Time Sliced Perturbation Theory (TSPT) for LSS

Diego Blas
w./ M. Garny, M. Ivanov, S. Sibiryakov 1512.05807, 1605.02149, 19xx.xxxx
check Ivanov+Sibiryakov (+ collaborators) 18/19 papers!
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O(40) pages/paper
Dynamics of distribution function(al)

\[ \delta \rightarrow P[\delta] \]

\[ \delta_L \rightarrow P[\delta_L] \]

SPT

Equal Time/Time Sliced (P)T
(Also Valageas 01, 04)
Dynamics of distribution function(al)

SPT

Dynamics

IR sens. in kernels!

initial time

$\delta$

$P[\delta]$

$\delta_L$

$P[\delta_L]$

Equal Time/Time Sliced (P)T

(also Valageas 01, 04)
TSPT is not only PT…

\[ \langle \delta_k(t)\delta_{k'}(t) \rangle = 2\pi^2 \delta^{(3)}(k + k')P(k, t) \]

from generating function(al) at fixed time (Time Sliced)

\[ \langle \delta_{k'}(t)\delta_k(t) \rangle = \frac{\delta^2 Z[J, t]}{\delta J_{-k} \delta J_{-k'}} \bigg|_{J=0} \]

primordial Gaussian field* evolved in time (adiabatic i.c.)

\[ Z[J, t] = \int [\mathcal{D}\delta_L(x)] \exp \left\{ - \int \frac{|\delta_L(k)|^2}{2P_L(k, t)} + \int J(x)\delta(x, t) \right\} \]

Perturbative techniques

NP techniques ("semiclassical")

*(PNG 1906.08697)
TSPT is not only PT...

\[
\langle \delta_k(t) \delta_{k'}(t) \rangle = 2\pi^2 \delta^{(3)}(k + k') P(k, t)
\]

from generating function(al) at fixed time (Time Sliced)

\[
\langle \delta_{k'}(t) \delta_k(t) \rangle = \left. \frac{\delta^2 Z[J, t]}{\delta J_{-k} \delta J_{-k'}} \right|_{J=0}
\]

primordial Gaussian field* evolved in time (adiabatic i.c.)

\[
Z[J, t] = \int [D\delta_L(x)] \exp \left\{ - \int \frac{\lvert \delta_L(k) \rvert^2}{2P_L(k, t)} + \int J(x)\delta(x, t) \right\}
\]

Perturbative techniques

NP techniques ("semiclassical")

IR res. of BAO \textbf{1605.02149}
IR res. primordial features \textbf{1906.08697}
RG flow ("EFT extension") \textbf{(in progress)}

Counts-in-cells \textbf{1811.07913} (in progress)

*(PNG 1906.08697)
TSPT: PT story

\[
Z[J, t] = \int [\mathcal{D} \delta_L(x)] \exp \left\{ - \int \frac{|\delta_L(k)|^2}{2P_L(k,t)} + \int J(x)\delta(x,t) \right\}
\]

\[
Z[J, t] = \int [\mathcal{D} \delta] \mathcal{P}[\delta, t] e^{\int [dk] J_k \delta_{-k}}
\]

from conservation of probability

\[
\frac{d}{dt} \{[\mathcal{D} \delta] \mathcal{P}(\delta, t)\} = 0
\]

solved in a perturbative recursive expansion around Gaussian

\[
\mathcal{P}(\delta, t) = \frac{e^{-\sum_{n=2} \frac{1}{n!} \int [dk] \Gamma_n(t, k_n) \Pi \delta_k^n}}{\mathcal{N}} \quad \Gamma_n(\Gamma_m, \dot{\delta})
\]

\[m < n\]
TSPT diagrams

PT* for $Z[J]$

$$Z[J,t] = \int [\mathcal{D}\delta] \mathcal{P}[\delta, t] e^{\int [dk] J_k \delta_{-k}}$$

$$= \int [\mathcal{D}\delta] e^{-\frac{e^{-2t}}{2}} \int [dk] \bar{\Gamma}_2^{tot}(k) \delta_{k} \delta_{-k} \left(1 + \sum_{n=3} e^{-2t} \frac{1}{n!} \int [dk_n] \bar{\Gamma}_n^{tot}(k_n) \prod \delta_{k_n} + \ldots \right)$$

gaussian integrals + Feynman diagrams

- propagator
  $$\frac{1}{\Gamma_2(t,k)} = e^{2t} P(k, t_0)$$
- vertices
  $$e^{-2t} \Gamma_n(t, \{k_n\})$$
- n-lines

* beyond PT studies for CiC
Ivanov et al 1811.07913
\[ \langle \delta \, \delta \rangle = \quad \quad + \quad \frac{\Gamma_3}{\Gamma_3} \quad \quad + \quad \frac{\Gamma_4}{\Gamma_4} \quad + \quad \ldots \]

\[ \langle \delta \, \delta \, \delta \rangle = \quad \quad + \quad \quad + \quad \quad + \quad \quad + \quad \quad + \quad \ldots \]

\[ \lim_{\epsilon \to 0} \frac{\Gamma_{n+m}}{\epsilon q_1, \ldots, \epsilon q_m, k_1, \ldots, k_n} < \infty \]
Note that one-loop tadpole graphs have been already taken care of, see (30).

The first graph is simply the linear power spectrum. The extra terms come from the so-called ‘sunrise’ diagram (see (B) for more details).

In terms of Feynman diagrams, at the order $\mathcal{P}_{\text{ren}}$, the tadpole graphs can be divided out, such that the error bars reflect the sub-leading contribution of the long wavelength modes to the cosmic variance. This issue, however, is not the main focus of this paper.

We are measuring power spectra and correlation functions in a fiducial model. To write the exponent in the above form, we have used a generic field obeying (4) in the previous sections.

$P_{w} = -\frac{1}{10} \frac{k^2}{k_{osc}^2} \int_{q \ll k} [dq] P_s(q) P_w(k) \sim P_w \left( \frac{k^2}{k_{osc}^2} \delta^2 \right)$
\[ P_{NL}^{\text{wiggles}} = P_{wiggles} + P_{\text{smooth}} \]

\[ P_0 = P_{wiggles} + P_{\text{smooth}} \]

\[ P_w = -\frac{1}{10} \frac{k^2}{k_{osc}^2} \int [dq] P_s(q) P_w(k) \sim P_w \left( \frac{k^2}{k_{osc}^2} \delta^2 \right) \]

\[ \sim P_w \left( \frac{k^2}{k_{osc}^2} \delta^2 \right)^2 \]

\[ \sim P_w \left( \frac{k^2}{k_{osc}^2} \delta^2 \right)^3 \]
The so-called 'sunrise' diagram (see (B) for more details).

The first graph is simply the linear power spectrum. The second diagram is formed by a sum over all daisy 3-vertices. This issue, however, is not the main goal of this paper and will be addressed in detail elsewhere.

Let us now focus on the 1-loop PS (e.g. including next to leading order corrections 2.3.1 1-loop results and comparison with SPT approach and their 'role' is to reproduce eventually the SPT result. In order to further introduce new counter-terms for the field, it is very instructive to perform one - loop computation, to which we proceed now.

To make the connection with the SPT approach, i.e. to write TSPT as a series in $C_{\text{osc}}^2$ and $C_{\text{osc}}^4$, we will use the identity in (93).

For Gaussian variables, we obtain our final expression. Even if a mode falls in this subset, it contributes roughly (0\(C_{\text{osc}}^2\))\(C_{\text{osc}}^4\)\(\delta^2\) while the above expressions are based on a rigid reality.

The symmetry factor for the 1-loop IR. (the combinatorial factor should only multiply the peak power through\(C_{\text{osc}}^2\))\(C_{\text{osc}}^4\)\(\delta^2\)

The field has to be identified with the velocity divergence \(\sigma\) and its 'role' is to reproduce eventually the SPT result. In order to further introduce new counter-terms for the field, it will be more convenient used to be a generic field obeying (4) in the previous sections.

However, in order to switch to the familiar notation of SPT, it will be more convenient to relabel this field as follows, $wiggles = P_{wiggles}$ $\propto P_{w} \left( \frac{k^2}{k_{osc}^2} \delta^2 \right)^{2}$

$P_{w}$ $\propto P_{w} \left( \frac{k^2}{k_{osc}^2} \delta^2 \right)^{3}$

$P_{0} = P_{wiggles} + P_{\text{smooth}}$

$P_{w}^N L$ $P_{wiggles} =$

$- \frac{1}{10} \frac{k^2}{k_{osc}^2} \int_{q \ll k} [dq] P_{s}(q) P_{w}(k) \sim P_{w} \left( \frac{k^2}{k_{osc}^2} \delta^2 \right)$
TSPT resummation of BAO

\[ \Sigma^2(k_{BAO}) = \frac{4\pi}{3} \int_0^{k_L} dq P_s(q) \left( 1 - j_0(q r_{BAO}) + 2 j_2(q r_{BAO}) \right) \]

\[ P(k) = P_s(k) + e^{-k^2\Sigma^2} P_w(k) \]

same damping as Baldauf et al. 15

(same effect as other IR resummations)
TSPT resummation of BAO

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same damping as Baldauf et al. 15

(linear)

DB, Garny, Ivanov, Sibiryakov 1605.02149

(same effect as other IR resummations)
TSPT resummation of BAO

- diagrammatic technique ‘easily’ extended beyond LO
- diagrammatic technique easily extended to n-point

needed? why not! (to get best precision!)

TSPT resummation in 2019

- including RSD + bias

small deviation wrt standard formalism

- resummation of oscillating features in PS

\[ P_L(k) = P_{L,\Lambda CDM} + A_{\text{lin}} \cos(\omega k) + A_{\text{log}} \cos(\gamma \log k/k_*) \]

\[ \gamma \gg 1 \]

damping of feature with \( \Sigma^2(k/\gamma) \)
\[ \mathcal{P} [\delta; t_0] = \mathcal{N}^{-1} \exp \left\{ - \int [dk] \frac{\delta_k \delta_{-k}}{2P_L(k)} - \int \frac{[dk]^3}{3!} \Gamma_3^{(3)} (k_1, k_2, k_3) \delta_{k_1} \delta_{k_2} \delta_{k_3} \right\} \]

for small PNG

\[ (2\pi)^3 \delta_D^{(3)} (k_{123}) B_L (k_1, k_2, k_3) = -\Gamma_3^{(3)} (k_1, k_2, k_3) P_L (k_1) P_L (k_2) P_L (k_3) \]

on can find the new PDF

\[ g(t) = e^t \]

\[ \Gamma_n (\eta; k_1, \ldots, k_n) = \frac{1}{g^2(\eta)} \bar{\Gamma}_n (k_1, \ldots, k_n) + \frac{1}{g^3(\eta)} \bar{\Gamma}_n^{NG} (k_1, \ldots, k_n) \]

extra vertices

\[ = - \frac{1}{g^3(\eta)} \frac{1}{3!} \bar{\Gamma}_3^{NG} (k_1, k_2, k_3) \]


**TSPT for PNG: axion monodromy**

Vasudevan et al 1906.08697

\[
B_\zeta (k_1, k_2, k_3) = f_{NL}^{\text{res}} \frac{4 \mathcal{A}_\zeta^4}{k_1^2 k_2^2 k_3^2} \left[ \sin \left( \gamma \ln \frac{k_t}{k_*} \right) + \frac{1}{\gamma} \sum_{i \neq j} \frac{k_i}{k_j} \cos \left( \gamma \ln \frac{k_t}{k_*} \right) + O \left( \frac{1}{\gamma^2} \right) \right]
\]

Flauger, Pajer 1002.0833

\[
B_{\delta \delta \delta}^{NG,LO} (\eta; k_1, k_2, k_3) = g^3(\eta) e^{-g^2(\eta) \Sigma_B (k_1, k_2, k_3)} B_L (k_1, k_2, k_3)
\]

\[B_{\delta \delta \delta}^{NG,LO} (\eta; k_1, k_2, k_3) =
\]

+ new (small) term for the squeezed limit after IR resummation!

also Beutler et al 1906.08758
What about the UV?

\[ \delta_{k, \Lambda} = \delta_k \Theta(\Lambda - k) \]

\[ Z_\Lambda[J, t] = \int [\mathcal{D}\delta_\Lambda] \mathcal{P}_\Lambda[\delta_\Lambda, t] \exp \left\{ \int_0^\Lambda [dk] J_k \delta_{-k} \right\} \]

\[ \int_0^{\Lambda_1} [\mathcal{D}\delta] \int_{\Lambda_1}^\Lambda [\mathcal{D}\delta] \mathcal{P}_\Lambda \equiv \int_0^{\Lambda_1} [\mathcal{D}\delta] \mathcal{P}_{\Lambda_1} \]

The distribution acquires \textbf{counterterms} to compensate cut-off dependent integrals

\[ \mathcal{P}_\Lambda \equiv e^{-\Gamma_\Lambda} = e^{-\Gamma^{TSP}} - \sum \alpha_n(\Lambda, t) C_n(k) \]

Wilsonian/Polchinski RG:

\[ \frac{d}{d\Lambda} \Gamma_\Lambda = \mathcal{F}(\Gamma_\Lambda) \]

New insights on counterterms of EFTofLSS?
Conclusions

\[ \delta \rightarrow \mathcal{P}[\delta] \]

\[ \delta_L \rightarrow \mathcal{P}[\delta_L] \]

SPT \quad \text{dynamics} \quad \text{initial time}

statistics

TSPT
Conclusions

- TSPT alternative to SPT/LPT for equal time observables

- So far useful for:
  - IR resummation of primordial features (BAO/Primordial)
  - New insights on counts-in-cells statistics
The near (?) future

- **PT-like**: More on PNG: squeezed limit + other PNG

- **EFT-like**: Wilsonian RG flow

- **Non-perturbative**: Continue exploration of counts-in-cells. Other non-perturbative ideas?

- **Connection to**: Gonzalo Palma's and Giovanni Cabass's talks
Counts in Cells statistics

Ivanov et al 1811.07913

\[ \mathcal{P}(\delta_*) = \mathcal{N}^{-1} \int \mathcal{D}\delta_L \exp \left\{ - \int_k \frac{|\delta_L(k)|^2}{2g^2P(k)} \right\} \delta_D^{(1)}(\delta_* - \bar{\delta}_W[\delta_L]) \]