Cosmological Bootstrap,
With and Without Spin

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In collaboration with:

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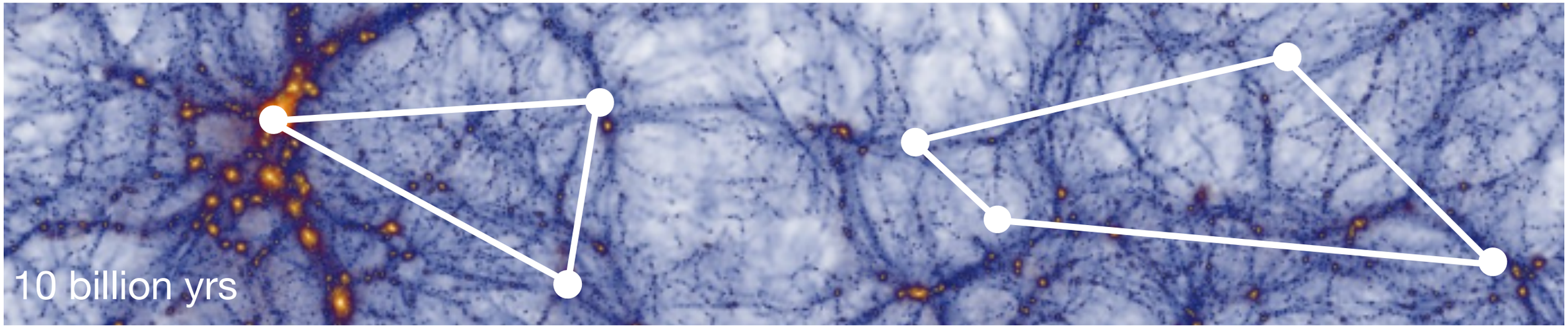
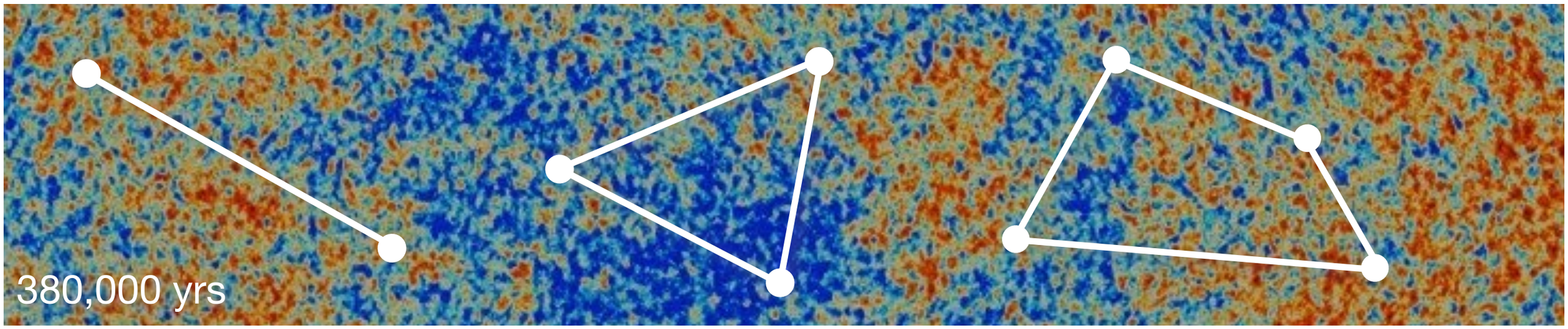
+ Carlos Duaso Pueyo, Austin Joyce (to appear)

University of Amsterdam

Primordial Archeology





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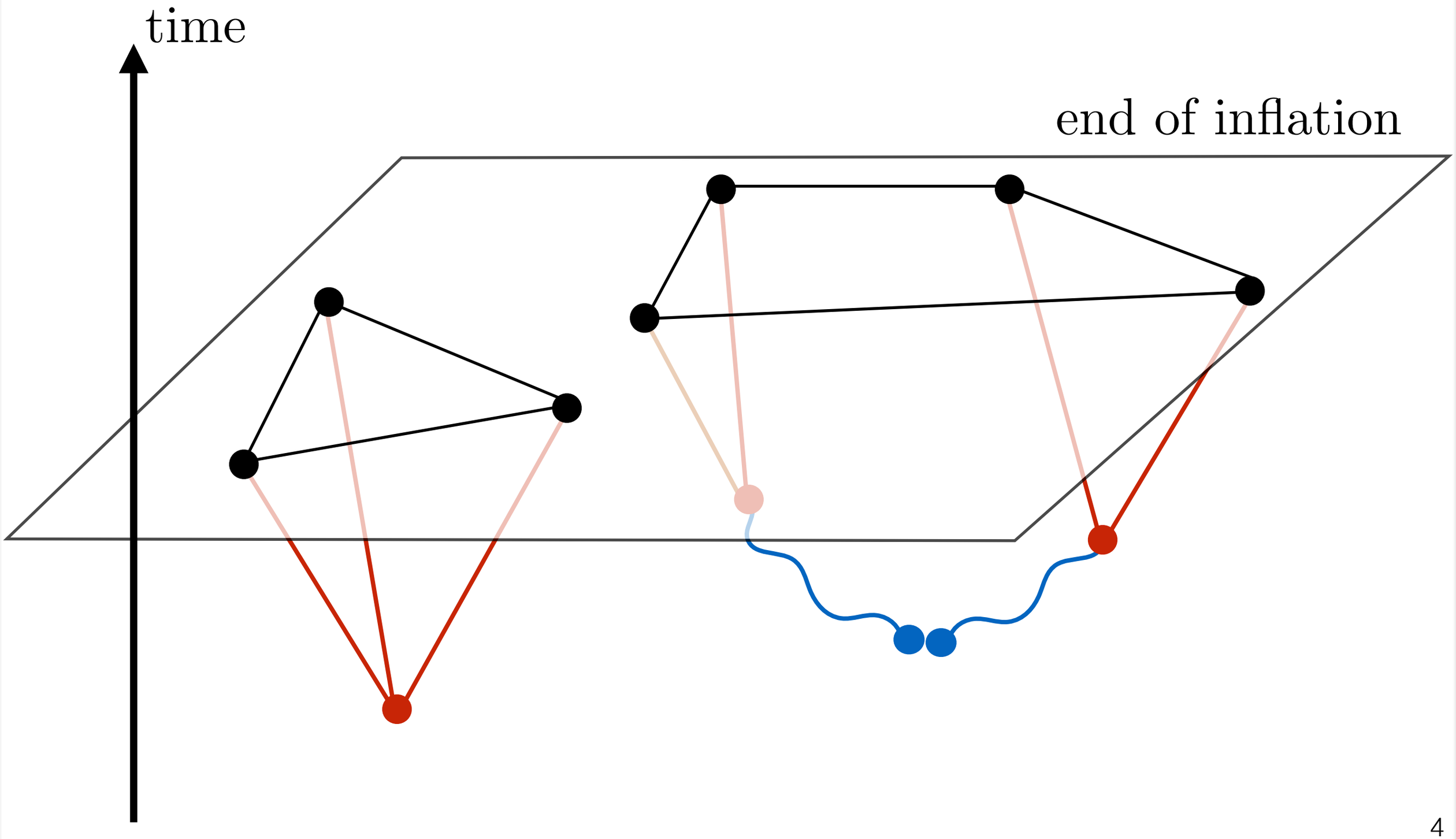
Goal

A menu of templates of non-Gaussianity,
compatible with general principles:

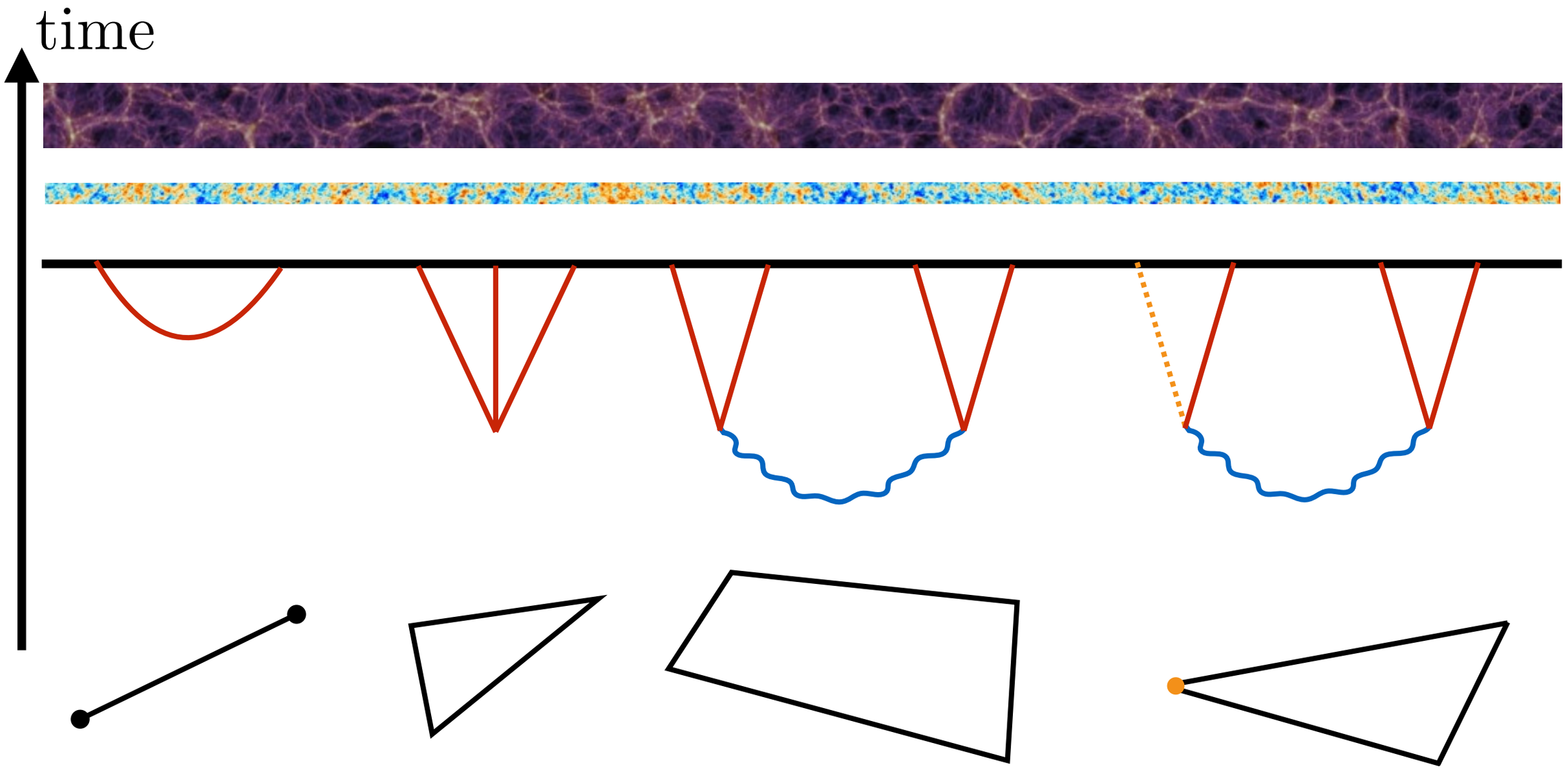
1. Symmetry
2. Locality
3. Unitarity

This would be a theory of shapes.
Model would tell sizes of those shapes.

Strategy: Time Without Time



What's in the Menu?

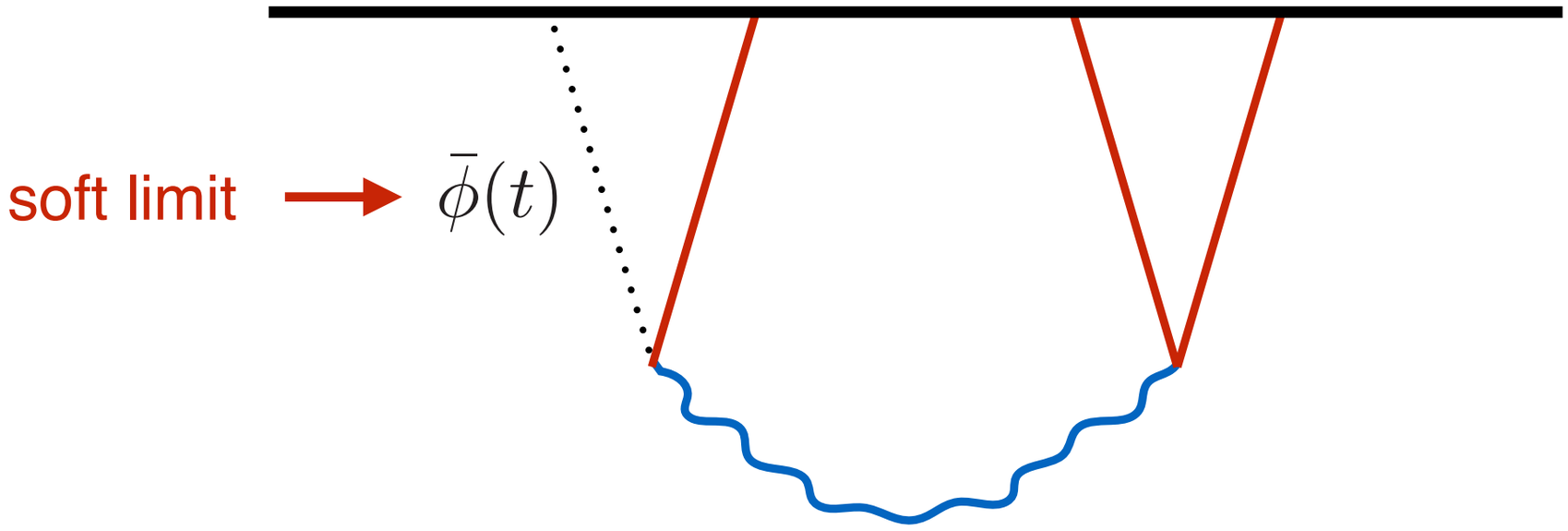


There is a very small* menu of histories!
New templates of non-gaussianity.
Time evolution is emergent, and constraining!

*under the assumption of weak coupling, very softly broken de Sitter symmetry (f_{NL}~1)

Inflationary Bispectra

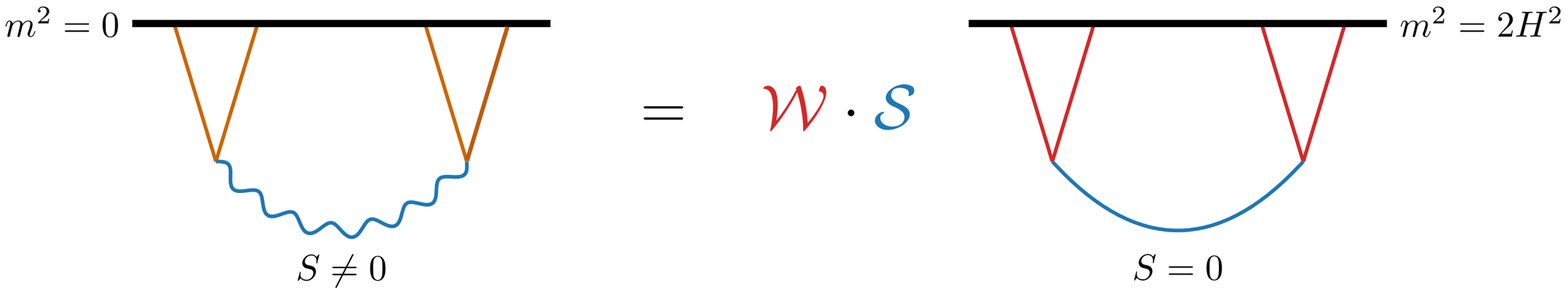
Slow-roll inflationary bispectra can be obtained from de Sitter four-point functions by a soft limit:



This motivates us to study four-point functions in de Sitter as a building block.

Creminelli, 2003
Kundu, Shukla, Trivedi, 2014
Arkani-Hamed, Maldacena, 2015
Arkani-Hamed, Baumann, Lee, GLP, 2018
Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

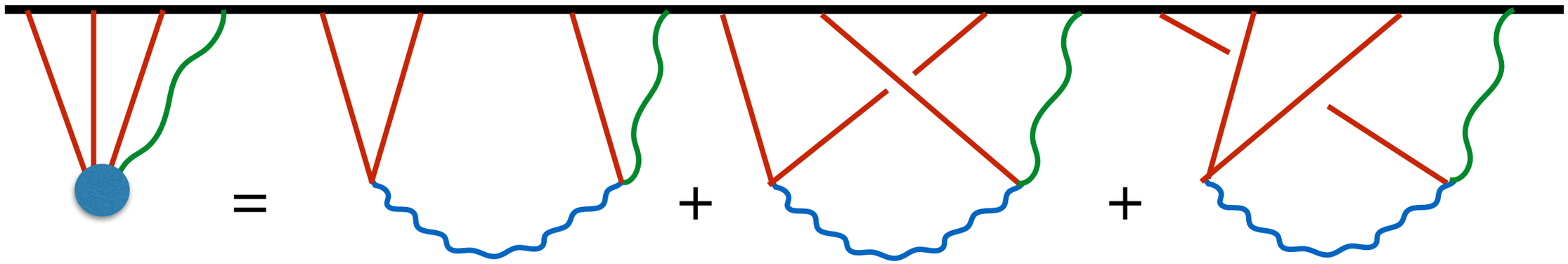
Weight-Shifting



All the dynamics contained in a single diagram!
Other possibilities can be kinematically bootstrapped.
We can even change the spin of external legs.

"Weight": Mass and Spin of the particles

Weight-Shifting, with a Spin

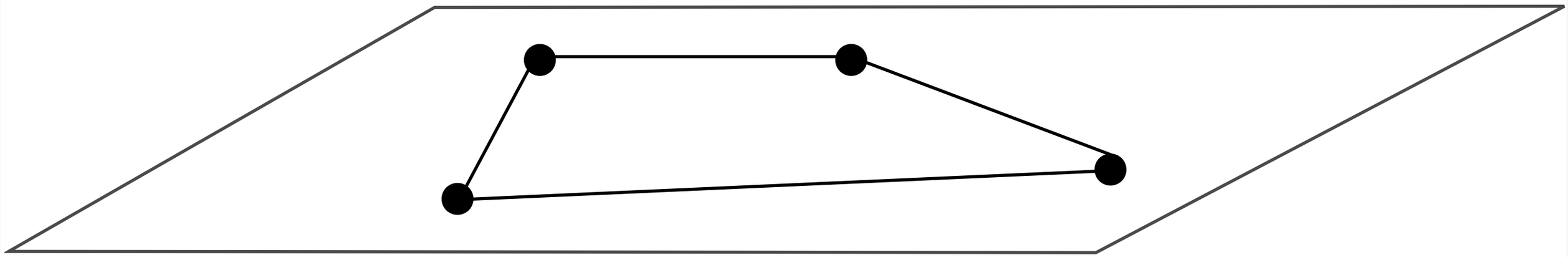


Spinning correlation functions can be compatible with symmetry, but incompatible with locality.
Weakly coupled constraints on quantum gravity!

The Cosmological Bootstrap

Arkani-Hamed, Baumann, Lee, GLP, 2018

I. Symmetries

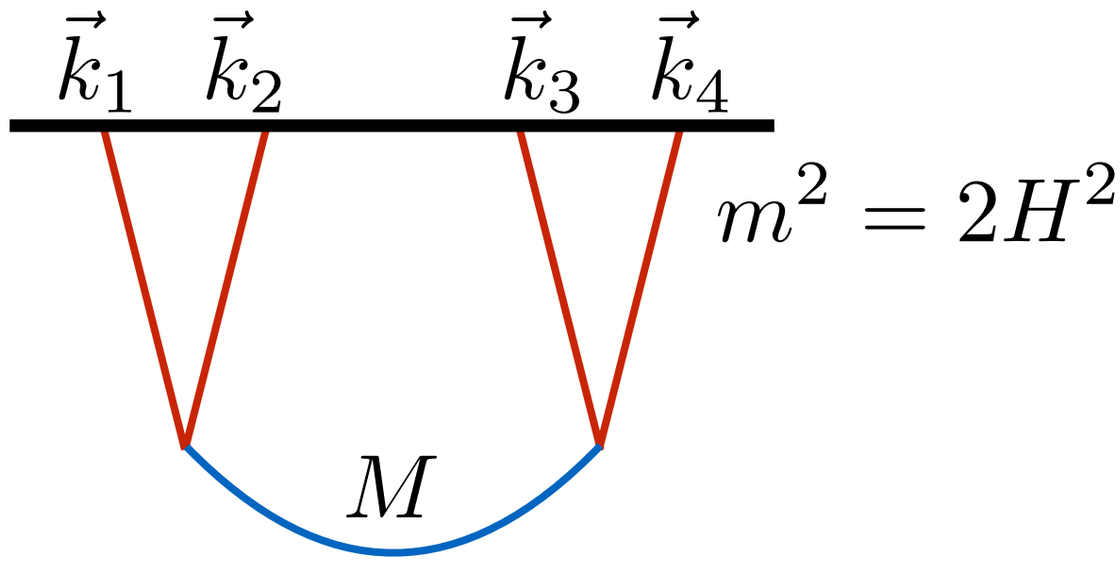


3 Translations
3 Rotations
1 Dilation
} Easy to diagonalize

3 dS Boosts $\Longrightarrow$ Conformal Ward identities (depend on mass and spin)

Arkani-Hamed, Baumann, Lee, GLP, 2018

II. Kinematics



$$m^2 = 2H^2$$

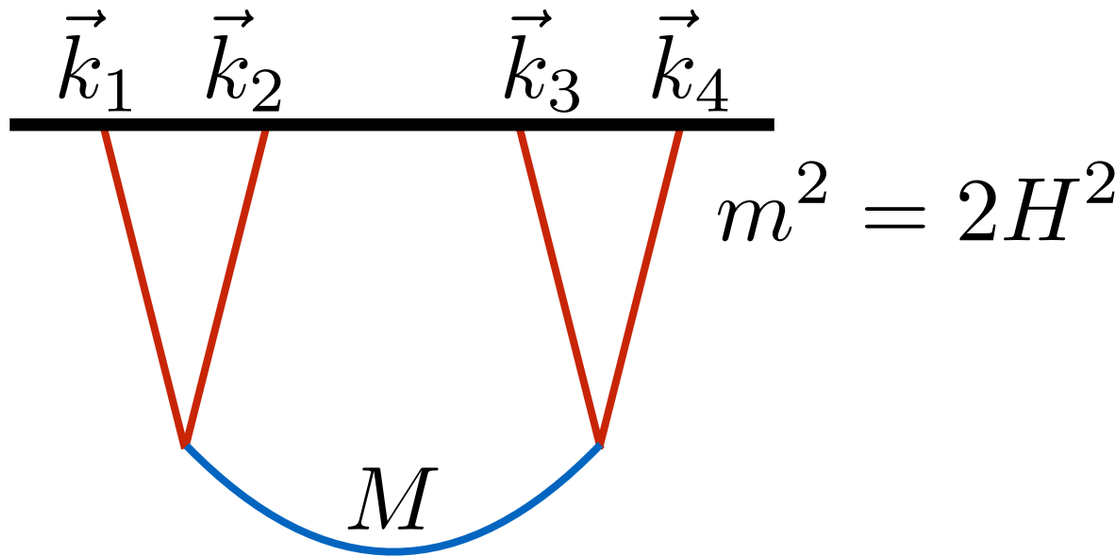
$$s = |\vec{k}_1 + \vec{k}_2|, \;\; u = \frac{s}{|\vec{k}_1| + |\vec{k}_2|}, \;\; v = \frac{s}{|\vec{k}_3| + |\vec{k}_4|}$$

Translations, Rotations, Dilation:

$$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle = \delta^3 \left(\sum_i \vec{k}_i \right) \frac{1}{s} \hat{F}\left(u, v\right)$$

Arkani-Hamed, Baumann, Lee, GLP, 2018

II. Kinematics



$$m^2 = 2H^2$$

Conformal Ward identity:

$$(\Delta_u - \Delta_v)\hat{F} = 0$$

No dynamics yet.

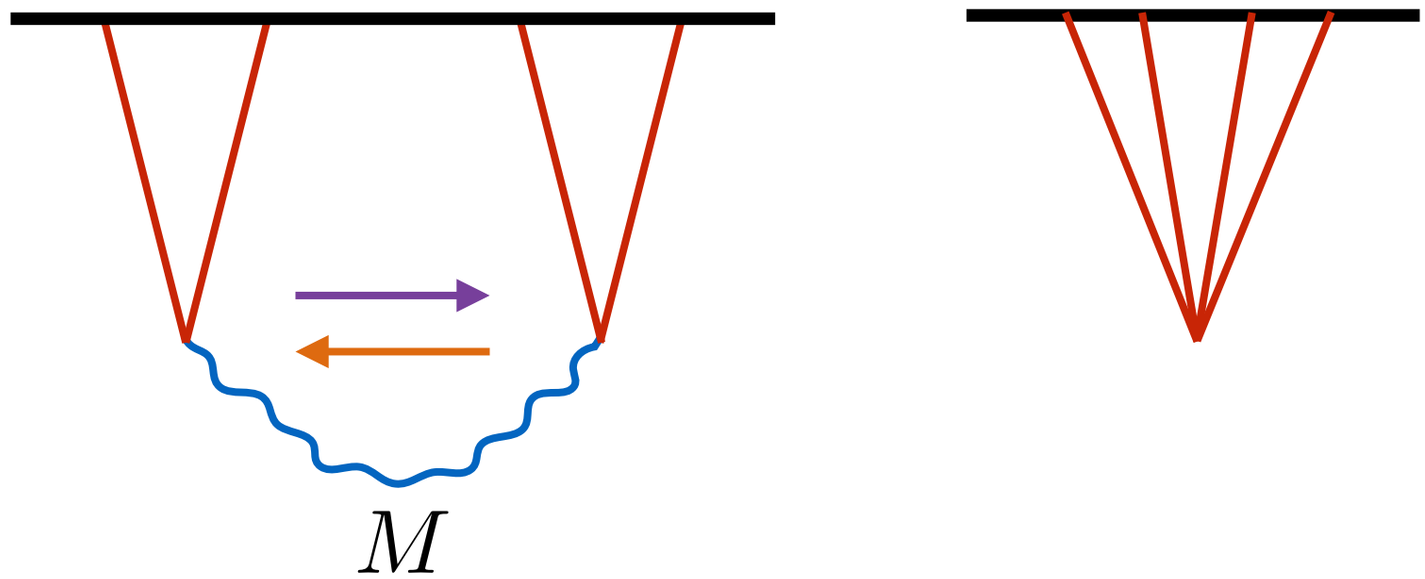
Δ_u is a second-order, hypergeometric differential operator.

Arkani-Hamed, Baumann, Lee, GLP, 2018

III. Dynamics

$$[\Delta_u + (M/H)^2]F_E(u,v) = F_C(u,v)$$

$$[\Delta_v + (M/H)^2]F_E(u,v) = F_C(u,v)$$

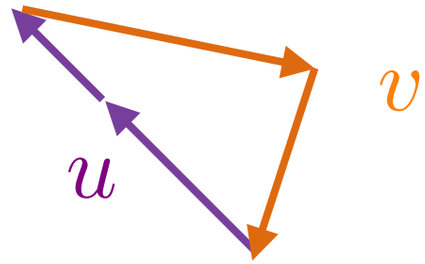


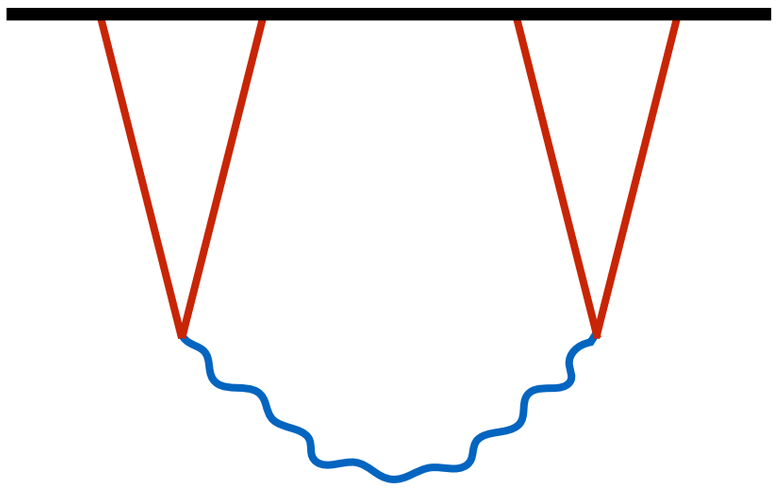
Each operator encodes consistent
time evolution of the two separate vertices.
Contact interactions can be easily classified.

Arkani-Hamed, Baumann, Lee, GLP, 2018

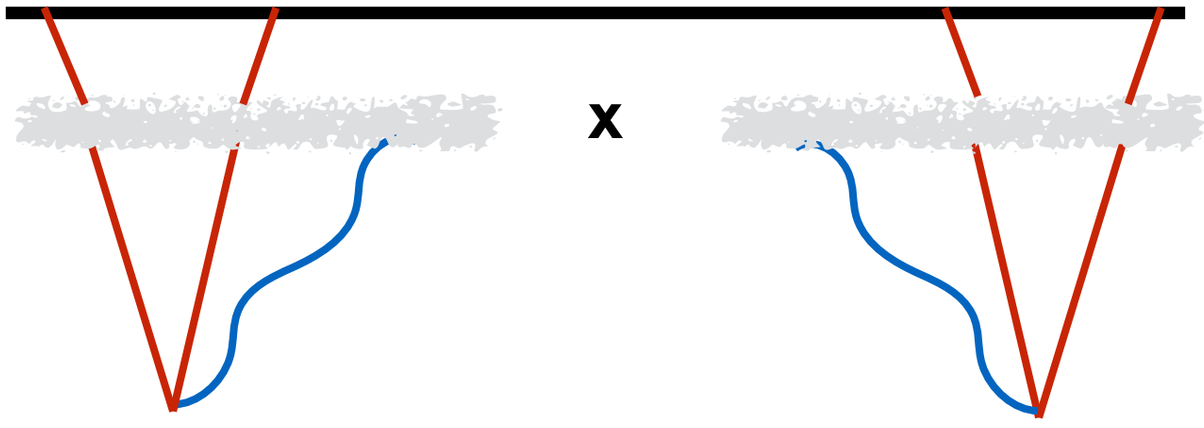
IV. Boundary Conditions

1.Smooth in "folded" limit



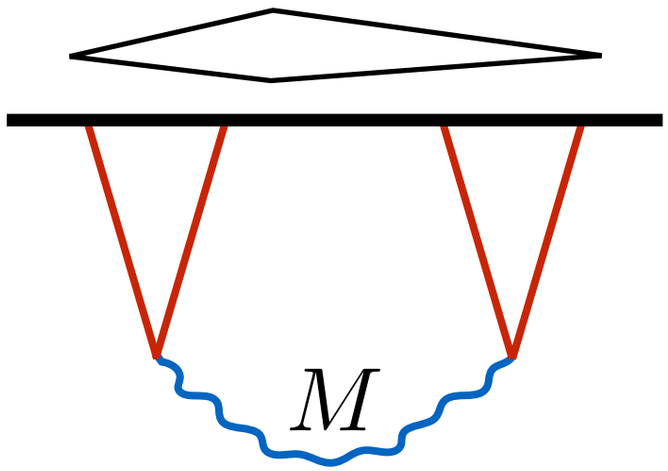


2.Factorizes in "early times" limit

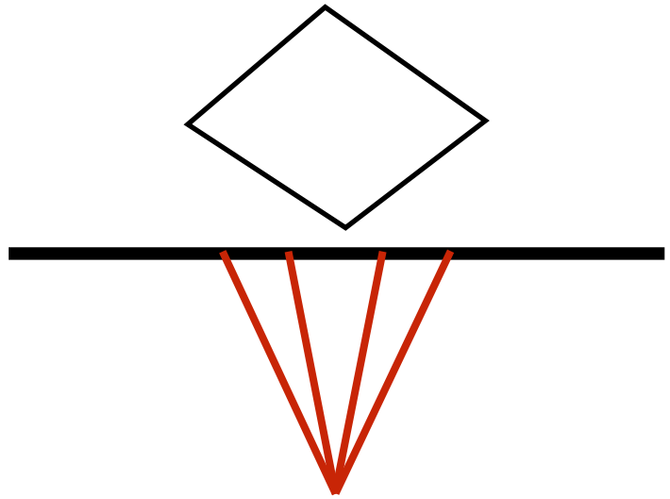


Arkani-Hamed, Baumann, Lee, GLP, 2018

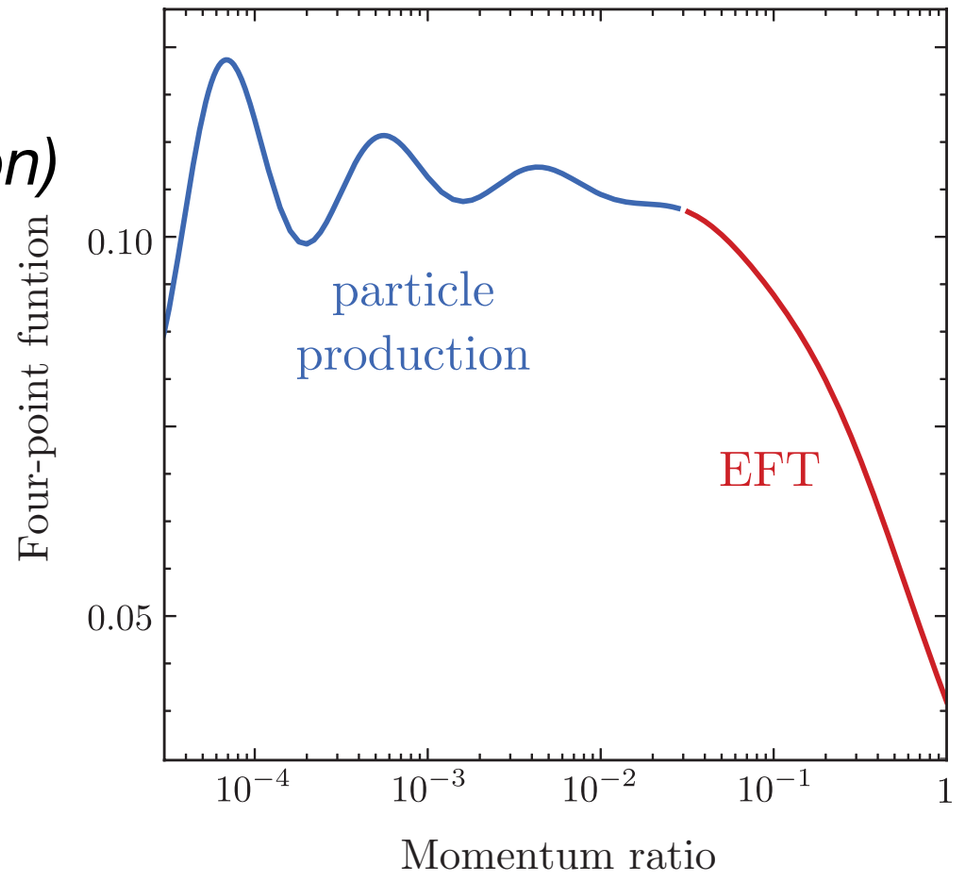
Four-Point Function



Collapsed
(particle production)

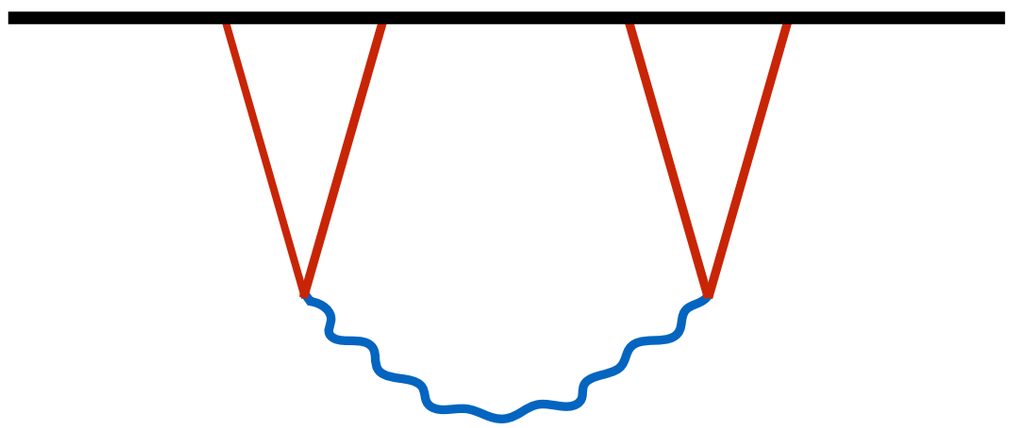


Equilateral
(EFT)

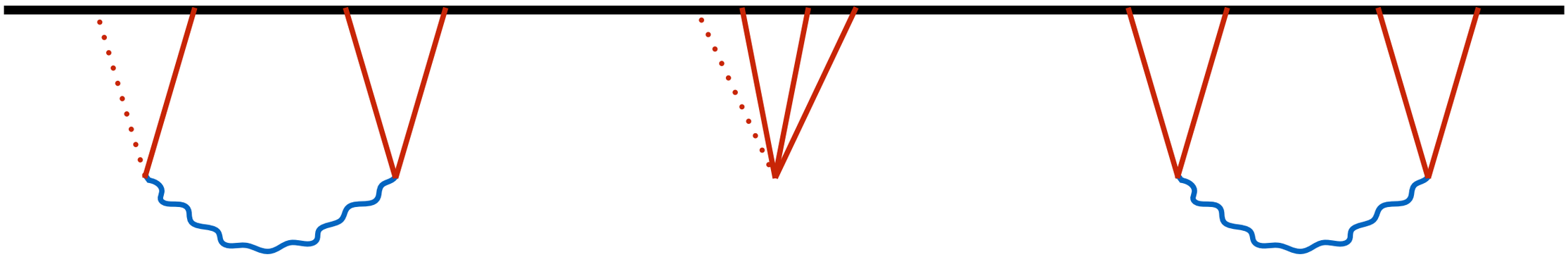


Arkani-Hamed, Maldacena, 2015
Lee, Baumann, GLP, 2016
Baumann, Goon, Lee, GLP, 2017
Arkani-Hamed, Baumann, Lee, GLP, 2018

Examples



M, S



$M = 0, S = 2$

Maldacena, 2002

Creminelli, 2003

$M = 0, S = 2$

Seery, Lidsey, Sloth, 2006
Seery, Sloth, Vernizzi, 2008
Arroja, Koyama, 2008

Tensor Correlators

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

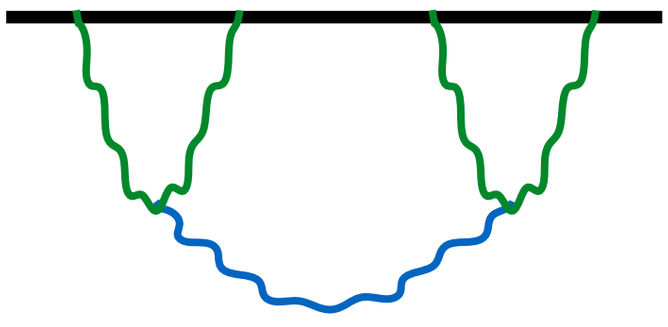
Gravitational Waves

Graviton is other guaranteed degree of freedom

Maldacena, GLP, 2011

Very little is known beyond three points

In flat space, consistent S-matrix of gravitons is very constrained



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Tensors are hard to compute using Lagrangians

Strategy

1. A massless spinning field must satisfy constraints to propagate right number of degrees of freedom (Ward-Takahashi identity)

2. Weight-Shift to right quantum numbers

3. Write WT Identity for correlation of interest

4. Find particular solution of the WI

5. All homogeneous solutions obtained by Weight-Shifting into conserved tensors.

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

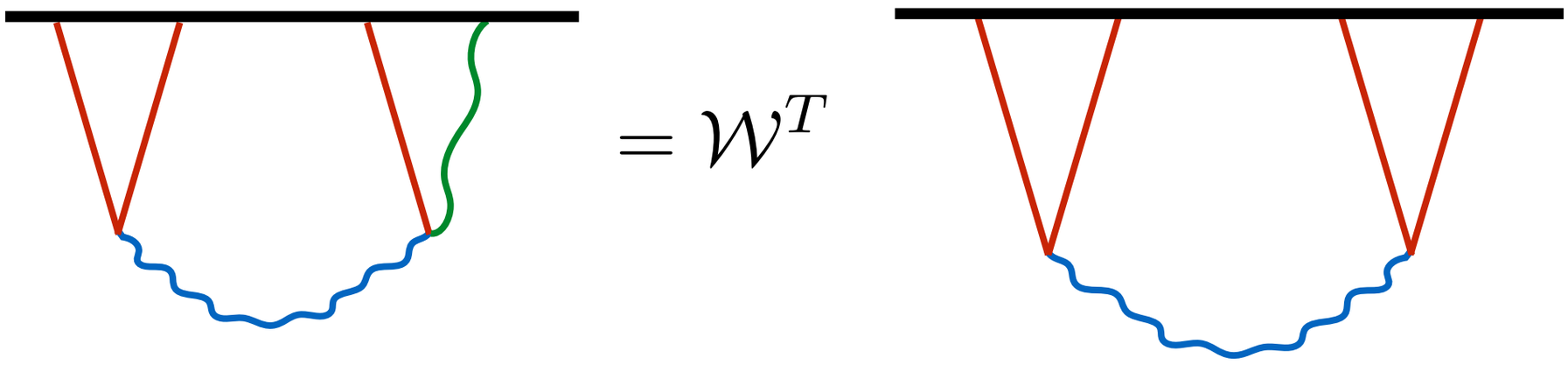
Weight Shifting Tensors

Other than having proper quantum numbers, light spinning fields must propagate correct number of degrees of freedom:

$$A_\mu \longrightarrow J_i, \qquad k_i J_i = 0$$

$$g_{\mu\nu} \longrightarrow T_{ij}, \quad T_{ii} = 0, \; k_i T_{ij} = 0$$

We can build WS operators that take scalars to tensors:



However…

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

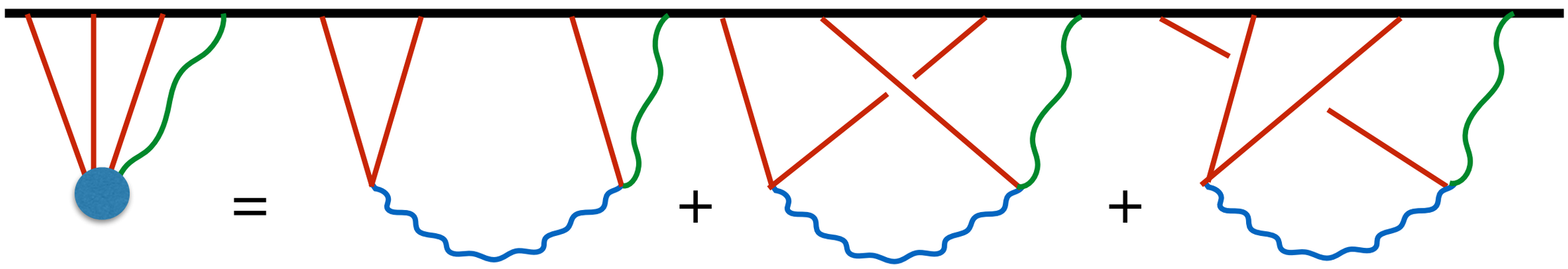
Conservation is Not Enough

Sometimes, demanding strict conservation is too strong.
Try Ward-Takahashi identity instead

$$k_i \langle J_i \mathcal{O}\mathcal{O}\mathcal{O} \rangle \sim \langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle$$

Hinterbichler, Hui, Khoury, 2012
Goldberger, Hui, Nicolis, 2013
GLP, 2013
Berezhiani, Khoury, 2013

In EFT, these happen to be the most interesting theories!



If we can’t solve WT identities - theory is ruled out.

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

Weight Shifting Tensors

Once we solved WT identity, all we can do is add "homogeneous solutions" with identically conserved currents. In that case, Weight-Shifting is easier.

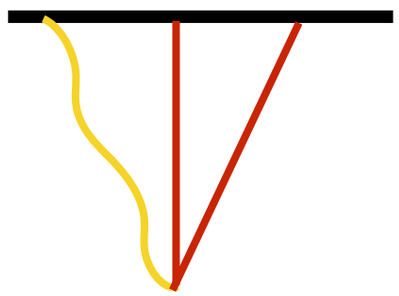
Example: Weight-Shifting to a conserved current (Photon)

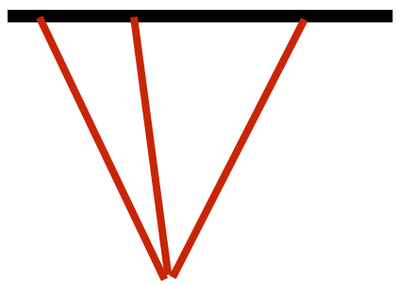
$$\mathcal{O} \to A_i = \mathcal{W}_1 \mathcal{O}$$

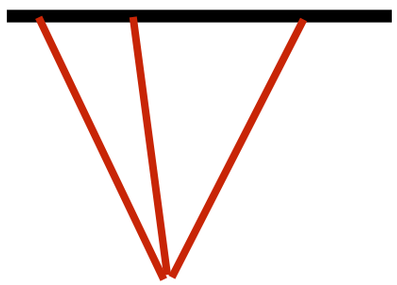
$$A_i \to J_i = \epsilon_{ijk} \partial_j A_k$$

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

Three-Point Example

$$\langle JOO \rangle =$$ 

$\mathcal{W}^J$  $= 0$ with $k_i \langle J_i O_2 O_3 \rangle = 0$

$\mathcal{O}$  $\neq 0$, but now

$$k_i \langle J_i O_2 O_3 \rangle = -e_2 \langle O_{1+2} O_3 \rangle - e_3 \langle O_2 O_{1+3} \rangle$$

Solution only exists if $e_2 + e_3 = 0$!

Arkani-Hamed, Baumann, Duaso Pueyo, Joyce, Lee, GLP, in prep.

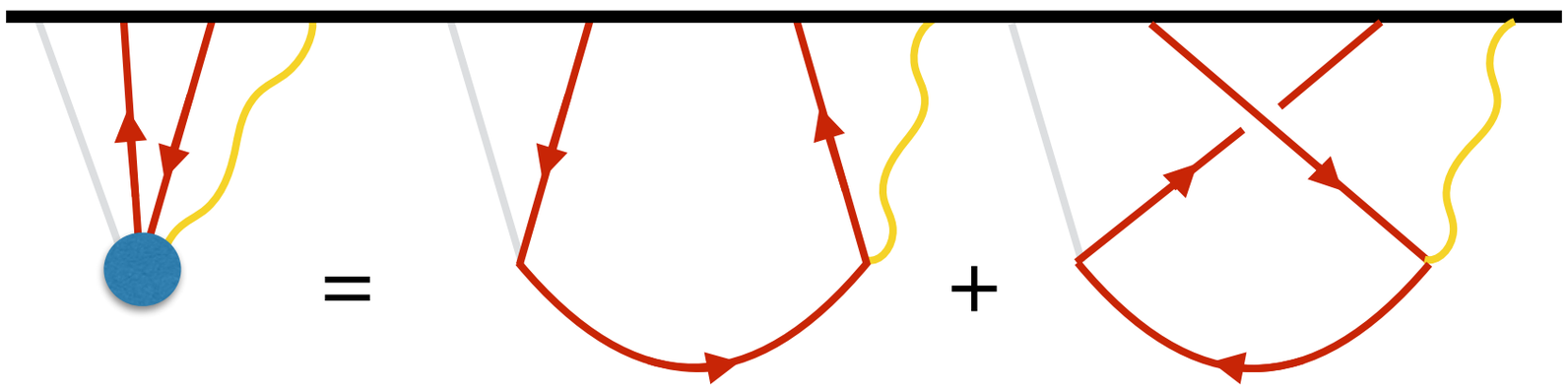
Four-Point Example

$$\langle J_1 O_2^+ O_3^- O_4^0 \rangle$$

Only combination of two channels will satisfy Ward-Takahashi identity:

$$k_1^i \langle J_1^i O_2^+ O_3^- O_4^0 \rangle = +\langle O_{1+2}^+ O_3^- O_4^0 \rangle - \langle O_2^+ O_{1+3}^- O_4^0 \rangle$$

We can apply WS to obtain each diagram, but only sum is physical!

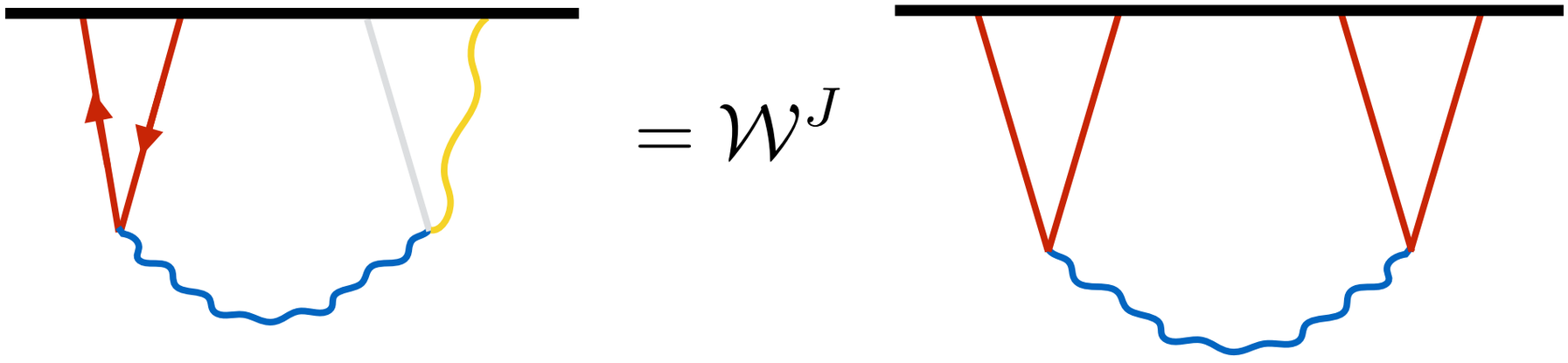


Arkani-Hamed, Baumann, Duaso, Joyce, Lee, GLP, in prep.

Four-Point Example

$$\langle J_1 O_2^+ O_3^- O_4^0 \rangle$$

Now that WT identity is solved, other possibilities are obtained through identically conserved Weight-Shifting operator

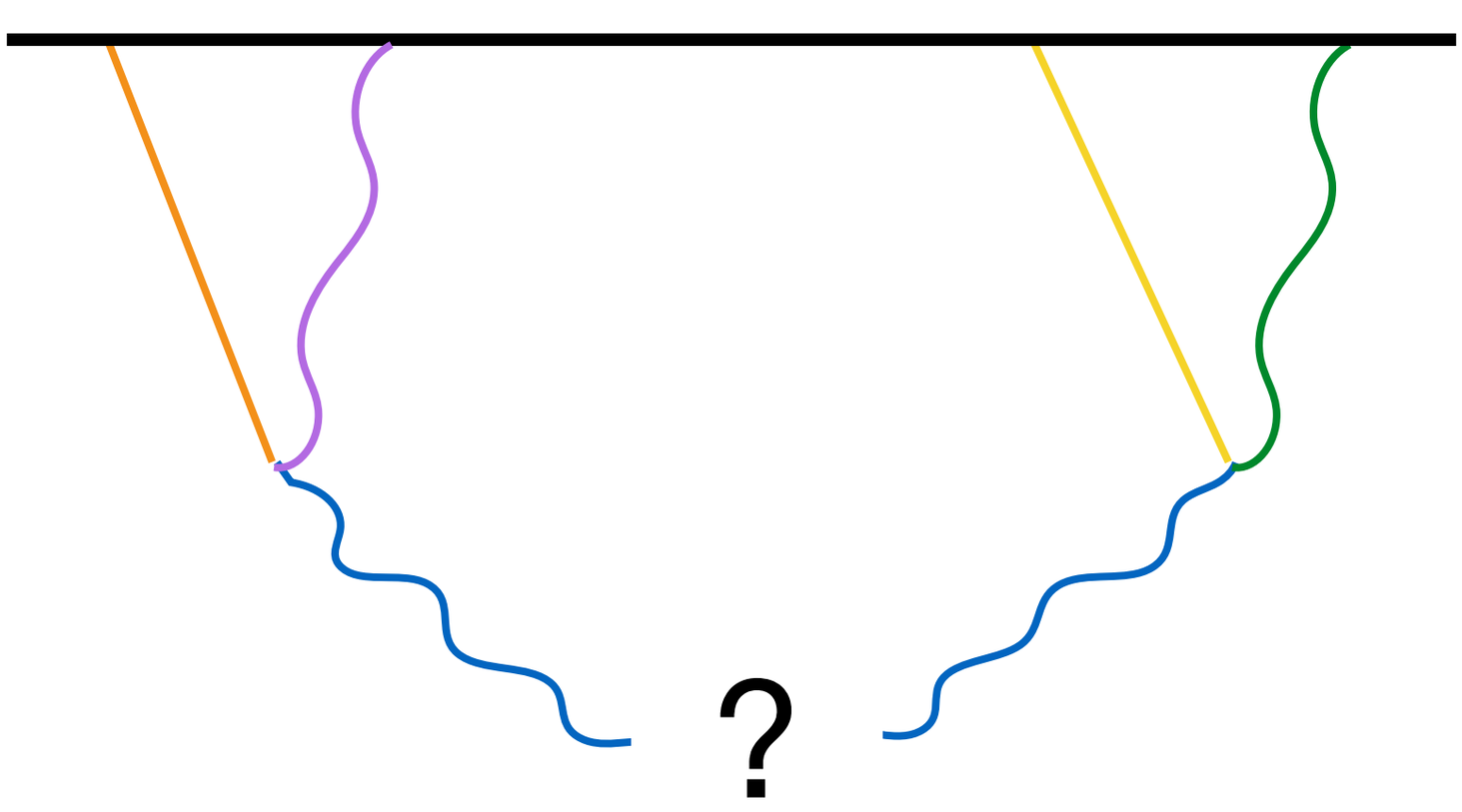


Arkani-Hamed, Baumann, Duaso, Joyce, Lee, GLP, in prep.

Future

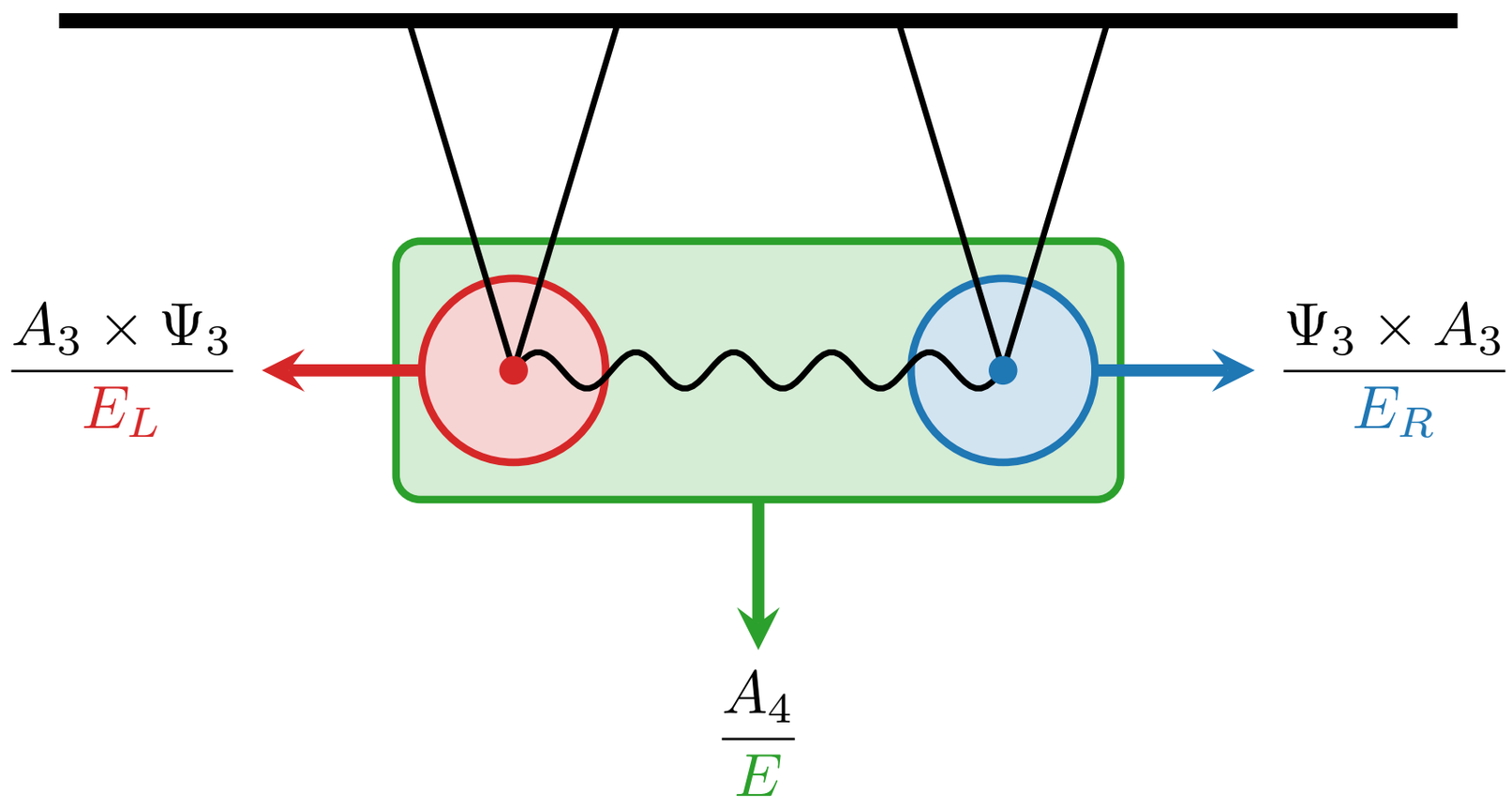
Consistent Gluing…

We can classify all three-point functions.
Can we glue them together
to get a consistent four-point function?



… From Factorization

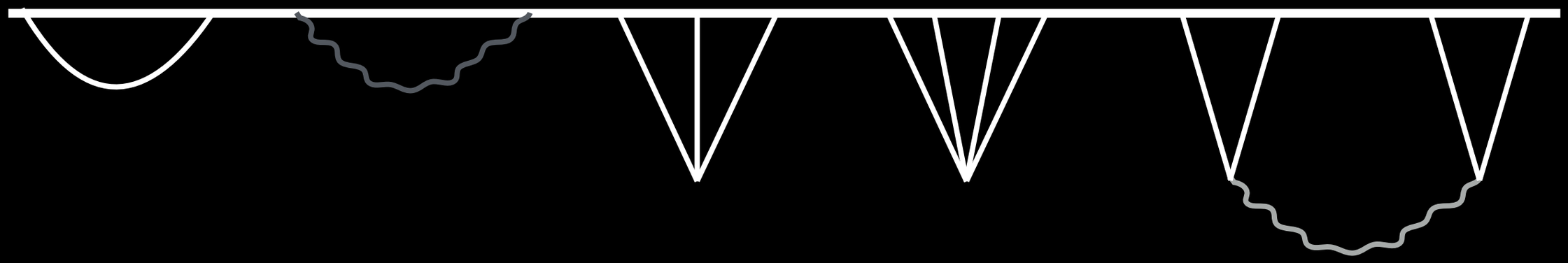
The four-point “diagrams” are more on-shell than Feynman diagrams. They satisfy non-trivial factorization rules



Our approach is a hybrid of S-matrix gauge-invariance and on-shell construction.

Benincasa, Cachazo, 2007
McGady, Rodina, 2013
Arkani-Hamed, Rodina, Trnka, 2017

Take Away



Bootstrap eliminates Lagrangians and fields
Time evolution is encoded in the correlator

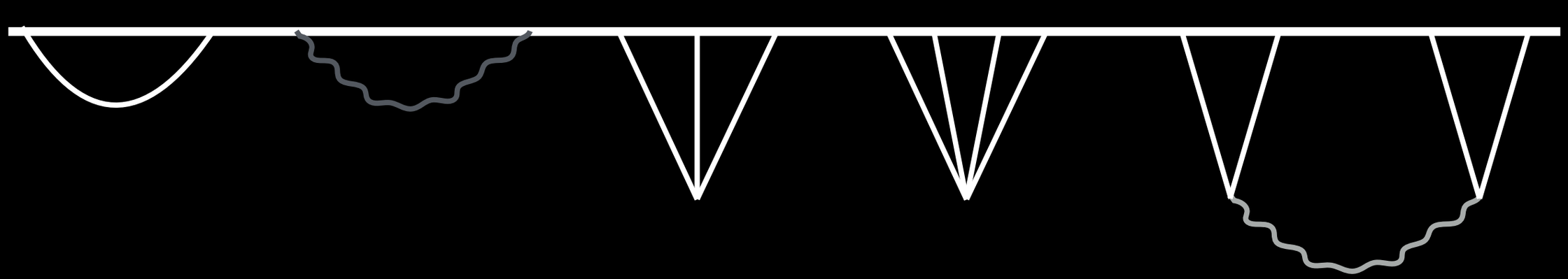
There is only a small menu of possibilities!

Much left to do!

A **necessity** towards better understanding of inflation and the big bang!

Cosmological Bootstrap,
With and Without Spin

Thank You For Your Attention!



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