LSS Consistency Relations

(ISW and kSZ effects)

The Non-Gaussian Universe
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Going beyond PT: e.g., Consistency Relations

Go beyond PT and phenomenological models by deriving exact results without explicitly solving the dynamics.

Consistency relations: use symmetries of the system

More general ⟹

- lose details of the dynamics
- also applies to biased tracers
- remains valid whatever baryonic effects

⟹ test of general physical principles.

constrain models.

Kinematic consistency relations

Kehagias & Riotto (2013), Kehagias et al(2013), Peloso & Pietroni (2013a,b), Creminelli et al. (2013a,b,c), P.V. (2013), P.V., Taruya and Nishimichi (2017)

A) Correlation and response functions

1311.1236

A general property for systems parameterized by a Gaussian field:

1) a Gaussian field: $\varphi(x)$

2) nonlinear functionals: $\rho_1, \quad \rho_2, ..., \rho_n$

We consider the mixed correlation:

$$C^{\ell,n} = \langle \varphi(x_1) \dots \varphi(x_\ell)\, \rho_1 \dots \rho_n \rangle = \int \mathcal{D}\varphi \; e^{-(1/2)\varphi \cdot C_0^{-1} \cdot \varphi} \; \varphi(x_1) \dots \varphi(x_\ell) \; \rho_1 \dots \rho_n$$

integrations by parts

$$C^{\ell,n} = C_0(x_1, x_1') \dots C_0(x_\ell, x_\ell') \cdot R^{\ell,n}(x_1', .., x_\ell')$$

Response function:

$$R^{\ell,n}(x_1, .., x_\ell) = \langle \frac{\mathcal{D}^\ell [\rho_1 \dots \rho_n]}{\mathcal{D}\varphi(x_1)..\mathcal{D}\varphi(x_\ell)} \rangle$$

In the cosmological case, we consider the density field:

$$\langle \tilde{\delta}_{L0}(\mathbf{k}'_1)..\tilde{\delta}_{L0}(\mathbf{k}'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle = P_{L0}(k'_1)..P_{L0}(k'_\ell)\;\left\langle \frac{\mathcal{D}^\ell[\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)]}{\mathcal{D}\tilde{\delta}_{L0}(-\mathbf{k}'_1)..\mathcal{D}\tilde{\delta}_{L0}(-\mathbf{k}'_\ell)} \right\rangle$$

On large scales, or at early times, we recover the linear regime

$$k'_i \ll k_L : \qquad \langle \tilde{\delta}_{L0}(\mathbf{k}'_1)..\tilde{\delta}_{L0}(\mathbf{k}'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle \to \langle \tilde{\delta}(\mathbf{k}'_1,t'_1)..\tilde{\delta}(\mathbf{k}'_\ell,t'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle$$

We obtain the squeezed density correlation if we can evaluate the response function

$\Longrightarrow$ "CONSISTENCY RELATIONS"

B) Derivation of the kinematic consistency relations

A consequence of a symmetry of the system associated with the equivalence principle:

all particles/structures fall in the same fashion in a gravitational potential.

From a solution $\{\delta(\mathbf{x},t),\mathbf{v}(\mathbf{x},t),\Phi(\mathbf{x},t)\}$ we can build a new solution that corresponds to a uniform time-dependent translation,

$$\mathbf{x}' = \mathbf{x} - \mathbf{n}(\tau), \quad \mathbf{v}' = \mathbf{v} - \dot{\mathbf{n}}(\tau), \quad \delta' = \delta, \quad \Phi' = \Phi + (\ddot{\mathbf{n}} + \mathcal{H}\dot{\mathbf{n}}) \cdot \mathbf{x}'$$

We can absorb in this fashion, through a change of variable, the impact of a large-scale gravitational potential, which has a constant gradient at lowest order.

$k' \to 0$ $\quad\quad \delta(\mathbf{x},t) \to \delta(\mathbf{x}+\Delta\mathbf{x},t)$ $\quad$ with $\quad \Delta\mathbf{x} = D_+(t) \int d\mathbf{k}' \Delta\tilde{\delta}_{L0}(\mathbf{k}') \frac{i\mathbf{k}'}{k'^2}$

$\tilde{\delta}(\mathbf{k}) \to \tilde{\delta}(\mathbf{k}) e^{-i\mathbf{k}\cdot\Delta\mathbf{x}}$

$$R^{1,n}_{k'\to 0} = \langle \tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle \sum_{i=1}^{n} \frac{\mathbf{k}_i \cdot \mathbf{k}'}{k'^2} \bar{D}_+(t_i)$$

From the response function, we obtain the consistency relations:

$$\langle\tilde{\delta}(\mathbf{k}'_1,t'_1)..\tilde{\delta}(\mathbf{k}'_\ell,t'_\ell)\,\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle'_{k'_j\to 0} = P_L(k'_1,t'_1)...P_L(k'_\ell,t'_\ell)\langle\tilde{\delta}(\mathbf{k}_1,t_1)..\tilde{\delta}(\mathbf{k}_n,t_n)\rangle' \times \prod_{j=1}^{\ell}\left(-\sum_{i=1}^{n}\frac{\mathbf{k}_i\cdot\mathbf{k}'_j}{k'^2_j}\frac{\bar{D}_+(t_i)}{\bar{D}_+(t'_j)}\right)$$

Lowest-order case, bispectrum,

$$\lim_{k'\to 0} B(k',t';k_1,t_1;k_2,t_2) = -P_L(k',t')P(k_1;t_1,t_2)\left(\frac{\mathbf{k}_1\cdot\mathbf{k}'}{k'^2}\frac{\bar{D}_+(t_1)}{\bar{D}_+(t')}+\frac{\mathbf{k}_2\cdot\mathbf{k}'}{k'^2}\frac{\bar{D}_+(t_2)}{\bar{D}_+(t')}\right)$$

These relations vanish at equal times, because they merely express how small scales are uniformly transported by large-scale modes

These exact relations can be generalized to multi-fluid cases.

They remain valid for baryons, galaxies, ..., independently of small-scale physics.

These consistency relations rely on the following conditions:

- Gaussian initial conditions
- equivalence principle
- separation of scales

Exact results $\Rightarrow$ test of Gaussianity, of General Relativity, constraints on models

Null test: $0 = 0$ at equal times, if Gaussian initial conditions and GR.

C) Non-Gaussian initial conditions

If the initial density field is a nonlinear function of a Gaussian field:

$$\delta_{L0}(\mathbf{k}) = \chi_0(\mathbf{k}) + \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} d\mathbf{k}_i \delta_D\left(\mathbf{k} - \sum_{i=1}^{n} \mathbf{k}_i\right) \times f_{\mathrm{NL}\,0}^{(n)}(\mathbf{k}_1, \ldots, \mathbf{k}_n) \prod_{i=1}^{n} \chi_0(\mathbf{k}_i),$$

or its PDF is non-Gaussian:

$$\mathcal{P}(\delta_{L0}) = e^{-\int d\mathbf{k}\, \delta_{L0}(\mathbf{k})\delta_{L0}(-\mathbf{k})/2P_{\chi_0}(k)} \left[1 + \sum_{n=2}^{\infty} \int \prod_{i=1}^{n} d\mathbf{k}_i \times \delta_D\left(\sum_{i=1}^{n} \mathbf{k}_i\right) S_n(\mathbf{k}_1, \ldots, \mathbf{k}_n) \prod_{i=1}^{n} \delta_{L0}(\mathbf{k}_i)\right]$$

the consistency relations take a more complicated form:

$$\begin{aligned}\langle \delta_{L0}(\mathbf{k}') \prod_{j=1}^{m} \delta(\mathbf{k}_j, \tau_j) \rangle'_{k' \to 0} = &-P_{\chi_0}(k') \left\langle \prod_{j=1}^{m} \delta(\mathbf{k}_j, \tau_j) \right\rangle' \sum_{j=1}^{m} D_+(\tau_j) \frac{\mathbf{k}_j \cdot \mathbf{k}'}{k'^2} + P_{\chi_0}(k') \sum_{n=2}^{\infty} n \int \prod_{i=1}^{n-1} d\mathbf{k}'_i \\ &\times \delta_D\left(\mathbf{k}' - \sum_{i=1}^{n-1} \mathbf{k}'_i\right) S_n(-\mathbf{k}', \mathbf{k}'_1, \ldots, \mathbf{k}'_{n-1}) \left\langle \prod_{i=1}^{n-1} \delta_{L0}(\mathbf{k}'_i) \prod_{j=1}^{m} \delta(\mathbf{k}_j, \tau_j) \right\rangle'.\end{aligned}$$

new terms that do not vanish at equal times

Angular-averaged consistency relations

P.V. (2013), Kehagias et al.(2013), T. Nishimichi & P.V. (2014,2015)

A) Approximate symmetry

1311.4286

To go beyond the leading-order relations, or to obtain new relations, we must find additional symmetries.

If we make the change of variables $\eta = \ln D_+, \quad \mathbf{v} = \dot{a} f \mathbf{u}, \quad \Phi = (\dot{a} f)^2 \varphi$ where $f = \frac{d \ln D_+}{d \ln a}$

the equations of motion read as

$$\frac{\partial \delta}{\partial \eta} + \nabla \cdot [(1+\delta)\mathbf{u}] = 0, \qquad \frac{\partial \mathbf{u}}{\partial \eta} + \left(\frac{3\Omega_{\mathrm{m}}}{2f^2} - 1 \right) \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla \varphi, \qquad \nabla^2 \varphi = \frac{3\Omega_{\mathrm{m}}}{2f^2} \, \delta,$$

Within the approximation $\Omega_{\mathrm{m}}/f^2 \simeq 1$ all explicit dependence on cosmology disappears.

$\Rightarrow$ approximate symmetry

This remains true beyond shell crossing. The equation of motion of the particle trajectories reads as:

$$\frac{\partial^2 \mathbf{x}}{\partial \eta^2} + \left(\frac{3\Omega_{\mathrm{m}}}{2f^2} - 1 \right) \frac{\partial \mathbf{x}}{\partial \eta} = -\nabla \varphi$$

B) Angular averaging

To get rid of the leading-order kinematic effect, associated with the uniform motion of small-scale structures, we integrate over the angles of the soft modes.

$$\tilde{C}_W^n = \int d\mathbf{k}' \tilde{W}(k') \langle \tilde{\delta}_{L0}(\mathbf{k}') \tilde{\delta}(\mathbf{k}_1, t_1)..\tilde{\delta}(\mathbf{k}_n, t_n) \rangle = \int d\mathbf{k}' \tilde{W}(k') \, P_{L0}(k') \langle \frac{\mathcal{D}[\tilde{\delta}(\mathbf{k}_1, t_1)..\tilde{\delta}(\mathbf{k}_n, t_n)]}{\mathcal{D}\tilde{\delta}_{L0}(-\mathbf{k}')} \rangle$$

This means that we need to compute the impact of a large-scale spherical overdensity onto small-scale fluctuations.

This corresponds to a change of the background matter density: change of the cosmological parameter $\Omega_{\rm m}$

The approximate symmetry $\Omega_{\rm m}/f^2 \simeq 1$ allows us to describe the effect of a change of cosmological background

$$\tilde{\delta}_{\epsilon_0}(\mathbf{k}, t) = \tilde{\delta}[(1-\epsilon)\mathbf{k}, D_{+\epsilon_0}] + 3\epsilon \delta_D(\mathbf{k})$$

$$\left. \frac{\partial \tilde{\delta}(\mathbf{k}, t)}{\partial \epsilon_0} \right|_{\epsilon_0 = 0} = D_+(t) \left[\frac{13}{7} \frac{\partial \tilde{\delta}}{\partial \ln D_+} - \mathbf{k} \cdot \frac{\partial \tilde{\delta}}{\partial \mathbf{k}} \right]$$

This eventually leads to the angular-averaged consistency relations:

$$\int \frac{d\mathbf{\Omega}_{\mathbf{k}'}}{4\pi} \langle \tilde{\delta}(\mathbf{k}',t)\tilde{\delta}(\mathbf{k}_1,t)..\tilde{\delta}(\mathbf{k}_n,t)\rangle'_{k'\to 0} = P_L(k',t)\left[1+\frac{13}{21}\frac{\partial}{\partial \ln D_+} - \frac{1}{3}\sum_{i=1}^{n}\frac{\partial}{\partial \ln k_i}\right] \langle \tilde{\delta}(\mathbf{k}_1,t)..\tilde{\delta}(\mathbf{k}_n,t)\rangle'$$

These relations no longer vanish at equal times.

The lowest-order relation, for the bispectrum, reads as:

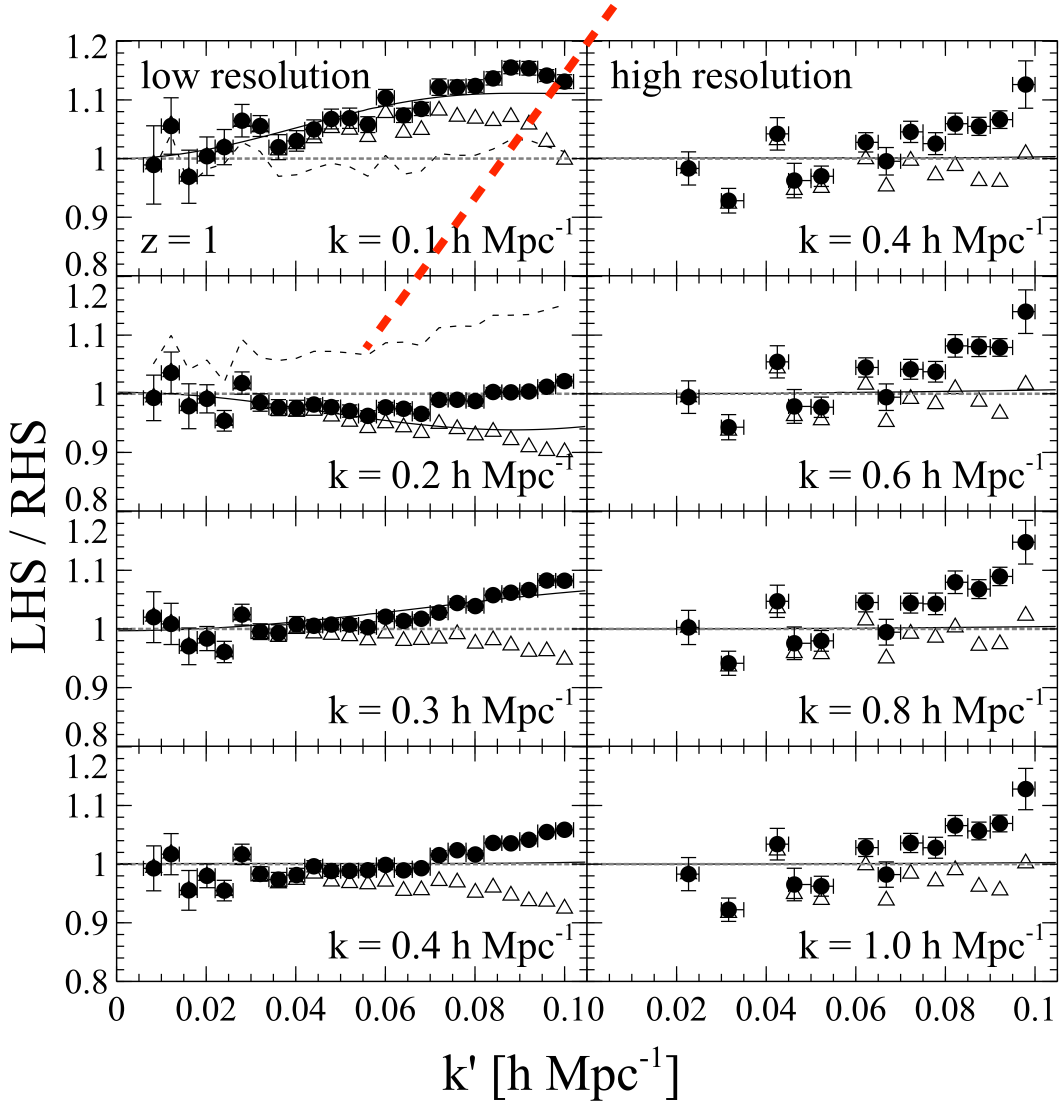
$$\int \frac{d\mathbf{\Omega}_{\mathbf{k}'}}{4\pi}\; B\left(\mathbf{k}', \mathbf{k}-\frac{\mathbf{k}'}{2}, -\mathbf{k}-\frac{\mathbf{k}'}{2}; t\right)_{k'\to 0} = P_L(k',t)\left[1+\frac{13}{21}\frac{\partial}{\partial \ln D_+} - \frac{1}{3}\frac{\partial}{\partial \ln k}\right] P(k,t)$$

Numerical check:

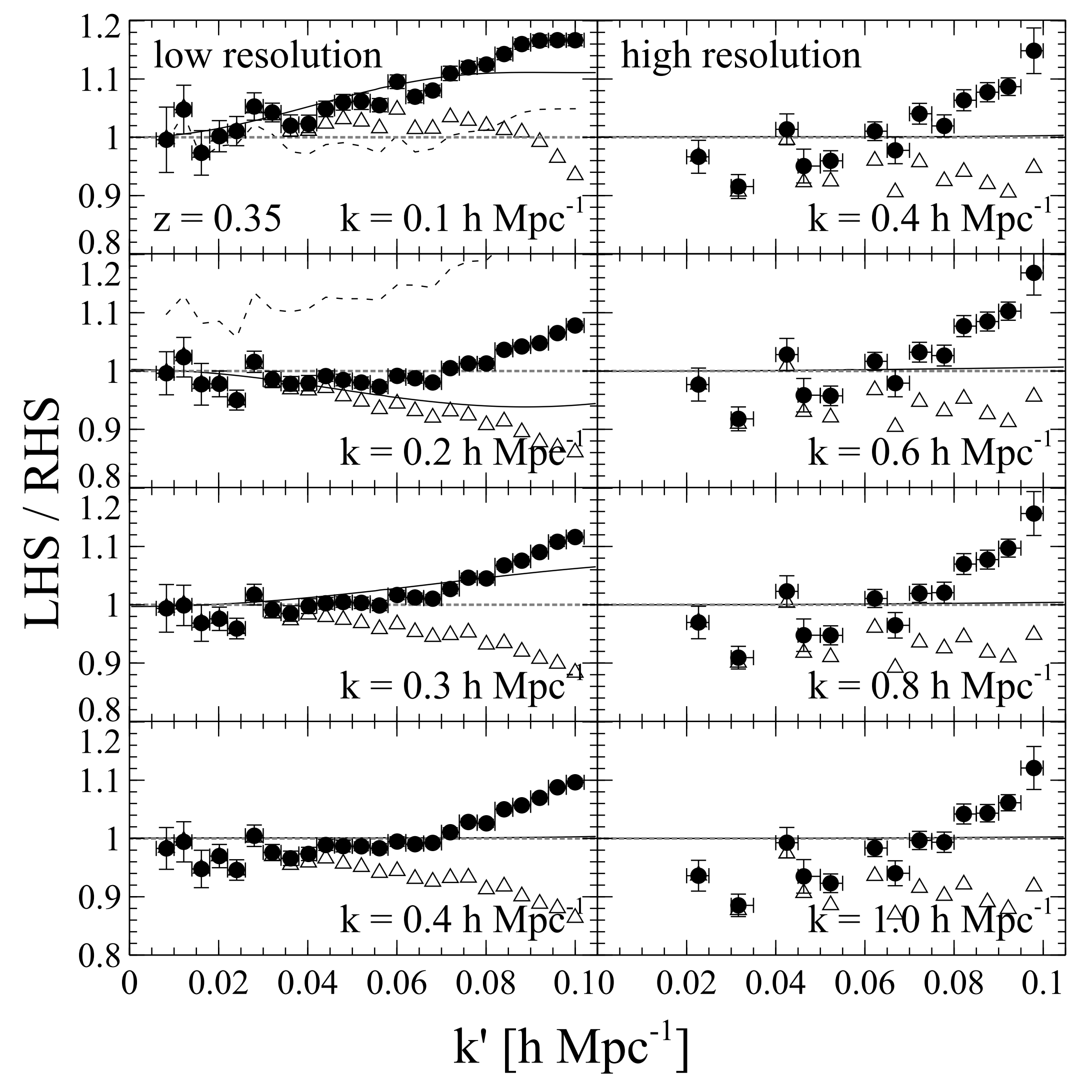
ratio of the nonlinear bispectrum to the consistency relation result, given by a product of one linear power spectrum and one nonlinear power spectrum.

consistency relation = 1

lowest-order PT result (out of the plot otherwise)



this approximate consistency relation significantly improves over lowest-order PT result, and goes up to k~1 h/Mpc



C) Redshift space

In redshift space the relations are more intricate:

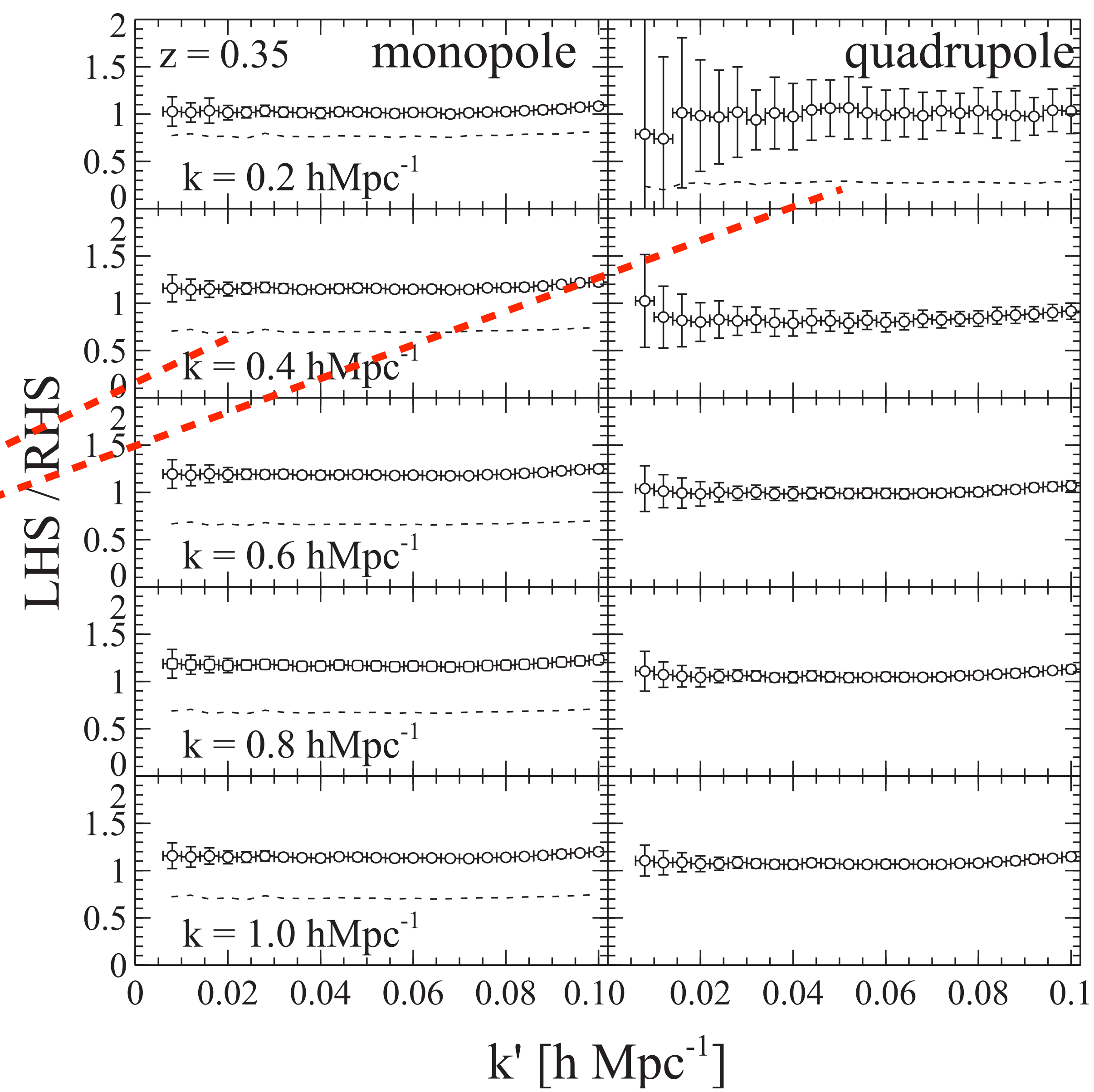
$$\int \frac{\mathrm{d}\Omega_{k'}}{4\pi} \left\langle \frac{\tilde{\delta}^s(\boldsymbol{k}')}{1+f\mu'^2} \tilde{\delta}^s(\boldsymbol{k}_1)\ldots\tilde{\delta}^s(\boldsymbol{k}_n) \right\rangle'_{k'\to 0} = P_L(k') \left[1 + \frac{f}{3} + \frac{13}{21}\frac{\partial}{\partial \ln D_+} + \left(\frac{13}{7}+f\right)\frac{f}{3}\frac{\partial}{\partial f} - \sum_{i=1}^{n} \frac{k_i}{3}\frac{\partial}{\partial k_i} - f \sum_{i=1}^{n} \frac{k_{ri}}{3}\frac{\partial}{\partial k_{ri}} \right] \langle \tilde{\delta}^s(\boldsymbol{k}_1)\ldots\tilde{\delta}^s(\boldsymbol{k}_n)\rangle'.$$

Bispectrum monopole:

$$\int_{-1}^{1} \frac{\mathrm{d}\mu}{2} \int \frac{\mathrm{d}\Omega_{k'}}{4\pi} \frac{B^s_{k'\to 0}}{1+f\mu'^2} = P_L(k') \left\{ \left[1 + \frac{f}{3} + \frac{13}{21}\frac{\partial}{\partial \ln D_+} + \left(\frac{13}{7}+f\right)\frac{f}{3}\frac{\partial}{\partial f} - \frac{1}{3}\frac{\partial}{\partial \ln k} \right] P^s_0(k) - \frac{2f}{15} P^s_2(k) - \frac{f}{3}\frac{\partial}{\partial \ln k}\left[\frac{1}{3} P^s_0(k) + \frac{2}{15} P^s_2(k) \right] \right\}$$

consistency relation = 1

lowest-order PT result



this approximate consistency relation significantly improves over lowest-order PT result, and goes up to k~1 h/Mpc

Density-velocity consistency relations

L. Rizzo, D. Mota and P.V. (2016,2017)

A) Non-zero equal-time relation

Let us go back to the exact kinematic consistency relations.

For a long-wavelength perturbation, we had the transformation:

$$\mathbf{x}(\mathbf{q},\tau) \to \hat{\mathbf{x}}(\mathbf{q},\tau) = \mathbf{x}(\mathbf{q},\tau) + D_+(\tau)\Delta\Psi_{L0}(\mathbf{q}),$$

This gave us for the density contrast:

$$\tilde{\delta}(\mathbf{k},\tau) \to \hat{\tilde{\delta}}(\mathbf{k},\tau) = \tilde{\delta}(\mathbf{k},\tau) e^{-i\mathbf{k}\cdot D_+ \Delta\Psi_{L0}} = \tilde{\delta}(\mathbf{k},\tau) - iD_+(\mathbf{k}\cdot\Delta\Psi_{L0})\tilde{\delta}(\mathbf{k},\tau),$$

This also gives for the velocity field:

$$\tilde{\mathbf{v}}(\mathbf{k},\tau) \to \hat{\tilde{\mathbf{v}}}(\mathbf{k},\tau) = \tilde{\mathbf{v}}(\mathbf{k},\tau) - iD_+(\mathbf{k}\cdot\Delta\Psi_{L0})\tilde{\mathbf{v}}(\mathbf{k},\tau) + \frac{dD_+}{d\tau}\Delta\Psi_{L0}\delta_D(\mathbf{k}),$$

uniform translation

change of the velocity amplitude

this effect will not disappear in equal-time statistics !

This Dirac term at k=0 will be relevant in composite operators (momentum, ..):

$$\mathbf{p} = (1+\delta)\mathbf{v}, \qquad \tilde{\mathbf{p}}(\mathbf{k}) = \tilde{\mathbf{v}}(\mathbf{k}) + \int d\mathbf{k}_1 d\mathbf{k}_2 \delta_D(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k})\tilde{\delta}(\mathbf{k}_1)\tilde{\mathbf{v}}(\mathbf{k}_2).$$

This leads to:

$$k' \to 0: \quad \frac{\mathcal{D}\tilde{\mathbf{p}}(\mathbf{k})}{\mathcal{D}\tilde{\delta}_{L0}(\mathbf{k}')} = D_+ \frac{\mathbf{k}\cdot\mathbf{k}'}{k'^2}\tilde{\mathbf{p}}(\mathbf{k}) + \frac{dD_+}{d\tau} i \frac{\mathbf{k}'}{k'^2}[\delta_D(\mathbf{k}) + \tilde{\delta}(\mathbf{k})].$$

Nonzero consistency relation at equal times:

$$\left\langle \tilde{\delta}(\mathbf{k}') \prod_{j=1}^{n} \tilde{\delta}(\mathbf{k}_j) \prod_{j=n+1}^{n+m} \tilde{\mathbf{p}}(\mathbf{k}_j) \right\rangle'_{k'\to 0} = -iP_L(k') \frac{d\ln D_+}{d\tau} \sum_{i=n+1}^{n+m} \left\langle \prod_{j=1}^{n} \tilde{\delta}(\mathbf{k}_j) \prod_{j=n+1}^{i-1} \tilde{\mathbf{p}}(\mathbf{k}_j) \left(\frac{\mathbf{k}'}{k'^2} [\delta_D(\mathbf{k}_i) + \tilde{\delta}(\mathbf{k}_i)] \right) \prod_{j=i+1}^{n+m} \tilde{\mathbf{p}}(\mathbf{k}_j) \right\rangle',$$

For the bispectrum:

$$\langle \tilde{\delta}(\mathbf{k}') \tilde{\delta}(\mathbf{k}) \tilde{\mathbf{p}}(-\mathbf{k}) \rangle'_{k'\to 0} = -i \frac{\mathbf{k}'}{k'^2} \frac{d\ln D_+}{d\tau} P_L(k') P(k),$$

Again, it also applies to biased tracers, independently of baryonic physics etc.....

$$\langle \tilde{\delta}(\mathbf{k}') \tilde{\delta}_g(\mathbf{k}) \tilde{\mathbf{p}}_g(-\mathbf{k}) \rangle'_{k'\to 0} = -i \frac{\mathbf{k}'}{k'^2} \frac{d\ln D_+}{d\tau} P_L(k') P_{\delta_g \delta_g}(k),$$

We also obtain for the divergence of the momentum field:

$$\lambda \equiv \nabla \cdot [(1+\delta)\mathbf{v}], \qquad \tilde{\lambda}(\mathbf{k}) = i\mathbf{k} \cdot \tilde{\mathbf{p}}(\mathbf{k}).$$

$$\langle \tilde{\delta}(\mathbf{k}') \tilde{\delta}_g(\mathbf{k}) \tilde{\lambda}_g(-\mathbf{k}) \rangle'_{k'\to 0} = -\frac{\mathbf{k} \cdot \mathbf{k}'}{k'^2} \frac{d\ln D_+}{d\tau} P_L(k') P_{\delta_g \delta_g}(k),$$

B) Link with observable quantities

1) ISW

Secondary CMB anisotropy due to the Integrated Sachs-Wolfe effect:

$$\Delta_{\rm ISW}(\boldsymbol{\theta}) = 2 \int \mathrm{d}\eta \, \mathrm{e}^{-\tau(\eta)} \frac{\partial \Psi}{\partial \eta}[r, r\boldsymbol{\theta}; \eta],$$

This can be expressed in terms of the density field and its time derivative through the Poisson equation:

$$\frac{\partial \tilde{\Psi}}{\partial \eta} = \frac{4\pi \mathcal{G}_{\rm N} \bar{\rho}_0}{k^2 a} (\tilde{\lambda} + \mathcal{H}\tilde{\delta}),$$

with:

$$\lambda \equiv \nabla \cdot [(1+\delta)\boldsymbol{v}] = -\frac{\partial \delta}{\partial \eta}.$$

2) kSZ

Secondary CMB anisotropy due to the kinematic SZ effect:

$$\Delta_{\rm kSZ}(\boldsymbol{\theta}) = -\int \mathrm{d}\boldsymbol{l} \cdot \boldsymbol{v}_{\rm e} \sigma_{\rm T} n_{\rm e} \mathrm{e}^{-\tau} = \int \mathrm{d}\eta \, I_{\rm kSZ}(\eta) \boldsymbol{n}(\boldsymbol{\theta}) \cdot \boldsymbol{p}_{\rm e},$$

with:

$$I_{\rm kSZ}(\eta) = -\sigma_{\rm T} \bar{n}_{\rm e} a \mathrm{e}^{-\tau},$$

$$n_{\rm e} \boldsymbol{v}_{\rm e} = \bar{n}_{\rm e}(1+\delta_{\rm e})\boldsymbol{v}_{\rm e} = \bar{n}_{\rm e} \boldsymbol{p}_{\rm e}.$$

C) ISW consistency relations for 3-pt correlations

I) Galaxy-galaxy-ISW correlation

$$\xi_3(\delta_{\rm g}^{\rm s}, \delta_{\rm g_1}^{\rm s}, \Delta_{\rm ISW_2}^{\rm s}) = \langle \delta_{\rm g}^{\rm s}(\boldsymbol{\theta})\, \delta_{\rm g_1}^{\rm s}(\boldsymbol{\theta}_1)\, \Delta_{\rm ISW_2}^{\rm s}(\boldsymbol{\theta}_2)\rangle.$$

$$\Theta \gg \Theta_{\rm L}, \quad \Theta \gg \Theta_j, \quad |\boldsymbol{\theta} - \boldsymbol{\theta}_j| \gg |\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|. \quad \Rightarrow \quad k \ll k_{\rm L}, \quad k \ll k_j,$$

$$\xi_3 = \frac{(\boldsymbol{\theta} - \boldsymbol{\theta}_2)\cdot(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2)}{|\boldsymbol{\theta} - \boldsymbol{\theta}_2||\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|}(2\pi)^4 \int {\rm d}\eta\, b_{\rm g} I_{\rm g} I_{\rm g_1} I_{\rm ISW_2} \frac{{\rm d}\ln D}{{\rm d}\eta} \int_0^\infty {\rm d}k_\perp {\rm d}k_{1\perp}\, \tilde{W}_\Theta(k_\perp r)\tilde{W}_{\Theta_1}(k_{1\perp} r)\tilde{W}_{\Theta_2}(k_{1\perp} r)$$

$$\times P_{\rm L}(k_\perp,\eta) P_{\rm g_1,m}(k_{1\perp},\eta) J_1(k_\perp r|\boldsymbol{\theta} - \boldsymbol{\theta}_2|)\; J_1(k_{1\perp} r|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|),$$

- This is an explicit expression of the form: $\langle \delta_g \delta_g \Delta_{\rm ISW} \rangle = \langle \delta_g \delta \rangle_L \; \langle \delta_g \delta \rangle$

- Specific angular dependence (can be understood from symmetry).

2) Lensing-lensing-ISW correlation

Three-point correlation with the lensing convergence:

$$\xi_3(\kappa^{\mathrm{s}}, \kappa_1^{\mathrm{s}}, \Delta^{\mathrm{s}}_{\mathrm{ISW}_2}) = \langle \kappa^{\mathrm{s}}(\boldsymbol{\theta})\, \kappa_1^{\mathrm{s}}(\boldsymbol{\theta}_1)\, \Delta^{\mathrm{s}}_{\mathrm{ISW}_2}(\boldsymbol{\theta}_2) \rangle,$$

$$\xi_3 = \frac{(\boldsymbol{\theta}-\boldsymbol{\theta}_2)\cdot(\boldsymbol{\theta}_1-\boldsymbol{\theta}_2)}{|\boldsymbol{\theta}-\boldsymbol{\theta}_2||\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|}(2\pi)^4 \int \mathrm{d}\eta\, I_\kappa I_{\kappa_1} I_{\mathrm{ISW}_2} \frac{\mathrm{d}\ln D}{\mathrm{d}\eta} \int_0^\infty \mathrm{d}k_\perp \mathrm{d}k_{1\perp}\, \tilde{W}_\Theta(k_\perp r)\tilde{W}_{\Theta_1}(k_{1\perp} r)\tilde{W}_{\Theta_2}(k_{1\perp} r)$$

$$\times P_{\mathrm{L}}(k_\perp,\eta)P(k_{1\perp},\eta)J_1(k_\perp r|\boldsymbol{\theta}-\boldsymbol{\theta}_2|)\, J_1(k_{1\perp} r|\boldsymbol{\theta}_1-\boldsymbol{\theta}_2|).$$

- This is an explicit expression of the form: $\langle \kappa\kappa\Delta_{\mathrm{ISW}} \rangle = P_L \; P$

- Specific angular dependence

C) kSZ consistency relations for 3-pt correlations

I) Galaxy-galaxy-kSZ correlation

$$\xi_3(\delta_{\rm g}^{\rm s}, \delta_{\rm g_1}^{\rm s}, \Delta_{\rm kSZ_2}^{\rm s}) = \langle \delta_{\rm g}^{\rm s}(\boldsymbol{\theta})\, \delta_{\rm g_1}^{\rm s}(\boldsymbol{\theta}_1)\, \Delta_{\rm kSZ_2}^{\rm s}(\boldsymbol{\theta}_2) \rangle,$$

$$\xi_{3\|}^{\|} = -(2\pi)^4 \int {\rm d}\eta\, \frac{\rm d}{{\rm d}\eta}\left[b_{\rm g} I_{\rm g} D\right] I_{\rm g_1} I_{\rm kSZ_2} \frac{{\rm d}D}{{\rm d}\eta} \int_0^\infty {\rm d}k_\perp {\rm d}k_{1\perp}\, \tilde{W}_\Theta(k_\perp r) \tilde{W}_{\Theta_1}(k_{1\perp} r) \tilde{W}_{\Theta_2}(k_{1\perp} r)$$
$$\times \frac{k_{1\perp}}{k_\perp} P_{L0}(k_\perp) P_{\rm g_1,e}(k_{1\perp},\eta) J_0(k_\perp r|\boldsymbol{\theta} - \boldsymbol{\theta}_2|)\, J_0(k_{1\perp} r|\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2|),$$

+....

This is an explicit expression of the form:

$$\langle \delta_g \delta_g \Delta_{\rm kSZ} \rangle = \langle \delta_g \delta \rangle_L \, \langle \delta_g \delta_e \rangle$$

which corresponds to:

$$\langle \delta_g(\vec{k}') \delta_{g1}(\vec{k}_1) [(1+\delta_e)\vec{v}_e](\vec{k}_2) \rangle'_{k'\to 0} = \langle \delta_{Lg}(\vec{k}) \vec{v}_{Le}(-\vec{k}) \rangle' \langle \delta_{g1}(\vec{k}_1) \delta_e(-\vec{k}_1) \rangle'$$

Conclusions

- Consistency relations can provide exact results in regimes that are difficult to model (bias, non-linear,...).

- Simplest cases vanish at equal times and provide null test of GR or primordial Gaussianity.

- Introducing the velocity or the time derivative of the density, one obtains relations that do not vanish at equal times. They can be related to ISW or kSZ statistics.

- It remains to be seen whether this can be competitive for practical applications.