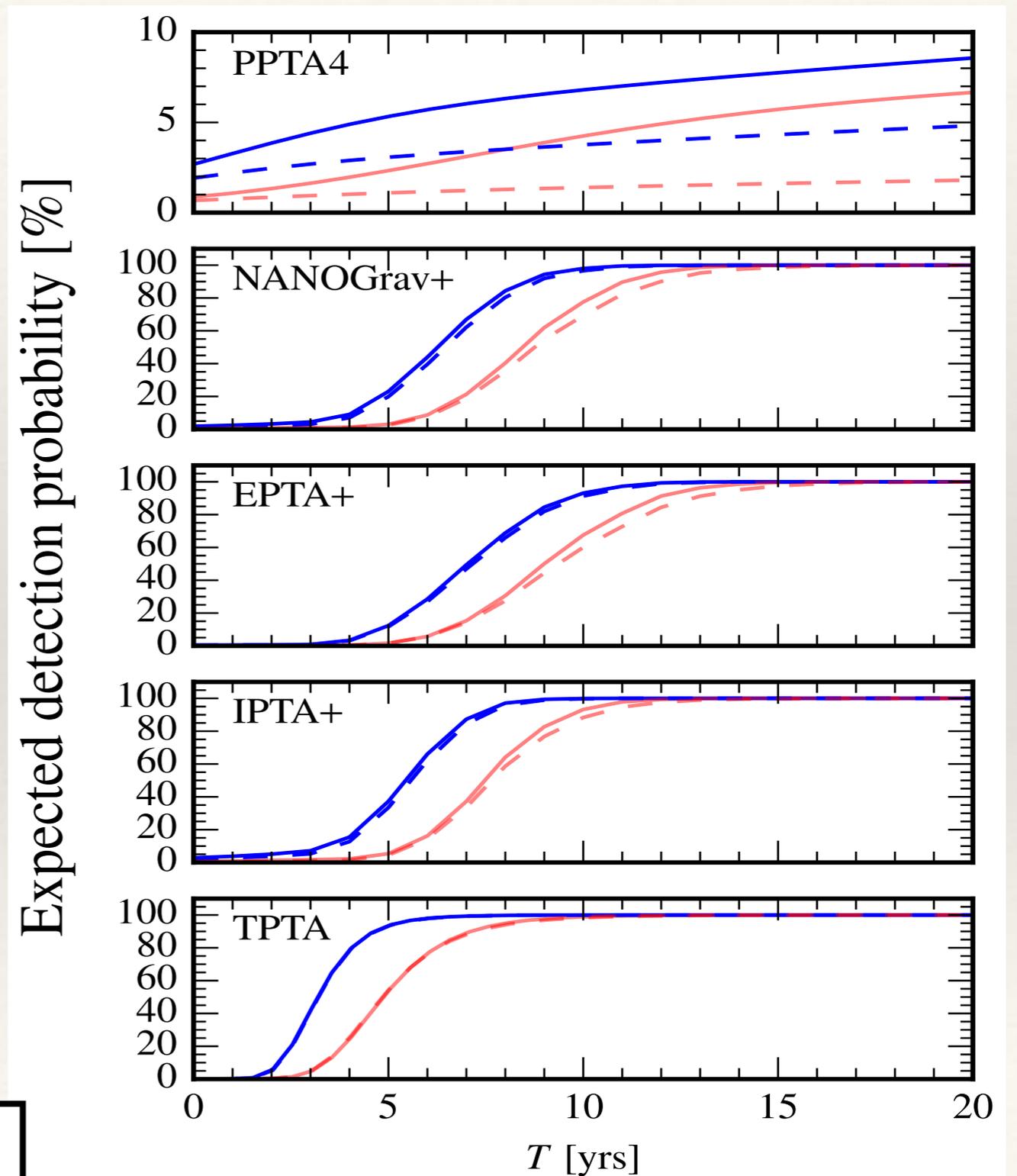
# Current PTA limits

- Astrophysical processes can diminish signal at low frequencies.



# PTA: time to detection

- ❖ Based on current theoretical understanding and observational results, expect detection in 5 to 10 years.
- ❖ Assumes existing pulsars continue to be observed and new pulsars are added.
- ❖ Hints will come earlier.

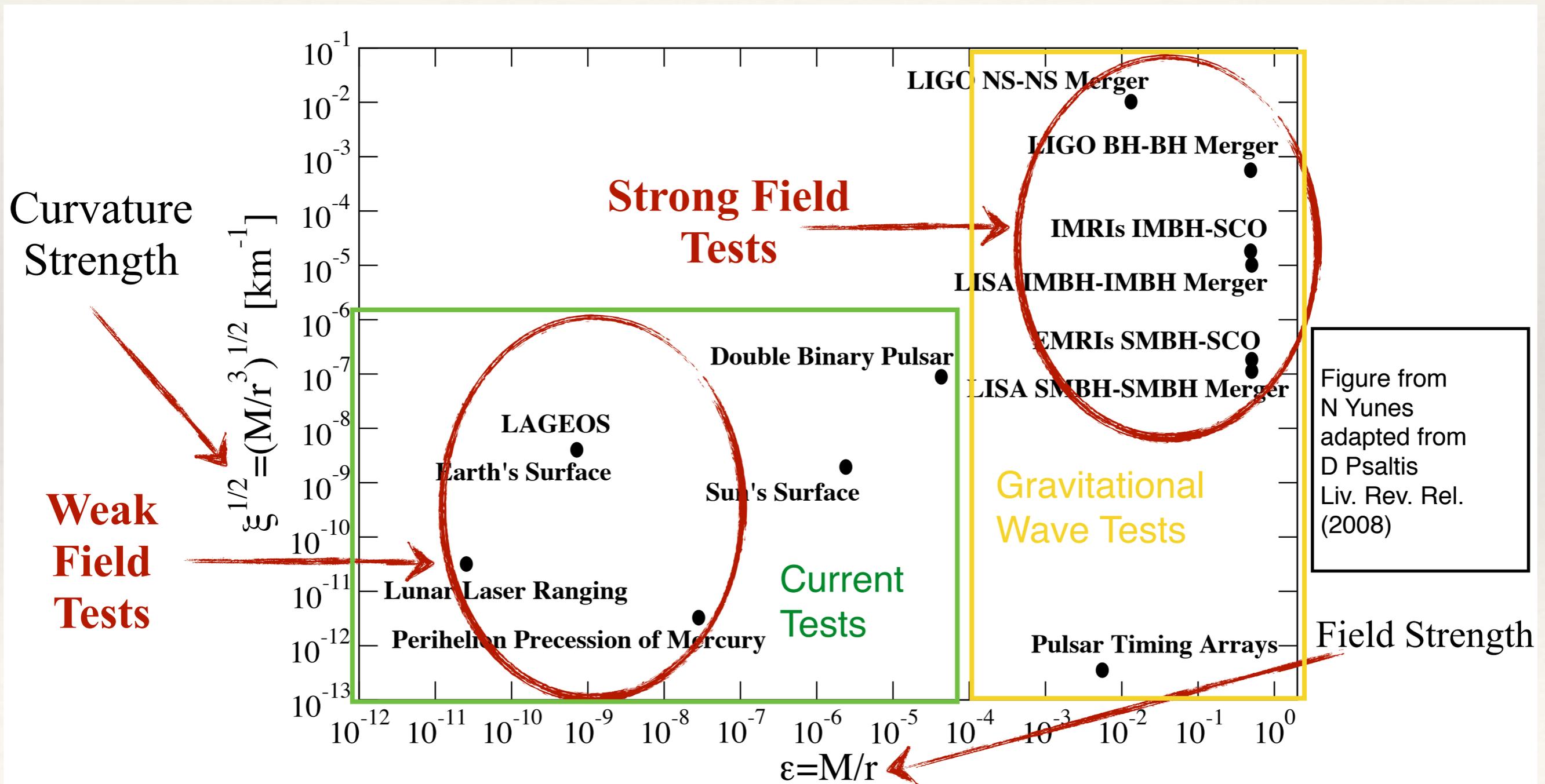


Taylor et al. (2016)

# Tests of gravitational physics with LISA Observations

# Fundamental physics with LISA

- Gravitational wave observations probe a regime of strong-field, non-linear and dynamical gravity that is inaccessible to other probes.



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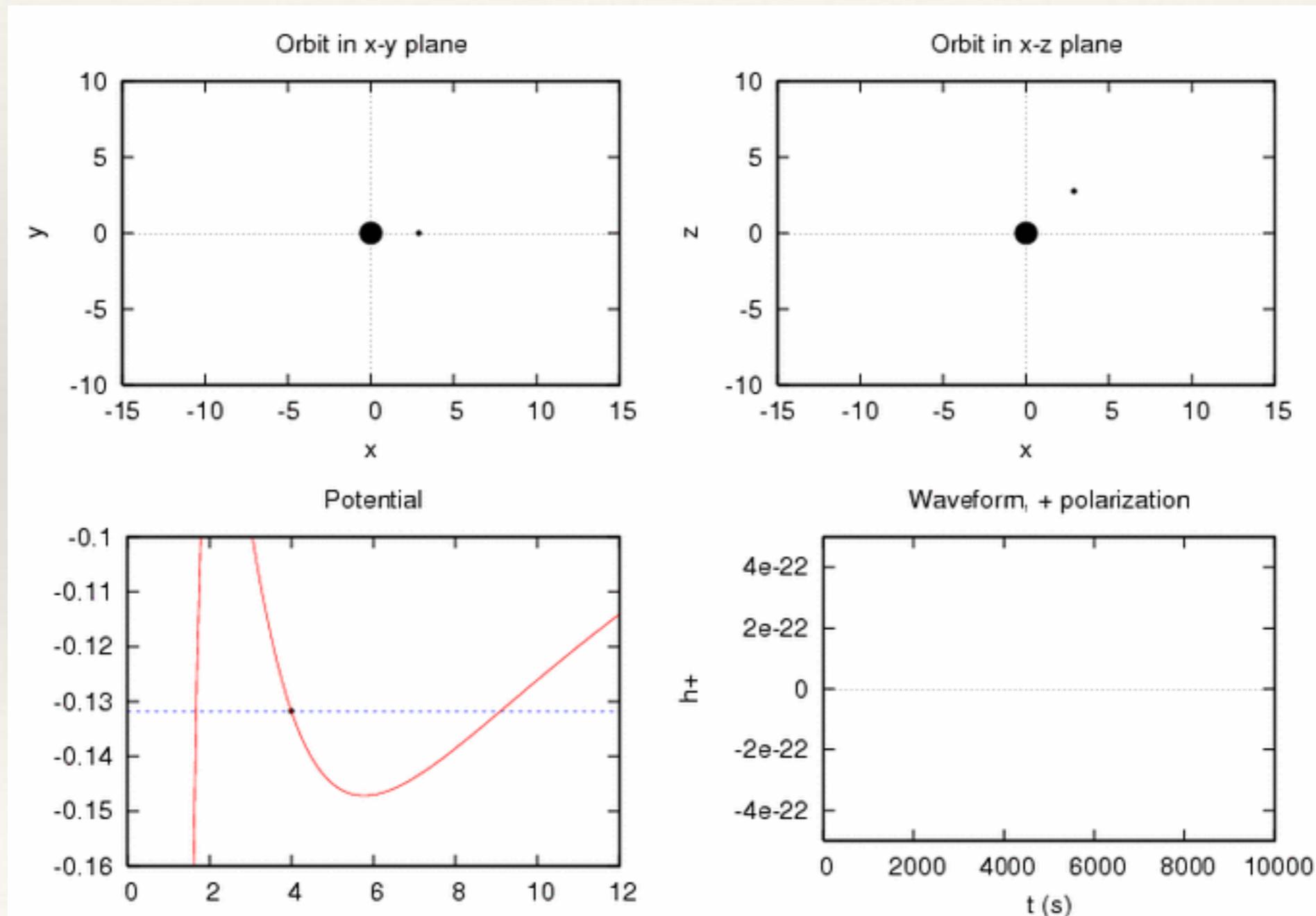
# Fundamental physics with LISA

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- ❖ Gravitational wave observations probe a regime of strong-field, non-linear and dynamical gravity that is inaccessible to other probes.
- ❖ All GW sources and detectors can be used to constrain fundamental physics.
- ❖ Space-based detectors are particularly good because
  - High SNR events: SNR of hundreds for MBH mergers.
  - Long duration signals: months to years in band; hundreds of thousands of cycles for typical EMRIs.
  - Clean systems: main sources are black hole binaries.
  - Rich dynamics: eccentricity and orbital inclination likely for EMRIs.
- ❖ Can test GW propagation, polarisation, energy loss, generic or specific deviations in alternative theories, constrain dark matter candidates etc.

# Probing the nature and structure of BHs

- ❖ GW emission from EMRIs encodes a map of the space-time structure outside the central massive black hole.



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# Probing the nature and structure of BHs

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- ❖ GW emission from EMRIs encodes a map of the space-time structure outside the central massive black hole. Can characterize a vacuum, axisymmetric spacetime in GR by its multipole moments. For Kerr BHs, these satisfy the ‘no-hair’ theorem:

$$M_l + iS_l = M(ia)^l$$

- ❖ Deviations from no-hair property can be indicative of violations of the Strong Equivalence Principle (implying violation of WEP or Local Lorentz Invariance or Local Positional Invariance). What are the observable consequences?
- ❖ Multipole moments are encoded in gravitational wave observables - precession frequencies & number of cycles spent near a given frequency (Ryan 95).

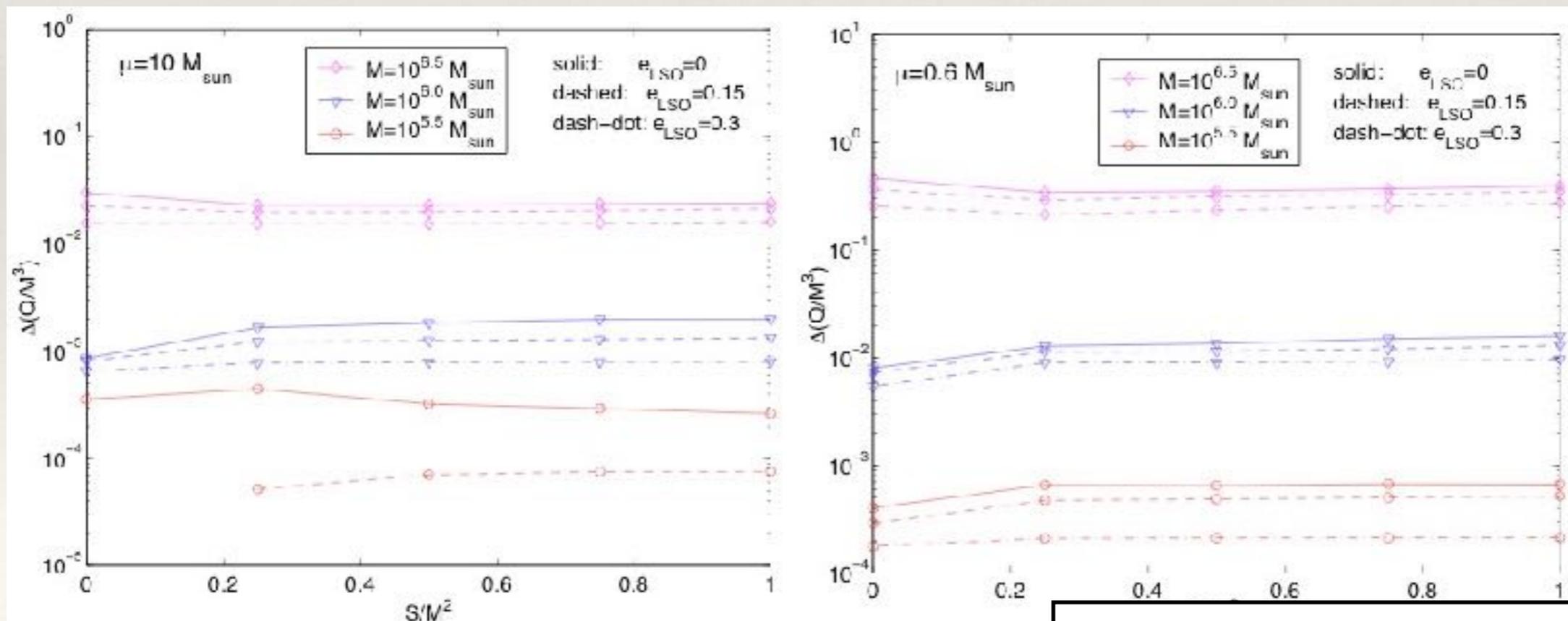
$$\Delta\mathcal{N}(f) = \frac{f^2}{df/dt} = f^2 \frac{dE/df}{dE/dt}$$

- ❖ Multipole moments enter at different orders in  $M\Omega$

$$\frac{\Omega_p}{\Omega} = 3(M\Omega)^{\frac{2}{3}} - 4\frac{S_1}{M^2}(M\Omega) + \left(\frac{9}{2} - \frac{3}{2}\frac{M_2}{M^3}\right)(M\Omega)^{\frac{4}{3}} + \dots$$

# Probing BH structure: the central object

- ❖ Need infinite number of multipoles to describe Kerr. Instead, consider “bumpy” black holes with small departures from Kerr.
  - Many studies, e.g., Collins & Hughes (2004), Glampedakis & Babak (2005), Barack & Cutler (2007), JG, Li & Mandel (2008), Sopuerta & Yunes (2009), Canizares, JG & Sopuerta (2012).
  - Can simultaneously measure  $M$ ,  $a$  to  $\sim 0.01\%$  and excess quadrupole to  $\sim 0.1\%$ .



Barack & Cutler (2007)

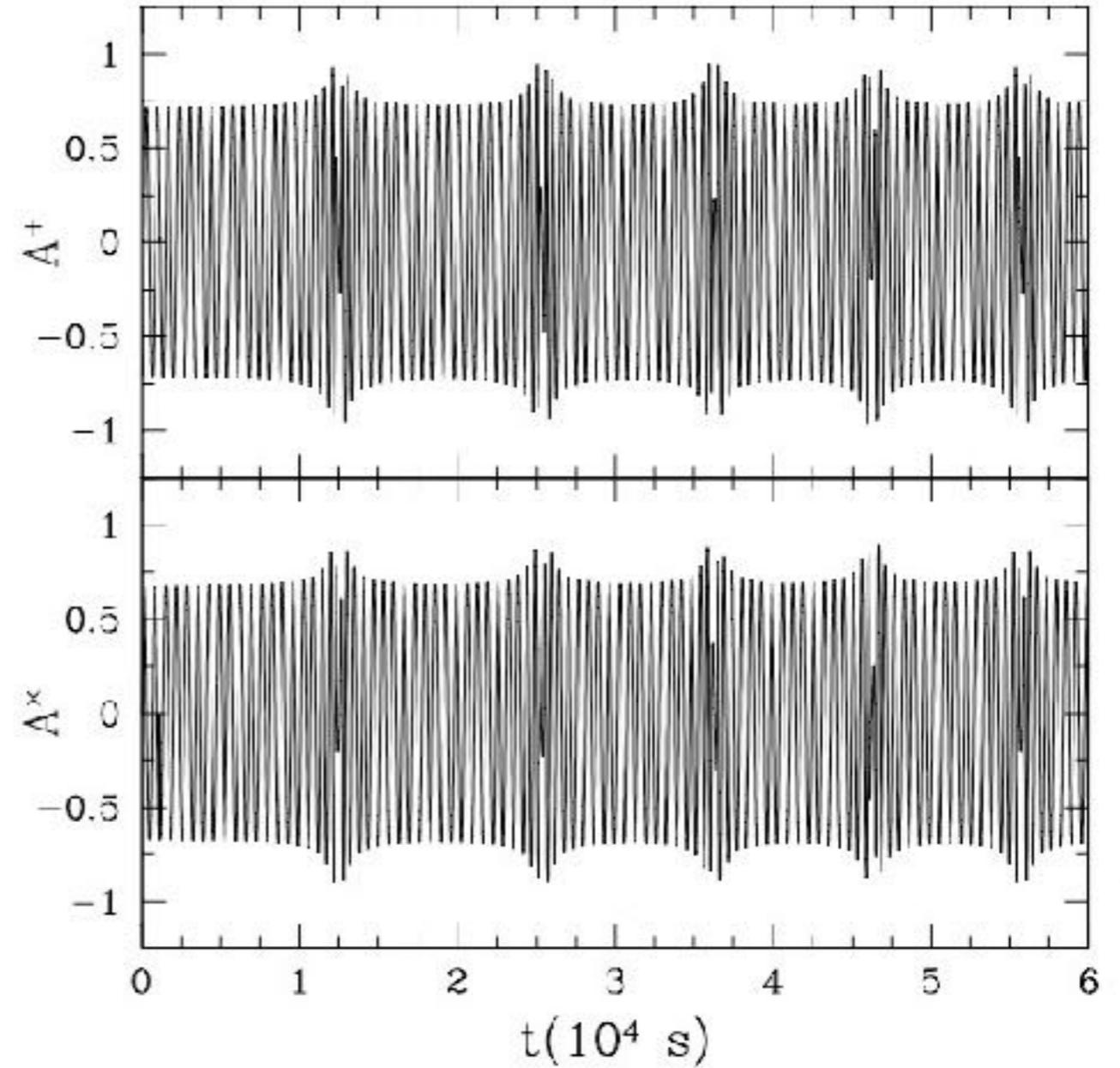
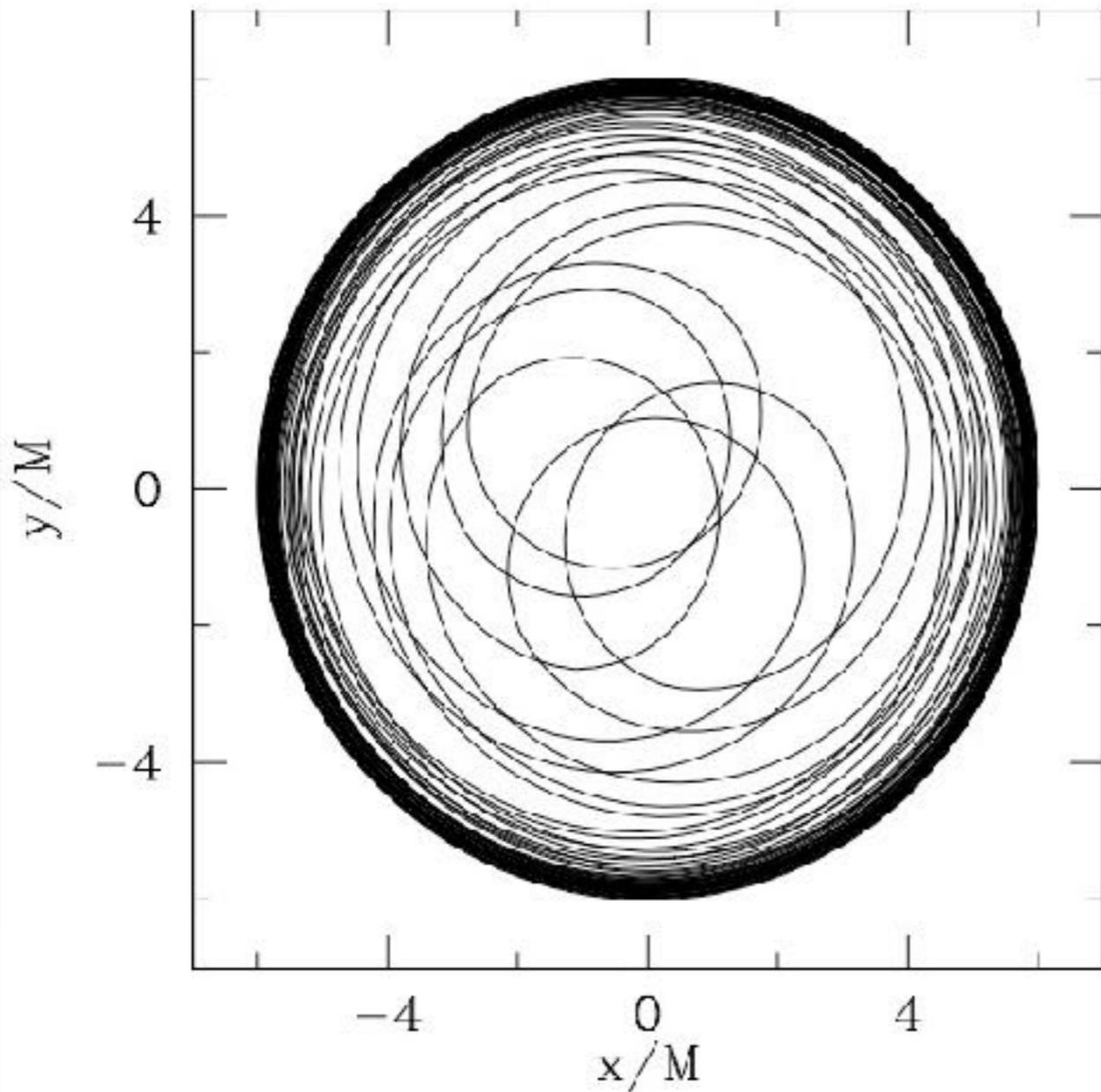
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  - Can simultaneously measure  $M$ ,  $a$  to  $\sim 0.01\%$  and excess quadrupole to  $\sim 0.1\%$ .
- ❖ Other information is also encoded in emitted GWs
  - **Tidal coupling:** Energy is lost ‘into the horizon’ through tidal heating. Infer strength of tidal interaction (Li & Lovelace 07).
  - **Presence of matter:** gas, accretion disc, second SMBH or exotic matter can leave measurable imprint on signal. Can’t be confused with no-hair violation.
  - **Horizon:** presence / absence of a horizon indicated by cut-off / continuation of emission at plunge, e.g., persistent emission for an inspiral into a Boson-Star.

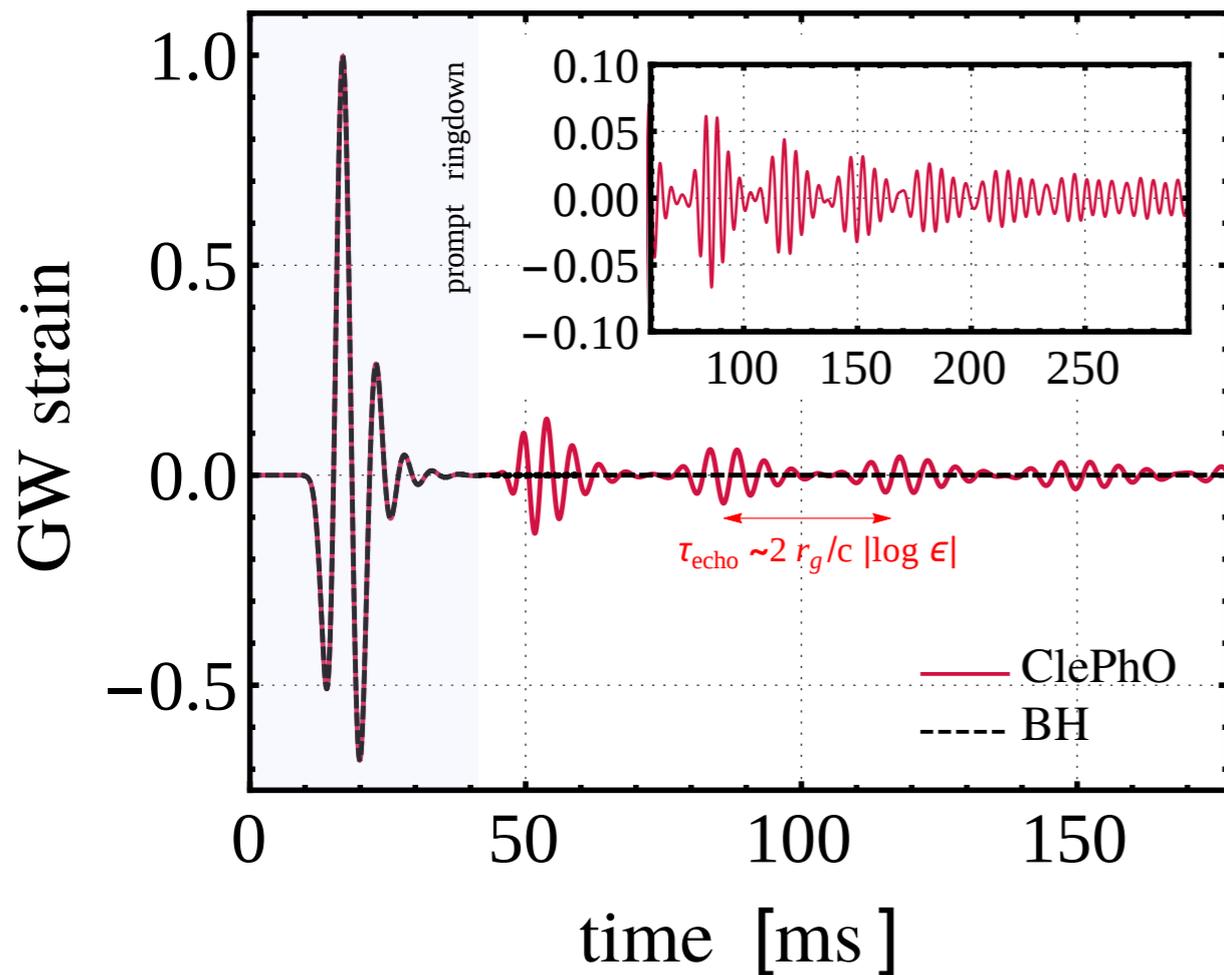
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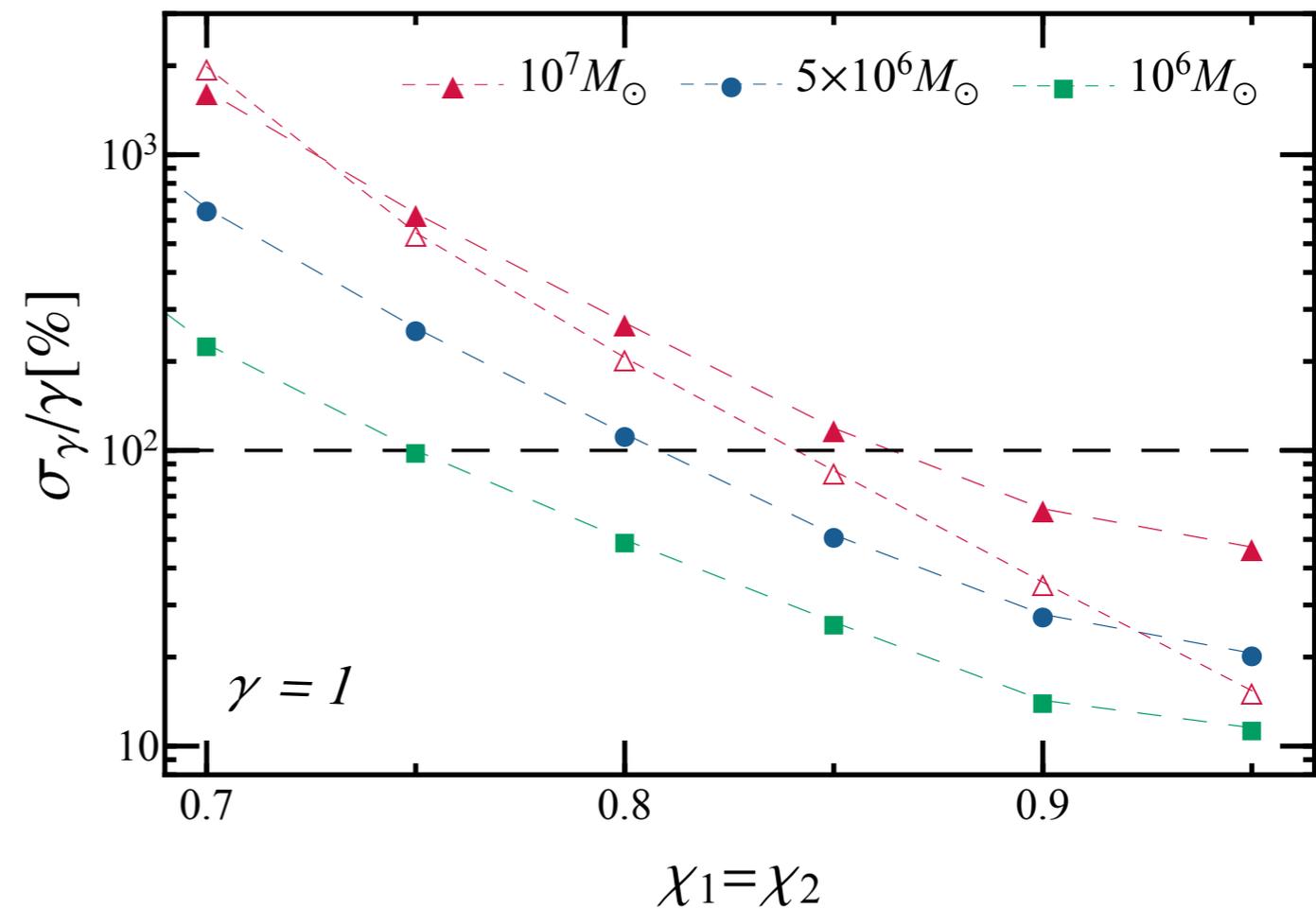
Kesden, Gair & Kamionkowski (2004)

# Signatures of Deviations - Horizons

- ❖ Other signatures of deviations from the black hole hypothesis include “echoes” from the vicinity of the horizon.



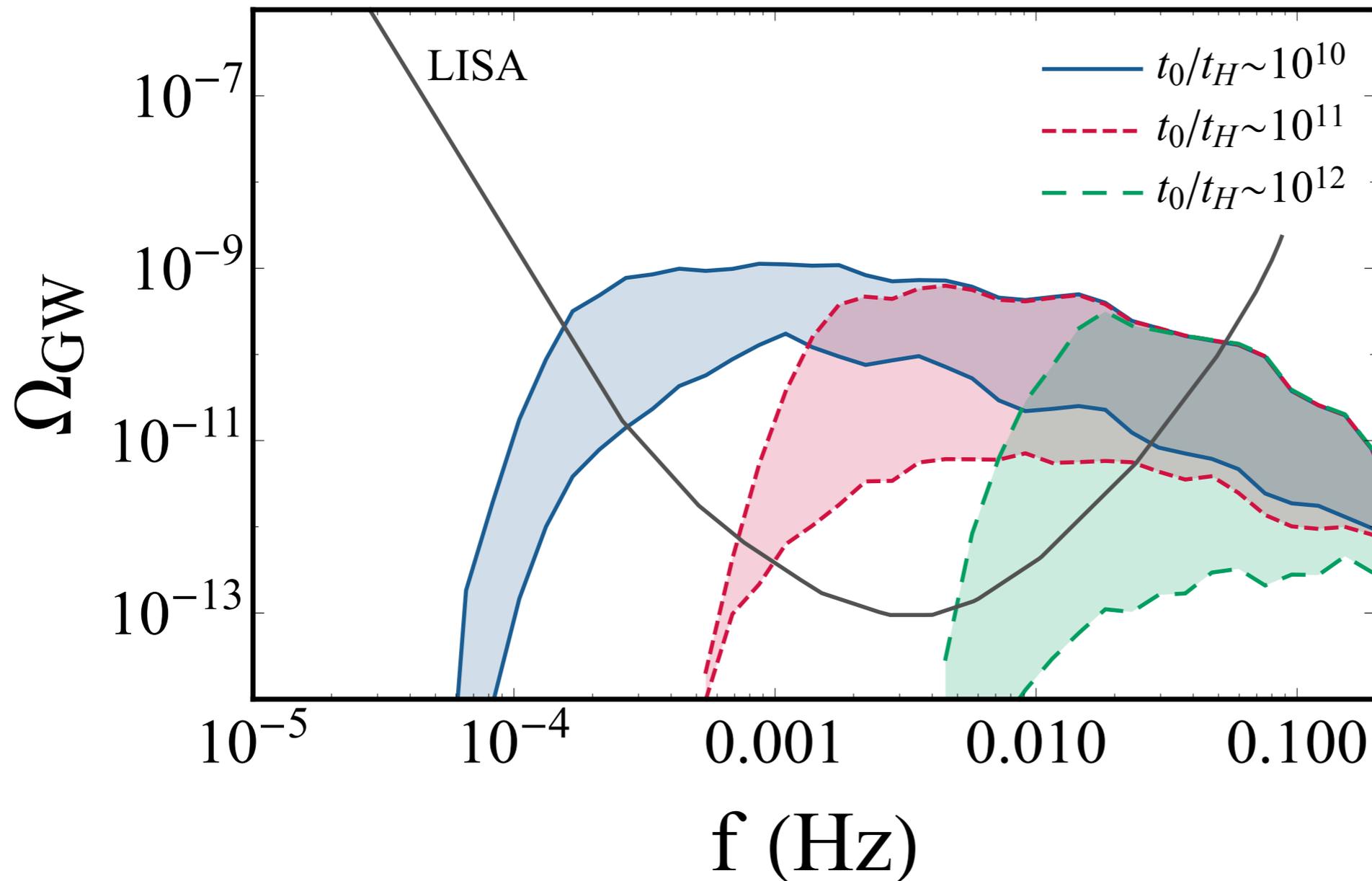
Cardoso & Pani (2017)



Maselli et al. (2017)

# Signatures of Deviations - Horizons

- ❖ Horizonless objects are generically unstable and would generate a stochastic background of gravitational waves.



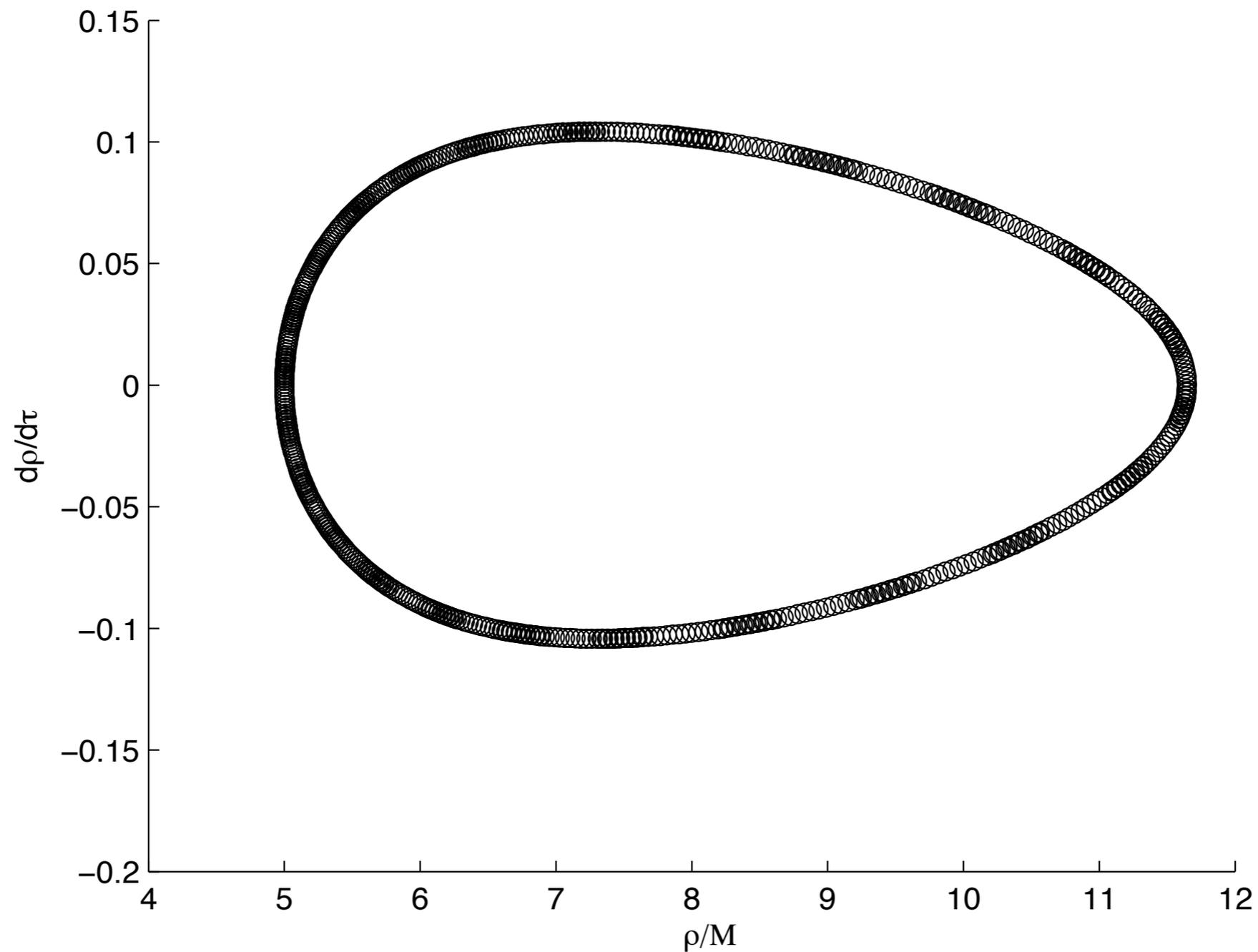
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# Deviations in the strong field

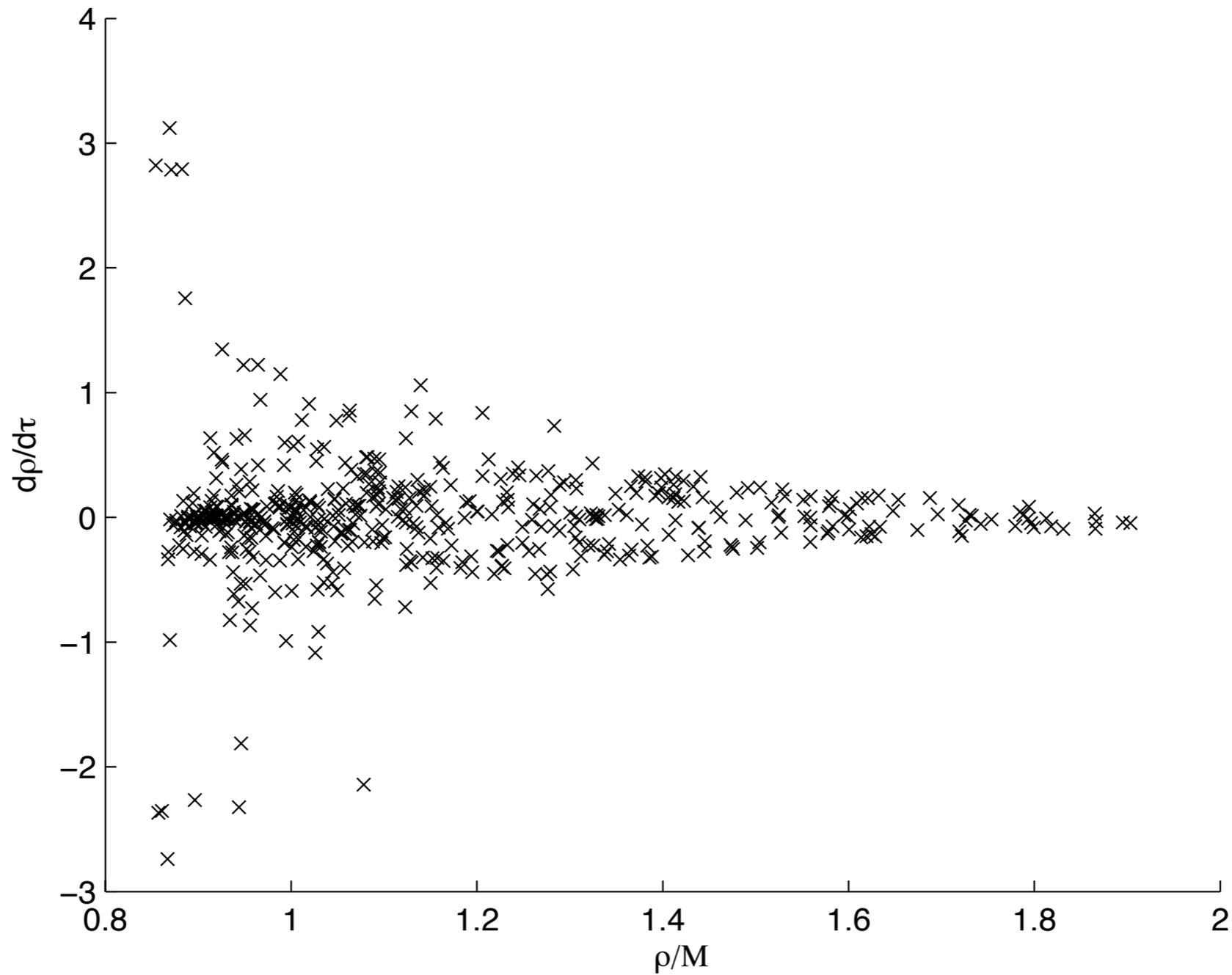
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- ❖ Strong field deviations can be more extreme. Kerr is special in having orbits with a complete set of integrals. This need not hold for other systems.
- ❖ Explore orbital properties using a Poincare map.
  - Many orbits show closed curves, indicating that they have an effective third integral.
  - Some orbits show space-filling maps, indicating ergodic behaviour - a ‘smoking gun’ for a non-Kerr spacetime.

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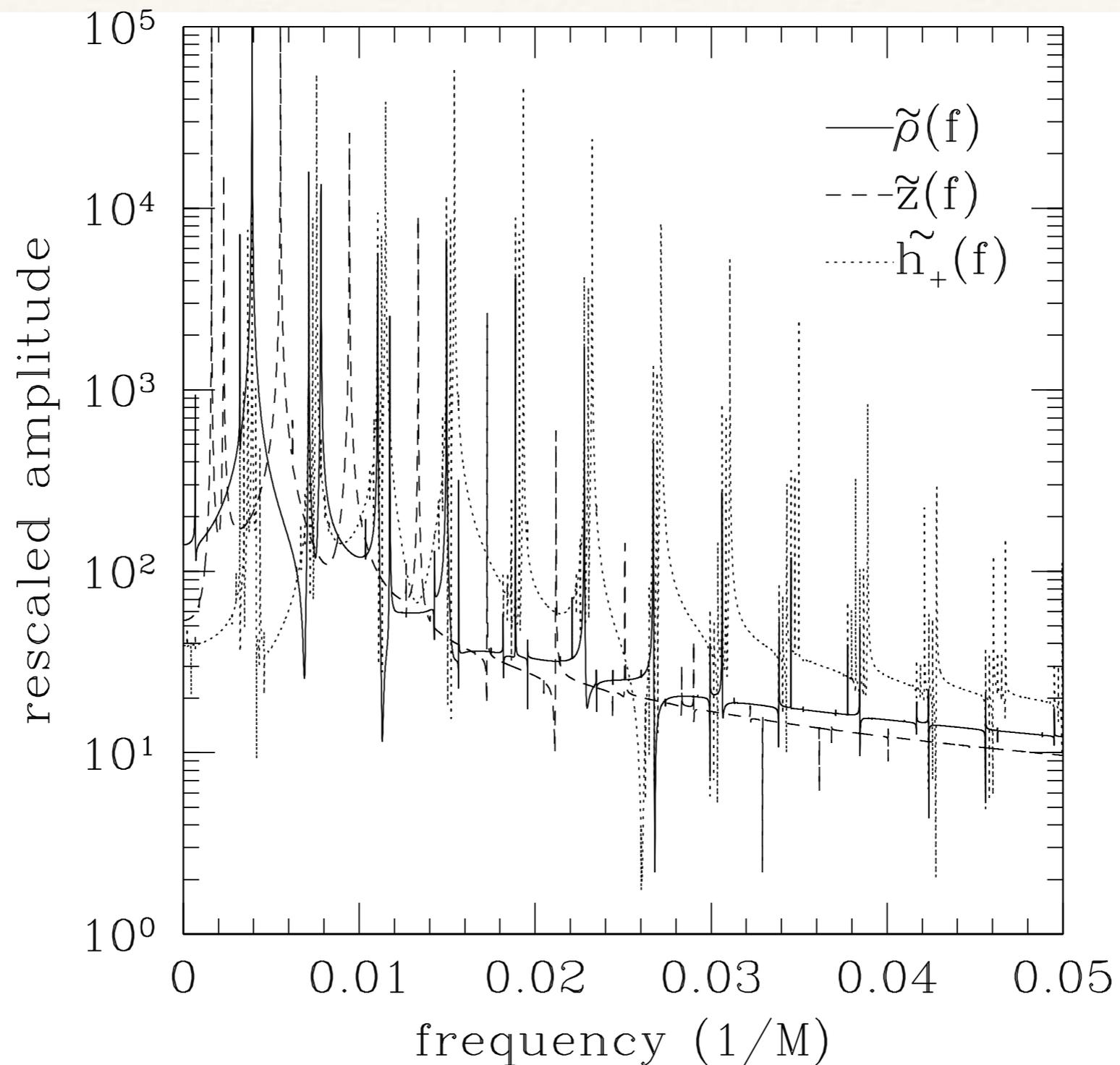
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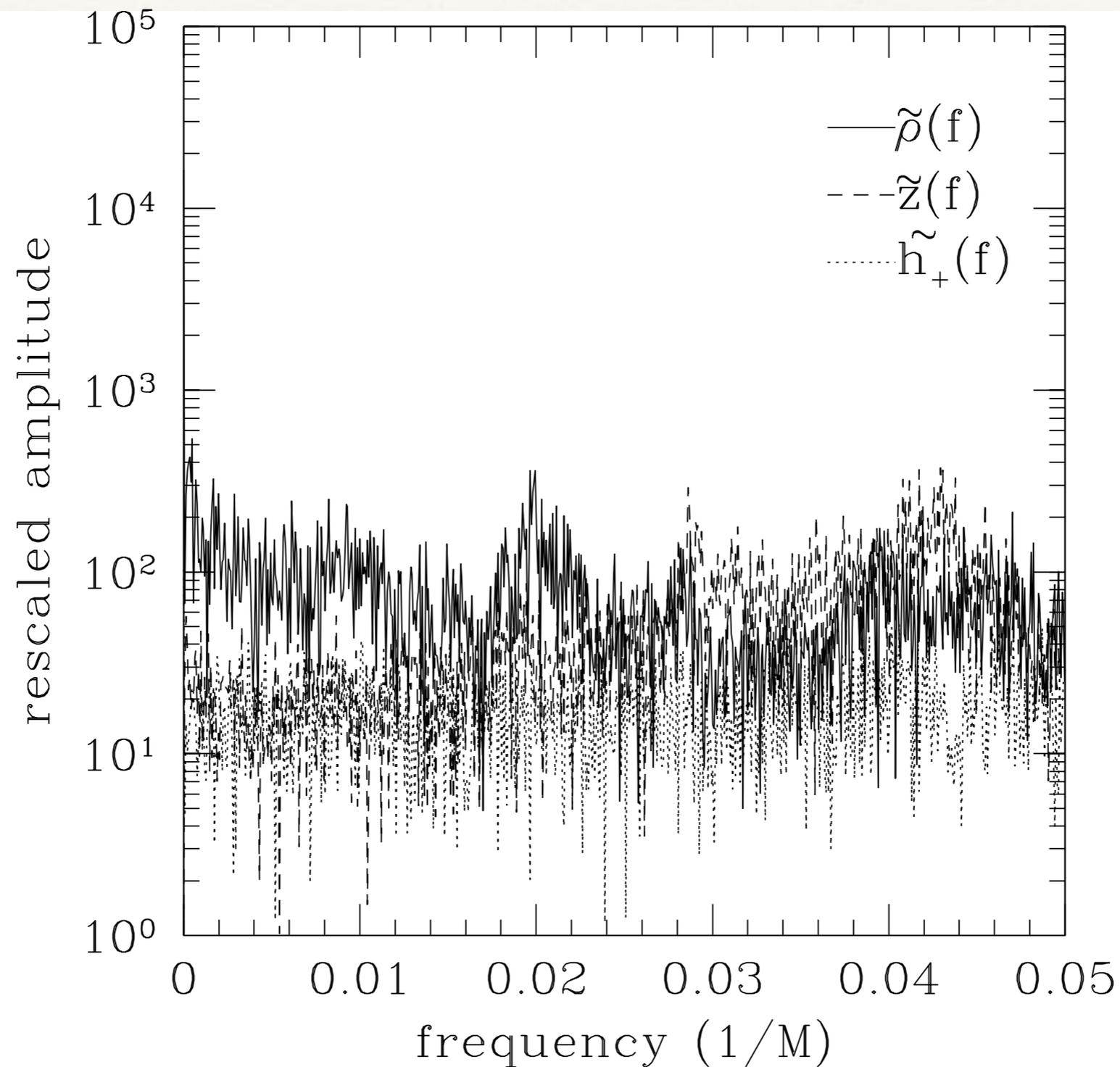
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- ❖ How to identify this in practice is not obvious.
  - Might detect this via a time-frequency analysis, or if waveform SNR stops accumulating prematurely.
  - Could even see intermittent ‘bursts’ of regular periodic radiation as orbit passes into and out of ergodic regime.

# Deviations in the strong field



# Deviations in the strong field



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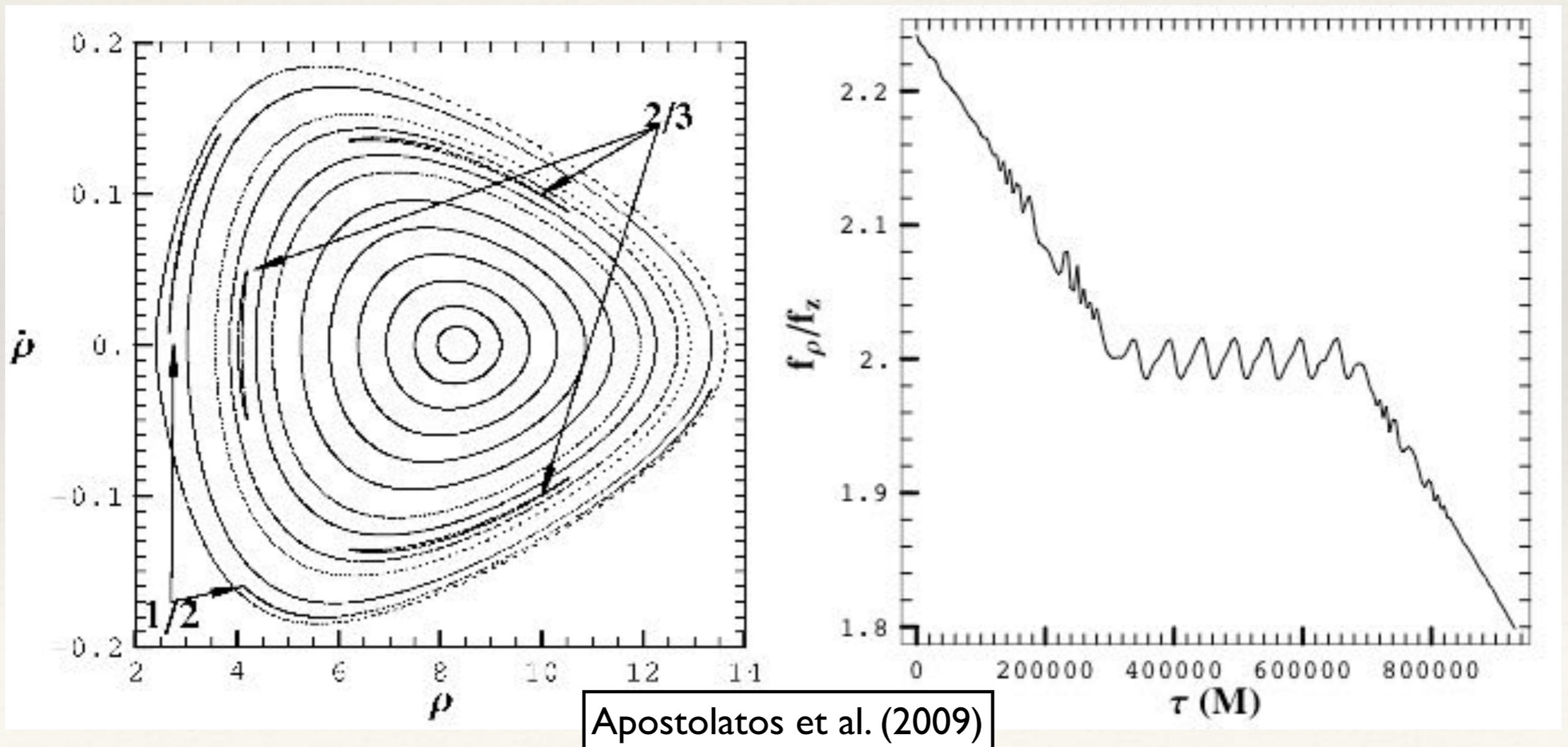
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- ❖ How to identify this in practice is not obvious.
  - Might detect this via a time-frequency analysis, or if waveform SNR stops accumulating prematurely.
  - Could even see intermittent ‘bursts’ of regular periodic radiation as orbit passes into and out of ergodic regime.
- ❖ Unlikely to be astrophysically relevant, as fine-tuning is needed.

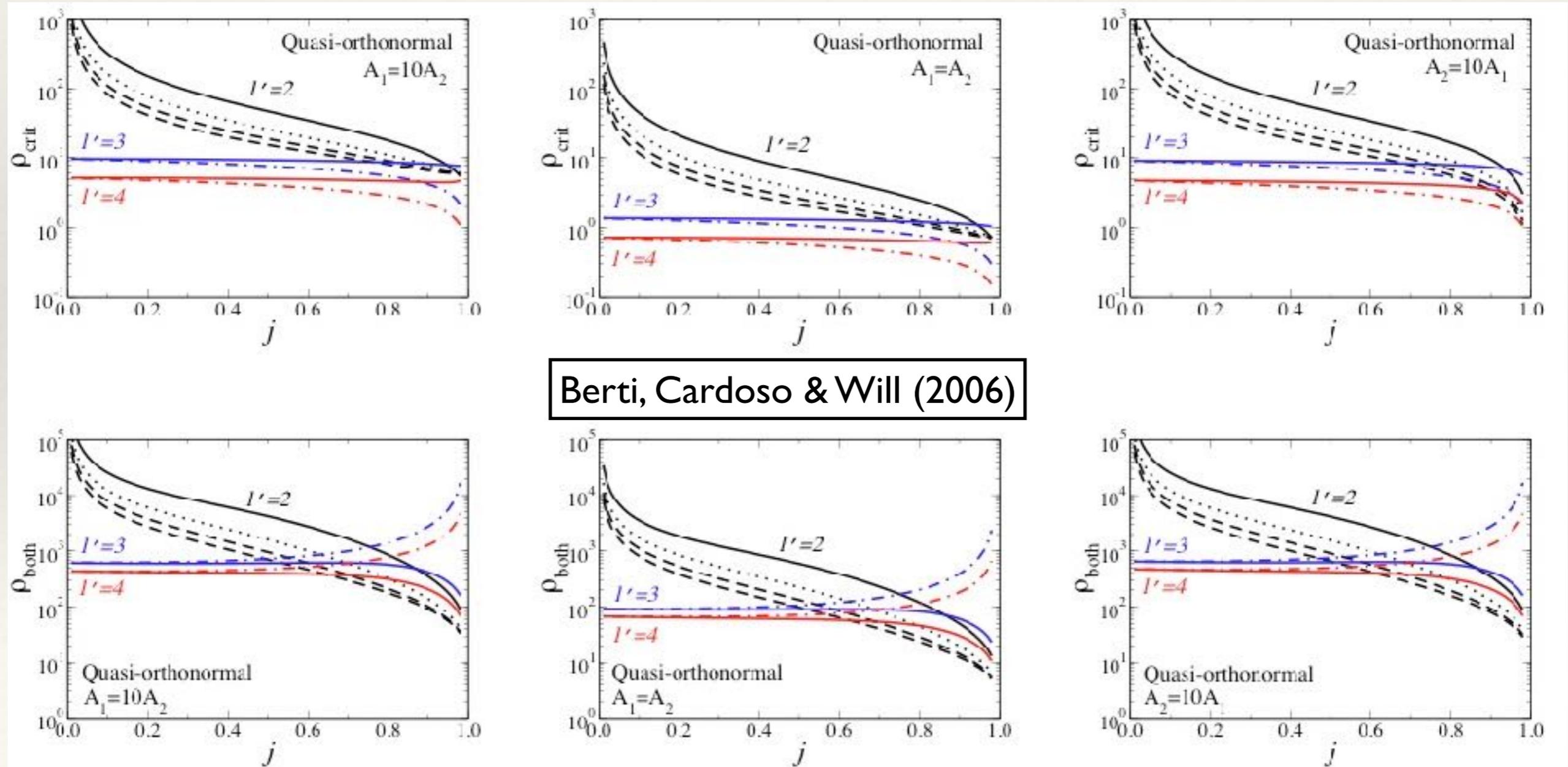
# Signatures of Deviations - Resonances

- ❖ When an integrable system is perturbed, resonant points become smeared out into resonant chains of islands (Poincare-Birkhoff theorem). Such deviation may therefore show up as a persistent resonance in the observed GWs.



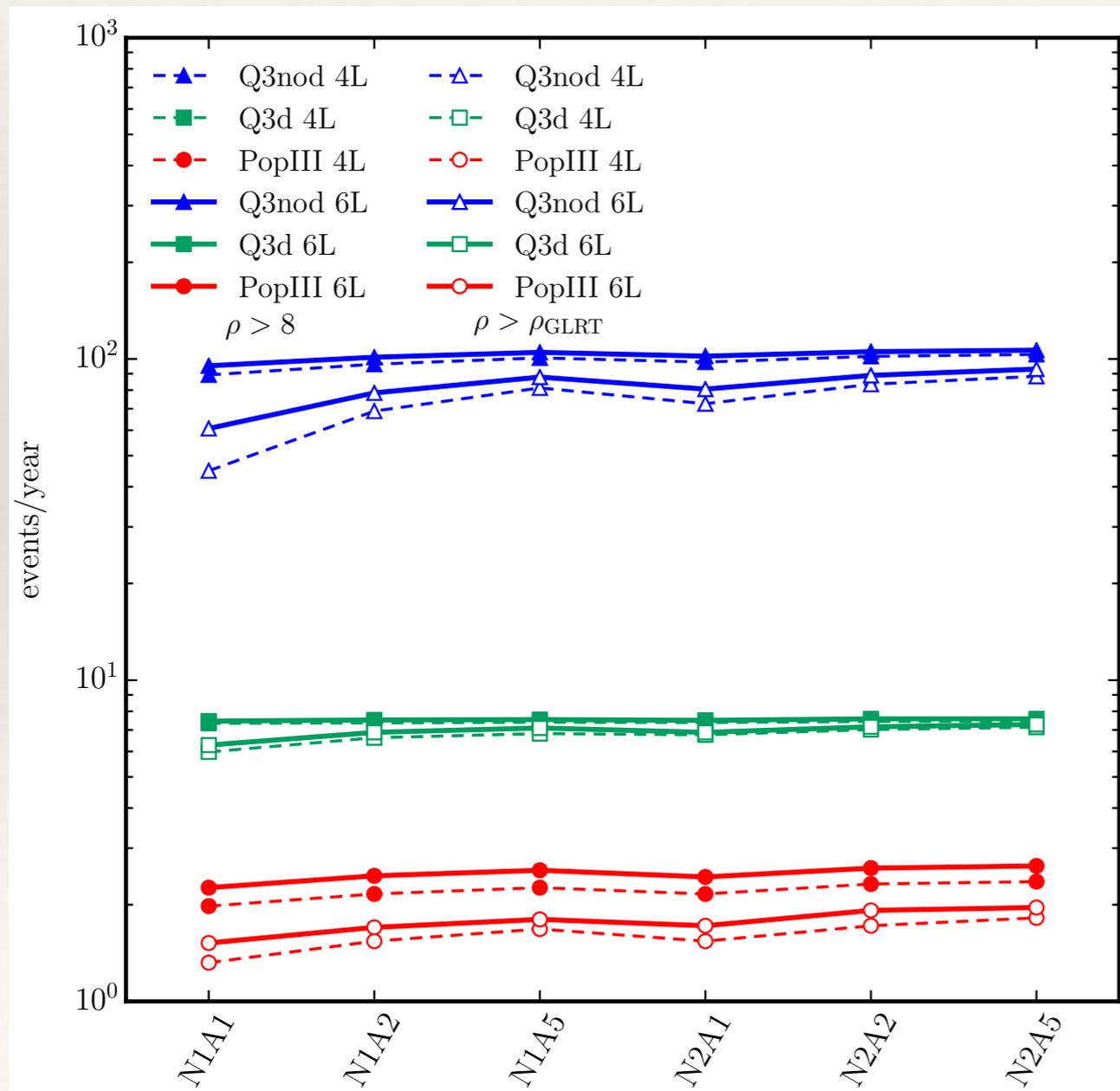
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Berti et al. (2016)

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# Tests of gravitational physics

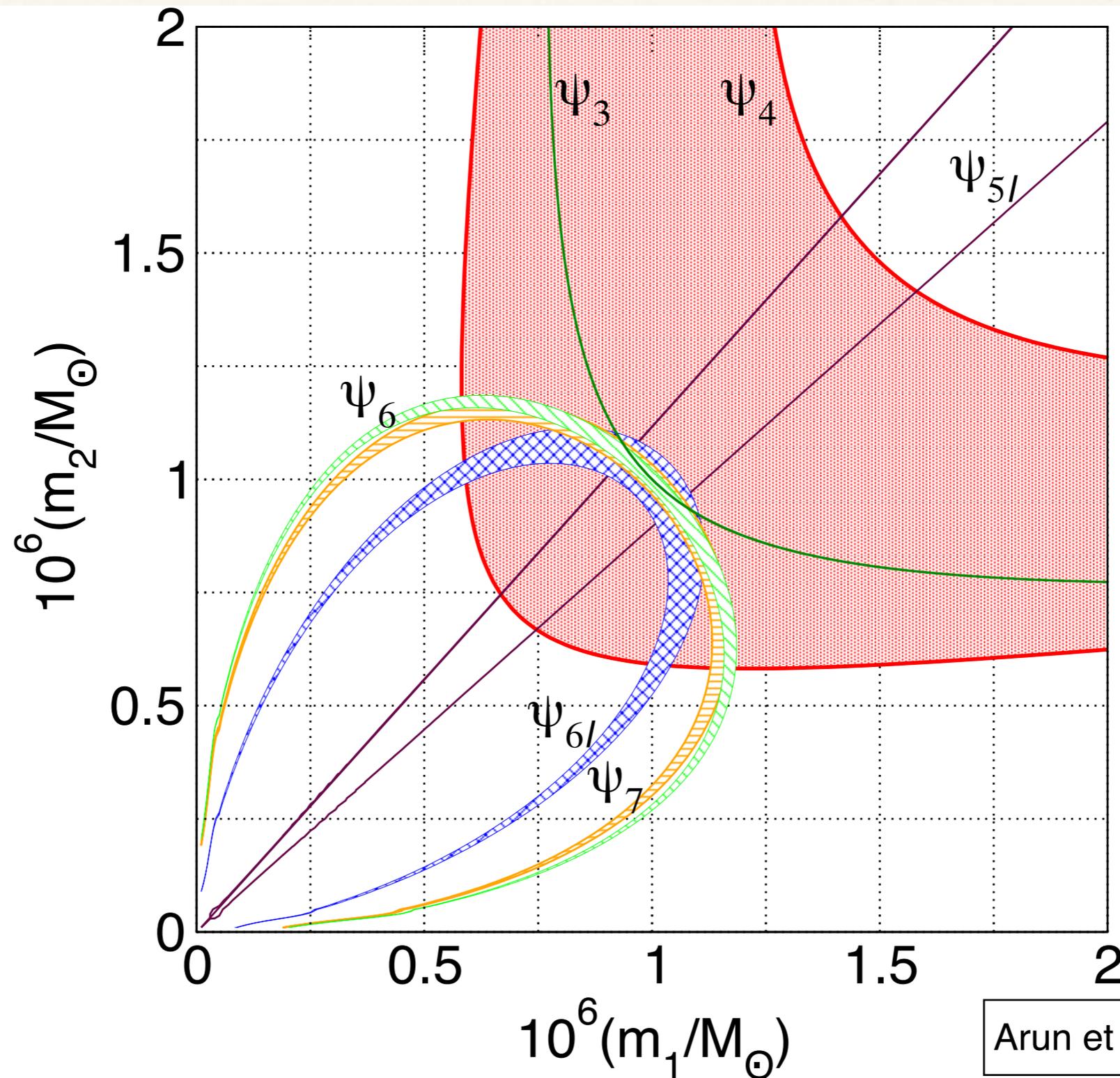
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- ❖ Inspiral phasing: in order to offer sensitivity to un-modelled deviations from GR, consider generic deviations to inspiral phase.
- ❖ Modify pN phase coefficients (Arun et al.)

$$\tilde{h}(f) = Af^{-\frac{7}{6}} \exp \left[ i\Psi(f) + i\frac{\pi}{4} \right]$$

$$\Psi(f) = 2\pi ft_c + \Phi_c + \sum_{k \in \mathbb{Z}} \left[ \psi_k + \psi_k^{\log} \log f \right] f^{(k-5)/3}$$

# Tests of gravitational physics



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# Tests of gravitational physics

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❖ LISA could measure  $\Delta\psi_0 \sim 0.1\%$ ,  $\Delta\psi_2, \Delta\psi_3 \sim 10\%$

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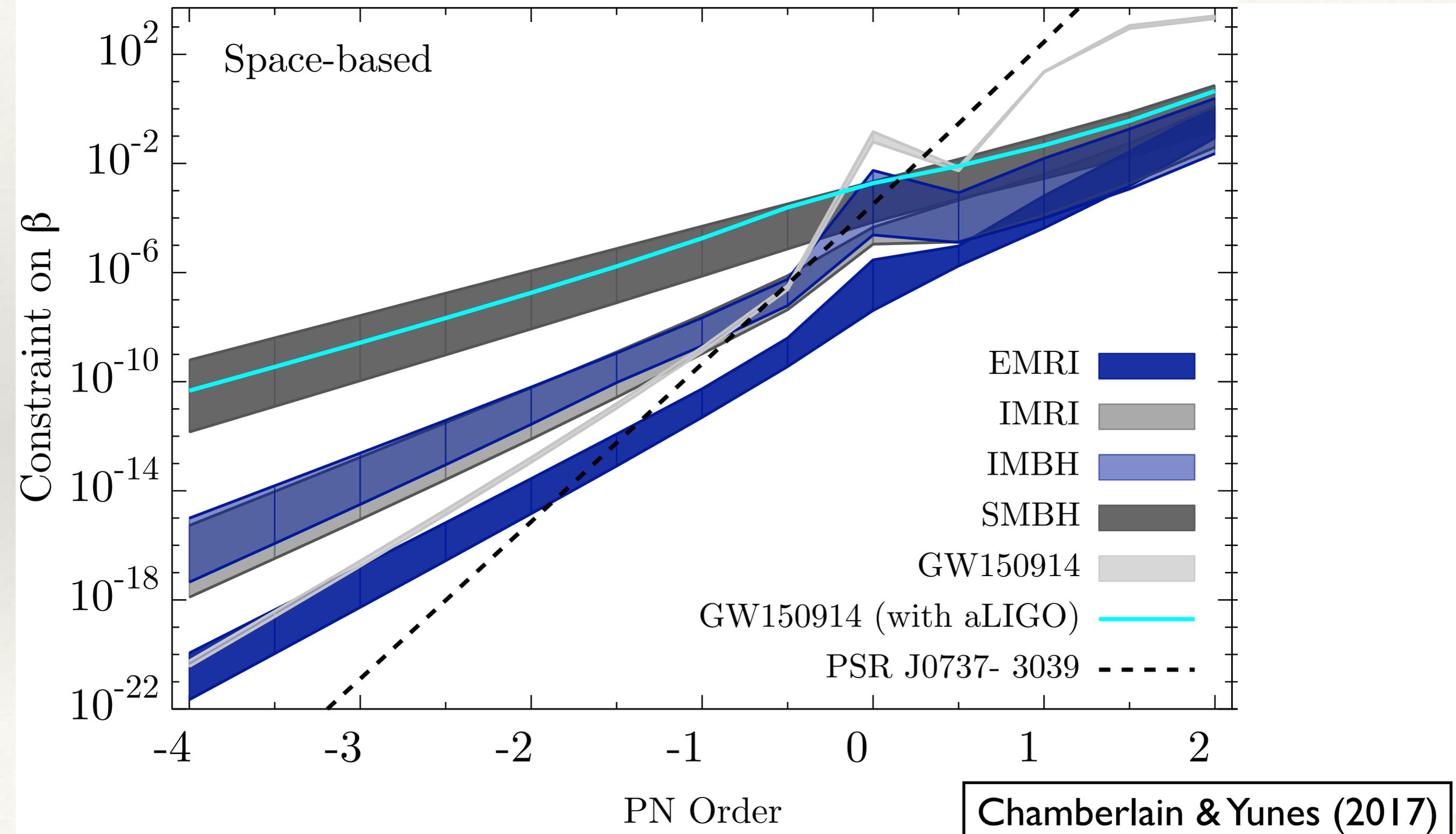
# Tests of gravitational physics

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- ❖ Parameterised post-Einsteinian formalism (Yunes & Pretorius 2009) uses

$$\tilde{h}^{\text{ppE}}(f) = \tilde{h}^{\text{GR}}(f) \times (1 + \alpha(\pi \mathcal{M} f)^a) \exp [i\beta(\pi \mathcal{M} f)^b]$$

# Tests of gravitational physics



# Tests of gravitational physics

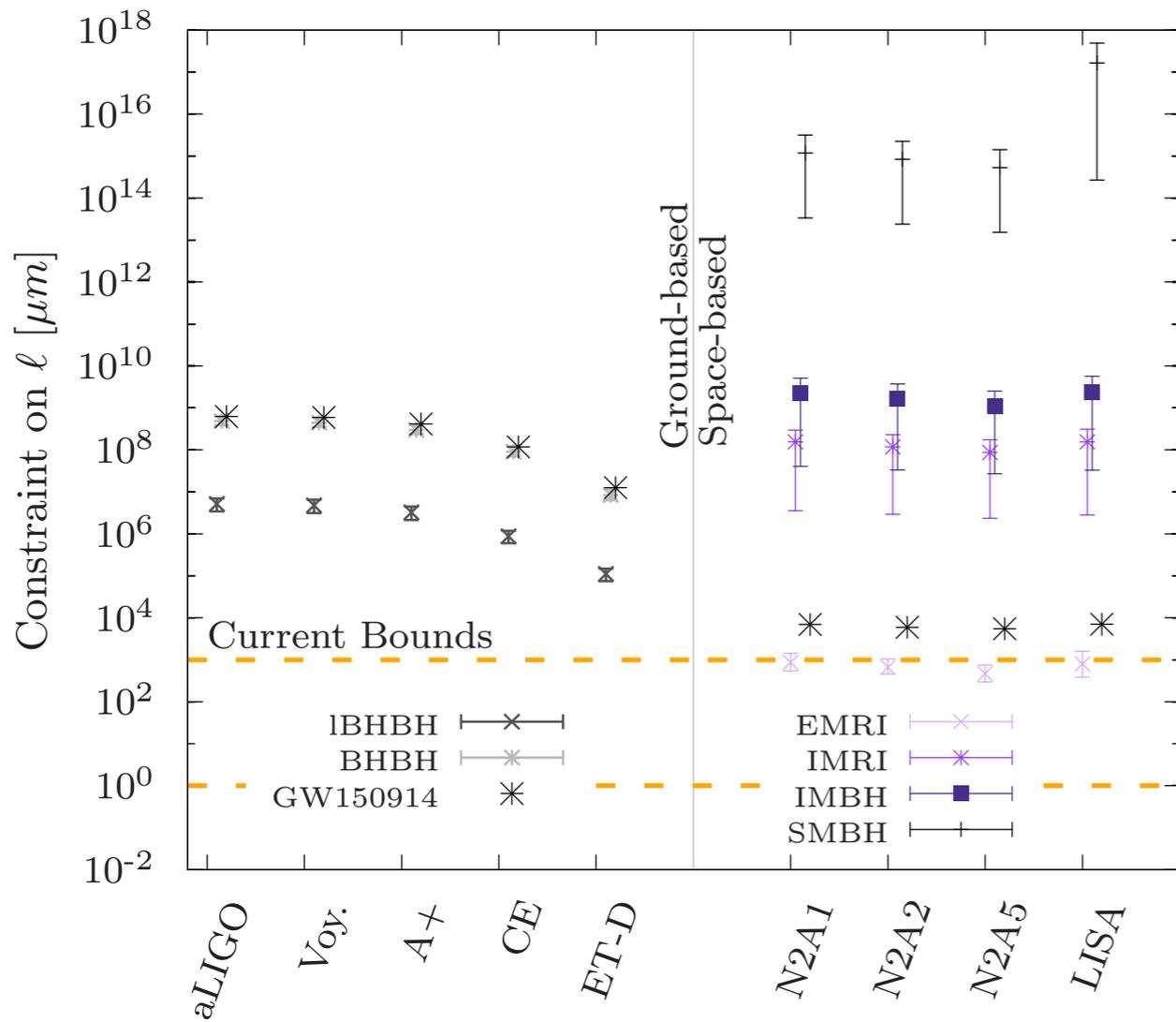
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- ❖ Many specific alternative theories can be directly mapped to the ppE parameterisation, allowing results to be interpreted physically.

GR Deviation	PN	Parameter	Best Space Const.	Best Ground Const.	Current Const.	Best Space Sys.	Best Ground Sys.
Dipole Radiation	-1	$\beta$	$4.9 \times 10^{-12}$	$1.9 \times 10^{-10}$	$4.4 \times 10^{-5}$	EMRI	NSNS
		$\delta \dot{E}_{\text{Dip}}$	$7.8 \times 10^{-8}$	$3.2 \times 10^{-8}$	$1.8 \times 10^{-3}$	EMRI/GW150914	NSNS
Large Extra-Dimension	-4	$\beta$	$2.2 \times 10^{-22}$	$6.4 \times 10^{-20}$	$9.1 \times 10^{-11}$	EMRI	NSNS
		$\ell [\mu m]$	$3.0 \times 10^2$	$7.5 \times 10^4$	$10 - 10^3$ [28–32]	EMRI/GW150914	BHBH
Time-Varying $G$	-4	$\beta$	$2.2 \times 10^{-22}$	$6.4 \times 10^{-20}$	$9.1 \times 10^{-11}$	EMRI	NSNS
		$\dot{G}$ [1/yr]	$6.8 \times 10^{-8}$	$1.1 \times 10^{-3}$	$10^{-12} - 10^{-13}$ [33–37]	EMRI	NSNS
Einstein-Æther Theory	0	$\beta$	$4.0 \times 10^{-8}$	$6.7 \times 10^{-5}$	$3.4 \times 10^{-3}$	EMRI	$\ell$ BHNS
		$(c_+, c_-)$	$(10^{-3}, 3 \times 10^{-4})$	$(10^{-2}, 4 \times 10^{-3})$	$(0.03, 0.003)$ [38, 39]	EMRI	NSNS
Khronometric Gravity	0	$\beta$	$4.0 \times 10^{-8}$	$6.7 \times 10^{-5}$	$3.4 \times 10^{-3}$	EMRI	$\ell$ BHNS
		$(\beta_{\text{KG}}, \lambda_{\text{KG}})$	$(10^{-4}, 10^{-2})/2$	$(10^{-2}, 10^{-1})/5$	$(10^{-2}, 10^{-1})/2$ [38, 39]	EMRI	GW150914
Graviton Mass	+1	$\beta$	$4.3 \times 10^{-5}$	$1.0 \times 10^{-3}$	$8.9 \times 10^{-2}$	EMRI/IMBH	$\ell$ BHBH
		$m_g$ [eV]	$9.0 \times 10^{-28}$	$9.9 \times 10^{-25}$	$10^{-29} - 10^{-18}$ [40–44]	SMBH/IMRI	GW150914

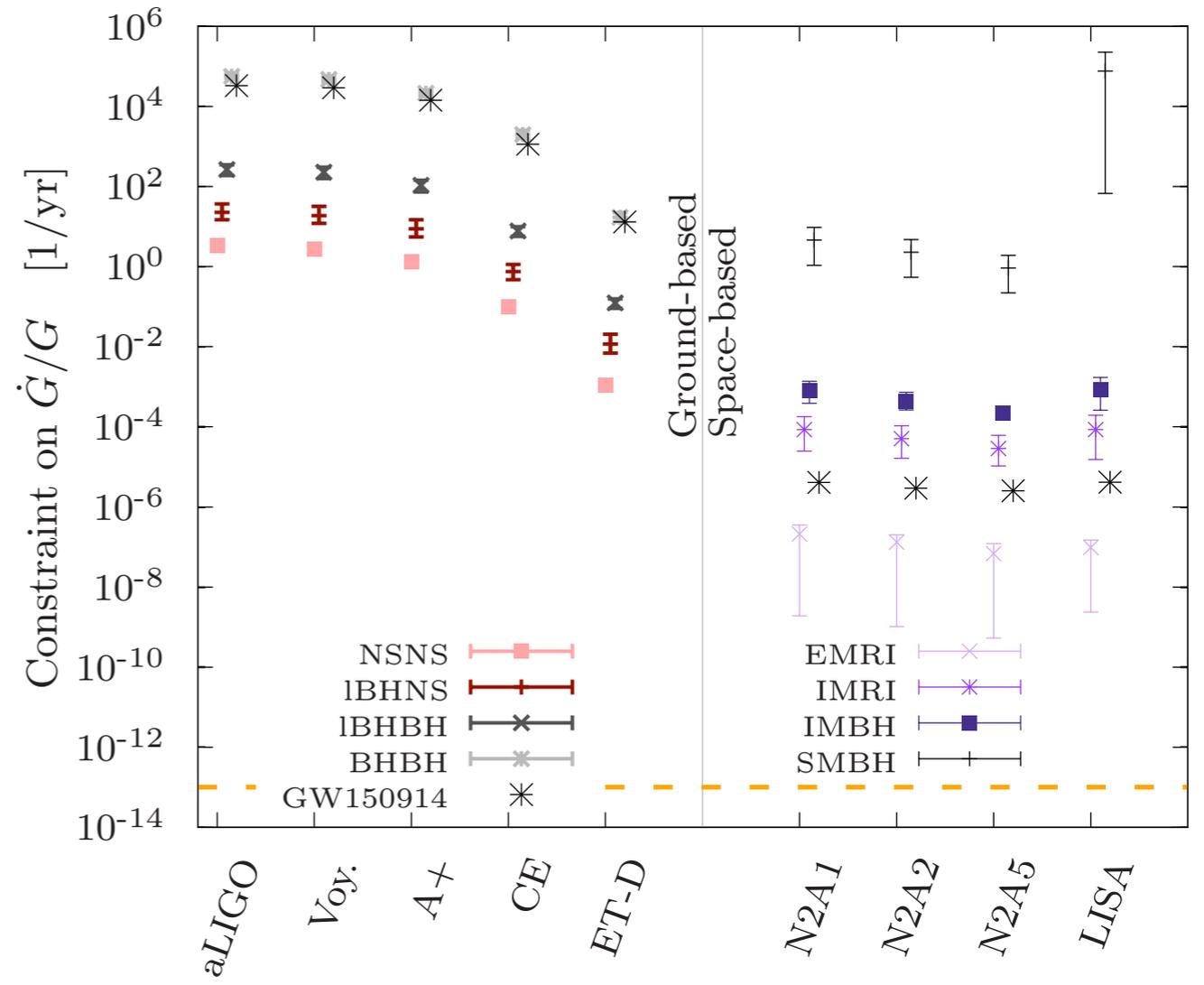
# Tests of gravitational physics



**Large extra dimensions**

$$\beta = \frac{dm}{dt} \frac{25}{851968} \left( \frac{3 - 26\eta + 34\eta^2}{\eta^{2/5}(1 - 2\eta)} \right)$$

$$\dot{m}_a = -2.8 \times 10^{-7} \left( \frac{M_\odot}{M_a} \right)^2 \left( \frac{\ell}{10\mu\text{m}} \right)^2 M_\odot \text{ yr}^{-1}$$

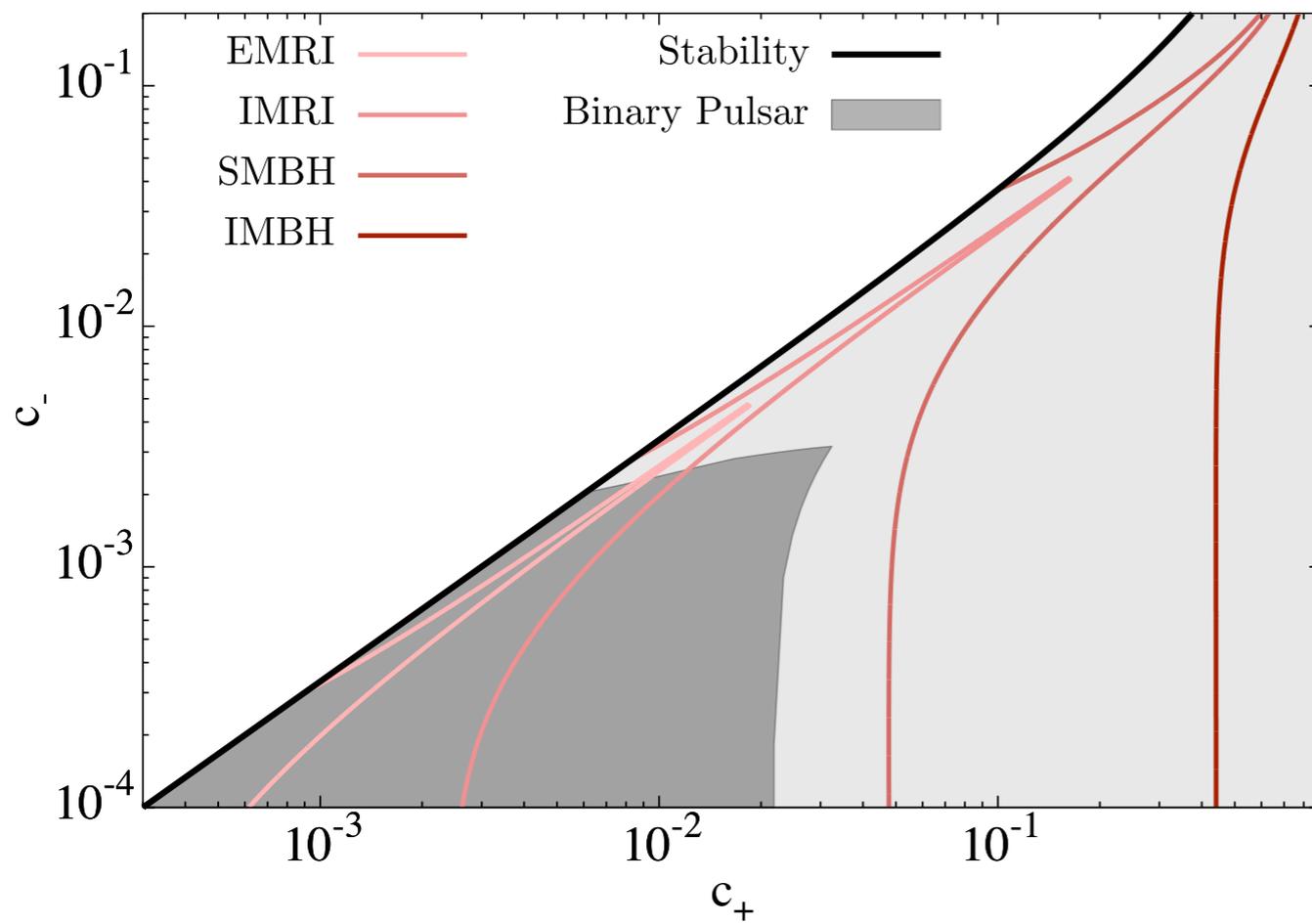


**$b = -13/3$**

**Varying Newton's constant**

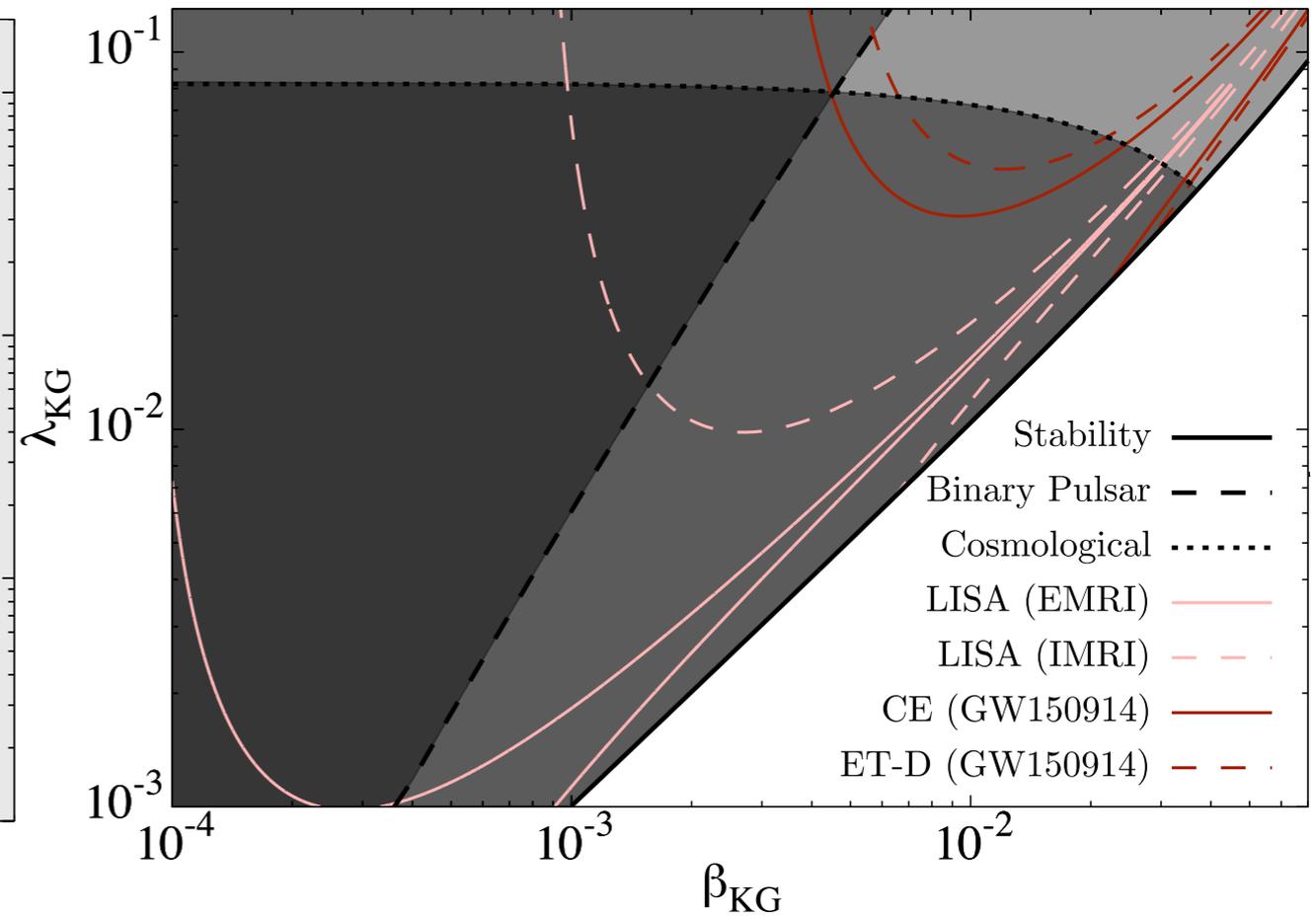
$$\beta = \frac{25}{65526} \frac{\dot{G}_z}{G} \mathcal{M}_z$$

# Tests of gravitational physics



Einstein-Aether gravity

$$\beta = -\frac{3}{128} \left[ \left(1 - \frac{c_{14}}{2}\right) (\mathcal{A}_{EA,1} + S\mathcal{A}_{EA,2} + S^2\mathcal{A}_{EA,3}) \right]$$



$b = -5/3$

Khronometric gravity

$$\beta = -\frac{3}{128} \left[ \left(1 - \frac{\alpha_{KG}}{2}\right) (\mathcal{A}_{KG,1} + S\mathcal{A}_{KG,2} + S^2\mathcal{A}_{KG,3}) \right]$$

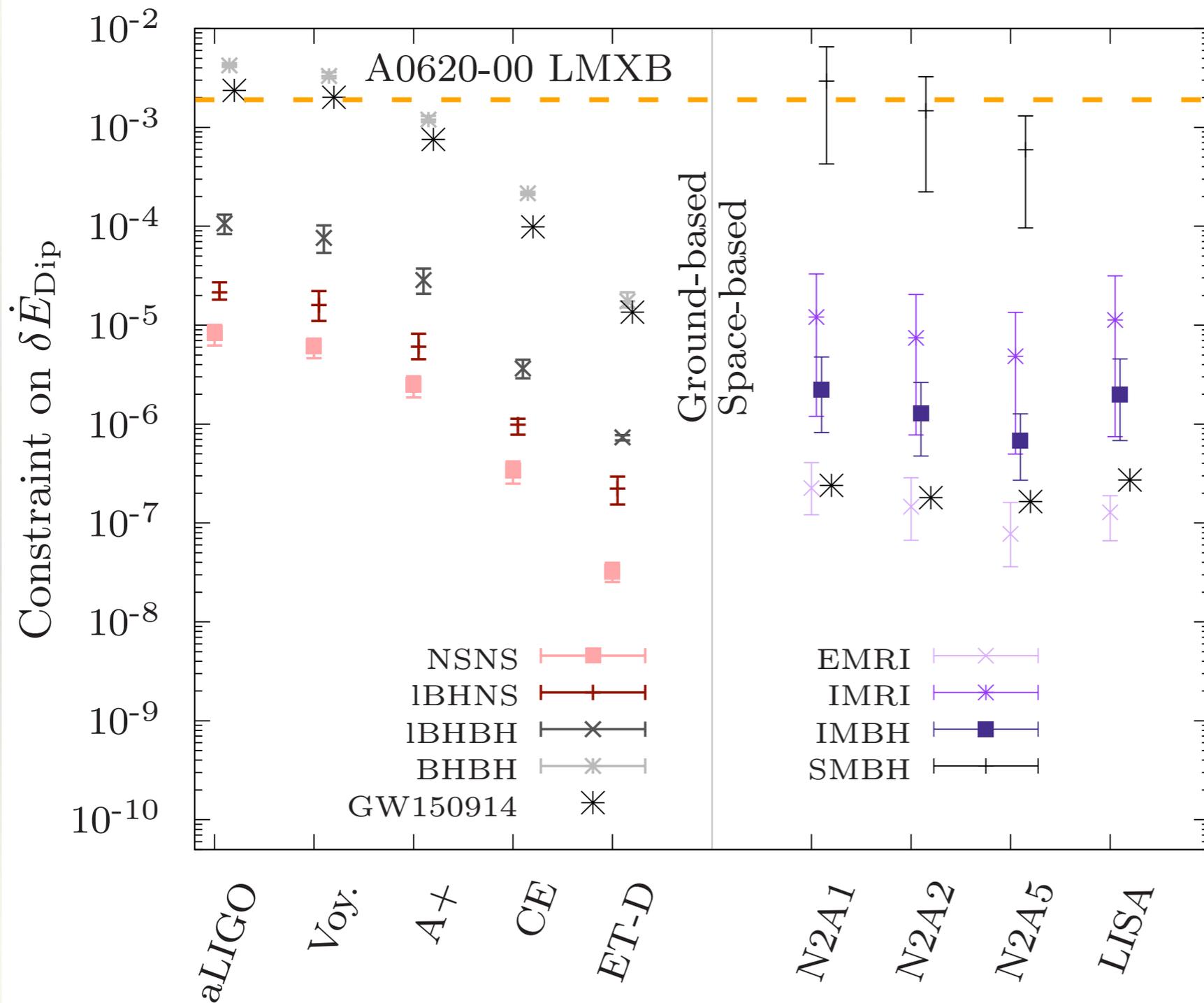
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# Dipole radiation

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- ❖ **Energy loss:** inspiral rate could differ from quadrupole formula prediction due to, e.g., dipole radiation in scalar-tensor gravity.

# Dipole radiation



Dipole radiation,  $b = -7/3$

$$\beta = -\frac{3}{224} \delta \dot{E}_{\text{Dip}} \eta^{2/5}$$

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# Dipole radiation

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- ❖ **Energy loss:** inspiral rate could differ from quadrupole formula prediction due to, e.g., dipole radiation in scalar-tensor gravity.
- ❖ Can translate into bound on Brans-Dicke theory. Best bounds from NS+MBH or SOBH inspirals.

$$\omega_{\text{BD}} > 2 \times 10^4 \left( \frac{S}{0.3} \right) \left( \frac{100}{\Delta\Phi_D} \right) \left( \frac{T}{1\text{yr}} \right)^{\frac{7}{8}} \left( \frac{10^4 M_{\odot}}{M_{\bullet}} \right)^{\frac{3}{4}}$$

---

# Modified propagation speed

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- ❖ **Propagation:** in GR GWs travel at the speed of light. Constrain “graviton mass” using GW observations.
- ❖ Can parameterise the modified dispersion relation in different ways, e.g.,

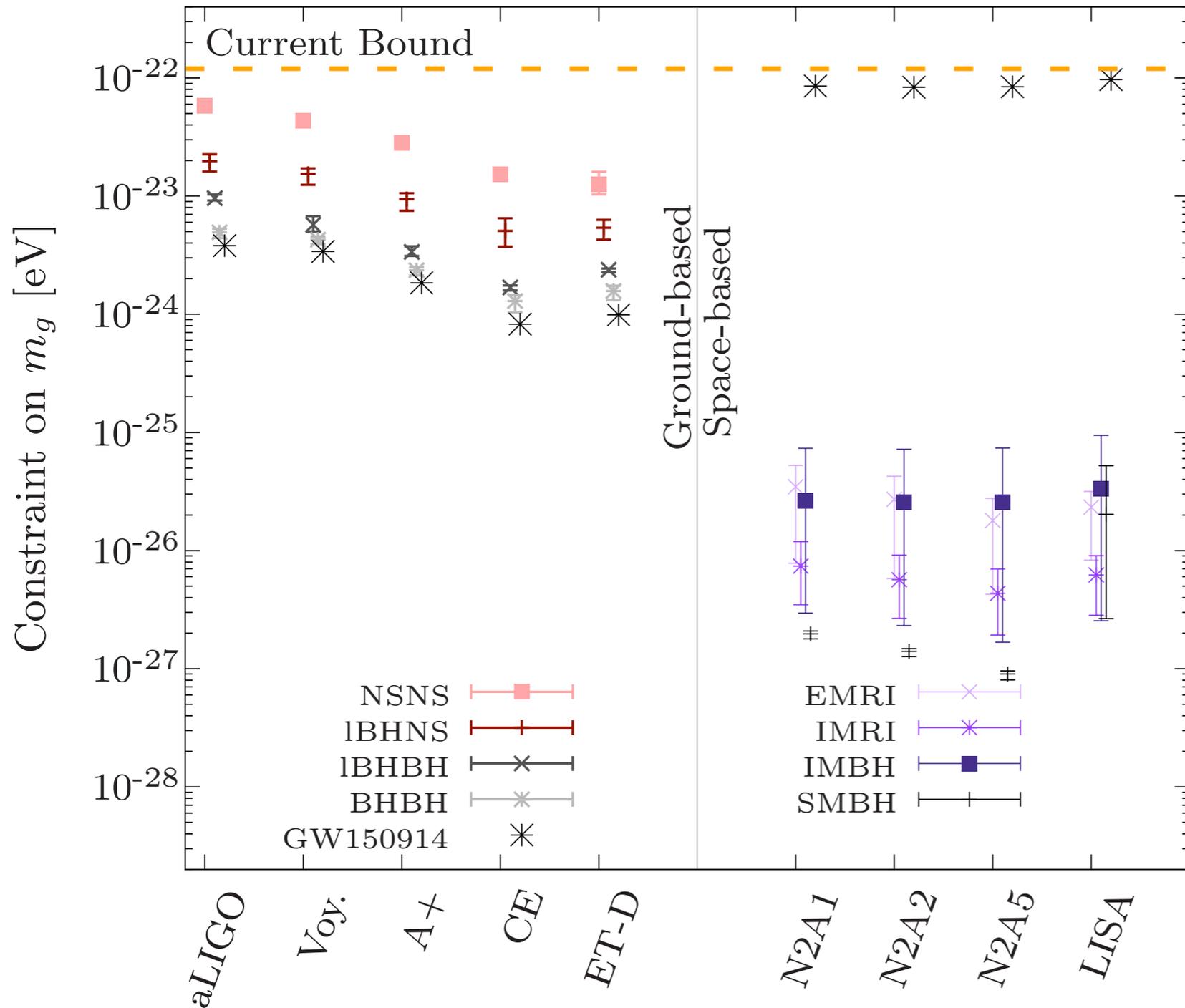
$$\omega^2 = k_i k^i + \frac{m_g^2}{\hbar^2} + A(k_i k^i)^\alpha$$

- ❖ Or the form popular in a cosmological setting

$$\omega^2 + iH\omega (3 + \alpha_M) = (1 + \alpha_T) k_i k^i$$

- ❖ Constraints come from observation of EM counterparts to GW observations and (lack of) dispersion in GW chirps. Current LIGO constraints from GW170817 and GW150914 are quite similar.

# Modified propagation speed

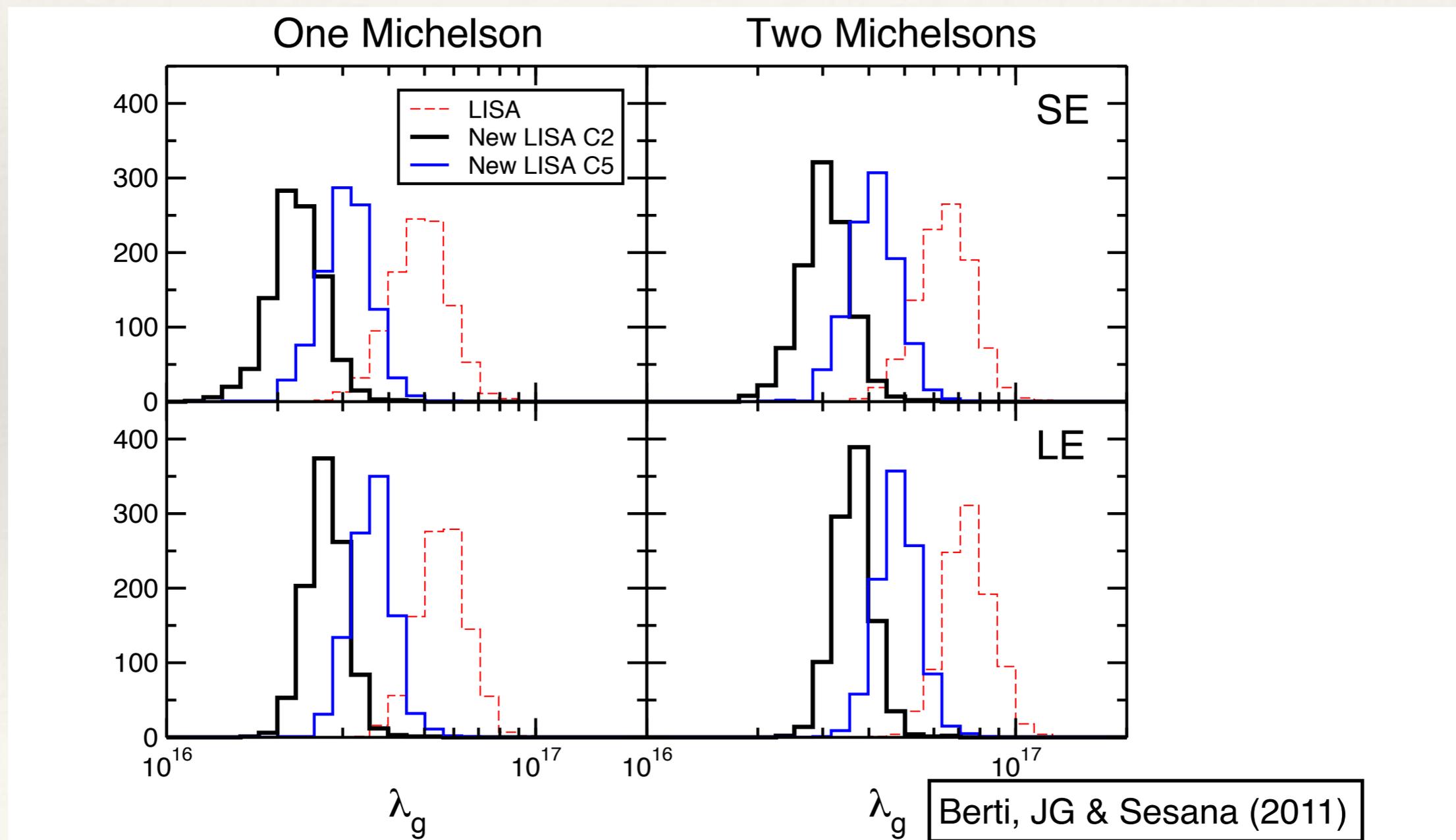


Massive graviton,  $b = -1$

$$\beta = \frac{\pi^2 D_0 \mathcal{M}_z}{\lambda^2}$$

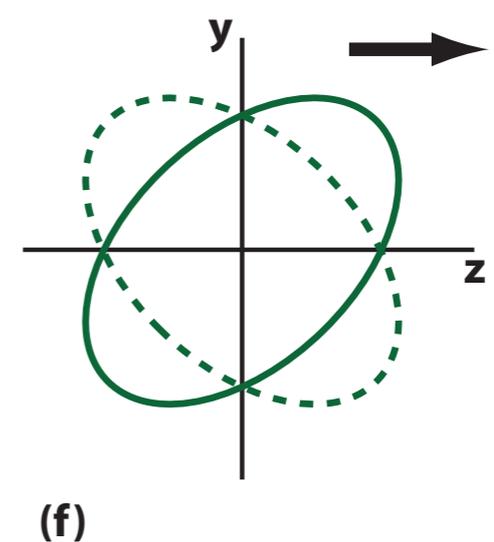
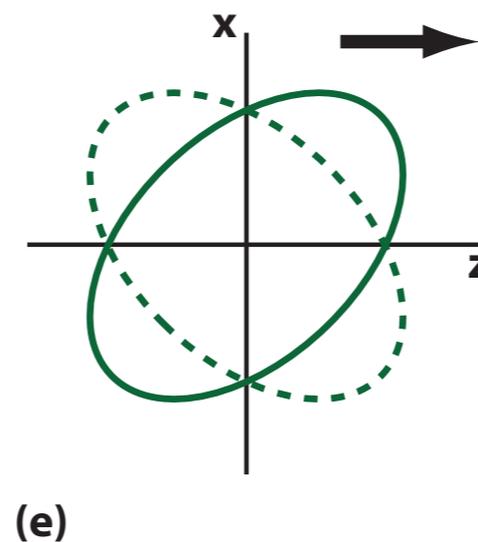
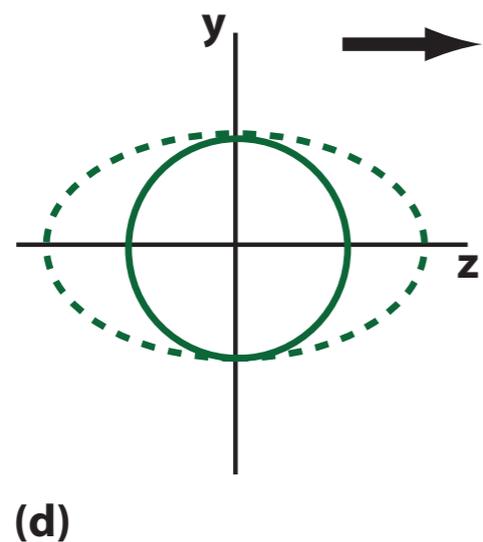
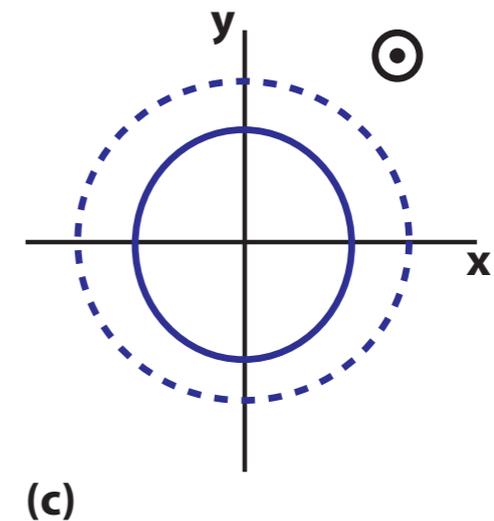
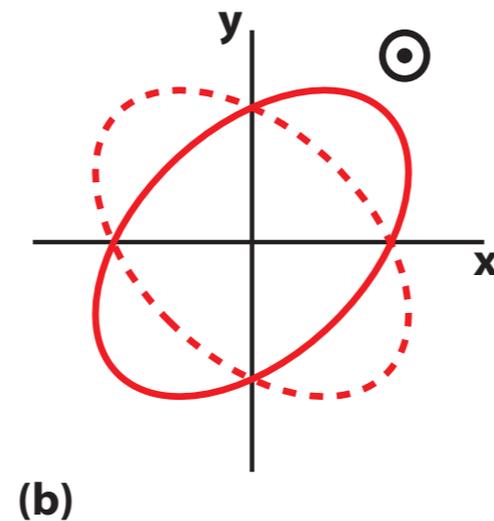
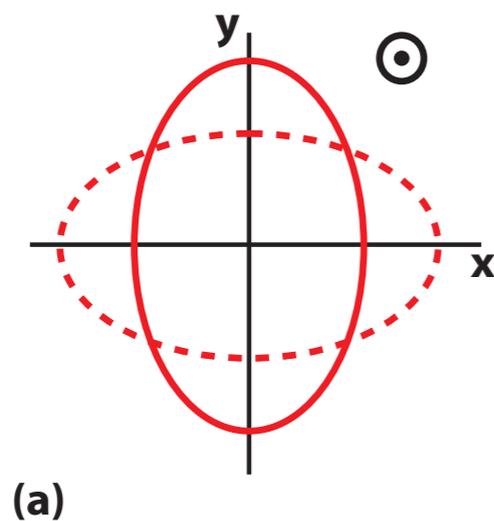
# Modified propagation speed

- ❖ Can also combine data from multiple events to strengthen the constraints. Bounds of the order  $\lambda_g = h/m_g > \text{few} \times 10^{16} \text{ km}$  are possible.



# Tests of gravitational physics

- ❖ **Polarisation:** in GR there are only two GW polarisation states - plus and cross, but four additional states are possible in metric theories.



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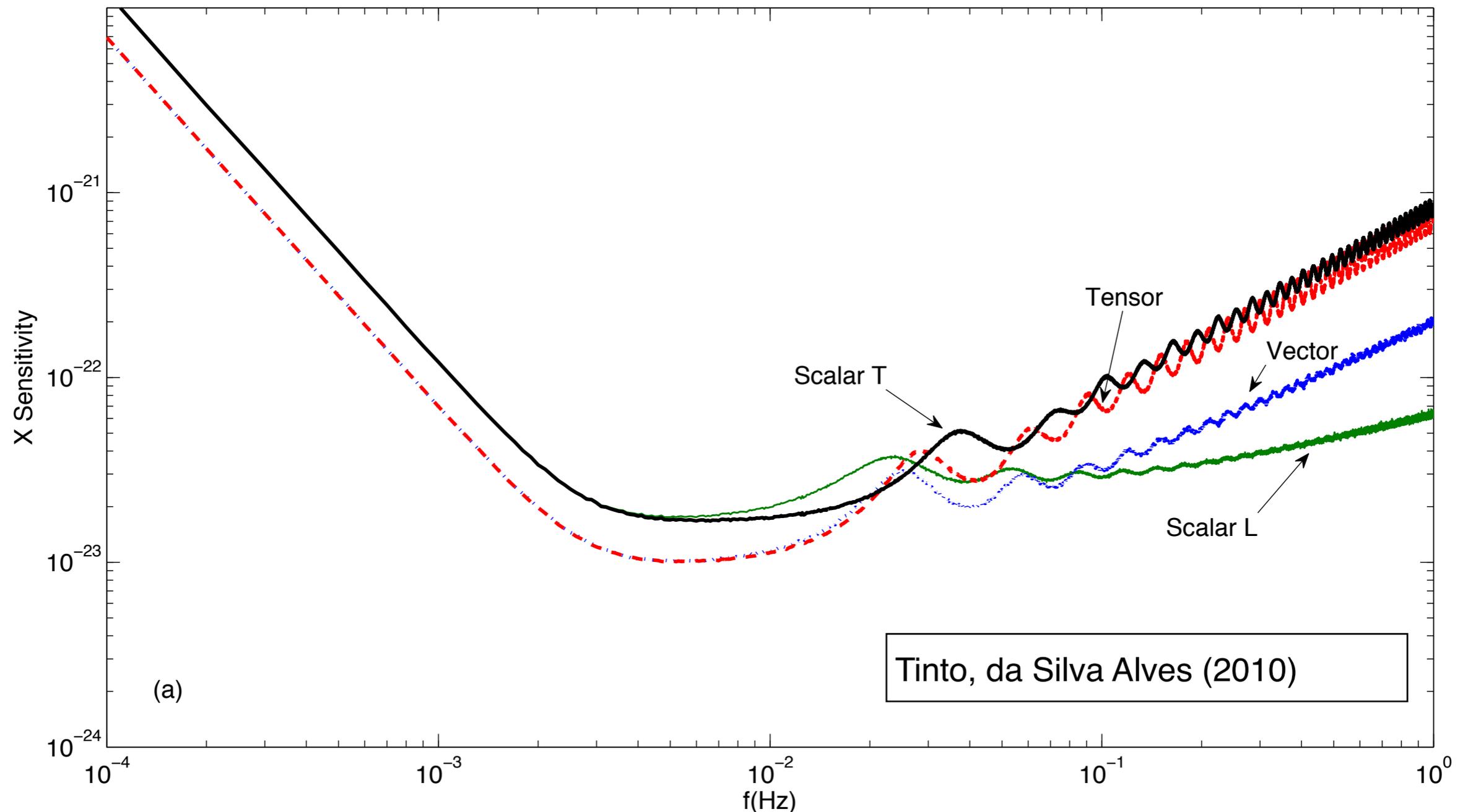
# Tests of gravitational physics

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- ❖ **Polarisation:** in GR there are only two GW polarisation states - plus and cross, but four additional states are possible in metric theories.
  - At frequencies greater than one over the light travel time, LISA is ten times more sensitive to scalar-longitudinal and vector modes than scalar-transverse and tensor modes.

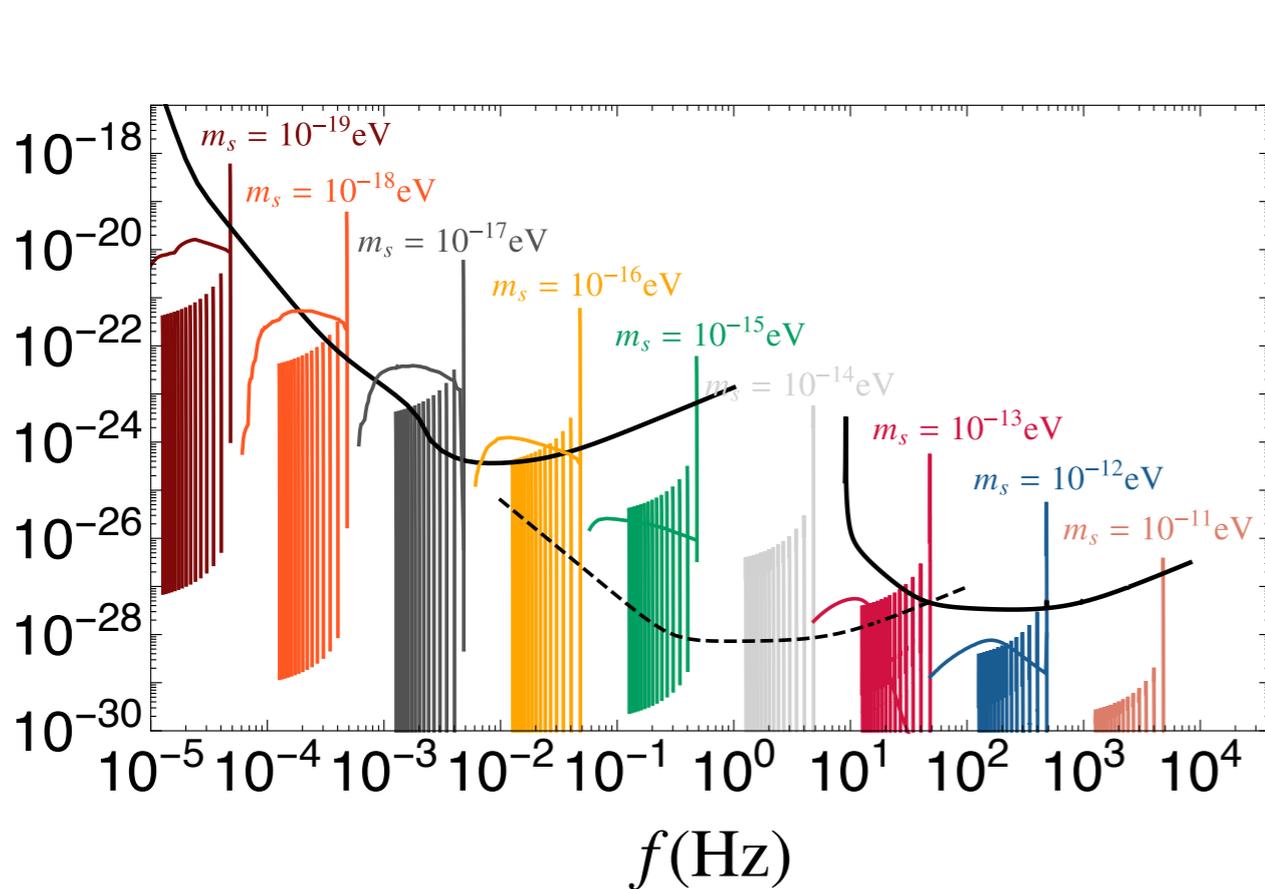
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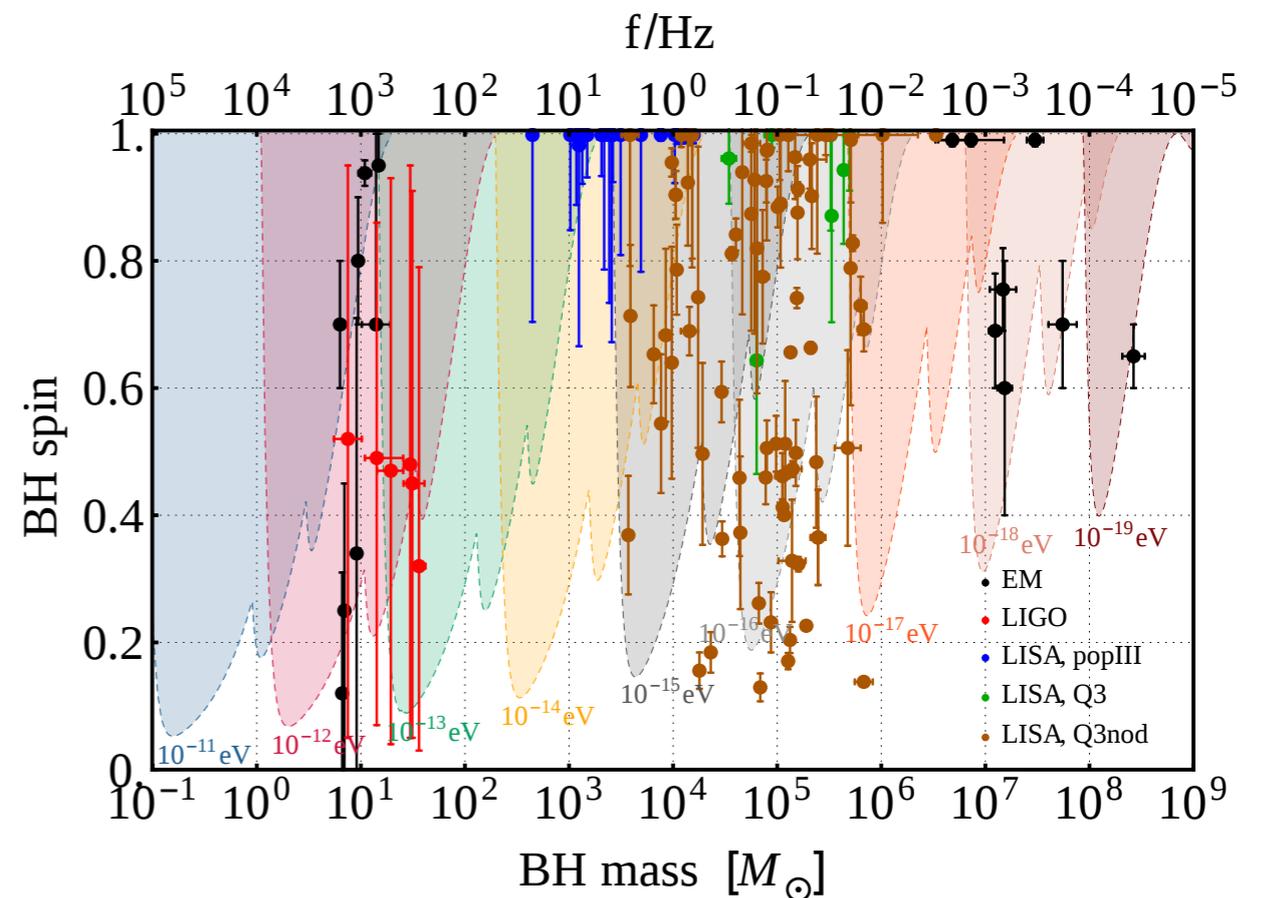


# Dark matter candidates: bosons

- ❖ Ultra-light boson fields  $\mu \sim \omega < m\Omega_H$  are subject to super-radiant instability, leading to formation of boson condensates outside black holes. These produce GW backgrounds when they dissipate.



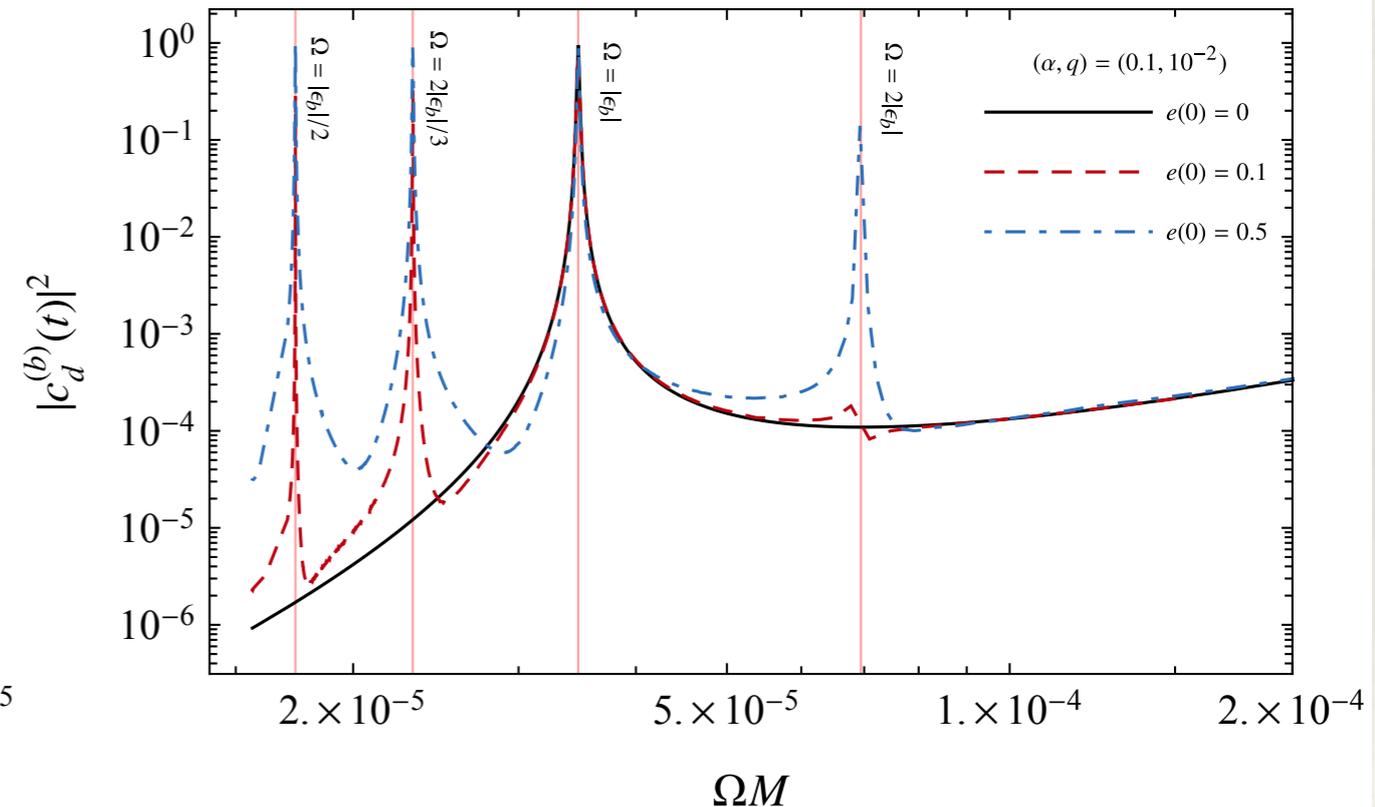
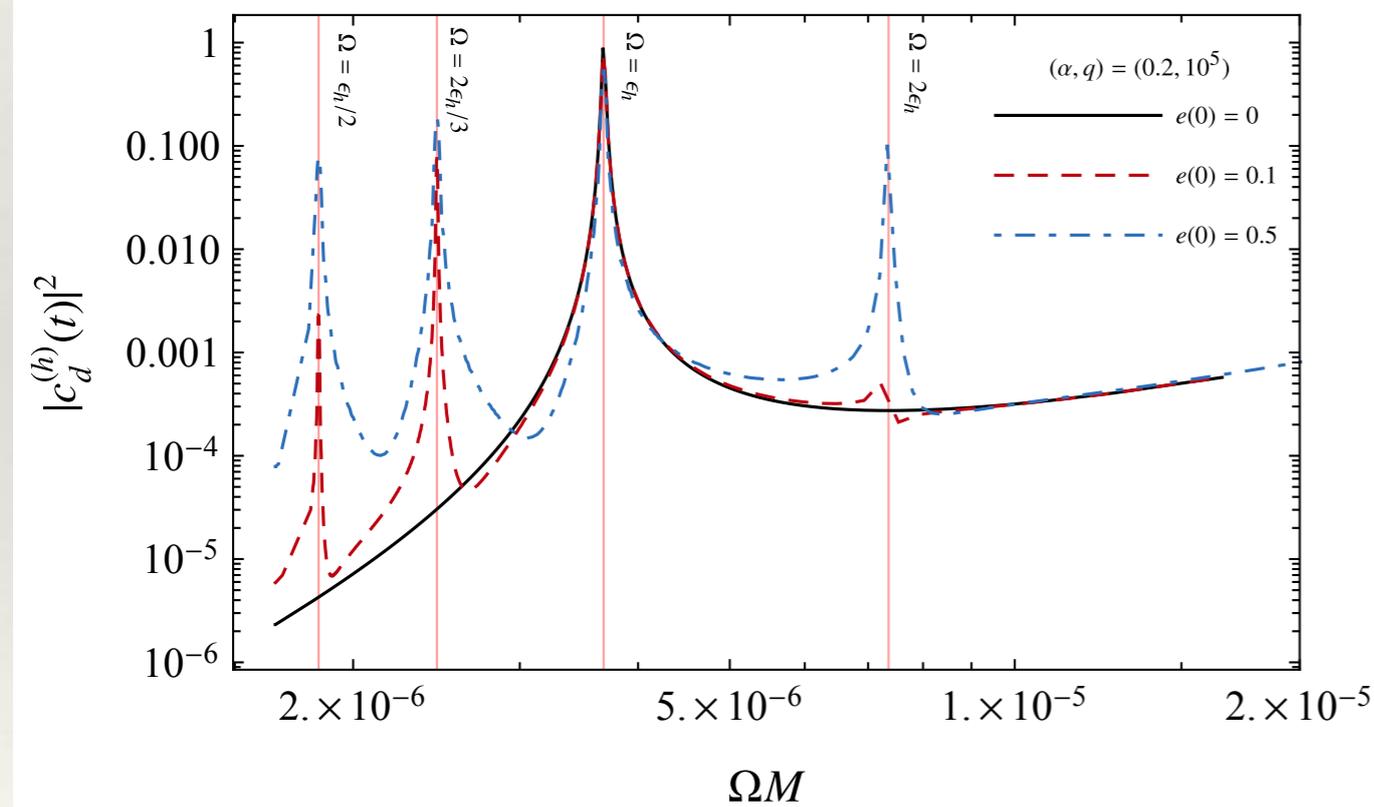
Brito et al. (2017a)



Brito et al. (2017b)

# Dark matter candidates: bosons

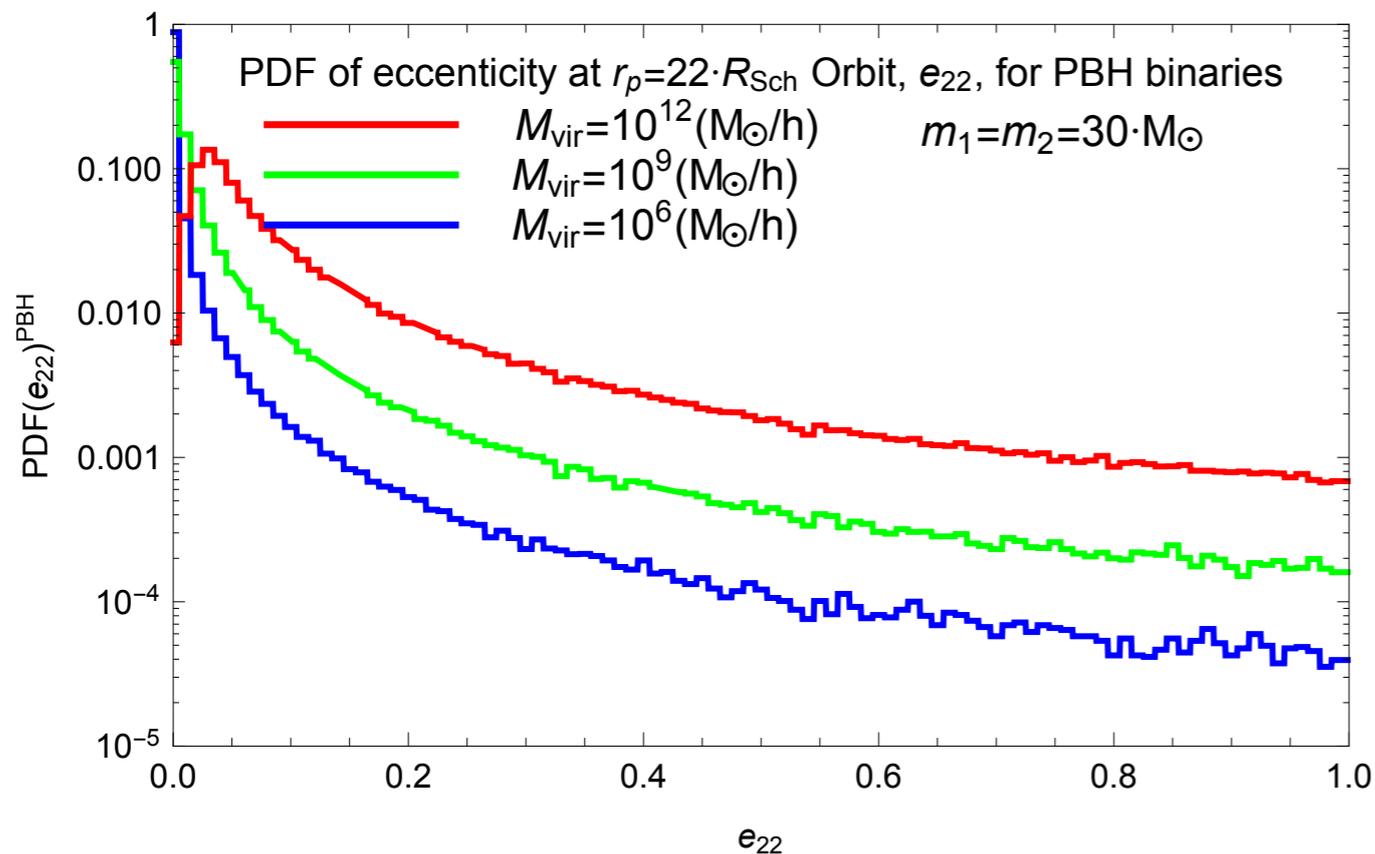
- ❖ Can also see lead resonant depletion of boson clouds during binary inspirals, e.g., EMRIs.



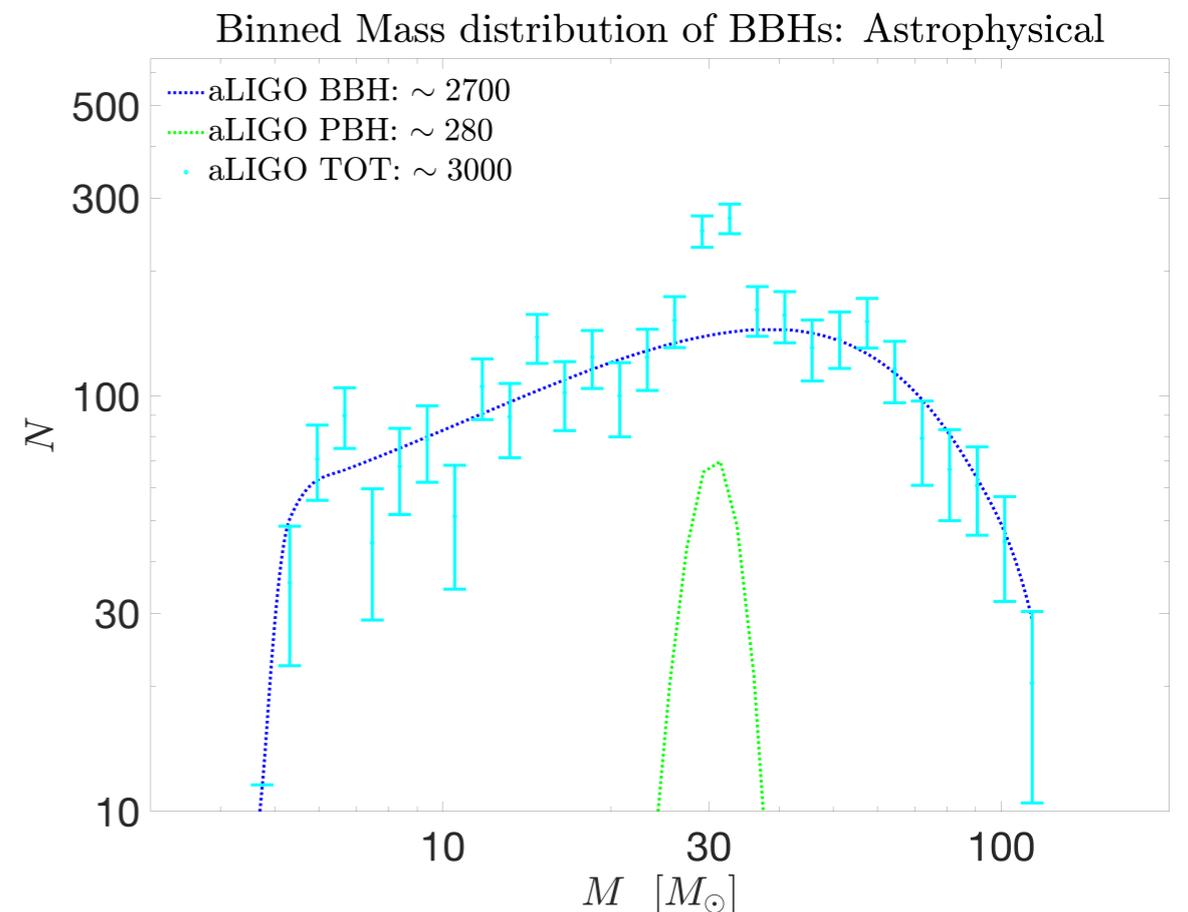
Brito et al. (2019)

# Dark matter candidates: primordial BHs

- ❖ Primordial BHs formed directly in the early Universe will generate GWs. Direct detection of these black holes possible as SOBHs. LISA measurements of eccentricity crucial for identifying primordial origin.



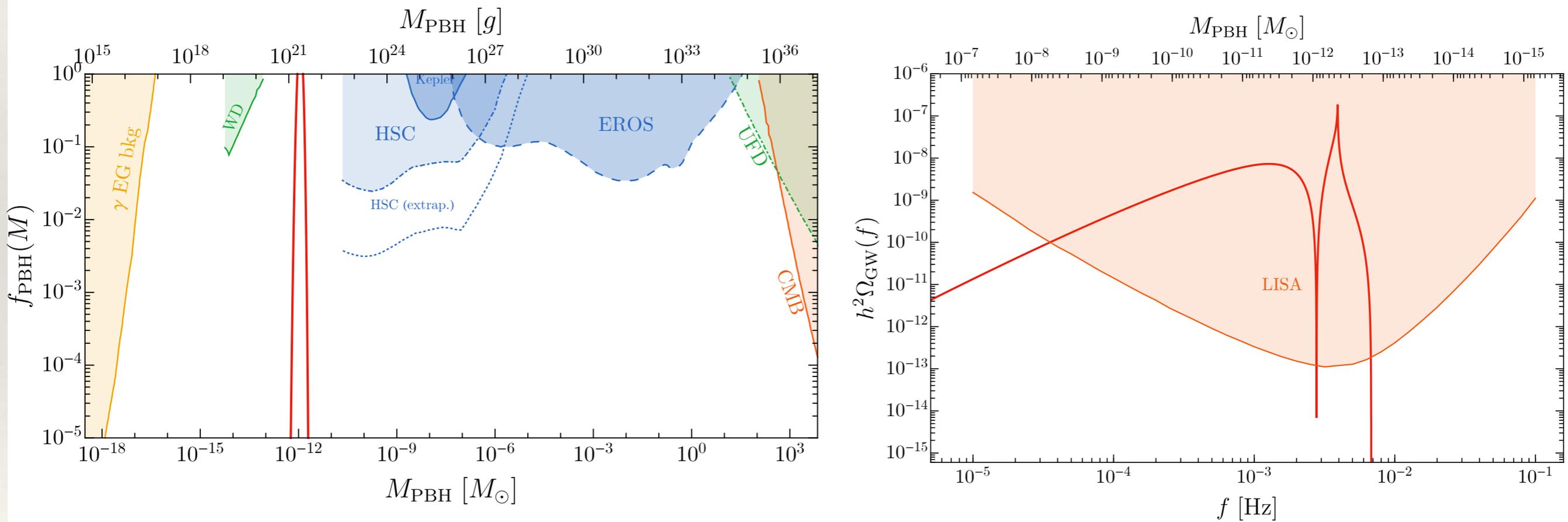
Cholis et al. (2016)



Kovetz et al. (2017)

# Dark matter candidates: primordial BHs

- ❖ LISA will probe a previously poorly constrained mass range by (non?)-observation of a stochastic GW background.



Bartolo et al. (2018)

# Cosmography with LISA Observations

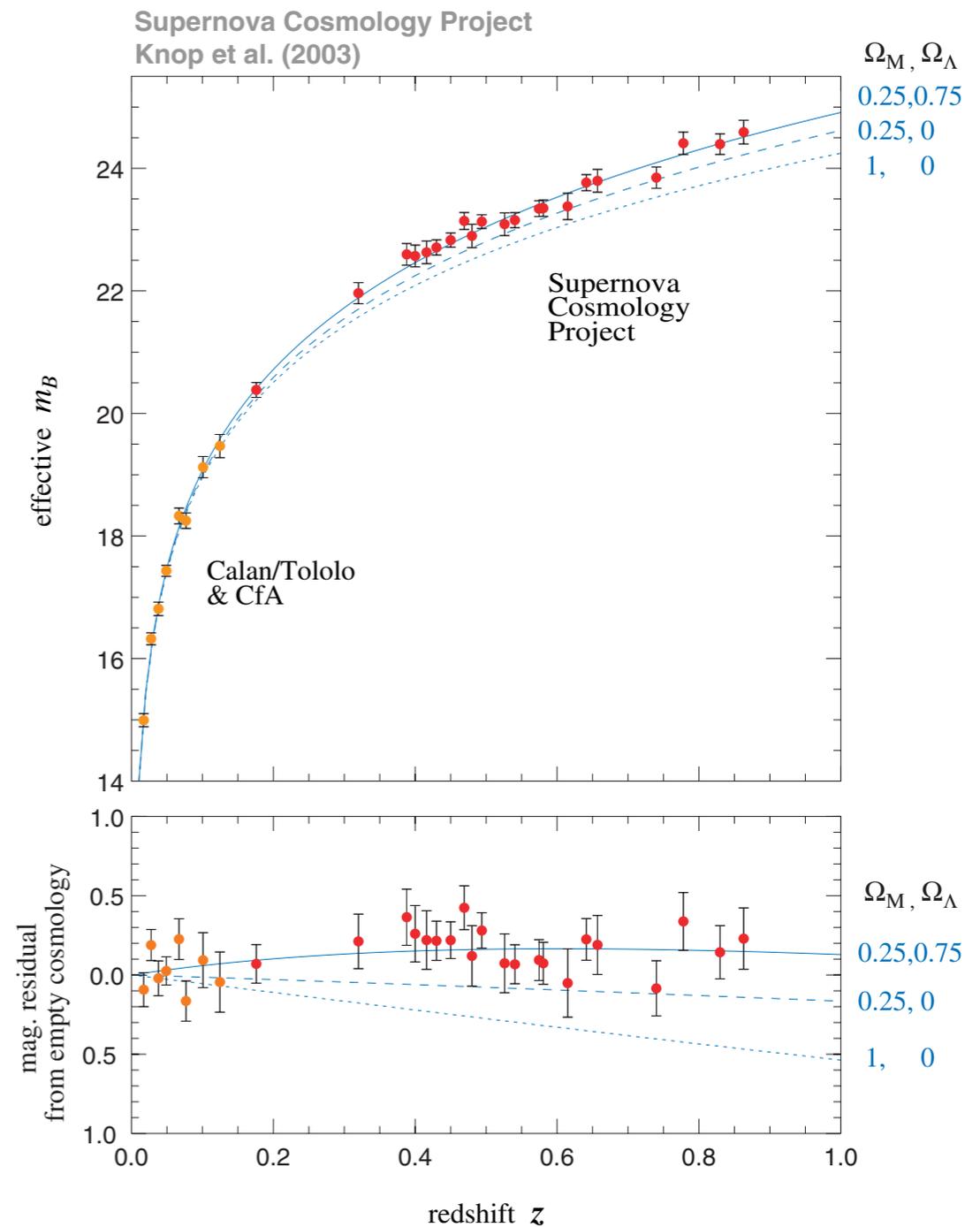
# Science: cosmography

- ❖ Dimensionless gravitational wave strain scales

as

$$h \sim \frac{\mathcal{M}}{\mathcal{D}} \sim \frac{(1+z)M}{D_L(z)}$$

- ❖ Phase evolution determines intrinsic parameters precisely. Amplitude then gives distance accurately (Schutz 1986).
- ❖ Need another way to break the mass / redshift degeneracy - electromagnetic counterpart.



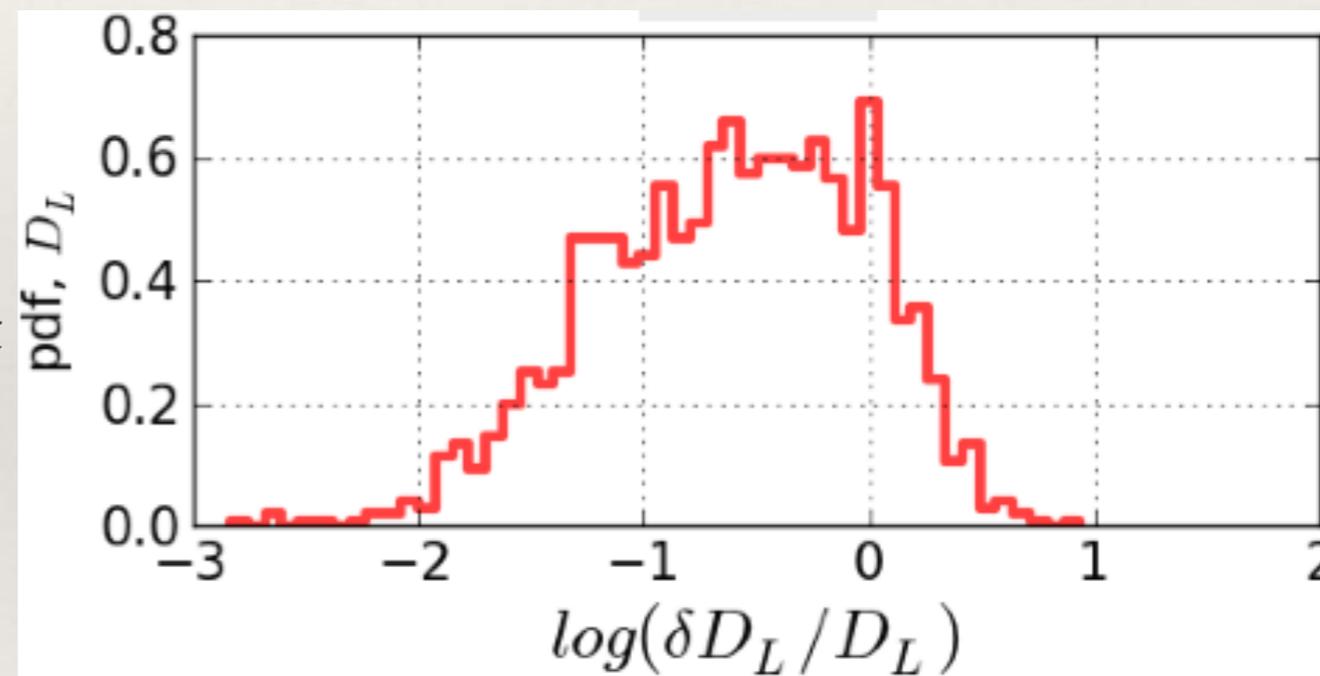
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- ❖ Phase evolution determines intrinsic parameters precisely. Amplitude then gives distance accurately (Schutz 1986).
- ❖ Need another way to break the mass/redshift degeneracy - electromagnetic counterpart.
- ❖ Massive black hole mergers were seen as promising candidates, but
  - counterpart mechanism is unclear.
  - weak lensing dominates errors for most sources.
  - LISA distance precision is poor.



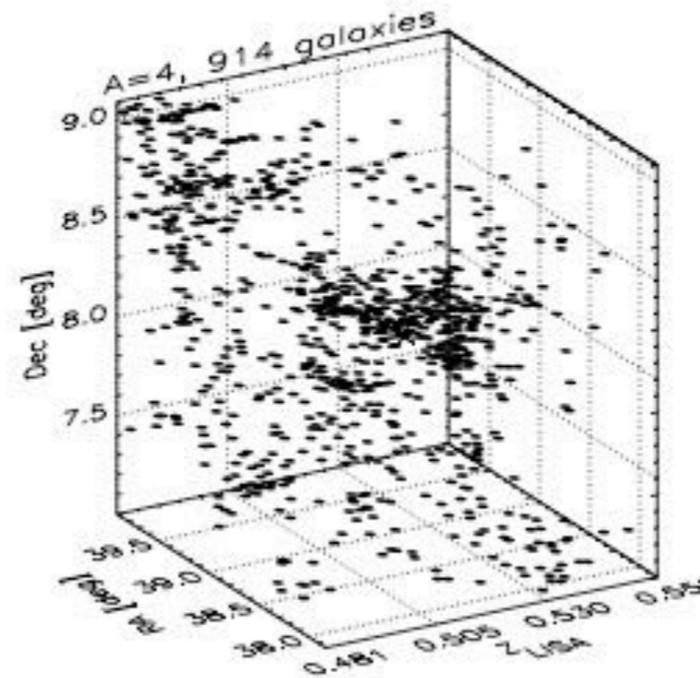
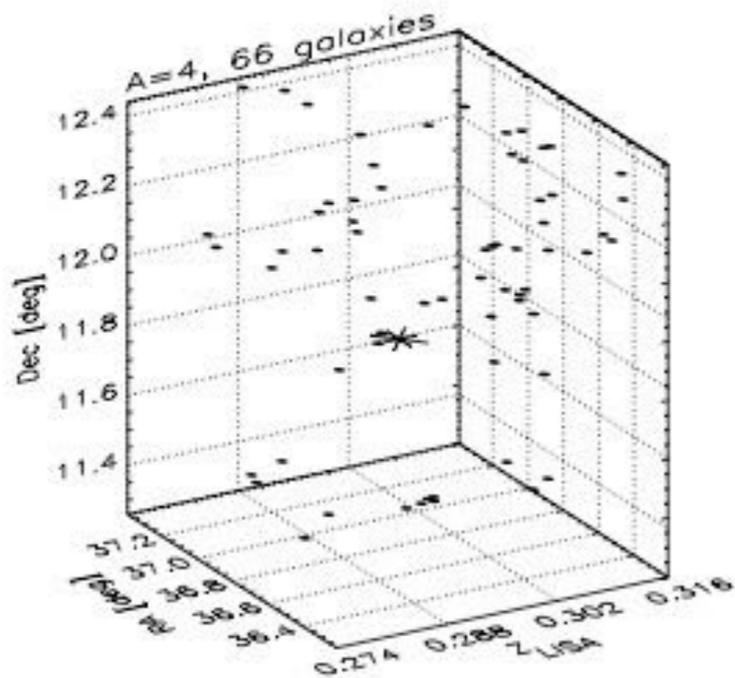
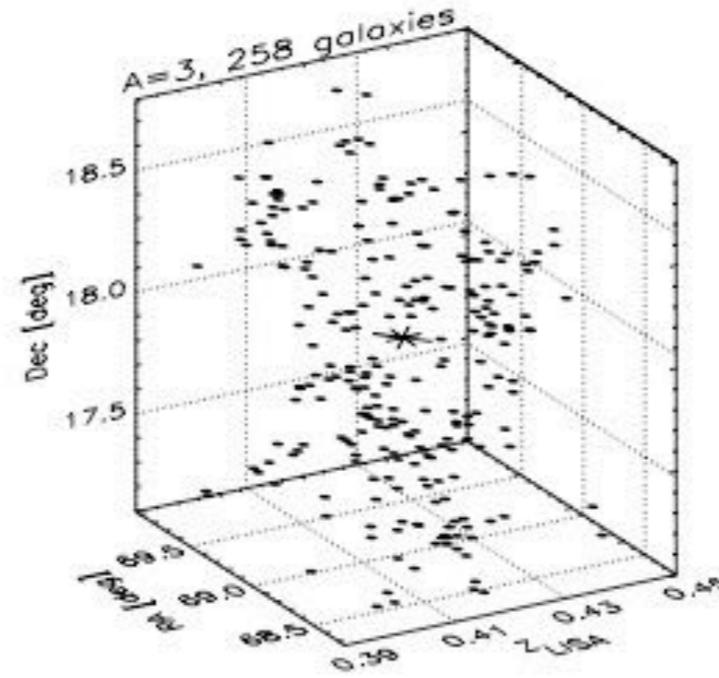
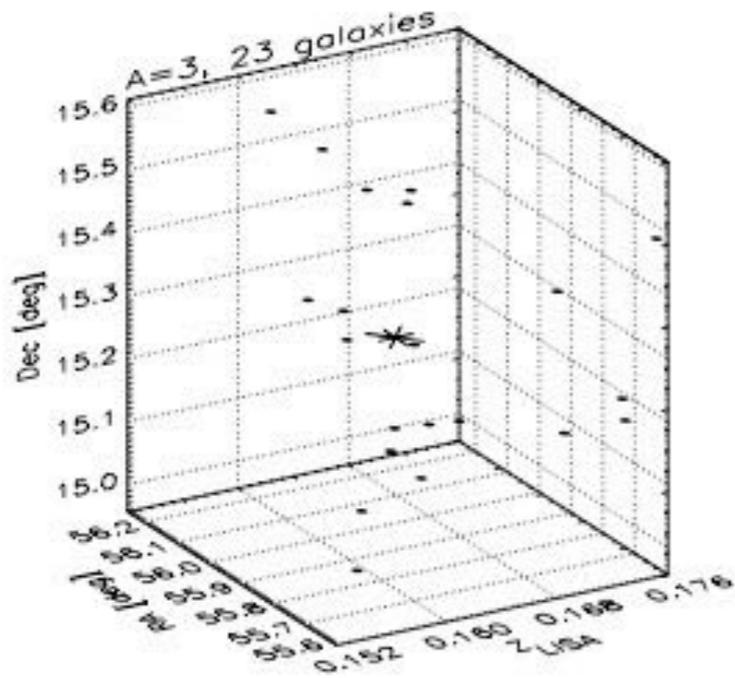
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# Science: cosmography

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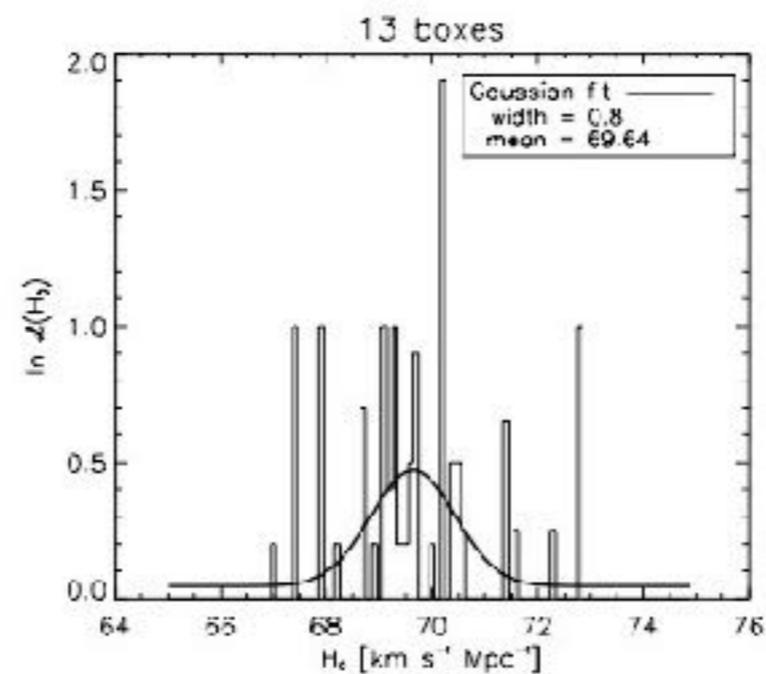
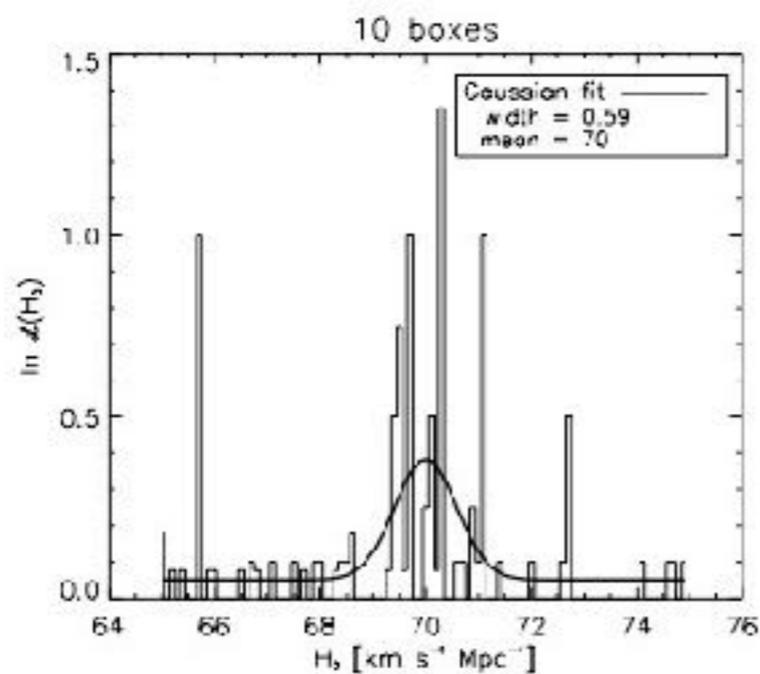
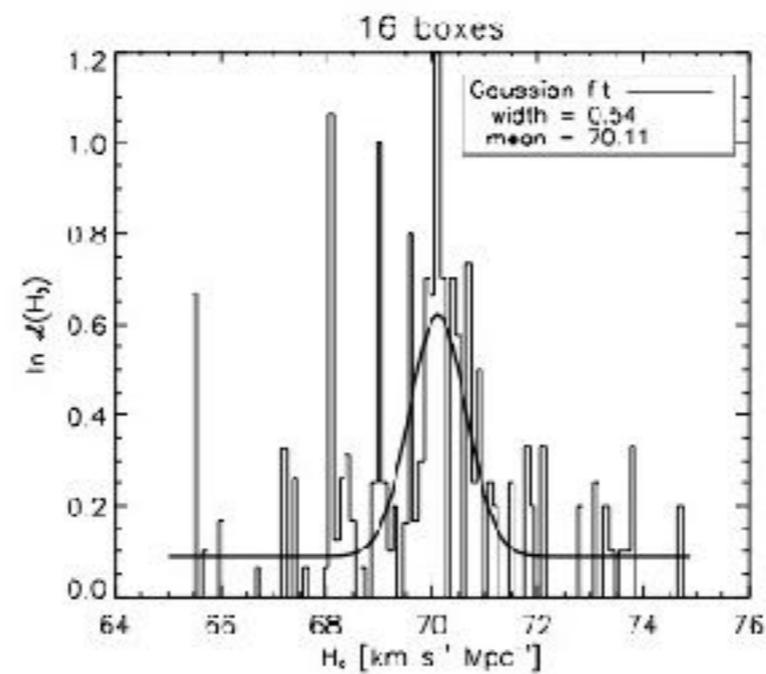
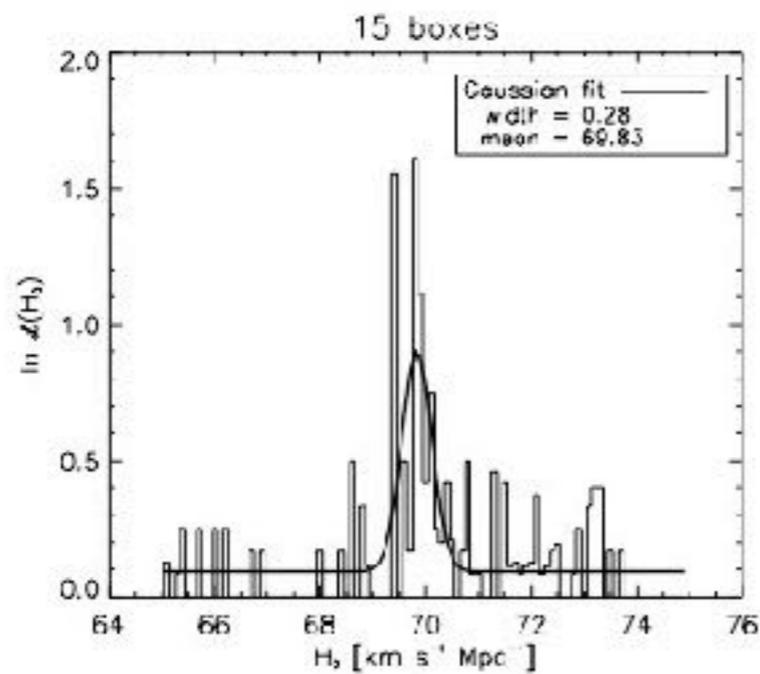
- ❖ Even without a counterpart, can estimate cosmological parameters statistically from GW observations.
- ❖ Use LISA observations of EMRIs to measure the Hubble constant (McLeod & Hogan 08)
  - Let every galaxy in the LISA error box “vote” on the Hubble constant.

# Science: cosmography



McLeod &  
Hogan (2008)

# Science: cosmography



McLeod &  
Hogan (2008)

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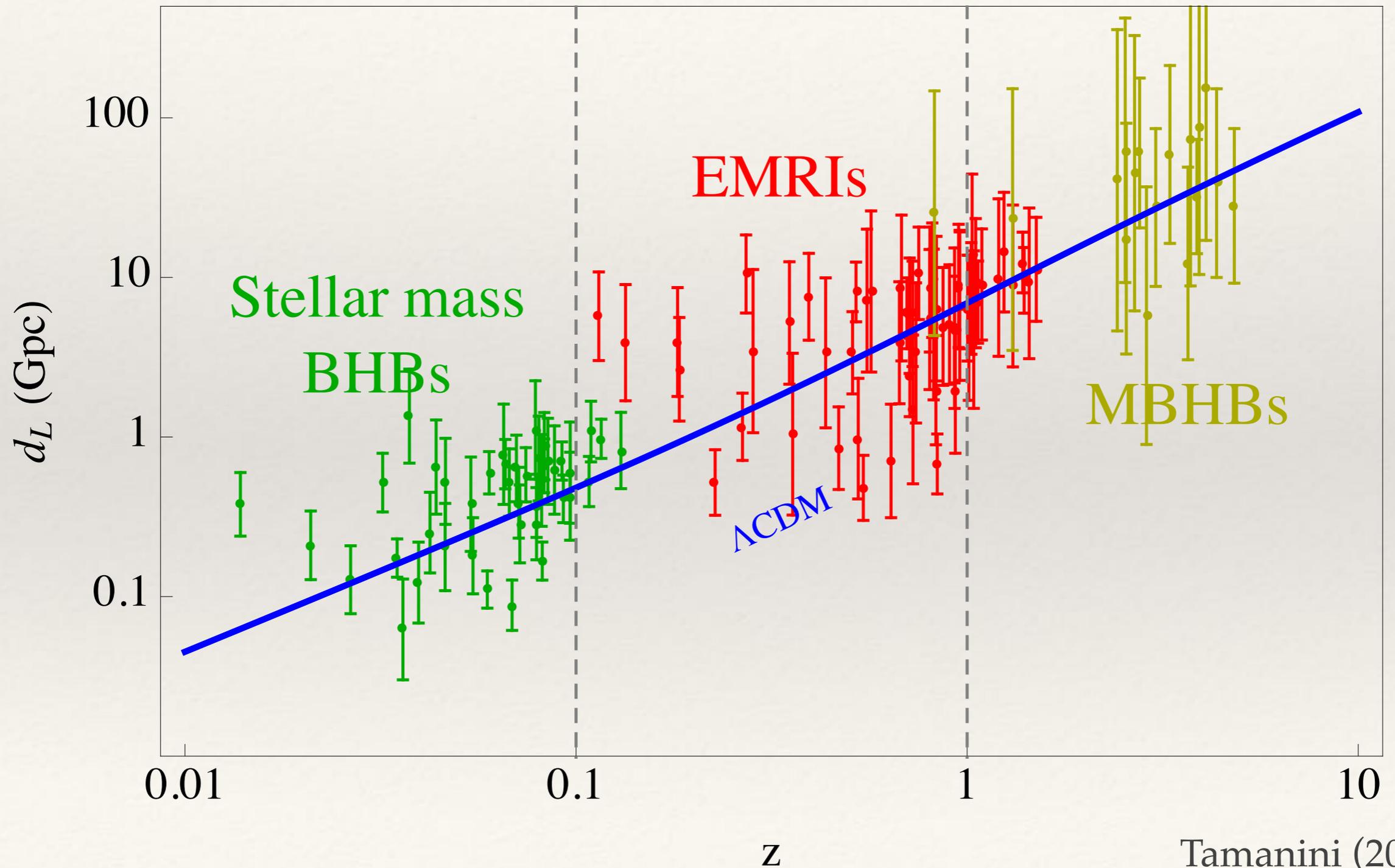
# Science: cosmography

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- ❖ Even without a counterpart, can estimate cosmological parameters statistically from GW observations.
- ❖ Use LISA observations of EMRIs to measure the Hubble constant (McLeod & Hogan 08)
  - Let every galaxy in the LISA error box “vote” on the Hubble constant.
  - If  $\sim 20$  EMRI events are detected at  $z < 0.5$ , classic LISA would determine the Hubble constant to  $\sim 1\%$ . Probably  $\sim 2\%$  with 20 events and new baseline.
  - LISA expected to observe a few tens of EMRIs per year, all at  $z < 0.5$ .
- ❖ Same analysis for SMBH mergers suggests classic LISA (5Gm, 6-link) could improve constraints on equation of state of dark energy by a factor of  $\sim 2-8$  (Babak et al. 2011). May not be possible with shorter configurations.

# Future $H_0$ measurements: LISA

Example of possible LISA cosmological data



# Future $H_0$ measurements: LISA

- ❖ Posteriors (best case)

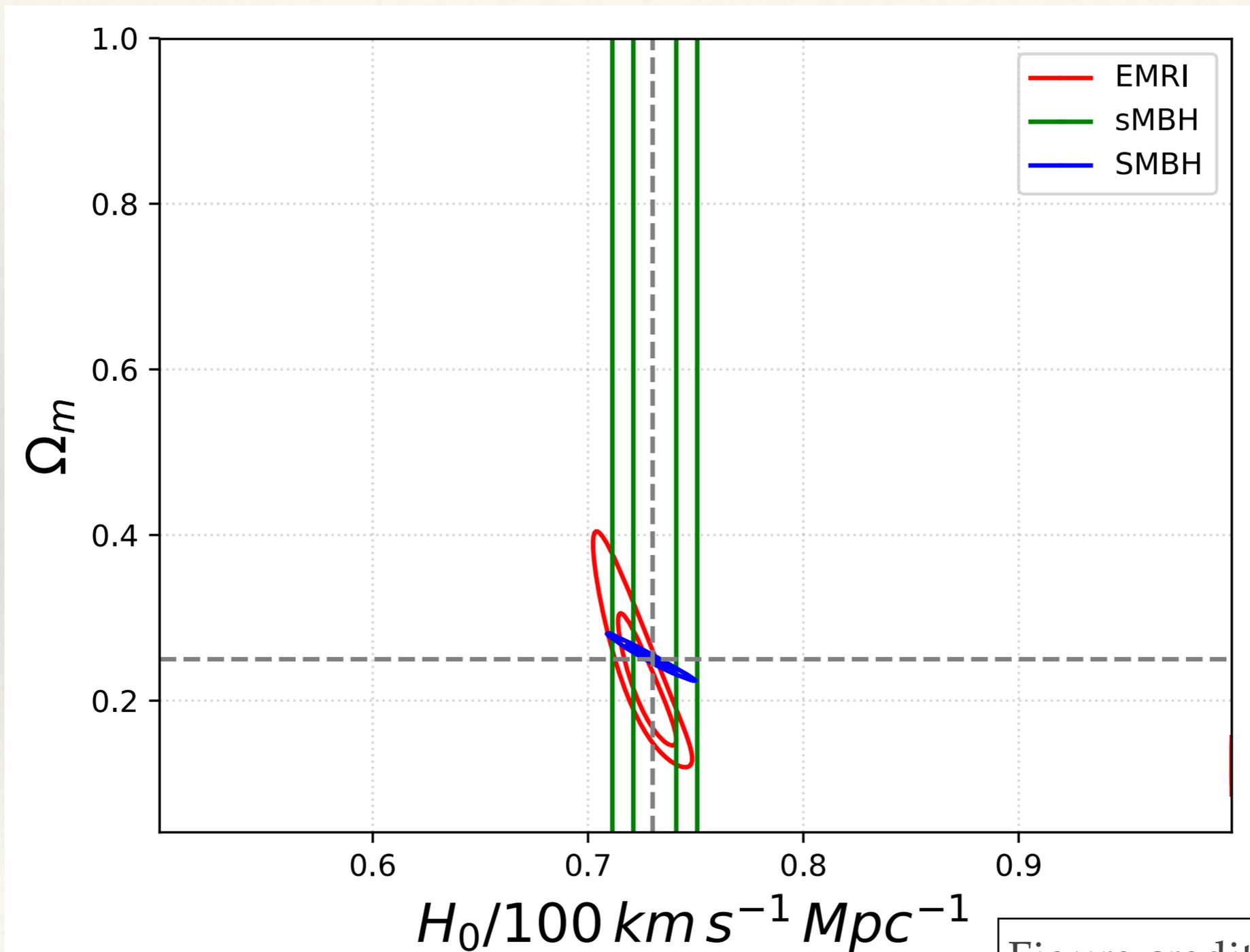


Figure credit: Nicola Tamanini

# Future $H_0$ measurements: LISA

- ❖ Posteriors (typical case)

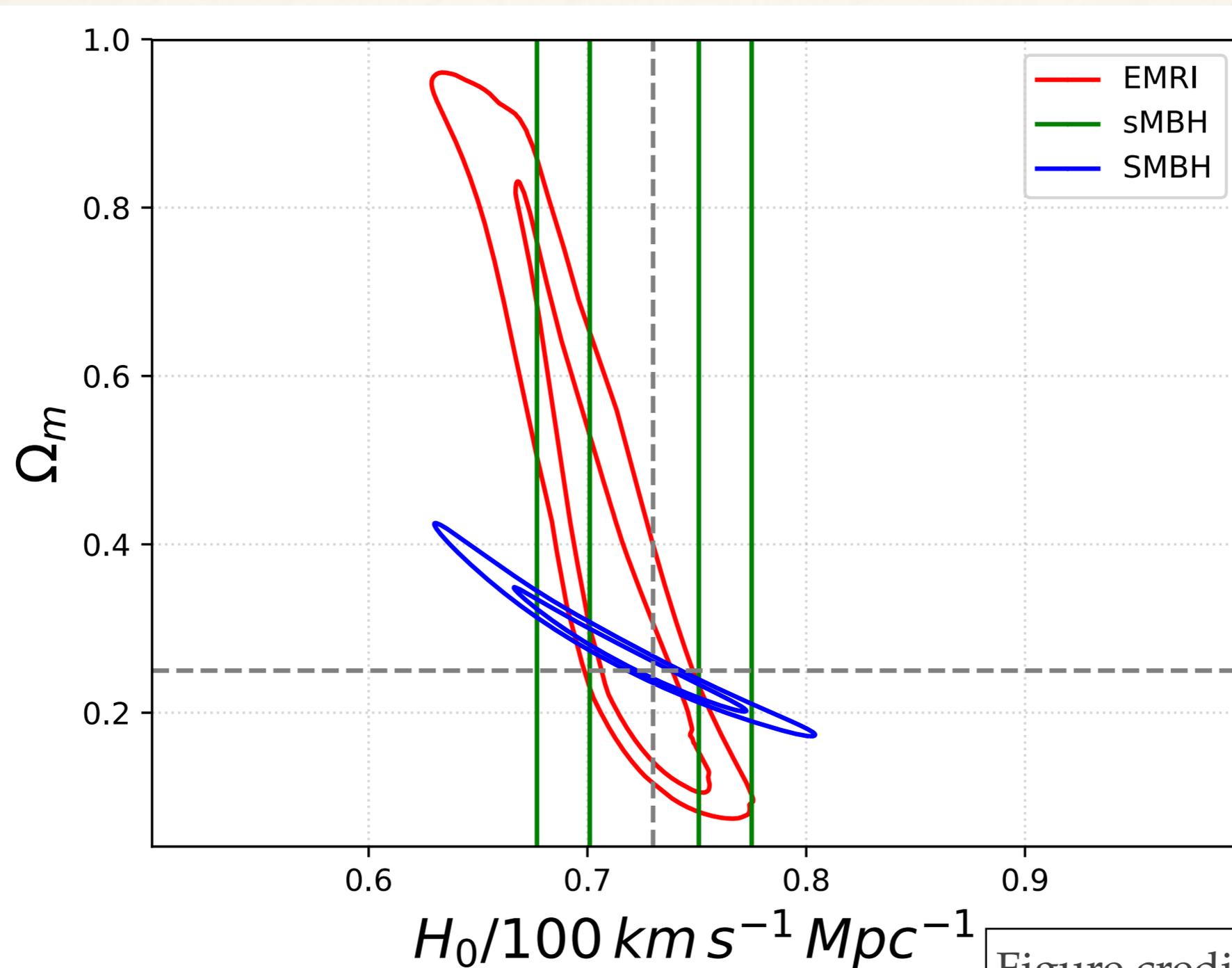
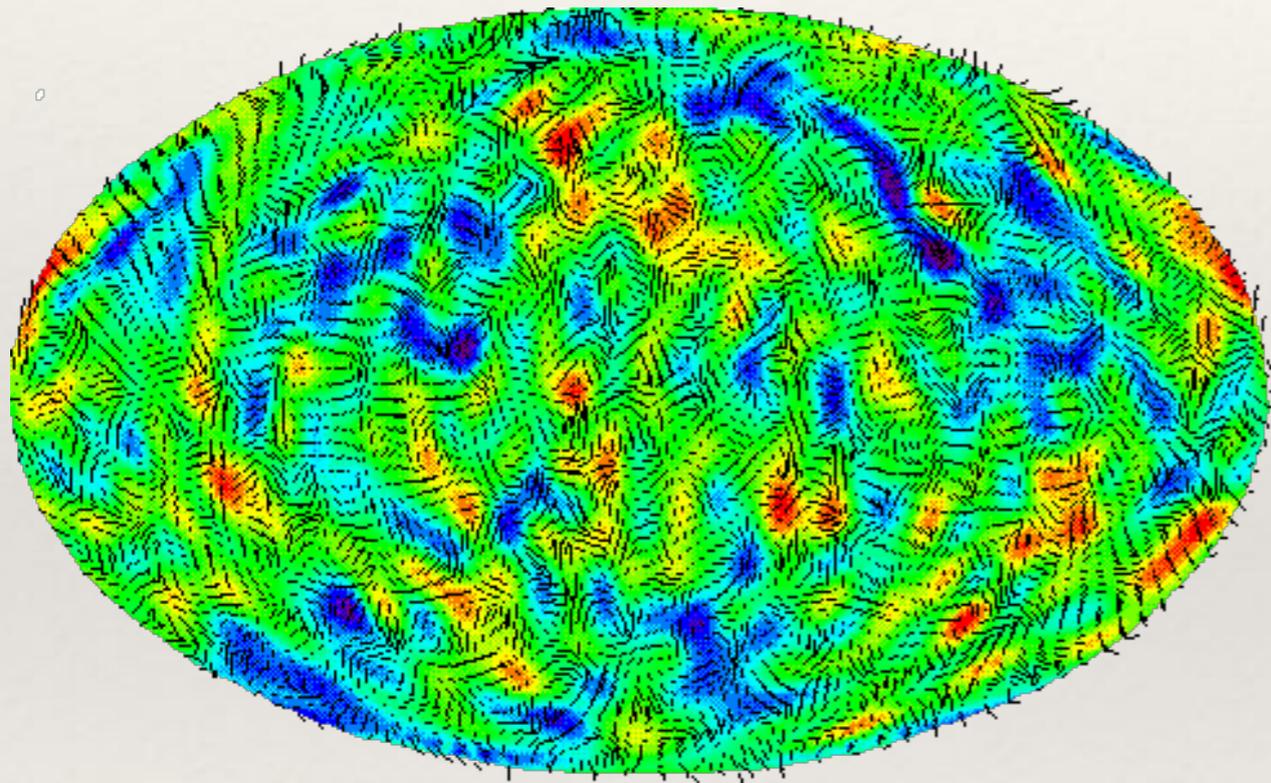


Figure credit: Nicola Tamanini

# Fundamental physics with PTA observations

# Gravitational wave backgrounds



- ❖ Primary source for PTAs is the astrophysical background of gravitational waves generated by supermassive black hole mergers.
- ❖ Natural to ask what (gravitational) information is encoded in such a background.
- ❖ GW background is a transverse-traceless tensor on the sky

$$h_{ab}^{\text{TT}} = \begin{pmatrix} h_{+} & h_{\times} \\ h_{\times} & -h_{+} \end{pmatrix}$$

- ❖ Analogous to polarisation of the cosmic microwave background.

# PTA response

- ❖ PTAs measure redshifts in pulsars

$$z(t, \hat{k}) \equiv \frac{\Delta v(t)}{\nu_0} = \frac{1}{2} \frac{\hat{u}^a \hat{u}^b}{1 + \hat{k} \cdot \hat{u}} \Delta h_{ab}(t, \hat{k})$$

- ❖  $\Delta h_{ab}$  is the difference in the metric perturbation between the Earth and the pulsar and can be written  $\Delta h_{ab} = \sum \Delta h_A e_{ab}^A$

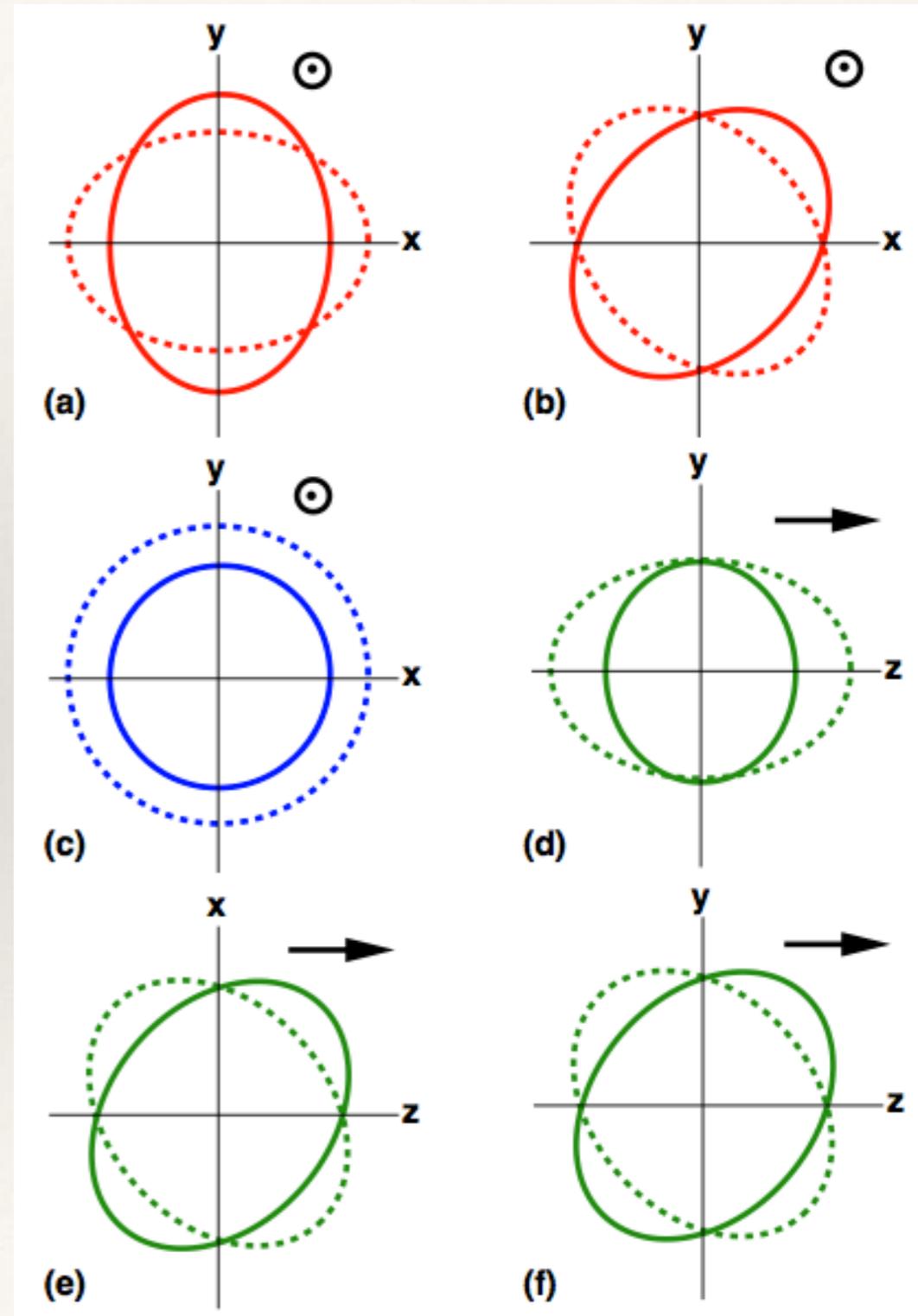
- ❖ There are 2 polarisation states in GR

$$e_{ij}^+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e_{ij}^\times = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- ❖ 4 additional states can exist in metric theories

$$e_{ij}^B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad e_{ij}^L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$e_{ij}^X = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad e_{ij}^Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



# PTA response

- ❖ The redshift induced by a GW background can be written as

$$\begin{aligned}
 z(t) &= \int_{-\infty}^{\infty} df \int_{S^2} d^2\Omega_{\hat{k}} \frac{1}{2} \frac{\hat{u}^a \hat{u}^b}{1 + \hat{k} \cdot \hat{u}} h_{ab}(f, \hat{k}) \left[ 1 - e^{-i2\pi f L(1 + \hat{k} \cdot \hat{u})/c} \right] e^{i2\pi f(t - \hat{k} \cdot \vec{x}/c)} \\
 &= \int_{-\infty}^{\infty} df \sum_{(lm)} \sum_P R_{(lm)}^P(f) a_{(lm)}^P(f) e^{i2\pi f t}
 \end{aligned}$$

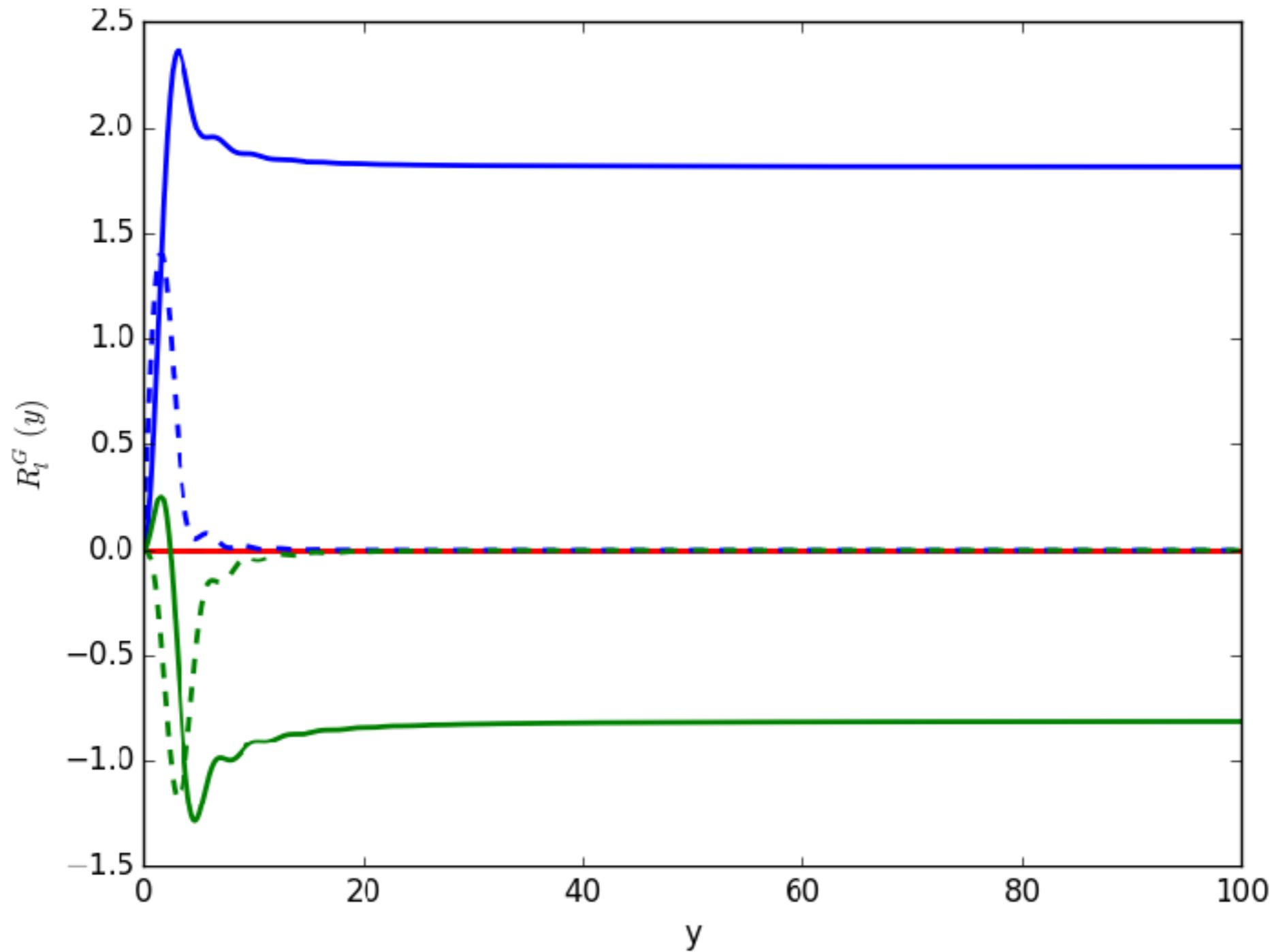
- ❖ where the response functions for individual modes are given by

$$R_{(lm)}^P(f) = \int_{S^2} d^2\Omega_{\hat{k}} \frac{1}{2} \frac{\hat{u}^a \hat{u}^b}{1 + \hat{k} \cdot \hat{u}} Y_{(lm)ab}^P(\hat{k}) e^{-i2\pi f \hat{k} \cdot \vec{x}/c} \left[ 1 - e^{-i2\pi f L(1 + \hat{k} \cdot \hat{u})/c} \right]$$

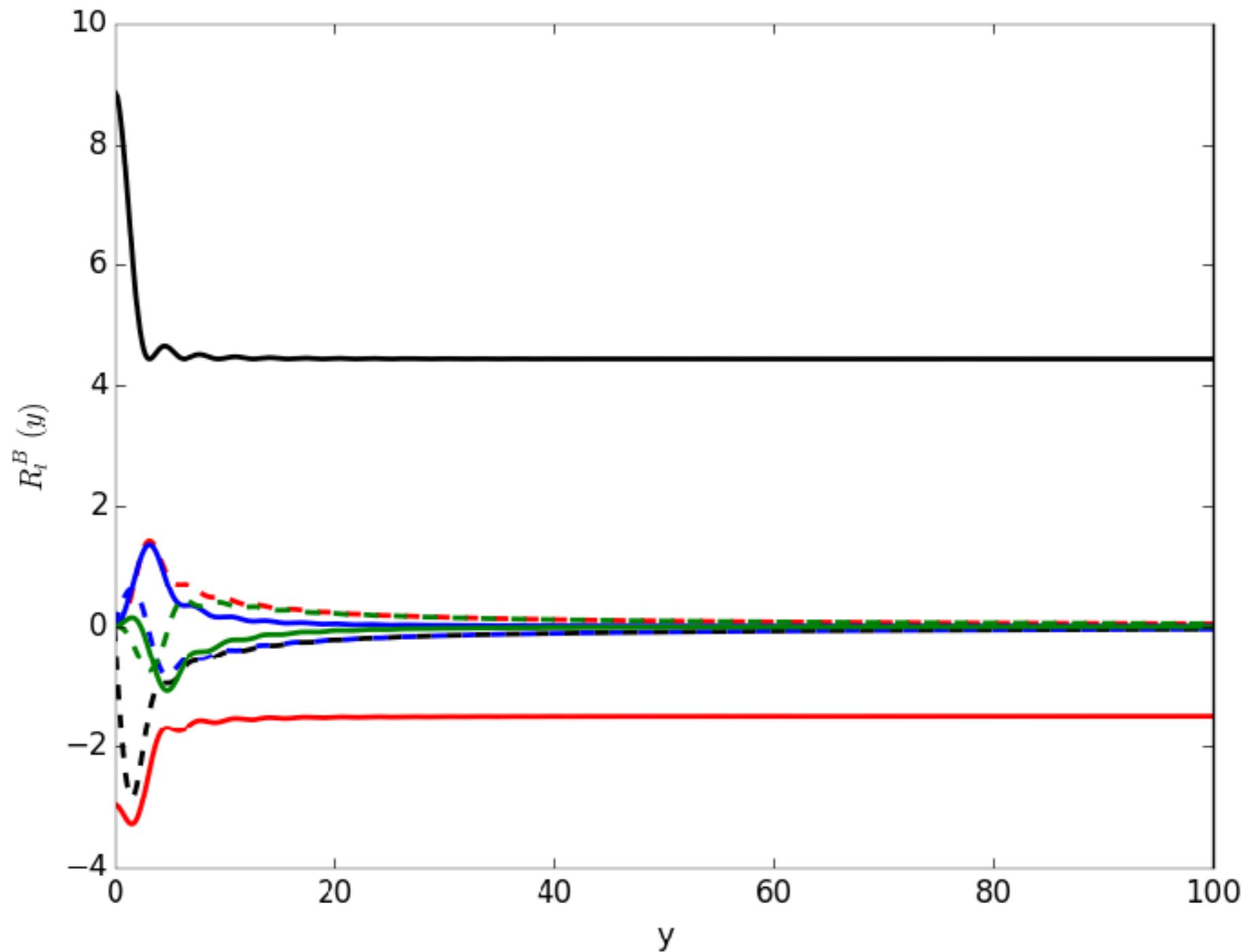
- ❖ Writing  $y \equiv 2\pi f L/c$  and working in the frame of the pulsar we find  $R_{I(lm)}^P(f) = Y_{lm}(\hat{u}_I) \mathcal{R}_l^P(y_I)$ . The total response takes the form

$$\begin{aligned}
 R_I(f) &= \sum_{lm} \left( a_{(lm)}^B(f) \mathcal{R}_l^B(y_I) + a_{(lm)}^L(f) \mathcal{R}_l^L(y_I) \right. \\
 &\quad \left. + a_{(lm)}^{VG}(f) \mathcal{R}_l^{VG}(y_I) + a_{(lm)}^G(f) \mathcal{R}_l^G(y_I) \right) Y_{lm}(\hat{u}_I)
 \end{aligned}$$

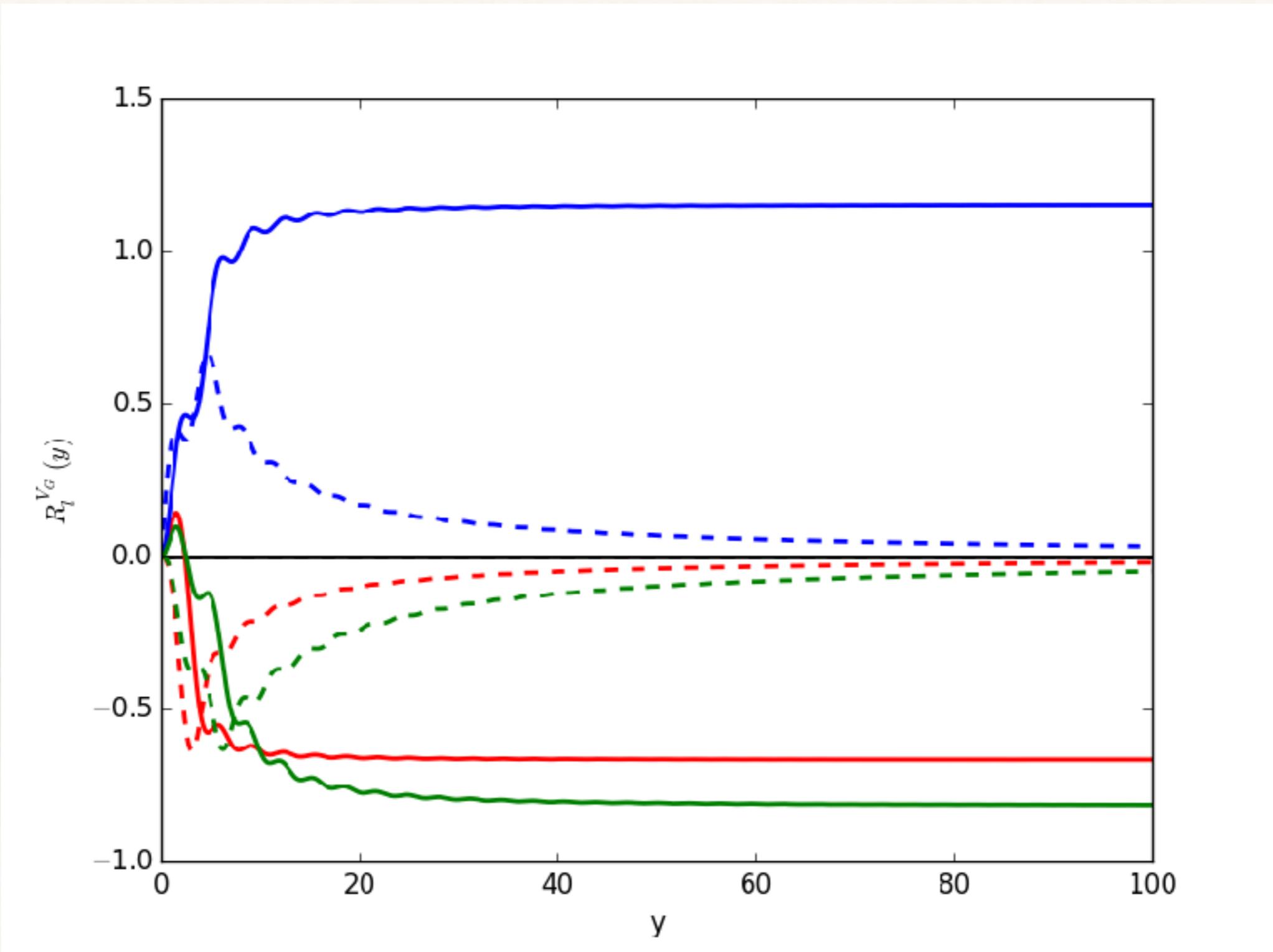
# PTA response to tensor modes



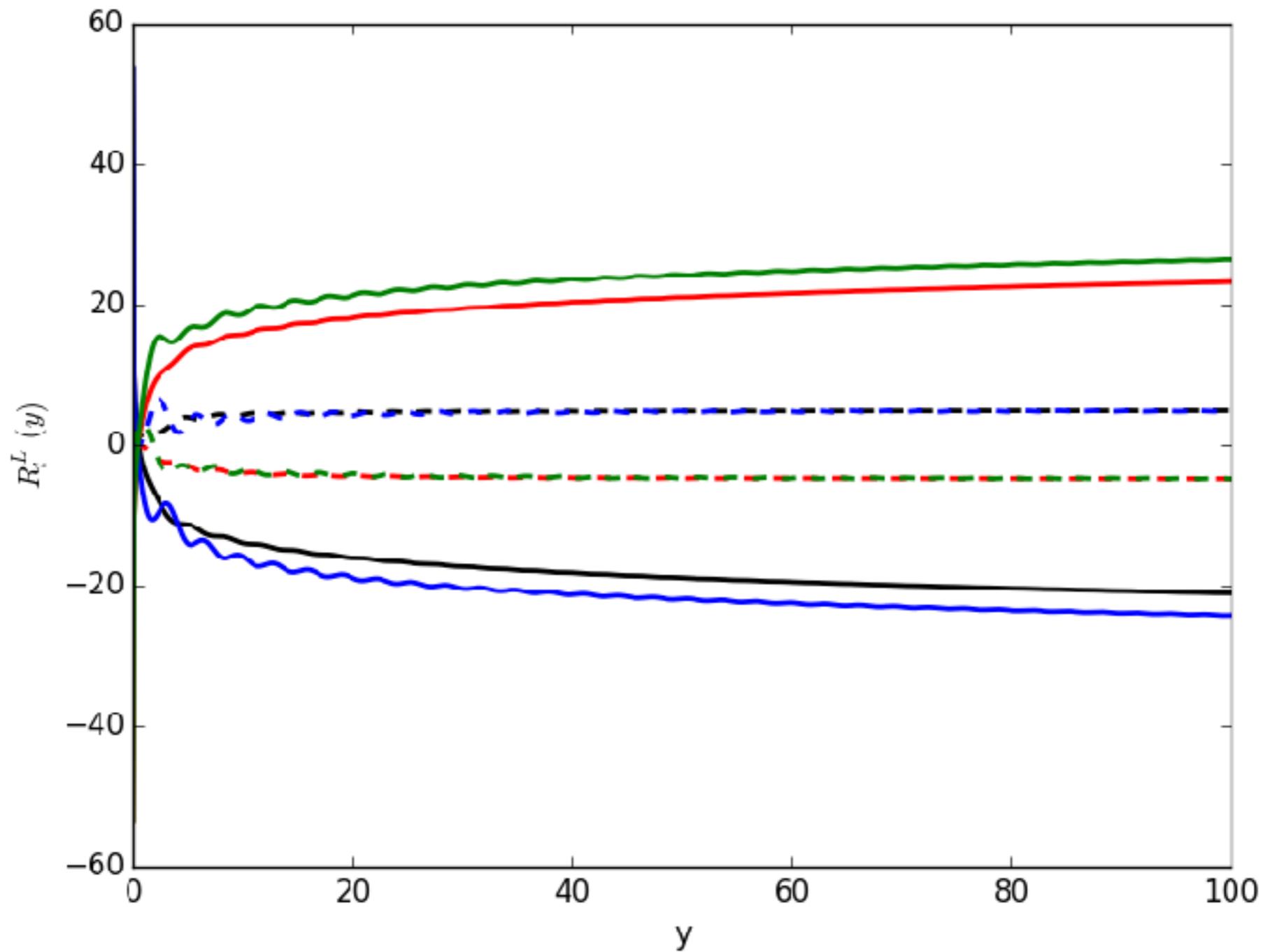
# PTA response to breathing modes



# PTA response to vector modes



# PTA response to scalar-longitudinal modes



# PTA background mapping

- ❖ If we have pulsars all over the sky, can decompose “pulsar response” map into spherical harmonic basis. Coefficients are linear combinations of different polarisations.
- ❖ No confusion between B and G modes due to range of  $l$ . Confusion with  $V^G$  and L possible unless have pulsars at several distances. But - can only measure  $N_p$  modes, i.e., equal to number of pulsars.

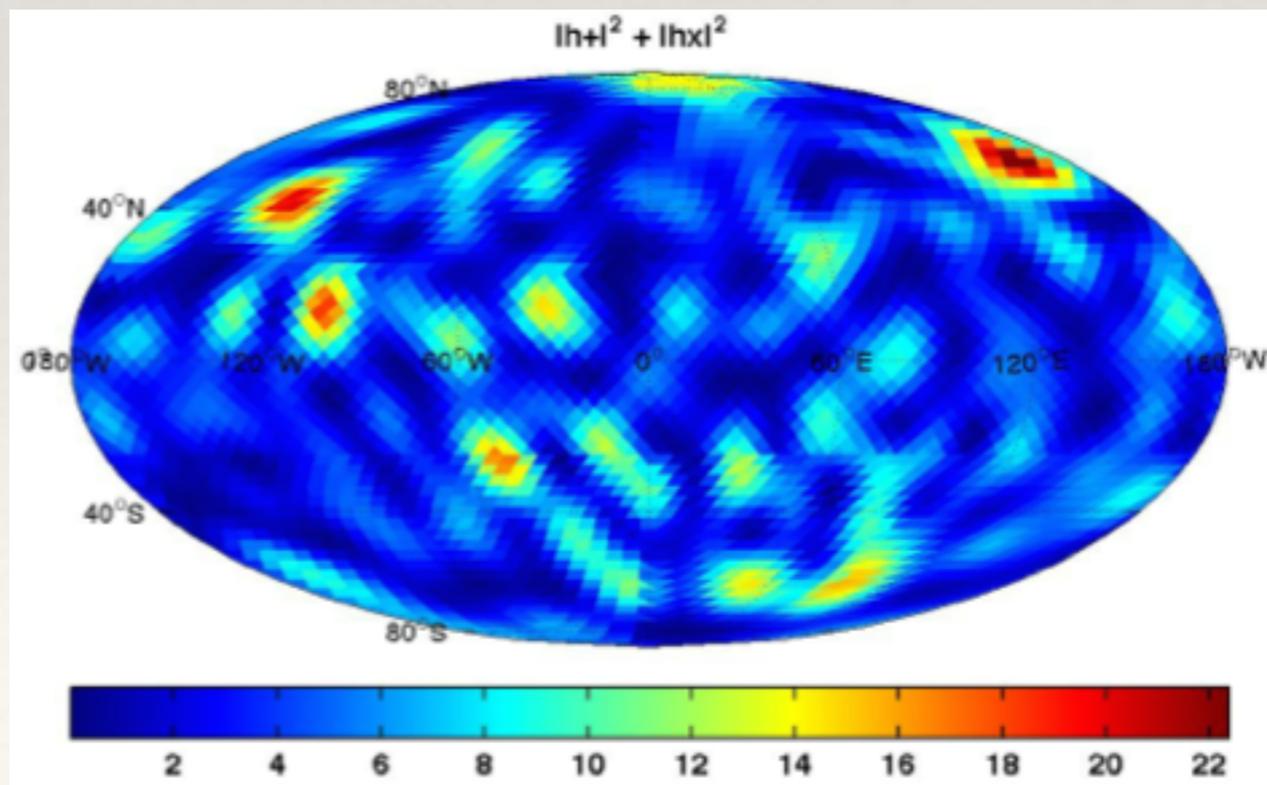
	$(l, m)$ mode								
	(0, 0)	(1, -1)	(1, 0)	(1, 1)	(2, -2)	(2, -1)	(2, 0)	(2, 1)	(2, 2)
$G$ : transverse-tensor (gradient)	—	—	—	—	0.44	0.38	0.32	0.38	0.44
$G$ : transverse-tensor (gradient)	—	—	—	—	0.49	0.39	0.37	0.39	0.49
$B$ : scalar-transverse (breathing)	0.16	0.53	0.46	0.53	—	—	—	—	—
$G$ : transverse-tensor (gradient)	—	—	—	—	16.2	10.5	11.4	10.5	16.2
$B$ : scalar-transverse (breathing)	4.36	16.1	14.1	16.1	—	—	—	—	—
$L$ : scalar-longitudinal	0.71	0.96	0.84	0.96	1.21	0.78	0.86	0.78	1.21
$G$ : transverse-tensor (gradient)	—	—	—	—	1.4e5	5.4e4	8.0e4	5.4e4	1.4e5
$B$ : scalar-transverse (breathing)	18.4	9.4e4	6.2e4	9.4e4	—	—	—	—	—
$L$ : scalar-longitudinal	3.08	11.5	8.68	11.5	20.9	7.51	11.9	7.52	20.9
$V_G$ : vector-longitudinal (gradient)	—	6.6e4	4.4e4	6.6e4	7.0e4	2.7e4	4.0e4	2.7e4	7.0e4

# Implications

- Individual modes of the background represent GW emission that is correlated between different points on the sky.

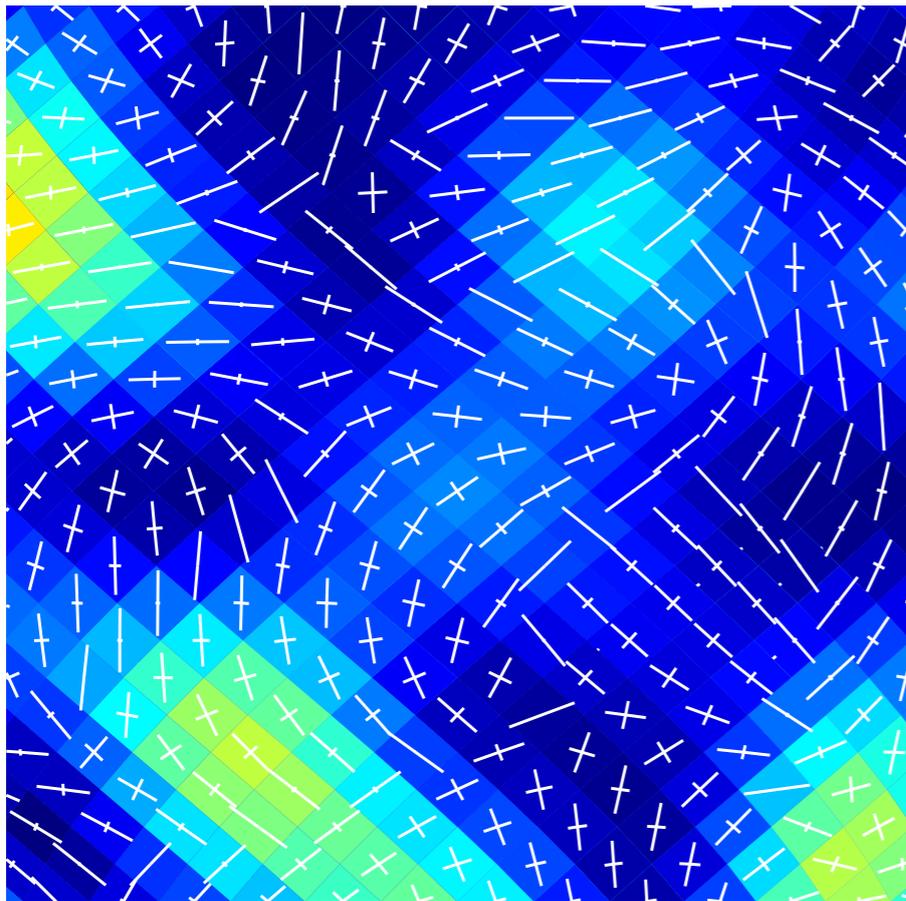
$$\langle h_+(f, \hat{k}) h_+^*(f', \hat{k}') \rangle_k = \frac{1}{2} \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} (N_l)^2 [C_l^{GG}(f) G_{l2}^+(\cos \theta) + C_l^{CC}(f) G_{l2}^-(\cos \theta)] \delta(f - f')$$

- No well-established physical mechanism to create such correlations - discovery of a correlated background would be a profound result.
- Mild anisotropy expected in power of GW background - could be consistent with either uncorrelated or correlated background.



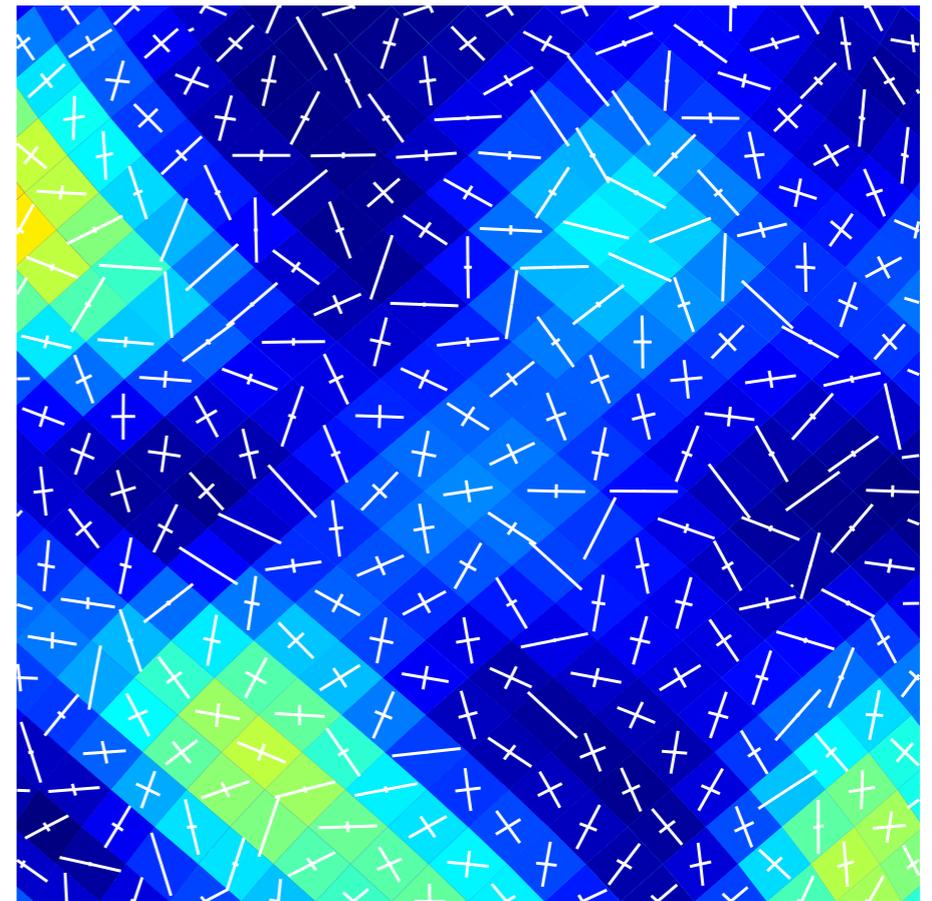
# PTA background measurements

- Polarization of background can distinguish correlated and uncorrelated origin.



2 4 6 8 10 12 14 16 18 20 22

Correlated

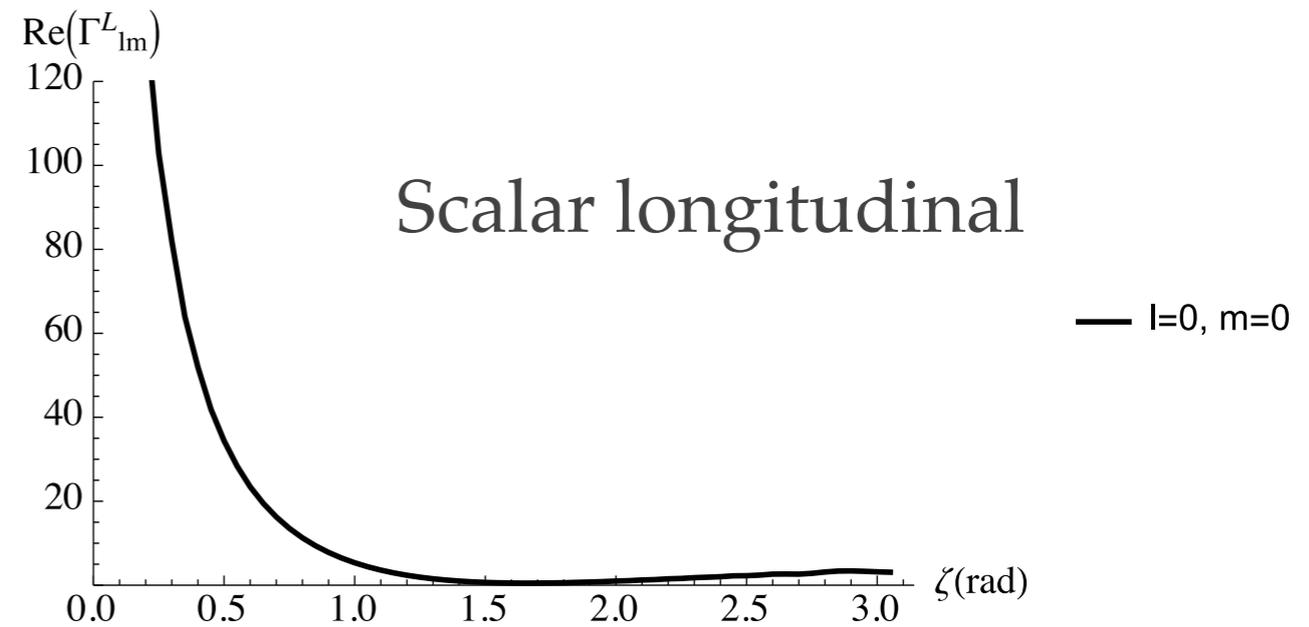
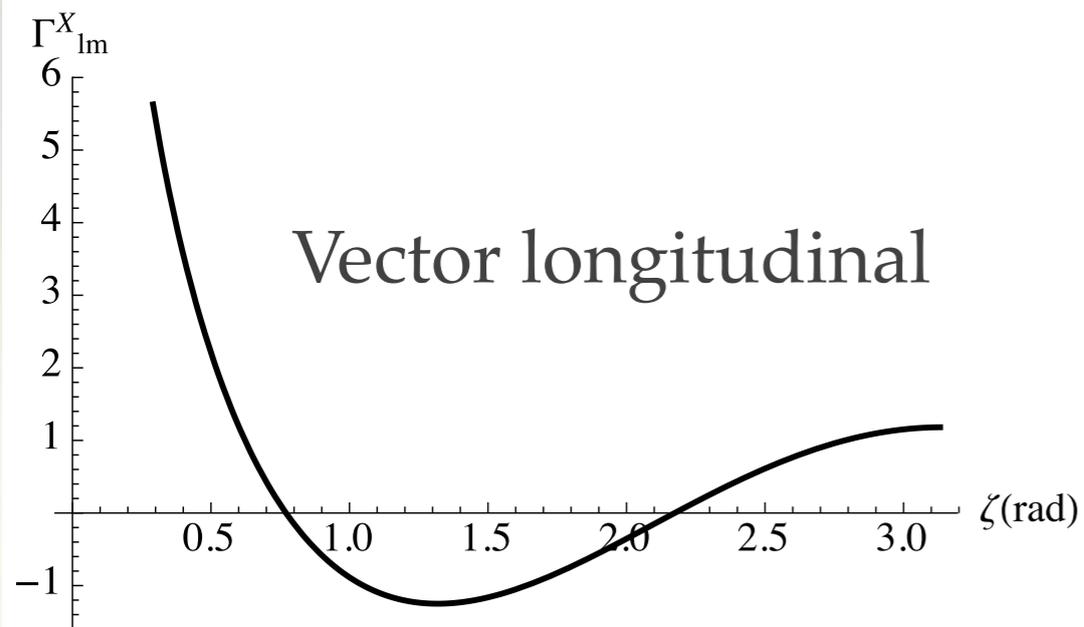
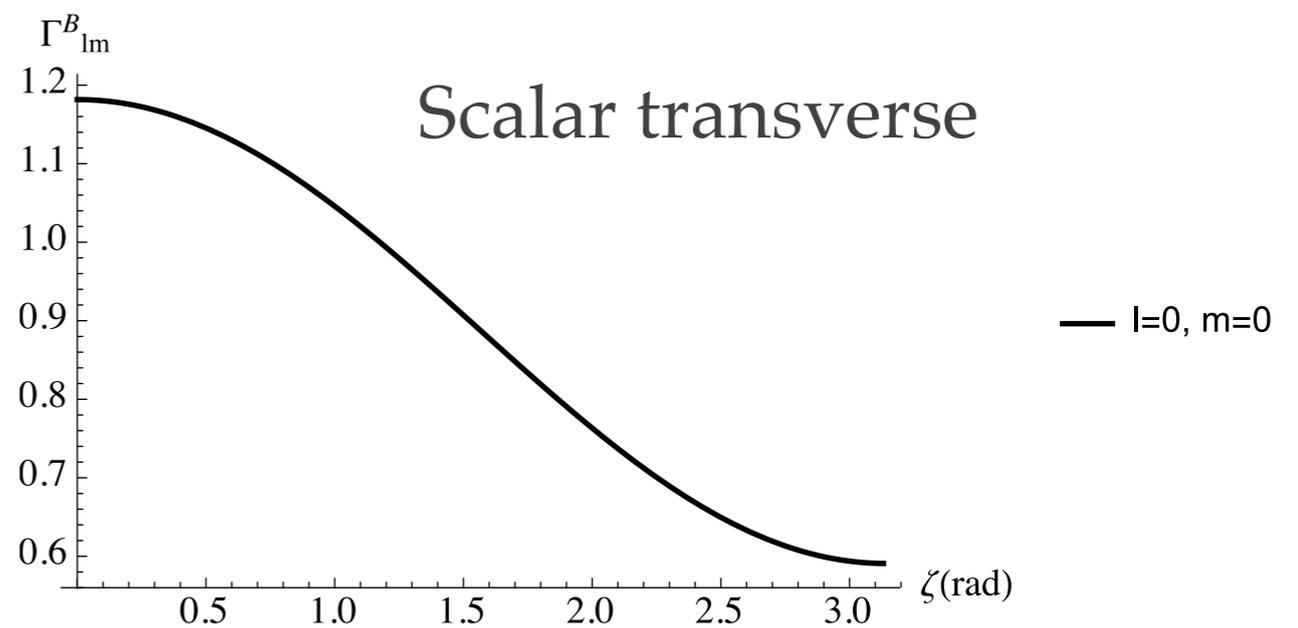
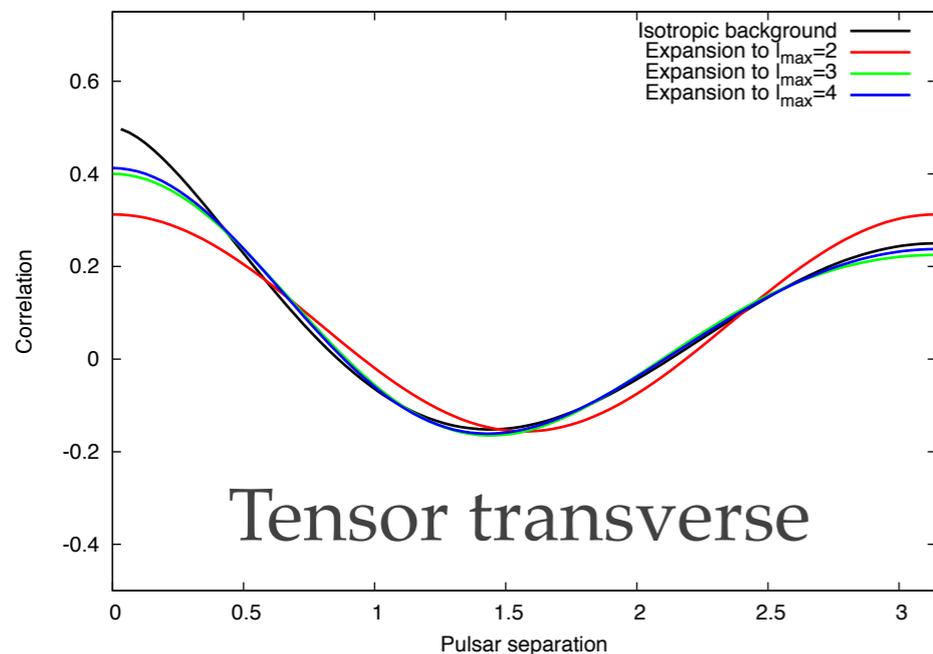


2 4 6 8 10 12 14 16 18 20 22

Uncorrelated

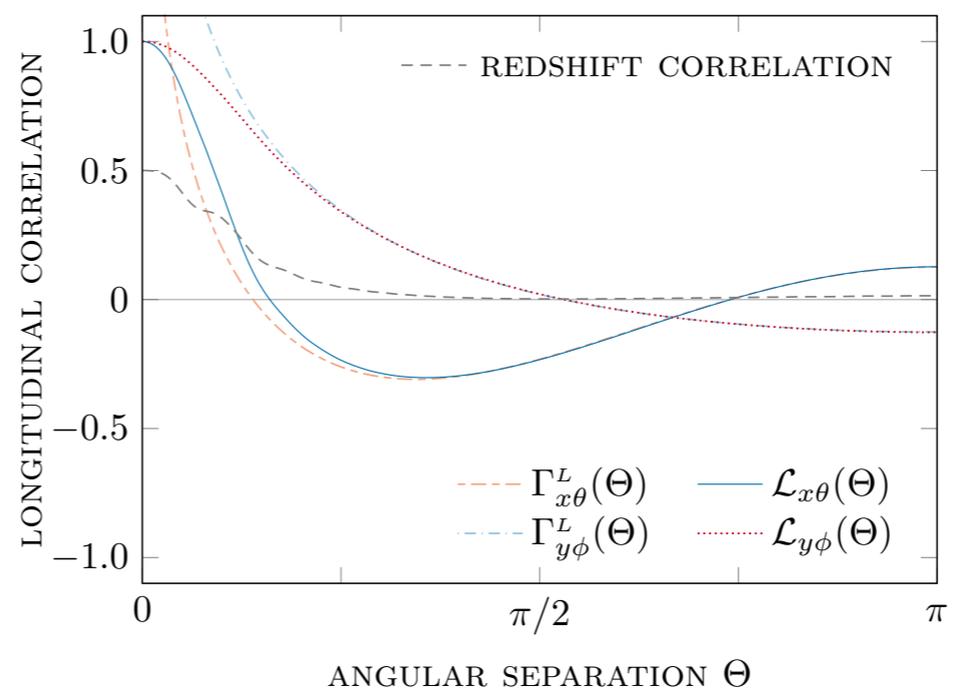
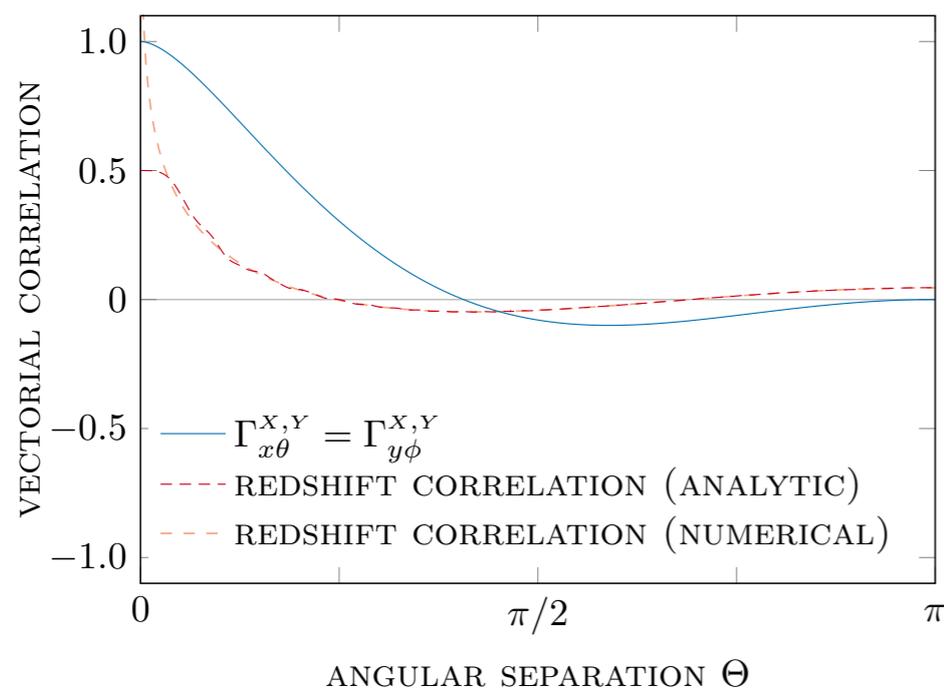
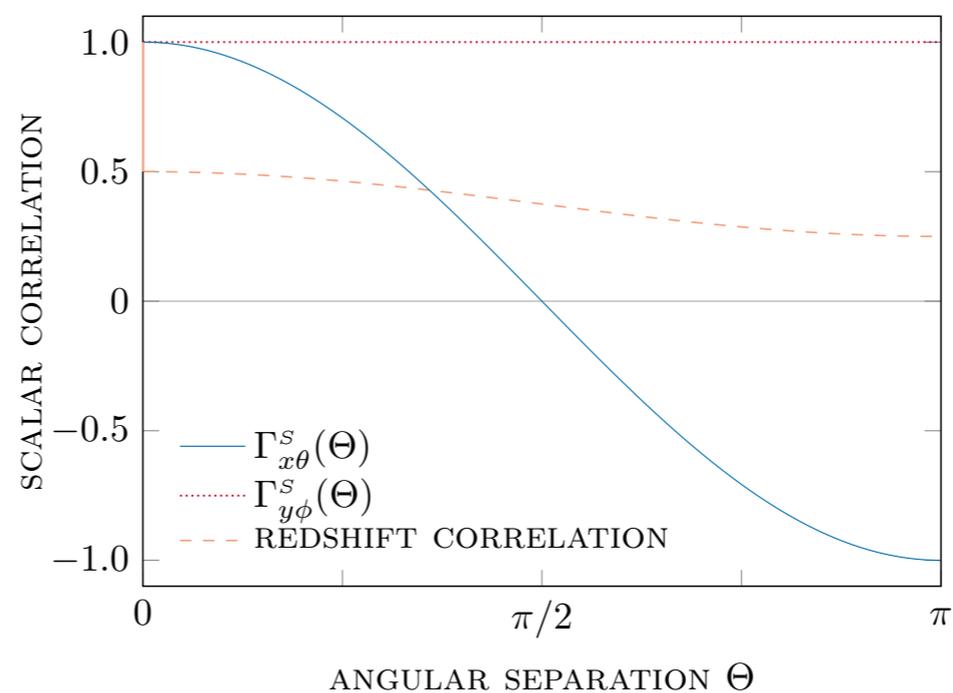
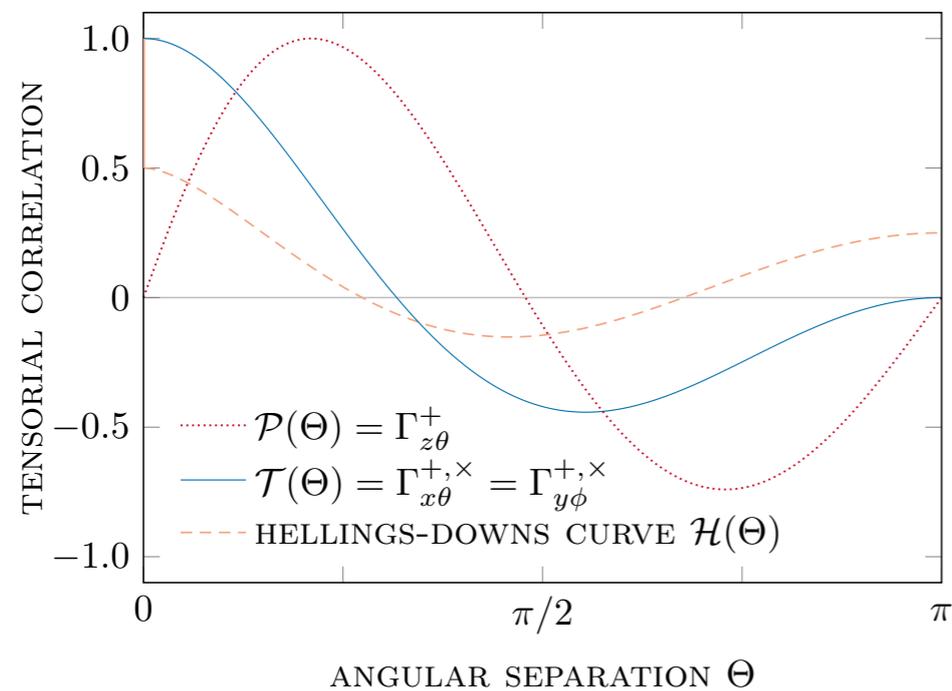
# Overlap reduction function: PTAs

- ❖ If we assume background is isotropic there are fewer parameters to fit.



# Overlap reduction function: astrometry

- ❖ Astrometric measurements (Gaia) help break degeneracies.



Mihaylov, JG  
et al. 2018

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# Massive Gravitons

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- ❖ Setting

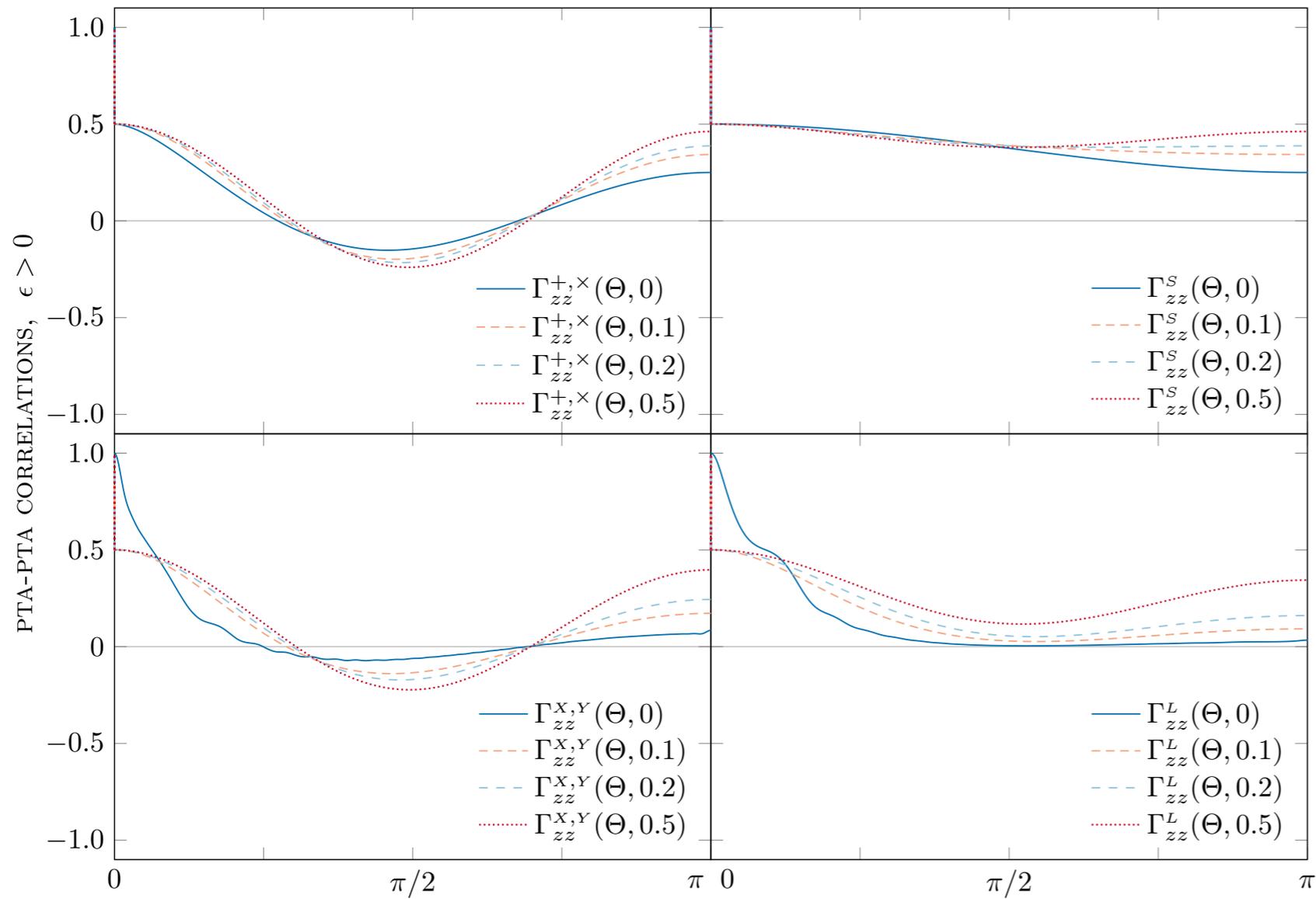
$$k_\mu = -\omega(1, (1 - \epsilon) \mathbf{q})$$

- ❖ in the pulsar response

$$z(t, \hat{k}) \equiv \frac{\Delta v(t)}{\nu_0} = \frac{1}{2} \frac{\hat{u}^a \hat{u}^b}{1 + \hat{k} \cdot \hat{u}} \Delta h_{ab}(t, \hat{k})$$

- ❖ corresponds to a change in propagation speed for the graviton. This modifies the correlation between pairs of pulsars on the sky and is therefore in principle detectable.

# Massive Gravitons



Mihaylov, JG et al. (2019)

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# Summary

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- ❖ LISA will launch in the early 2030s and will open a new window on the gravitational wave Universe, at mHz frequencies.
- ❖ LISA is expected to observe GWs from massive BH mergers, extreme-mass-ratio inspirals and (perhaps) cosmological stochastic backgrounds.
- ❖ LISA observations have massive potential for gravitational physics, including tests of the no-hair property of BHs, tests of GW propagation and polarisation, constraining dark matter candidates and constraining a variety of modified theories of gravity.
- ❖ Pulsar timing arrays are expected to make the first observations of the nHz GW background in the next 5-10 years.
- ❖ PTAs can in principle detect polarisation properties of the stochastic GW background which would be indicative of new physics.