Cosmological Bootstrap, With and Without Spin

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A central challenge of modern cosmology is to construct a consistent history of the universe that explains these correlations:

- 10 billion yrs
- 380,000 yrs
- $10^{-34}$ sec

Primordial Archeology
Goal

A menu of templates of non-Gaussianity, compatible with general principles:

1. Symmetry
2. Locality
3. Unitarity

This would be a theory of shapes. Model would tell sizes of those shapes.
Strategy: Time Without Time

time

end of inflation
What’s in the Menu?

There is a very small* menu of histories!
New templates of non-gaussianity.
Time evolution is emergent, and constraining!

*under the assumption of weak coupling, very softly broken de Sitter symmetry ($f_{\text{NL}} \sim 1$)
Inflationary Bispectra

Slow-roll inflationary bispectra can be obtained from de Sitter four-point functions by a soft limit:

This motivates us to study four-point functions in de Sitter as a building block.
Weight-Shifting

All the dynamics contained in a single diagram! Other possibilities can be kinematically bootstrapped. We can even change the spin of external legs.

“Weight”: Mass and Spin of the particles
Weight-Shifting, with a Spin

Spinning correlation functions can be compatible with symmetry, but incompatible with locality. Weakly coupled constraints on quantum gravity!
The Cosmological Bootstrap

Arkani-Hamed, Baumann, Lee, GLP, 2018
I. Symmetries

3 Translations
3 Rotations
1 Dilation
3 dS Boosts

\[
\begin{align*}
\text{Easy to diagonalize} \\
\text{Conformal Ward identities} \\
\text{(depend on mass and spin)}
\end{align*}
\]
II. Kinematics

\[ \vec{k}_1 \quad \vec{k}_2 \quad \vec{k}_3 \quad \vec{k}_4 \]

\[ m^2 = 2H^2 \]

\[ s = |\vec{k}_1 + \vec{k}_2|, \quad u = \frac{s}{|\vec{k}_1| + |\vec{k}_2|}, \quad v = \frac{s}{|\vec{k}_3| + |\vec{k}_4|} \]

Translations, Rotations, Dilation:

\[ \langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \delta^3 \left( \sum_i \vec{k}_i \right) \frac{1}{s} \hat{F} (u, v) \]
II. Kinematics

\[ \dot{\mathbf{k}}_1 \quad \dot{\mathbf{k}}_2 \quad \dot{\mathbf{k}}_3 \quad \dot{\mathbf{k}}_4 \]

\[ m^2 = 2H^2 \]

Conformal Ward identity:

\[ (\Delta_u - \Delta_v) \hat{F} = 0 \]

No dynamics yet.

\[ \Delta_u \] is a second-order, hypergeometric differential operator.
III. Dynamics

\[
[\Delta_u + (M/H)^2] F_E(u, v) = F_C(u, v)
\]

\[
[\Delta_v + (M/H)^2] F_E(u, v) = F_C(u, v)
\]

Each operator encodes consistent time evolution of the two separate vertices. Contact interactions can be easily classified.

Arkani-Hamed, Baumann, Lee, GLP, 2018
IV. Boundary Conditions

1. Smooth in “folded” limit

2. Factorizes in “early times” limit
Four-Point Function

Collapsed (particle production)

Equilateral (EFT)

M

Four-point function

Momentum ratio

particle production

EFT

Arkani-Hamed, Maldacena, 2015
Lee, Baumann, GLP, 2016
Baumann, Goon, Lee, GLP, 2017
Arkani-Hamed, Baumann, Lee, GLP, 2018
Examples

$M, S$

$M = 0, S = 2$

Maldacena, 2002
Creminelli, 2003

$M = 0, S = 2$

Seery, Lidsey, Sloth, 2006
Seery, Sloth, Vernizzi, 2008
Arroja, Koyama, 2008
Tensor Correlators

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.
Gravitational Waves

Graviton is other guaranteed degree of freedom

Very little is known beyond three points

In flat space, consistent S-matrix of gravitons is very constrained

Tensors are hard to compute using Lagrangians

Maldacena, GLP, 2011
Strategy

1. A massless spinning field must satisfy constraints to propagate right number of degrees of freedom (Ward-Takahashi identity)

2. Weight-Shift to right quantum numbers

3. Write WT Identity for correlation of interest

4. Find particular solution of the WI

5. All homogeneous solutions obtained by Weight-Shifting into conserved tensors.

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.
Weight Shifting Tensors

Other than having proper quantum numbers, light spinning fields must propagate correct number of degrees of freedom:

\[ A_\mu \rightarrow J_i, \quad k_i J_i = 0 \]
\[ g_{\mu\nu} \rightarrow T_{ij}, \quad T_{ii} = 0, \quad k_i T_{ij} = 0 \]

We can build WS operators that take scalars to tensors:

\[ = \mathcal{W}^T \]

However…

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.
Conservation is Not Enough

Sometimes, demanding strict conservation is too strong. 
Try Ward-Takahashi identity instead

$$k_i \langle J_i OOO \rangle \sim \langle OOO \rangle$$

In EFT, these happen to be the most interesting theories!

If we can’t solve WT identities - theory is ruled out.

Hinterbichler, Hui, Khoury, 2012
Goldberger, Hui, Nicolis, 2013
GLP, 2013
Berezhiani, Khoury, 2013

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.
Weight Shifting Tensors

Once we solved WT identity, all we can do is add “homogeneous solutions” with identically conserved currents. In that case, Weight-Shifting is easier.

Example: Weight-Shifting to a conserved current (Photon)

\[ \mathcal{O} \to A_i = W_1 \mathcal{O} \]
\[ A_i \to J_i = \epsilon_{ijk} \partial_j A_k \]
Three-Point Example

\[ \langle JOO \rangle = \]

\[ \mathcal{W}^J = 0 \quad \text{with} \quad k_i \langle J_i O_2 O_3 \rangle = 0 \]

\[ \mathcal{O} \neq 0 \quad \text{, but now} \]

\[ k_i \langle J_i O_2 O_3 \rangle = -e_2 \langle O_{1+2} O_3 \rangle - e_3 \langle O_2 O_{1+3} \rangle \]

Solution only exists if \( e_2 + e_3 = 0 \)!
Four-Point Example

\[ \langle J_1 O_2^+ O_3^- O_4^0 \rangle \]

Only combination of two channels will satisfy Ward-Takahashi identity:

\[ k_1^i \langle J_1^i O_2^+ O_3^- O_4^0 \rangle = +\langle O_{1+2}^+ O_3^- O_4^0 \rangle - \langle O_2^+ O_{1+3}^- O_4^0 \rangle \]

We can apply WS to obtain each diagram, but only sum is physical!
Four-Point Example

\[ \langle J_1 O_2^+ O_3^- O_4^0 \rangle \]

Now that WT identity is solved, other possibilities are obtained through identically conserved Weight-Shifting operator

\[ = \mathcal{W}^J \]

Arkani-Hamed, Baumann, Duaso, Joyce, Lee, GLP, in prep.
Future
We can classify all three-point functions. Can we glue them together to get a consistent four-point function?
The four-point “diagrams” are more on-shell than Feynman diagrams. They satisfy non-trivial factorization rules.

Our approach is a hybrid of S-matrix gauge-invariance and on-shell construction.

Benincasa, Cachazo, 2007
McGady, Rodina, 2013
Arkani-Hamed, Rodina, Trnka, 2017
Take Away

Bootstrap eliminates Lagrangians and fields
Time evolution is encoded in the correlator

There is only a small menu of possibilities!

Much left to do!

A necessity towards better understanding of inflation and the big bang!
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Thank You For Your Attention!

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