The different varieties of the Suyama-Yamaguchi consistency relation and its violation as a signal of statistical inhomogeneity

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Science: Theory vs Observation

- In Cosmology the predictions have a quantum input, which requires the notion of an ensemble of systems. However, we just observe one member of the ensemble: our own Universe. So, how can we compare theory with observations?

The CMB Temperature Map

- How do we study it?: through its statistical properties. We can obtain, among others, the average temperature and the dispersion of temperature values around the average. Indeed, at first order in cosmological perturbation theory, the dispersion can be parametrized by the spectrum $\mathcal{P}_\zeta(\vec{k})$ of the primordial curvature perturbation ζ.

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Comparing Theory and Experiment

- We have a very good theoretical framework: inflation + quantum mechanics.
- We have very good observational data: PLANCK satellite (1% precision).
- But science is done by comparing theory with observations, and the observations require an ensemble of universes. However, the experiment was performed only once in history!

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The Ergodic Theorem at Rescue!

- Well, the good part of the story is that, if there is statistical homogeneity, we can obtain information about the n-point correlators in the ensemble by performing n-point correlators in the set of universes created by spatial translations of our own Universe.
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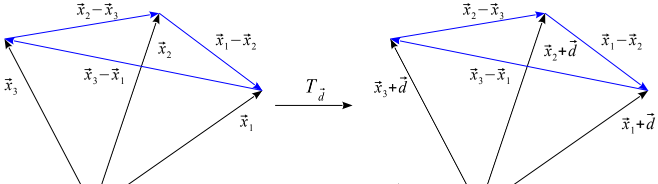
Statistical Homogeneity

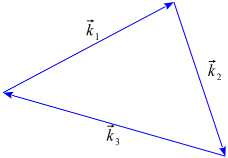
$$\langle \zeta(\vec{x}_1)\zeta(\vec{x}_2)\ldots\zeta(\vec{x}_n)\rangle \xrightarrow{T_{\vec{d}}} \langle \zeta(\vec{x}_1+\vec{d})\zeta(\vec{x}_2+\vec{d})\ldots\zeta(\vec{x}_n+\vec{d})\rangle$$



$$FT: \zeta(\vec{x}) = \int d^3k\, \zeta(\vec{k})\, e^{i\vec{k}.\vec{x}}$$

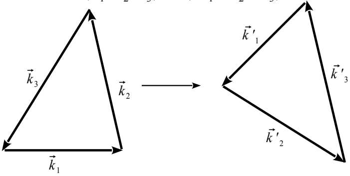
$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\ldots\zeta(\vec{k}_n)\rangle = \delta(\vec{k}_{12\ldots n}) F_\zeta(\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n)$$

$$\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$$

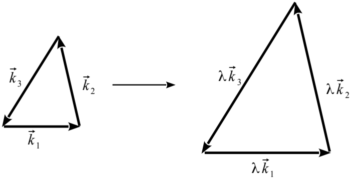


Statistical Isotropy and Scale Invariance

$$B(\vec{k}_1, \vec{k}_2, \vec{k}_3) = B(\vec{k}'_1, \vec{k}'_2, \vec{k}'_3)$$



$$B(\vec{k}_1, \vec{k}_2, \vec{k}_3) = B(\lambda\vec{k}_1, \lambda\vec{k}_2, \lambda\vec{k}_3)$$

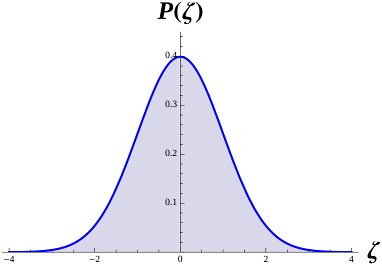


Gaussianity

$$\mathrm{P}[\zeta(\vec{x})] = \frac{1}{\sqrt{2\pi\langle\zeta^2(\vec{x})\rangle}} e^{-\zeta^2(\vec{x})/2\langle\zeta^2(\vec{x})\rangle}$$

The PDF is fully described with the two-point correlator, the *power spectrum* (PS):

$$\begin{aligned} \langle\zeta_{\vec{k}}\zeta_{\vec{k}'}\rangle &= (2\pi)^3\delta(\vec{k}+\vec{k}')P_\zeta(k) \\ &= \delta(\vec{k}+\vec{k}')\frac{2\pi^2}{k^3}A_\zeta\left(\frac{k}{k_0}\right)^{n_\zeta-1} \end{aligned}$$



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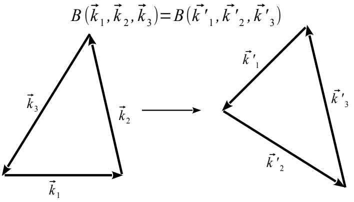

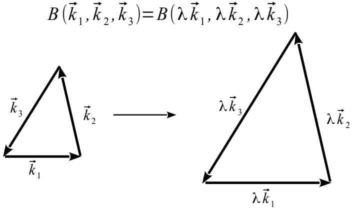

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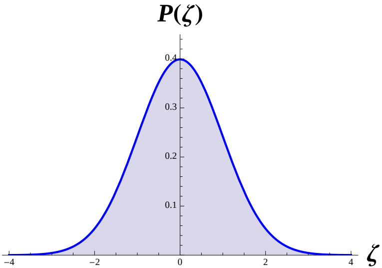

Statistical Descriptors for ζ

If the PDF is anisotropic and non-Gaussian, we have the following statistical descriptors

$$\text{SD:}\begin{cases} \text{Spectrum} & P_\zeta & \begin{cases} \text{Amplitude } A_\zeta, \\ \text{Spectral Index } n_\zeta, \\ \text{The level of statistical anisotropy } g_\zeta, \\ \text{The preferred direction } \hat{d}. \end{cases} \\ \text{Bispectrum} & B_\zeta & \begin{cases} \text{Products of the spectrum } P_\zeta, \\ \text{The level of non} - \text{gaussianity } f_{\rm NL}. \end{cases} \\ \text{Trispectrum} & T_\zeta & \begin{cases} \text{Products of the spectrum } P_\zeta, \\ \text{The level of non} - \text{gaussianity } \tau_{\rm NL}, \\ \text{The level of non} - \text{gaussianity } g_{\rm NL}. \end{cases} \\ . \\ . \\ . \\ (n-1) - \text{spectrum} & M_\zeta & \begin{cases} \text{Products of the spectrum } P_\zeta, \\ \text{Several levels of non} - \text{gaussianity}. \end{cases} \end{cases}$$

Statistical Descriptors for ζ

NG parameters in the BS and the TS

$$f_{\rm NL}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \frac{5}{6}\frac{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle}{P_\zeta(\vec{k}_1)P_\zeta(\vec{k}_2) + 2\text{perm.}}$$

$$\tau_{\rm NL}(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) = \frac{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle}{P_\zeta(\vec{k}_1)P_\zeta(\vec{k}_2)P_\zeta(\vec{k}_{12}) + 11\text{perm.}}$$

$$g_{\rm NL}(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) = \frac{25}{54}\frac{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\zeta_{\vec{k}_4}\rangle}{P_\zeta(\vec{k}_1)P_\zeta(\vec{k}_2)P_\zeta(\vec{k}_3) + 3\text{perm.}}$$

The Suyama-Yamaguchi (SY) consistency relations
The SY consistency relation relates two of the statistical descriptors (SD) for the primordial curvature perturbation ζ, namely the levels of non-gaussianity in the bispectrum $f_{\rm NL}$ and in the trispectrum $\tau_{\rm NL}$.

First Variety (Suyama and Yamaguchi, PRD 2008 - Sugiyama, JCAP 2012)

$$\tau_{\rm NL} \geqslant \left(\frac{6}{5} f_{\rm NL}\right)^2$$

Conditions

- *Condition 1*: The calculation of $f_{\rm NL}$ and $\tau_{\rm NL}$ is performed at all loop corrections in the diagrammatic approach of the δN formalism.
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- *Condition 3*: The fields involved are gaussian.
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$$\boxed{\int_{\vec{k}_1,\vec{k}_3\in b_S} \frac{d^3\vec{k}_1 d^3\vec{k}_3}{(2\pi)^6}\tau_{\rm NL}(\vec{k}_1,\vec{k}_3) \geqslant \left[\frac{6}{5}\int_{\vec{k}_2\in b_S} \frac{d^3\vec{k}_2}{(2\pi)^3} f_{\rm NL}(\vec{k}_2)\right]^2}$$

b_S is a narrow band of wavevectors which are very near some some $\vec{k}_S$.

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Third Variety (Assassi, Baumann, and Green, JCAP 2012)

$$\int \frac{d^3\vec{k}_1 d^3\vec{k}_3}{(2\pi)^6} \tau_{\rm NL}(\vec{k}_1,\vec{k}_3) P_\zeta(\vec{k}_1) P_\zeta(\vec{k}_3) \geqslant \left[\frac{6}{5}\int \frac{d^3\vec{k}_2}{(2\pi)^3} f_{\rm NL}(\vec{k}_2) P_\zeta(\vec{k}_2)\right]^2$$

The conditions for this variety are the same as in the previous one.

Fourth variety (Rodríguez, Beltrán Almeida, and Valenzuela-Toledo, JCAP 2013)

$$\tau_{\rm NL}(\vec{k}_1,\vec{k}_3) \geqslant \left(\frac{6}{5}\right)^2 f_{\rm NL}(\vec{k}_1) f_{\rm NL}(\vec{k}_3)$$

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$$\tau_{\rm NL}(\vec{k}_1,\vec{k}_3) \geqslant \left(\frac{6}{5}\right)^2 f_{\rm NL}(\vec{k}_1) f_{\rm NL}(\vec{k}_3)$$

This is a direct generalization of the first variety when there is no scale-invariance and whose form is easily inspired from the second and third varieties.

Third Variety (Assassi, Baumann, and Green, JCAP 2012)

$$\int \frac{d^3\vec{k}_1 d^3\vec{k}_3}{(2\pi)^6} \tau_{\rm NL}(\vec{k}_1,\vec{k}_3) P_\zeta(\vec{k}_1) P_\zeta(\vec{k}_3) \geqslant \left[\frac{6}{5}\int \frac{d^3\vec{k}_2}{(2\pi)^3} f_{\rm NL}(\vec{k}_2) P_\zeta(\vec{k}_2)\right]^2$$

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Conditions

- *Condition 1*: The calculation of $f_{\rm NL}$ and $\tau_{\rm NL}$ is performed at tree level in the diagrammatic approach of the δN formalism.
- *Condition 2*: The inflationary dynamics is driven by any number of slowly-rolling Lorentz-invariant scalar fields.
- *Condition 3*: The fields involved can be non-gaussian.
- *Condition 4*: The field perturbations, in principle, might not be scale-invariant.
- *Condition 5*: $f_{\rm NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ is evaluated in the squeezed limit ($\vec{k}_1 \rightarrow 0$) while $\tau_{\rm NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$ is evaluated in the collapsed limit ($\vec{k}_1 + \vec{k}_2 \rightarrow 0$).

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$$\tau_{\rm NL}(\vec{k}_1, \vec{k}_1) \geqslant \left(\frac{6}{5} f_{\rm NL}(\vec{k}_1)\right)^2$$

This comes from the second variety when $b_S \to 0$.

1. It is valid even if there is statistical anisotropy and even if there is strong scale dependence.
2. The only required condition is statistical homogeneity.
3. This is the first time this variety is reported in the literature.
4. An observed violation of this consistency relation would imply statistical inhomogeneity which, in turn, would imply the impossibility of comparing theory and observation, affecting the foundations on which cosmology is constructed as a science.

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$$\boxed{\tau_{\rm NL}^{\rm iso} \geqslant \left(\frac{6}{5} f_{\rm NL}^{\rm iso}\right)^2}$$

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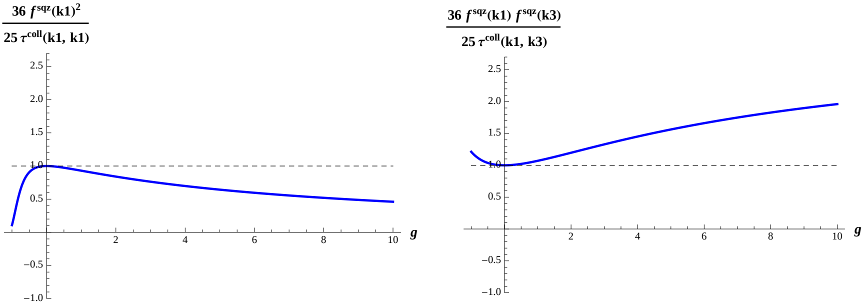
Results for the Fourth and Fifth variety in a Vector Curvaton-like Model



1. As expected, the fourth variety can be violated in several wavevector configurations.
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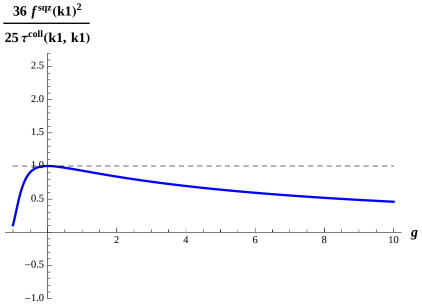

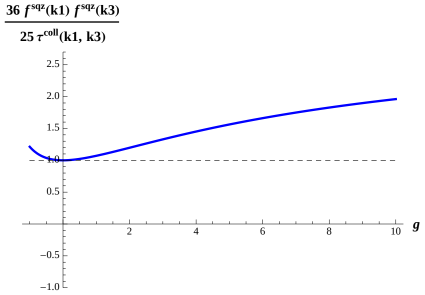

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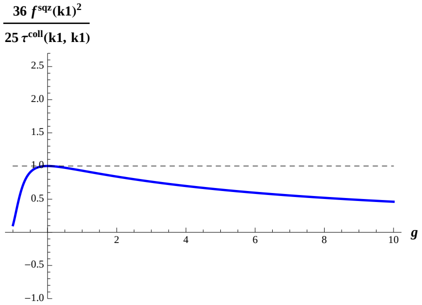

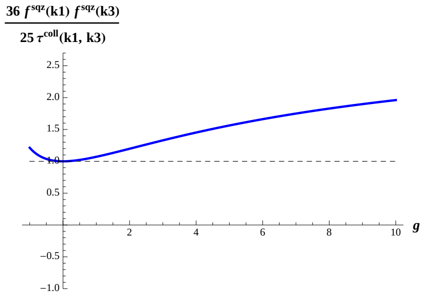

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Conclusions

- There are six different varieties of the Suyama-Yamaguchi consistency relation.
- The fourth variety of the SY consistency relation might be badly violated is there is strong scale dependence (models involving vector fields).
- The fifth variety of the SY consistency relation is a consequence of the fundamental assumption of statistical homogeneity.
- An observed violation of the fifth variety of the consistency relation would imply statistical inhomogeneity which, in turn, would imply the impossibility of comparing theory and observation,
affecting the foundations on which cosmology is constructed as a science.