CMB spectroscopy at third order in cosmological perturbations

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Linear CMB was successful

Density fluctuations are

- Almost scale invariant
- Adiabatic
- Gaussian
Small scales are not observed!
Neither is non-Gaussianity (Bispectrum)
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We know very little on

\[ k > 1 \text{ Mpc}^{-1} \]
How can we observe small scales?
CMB spectral distortions
COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE

Theory and observation agree

Intensity, 10^-4 ergs / cm^2 sr sec cm^-1

Waves / centimeter

http://lambda.gsfc.nasa.gov/product/cobe/cobe_image_table.cfm
Spectral distortions

Deviations from the isotropic Planck distributions.
Spectral distortions
Deviations from the isotropic Planck distributions.

\[ \delta T / T \]

Chemical potential \( \mu \)

200 400 600 800 1000 1200 GHz

Cosmic Microwave Background Spectrum from COBE

Theory and observation agree

Intensity, \( 10^{-4} \) ergs/cm^2 sr sec cm\(^{-1} \)

Waves / centimeter

http://lambda.gsfc.nasa.gov/product/cobe/cobe_image_table.cfm
Spectral distortions
Deviations from the isotropic Planck distributions.
Spectral distortions and energy release in the early Universe

\[ \mu = 1.4 \frac{\Delta \rho}{\rho} \quad \left| \begin{array}{c} 5 \times 10^4 < z < 2 \times 10^6 \\ \end{array} \right. \]

\[ y = \frac{1}{4} \frac{\Delta \rho}{\rho} \quad \left| \begin{array}{c} z < 5 \times 10^4 \\ \end{array} \right. \]

• Acoustic damping
• Annihilation of dark matter
• PBH evaporation
• etc...
\[
\frac{\Delta \rho}{\rho} \bigg|_{\text{Silk}} = 6 \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle \sim \langle \zeta^2 \rangle
\]
CMB at second order!

Short mode × Short mode → homogenous mode

\[ \frac{\Delta \rho}{\rho} \bigg|_{\text{Silk}} = 6 \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle \sim \langle \zeta^2 \rangle \]

Khatri Sunyaev (2013)
Summary at second order

• COBE FIRAS constraints on $0.1 < k \text{ Mpc} < 10^4$

\[
\mu, y < 10^{-5} \iff \mathcal{P}_\zeta < 10^{-5}
\]

• **Complementary constraints** on density perturbations.

• Future measurements: $\mu < 10^{-8}, y < 10^{-9}$
CMB at third order
→ Bispectrum?
Boltzmann equation

\[
\frac{\partial f_\gamma}{\partial t} + \frac{d}{dt} \frac{\partial f_\gamma}{\partial x} + \frac{dp_\gamma}{dt} \frac{\partial f_\gamma}{\partial p_\gamma} = \int_{p_{\gamma}',p_e,p_e'} \delta^{(3)}(p_{\text{tot.}}) \sum_{\text{spins}} |M|^2 
\times \left[ f_\gamma' f_e'(1 + f_\gamma)(1 - f_e) - f_\gamma f_e(1 + f_\gamma')(1 - f_e') \right]
\]

Expand this up to **third order** in perturbations...
\textbf{n}^{\text{th}} \text{ order Boltzmann equations}

\textit{If } \frac{T_e}{m_e} \ll 1, \text{ then derivative operators in the Compton scattering collision terms are } \left( \frac{p \partial}{\partial p} \right)^n \\
[AO 2016]
3rd order Boltzmann equations

Up to third order we only have

\[(p \partial / \partial p)^1, (p \partial / \partial p)^2, (p \partial / \partial p)^3\]

- Ansatz can be written as some derivatives of Planck dist.
- No need to follow full phase space evolution.
Solution at third order

Sum of $\delta T/T$, $y$, and $\kappa$ (ignore $\mu$ for simplicity)
Equation to solve

- Simple even at third order!

\[ \frac{d\langle\langle \kappa \rangle\rangle}{d\eta} = -2\langle\langle yA \rangle\rangle \]

First order temperature collision term:

\[ -\dot{\tau}^{-1} A = \frac{\Theta_{00}}{\sqrt{4\pi}} - \Theta + V + \frac{1}{10} \sum_{m=-2}^{2} Y_{2m} \Theta_{2m} , \]

[See textbook by Dodelson]
Results

- For scale invariant perturbations (local):
  \[ \kappa \approx 10^{-18} f_{NL}^{loc} \left( \frac{y}{4.0 \times 10^{-9}} \right) \]

- \( \kappa < 10^{-9} \) in Future space mission.

- Too small (?)
Results

• Normalization of nonlinear parameter:

\[ f_{NL} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \]

• No one knows the power on small scales

\[ 10^{-18} f_{NL}^{\text{loc.}} \left( \frac{y}{4.0 \times 10^{-9}} \right) \rightarrow \langle \zeta^2 \rangle^2 f_{NL}^{\text{loc.}} \]
Forecast in $P_\zeta$-$f_{NL}$ plane

$f_{NL}^\text{loc.} \langle \zeta^2 \rangle^2 < 10^{-10}$
Summary

• Third order distortion arises from non-Gaussianity.

• It is small for scale invariant perturbations.

• But, no one knows both bispectrum and powerspectrum on small scales.

• non-Gaussianity can be constrained for bigger powerspectrum.
Future works

• Are cluster contaminations (relativistic SZ effects) crucial?

• What about equilateral NG?
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\[ k_1 = k_2 = k_3 = 1 \sim 100 \text{Mpc}^{-1} \]