Modulated decay and the curvaton mechanism **(in the light of $F(\varphi)R$ Gravity)**

Tomohiro Matsuda (SIT) / COSMO13@Cambridge

Abstract

Modulation and the curvaton converts isocurvature perturbations into the curvature perturbation. In Einstein gravity, the isocurvature perturbation is a fraction perturbation on a uniform density (i.e, uni-Hubble) hypersurface

$$\delta\rho_\sigma = -\delta\rho_r \neq 0 \,(\text{iso}) \qquad \text{for} \qquad \rho_{\text{tot}} = \rho_\sigma + \rho_r = \text{constant}$$

$$\Downarrow \Downarrow \Downarrow \Downarrow \Downarrow$$

$\delta\rho_\sigma \neq 0$ causes the curvaton mechanism.

If radiation is diluted before the curvaton decay, we find

$$\zeta \simeq r_\sigma \zeta_\sigma = \left[\frac{3\rho_\sigma}{3\rho_\sigma + 4\rho_r}\right]\left(\frac{\delta\rho_\sigma}{\rho_\sigma}\right)$$

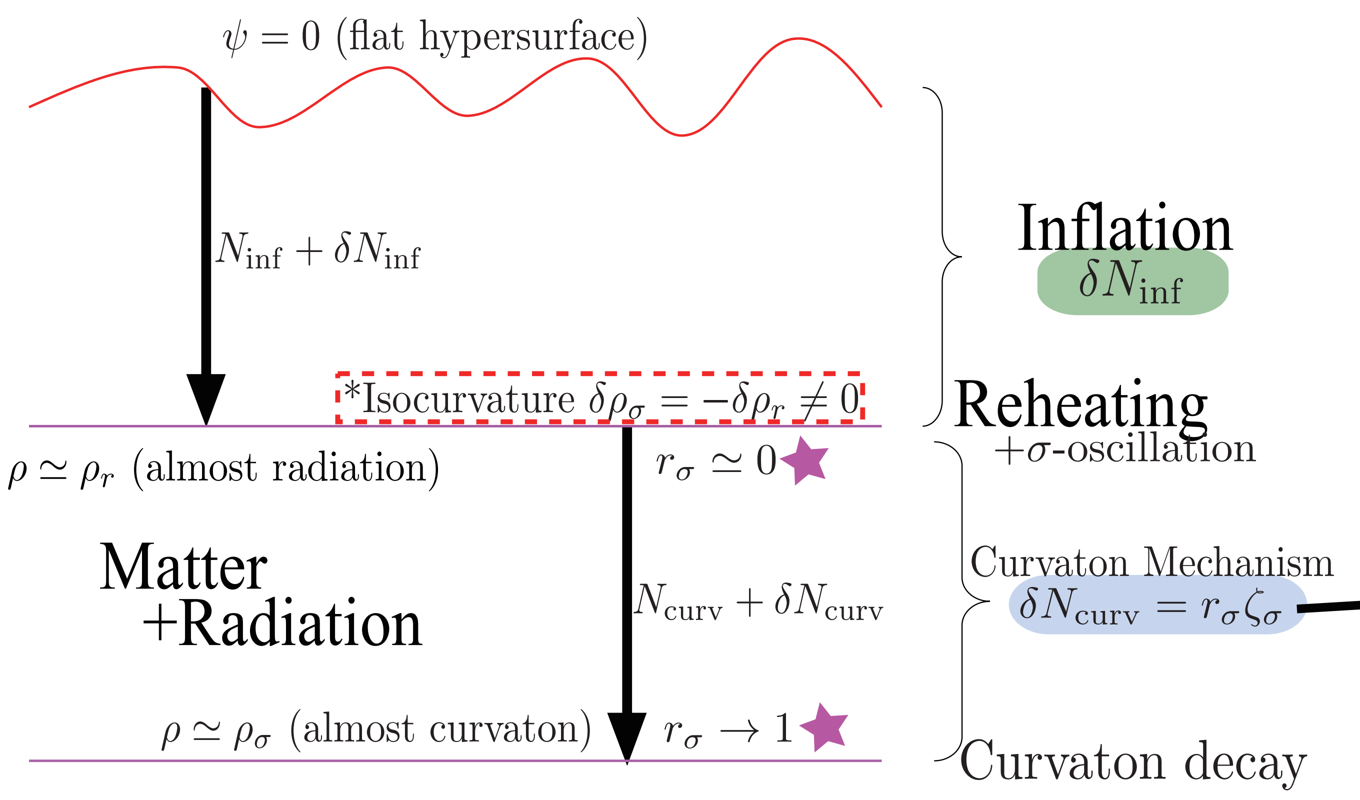
Now my question is:

"What could be the **Curvaton mechanism in FR Gravity**?"

"Is it New? If so, what is the counterpart in the Einstein Gravity after frame change"

Let me summarize the simple curvaton before "FR-curvaton".

The curvaton in Einstein Gravity



Component perturtbations

$$\zeta_\sigma = \delta N_{\rm inf} + \frac{\delta\rho_\sigma}{3\rho_\sigma}$$

$$\zeta_r = \delta N_{\rm inf} + \frac{\delta\rho_r}{4\rho_r}$$

The total ζ is

$$\zeta = r_\sigma\zeta_\sigma + (1 - r_\sigma)\zeta_r \to \delta N_{\rm inf} + r_\sigma\zeta_\sigma$$

r_σ is time-dependent

$\zeta_{\sigma,r}$ is a constant.

Idea : Two component densities scale differently $\to$ Shift of ζ ($\delta N_{\rm inf} \to \delta N_{\rm inf} + \delta N_{\rm curv}$)

There is A drastic change in FR gravity!

We consider a simplest action for FR gravity,

FR Gravity (toy model, $8\pi G = 1$)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(\varphi)R - X_{\rm kin}(\varphi,\phi_i) - V(\varphi,\phi_i)\right],$$

$$X_{\rm kin} \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}\sum_i g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i$$

After calculating the Energy-momentum tensor and the background equations, we have ($\hat\rho$ is used because it satisfies $\dot{\hat\rho} = -3H(\hat\rho + \hat P)$ in FR gravity)

$$H^2 = \frac{X + V - 3H\dot F}{3F} \equiv \frac{\hat\rho}{3}$$

$$\dot H = -\frac{2X + \ddot F - H\dot F}{2F} \equiv -\frac{\hat\rho + \hat P}{2}$$

On uniform H, $\delta\rho_E \equiv \delta(X + V)$ and δF sources isocurvature perturbation!
— No need to introduce Matter + Radiation.
—What we need is "Something that disappears" and δF".

Consider simplifications (see below) and the quantities

$\rho_E \equiv X + V,\ P_E \equiv X - V$

$\hat{\rho} \equiv (\rho_E - 3H\dot{F})/F,\ \hat{P} \equiv (P_E + 2H\dot{F} + \ddot{F})/F,$

the curvature perturbation is (neglect $\ddot{F}$ and $\delta\dot{F}$ for simplicity)

$$\begin{aligned}\zeta &= \frac{\delta\rho}{3(\rho + P)} \\ &= \frac{H}{F}\frac{\delta\rho_E - 3H\delta\dot{F} - \rho\delta F}{3H(\rho_E + P_E) - H^2\dot{F} + H\ddot{F}} \\ &\simeq r_E\zeta_E + r_F\zeta_F,\end{aligned}$$

where (discriminating the isocurvature perturbations: e.g, $\delta\rho \to \delta\rho^{\mathrm{adi}} + \delta\rho^{\mathrm{iso}}$)

$$\begin{aligned}r_E &\equiv \frac{(\rho_E + P_E)}{\rho + P} \qquad r_E + r_F \equiv 1 \\ \zeta_E &\equiv N_{\mathrm{inf}} + \frac{\delta\rho_E^{\mathrm{iso}}}{3(\rho_E + P_E)} \\ \zeta_F &\equiv N_{\mathrm{inf}} + H\frac{\delta F^{\mathrm{iso}}}{\dot{F}}\end{aligned}$$

Therefore, if $(\rho_E + P_E) = 2X$ (not identified at this moment) disappears during the cosmological evolution, one will find a creation of the curvature perturbation

$$\zeta \;\; = \;\; \delta N_{\rm inf} + H\frac{\delta F^{\rm iso}}{\dot{F}}.$$

This mechanism is typically valid for thermal inflation.

My second question is

** What we can see in the Einstein frame after the conformal transformation? **

FR-curvaton in the Einstein Frame

After conformal transformation, we have

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R - X_{\rm kin}(\chi_\varphi,\chi_i) - W(\chi,\chi_i)\right]$$

*All fields are rescaled and redefined. Not a simple substitution.
*The potential is calculated from $W(\chi_\varphi,\chi_i)=\frac{V(\varphi,\phi_i)}{\Omega^4}$, where $\Omega^2 = F/M_p^2$.
*The effective mass is rescaled to have χ_φ-dependence.

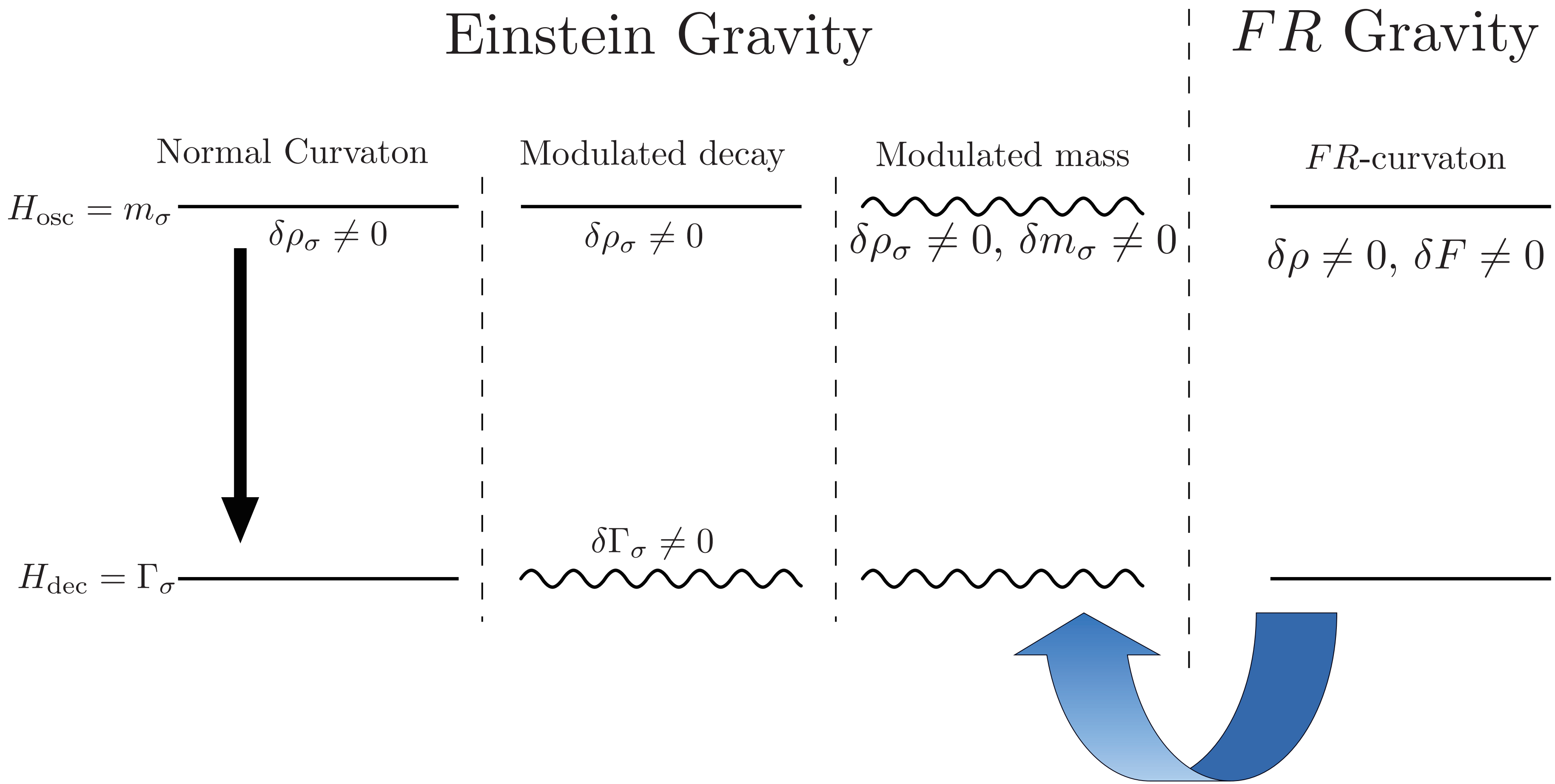
One thus inevitably finds Modulated mass for matter fields, which is originally $\delta F \neq 0$.

Mass-modulation in the multicomponent Universe is studied recently

- S. Enomoto, K. Kohri and T. Matsuda, 1301.3787 (non-linear formalism)
- K. Kohri, C. -M. Lin and T. Matsuda, 1303.2750 (familiar δN calculation)

*Similarly, modulated decay without mass-modulation is studied in

- D. Langlois and T. Takahashi, 1301.3319
- H. Assadullahi, H. Firouzjahi, M. H. Namjoo and D. Wands, 1301.3439



If what happens in FR-gravity is determined by a mass scale, FR-curvaton could be explained by the modulation in the Einstein Gravity; however the identity is far from trivial. (We can find a correspondence in Thermal Inflation, but this is an exceptionally easy example.)

Conclusion

Creation of the curvature perturbation in FR gravity has been studied in inflationary scenarios, such as
– Higgs inflation ($F(\phi) \propto 1 + \xi\phi^2$)
or
– induced-gravity inflation ($F(\phi) \sim \xi\phi^2$).

However, when we take a look at the evolution after inflation, they may have a somewhat distinguishable character.

* Work in progress.