ASYMPTOTIC STATE OF DE SITTER

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Prokopec, JCAP 1212 (2012) [arXiv:1110.3187 [gr-qc]]

Lazzari, Prokopec, arXiv:1304.0404 [hep-th] (2013)

COSMO 2013 , Cambridge, Sept 04, 2013

DE SITTER SPACE AND COSMIC INFLATION

- LINE ELEMENT (METRIC TENSOR):

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \quad or \quad g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}, \quad \eta_{\mu\nu} = diag(-1,\underbrace{1,1,..}_{D-1})$$

- FRIEDMANN (FLRW) EQUATION AND THE SCALE FACTOR

$$H^2 = \frac{\Lambda}{3} \quad \Rightarrow \; a(t) = a_0 e^{Ht}, \quad H = \sqrt{\frac{\Lambda}{3}}$$

- More generally, for a power law expansion scale factor reads:

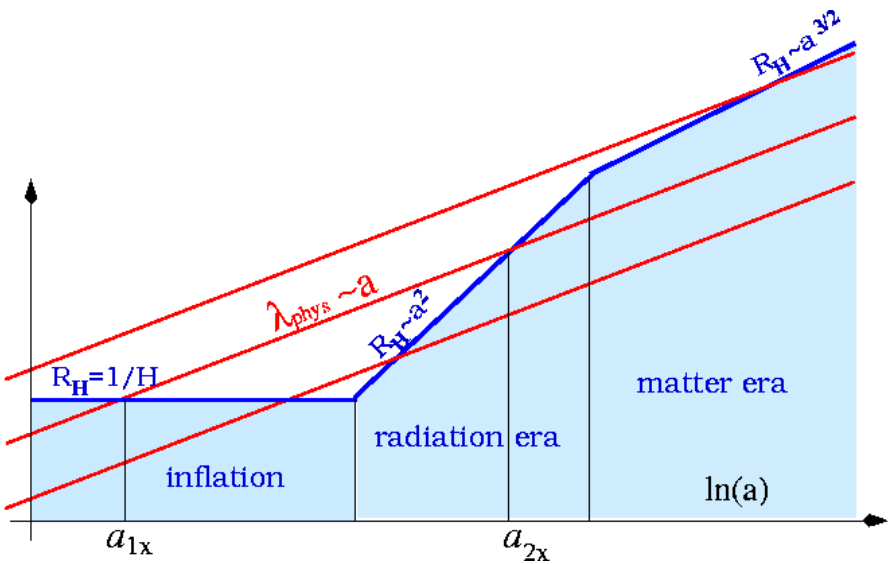
$$a = \left(\frac{t}{t_0}\right)^{1/\varepsilon} = \left[-(1-\varepsilon)H_0\eta\right]^{\frac{1}{1-\varepsilon}}, \quad H = H_0 a^{-\varepsilon}, \quad \varepsilon = -\frac{\dot{H}}{H^2} = \text{const.}$$

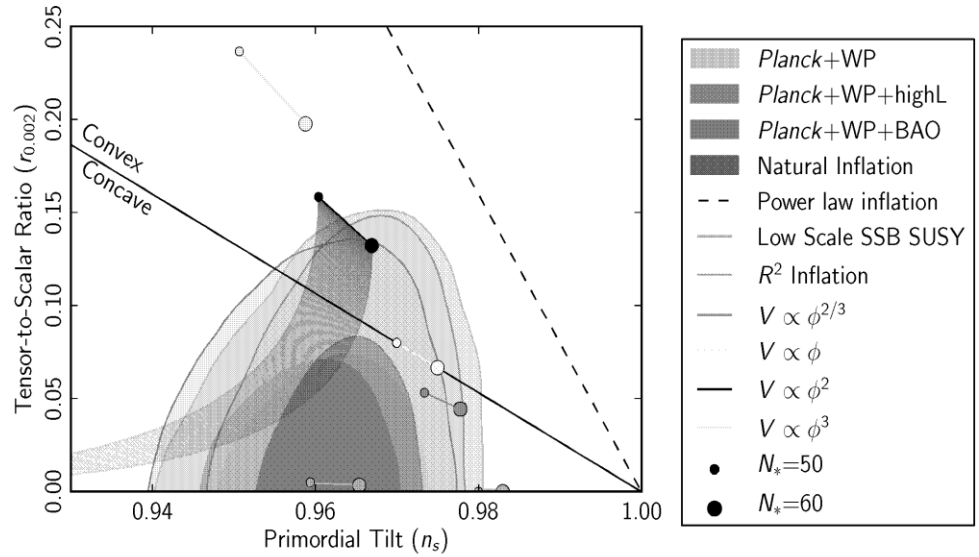
- True inflation: H(t) changes adiabatically in time
- ε=constant<<1 or $\varepsilon(t)$<<1 adiabatic in time (slow roll)

THIS IS "LOCALLY DE SITTER SPACE"

OBSERVATIONAL SUPPORT

- CMB OBSERVATIONS (Planck, WMAP)





Ade et al. XXII [Planck Collaboration] (2013), arxiv:1303.5082

Scalar and tensor (graviton) perturbations and amplified and stretched during inflation; they are observed at late times as CMB temperature fluctuations

CMB observations constrain the amplitude and spectral slope of scalar and tensor perturbations. Here, constraints on the <u>scalar spectral slope</u> and <u>tensor amplitude</u> compared with predictions in different inflationary models.

<u>NB:</u> Starobinsky's and Higgs inflation OK!

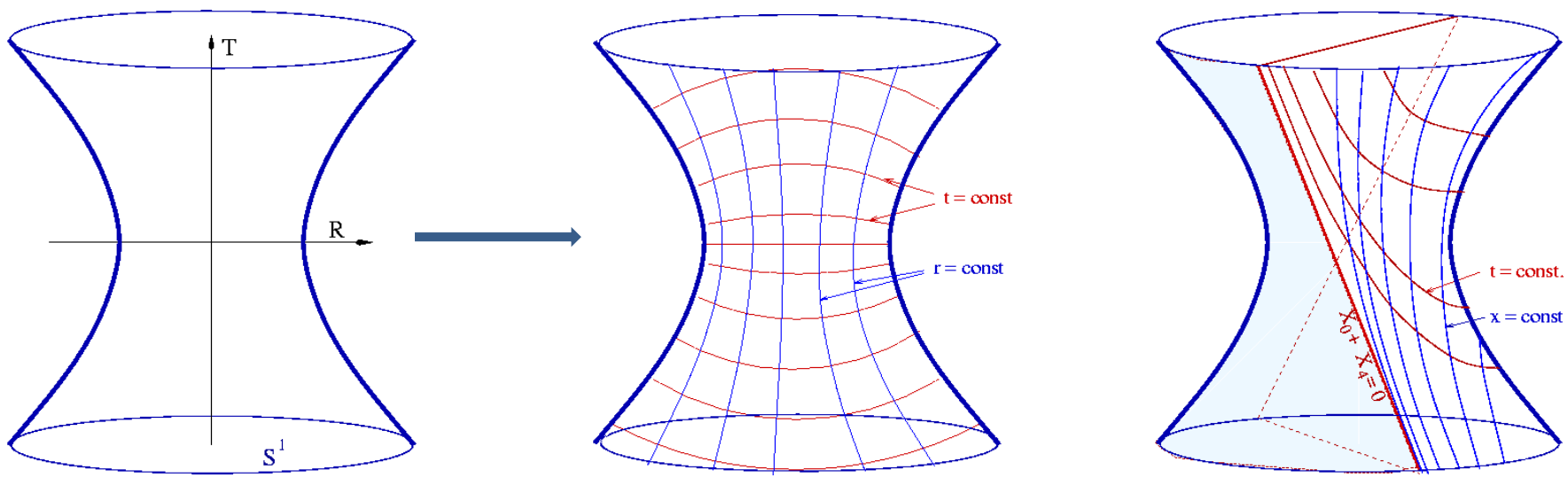
DE SITTER SPACE: CLASSICAL BACKGROUND

- 5-DIMENSIONAL FLAT EMBEDDING: de Sitter space is geometrically a hyperboloid, with SO(1,4) symmetry:

- SYMMETRY: Lorentz SO(1,4) with 10 generators, with Minkowski and anti-de Sitter: max symmetric spaces

$$-T^2 + R^2 = R_H^2 = \frac{1}{H^2}, \quad R^2 = X_1^2 + X_2^2 + X_3^2 + X_4^2$$

GLOBAL COORDINATES FLAT COORDINATES

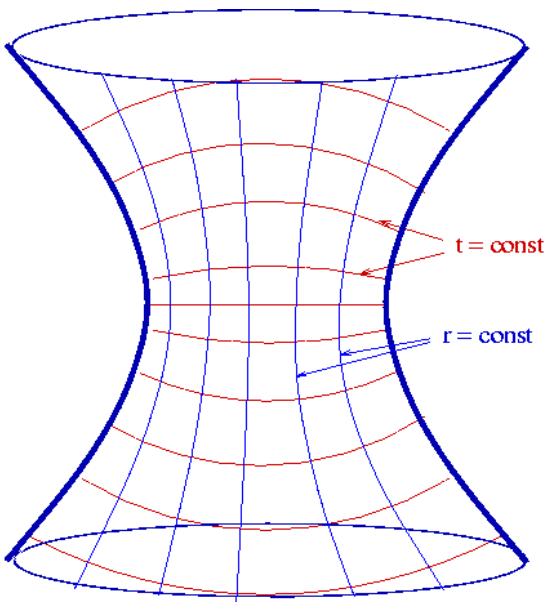


$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\chi^2 + H^2\sin^2(H\chi)d\Omega^2\right], \; a(\eta) = \frac{1}{\cos(H\eta)}, \qquad ds^2 = a^2(\eta)\left[-d\eta^2 + dr^2 + r^2 d\Omega^2\right], a(\eta) = -\frac{1}{H\eta}$$

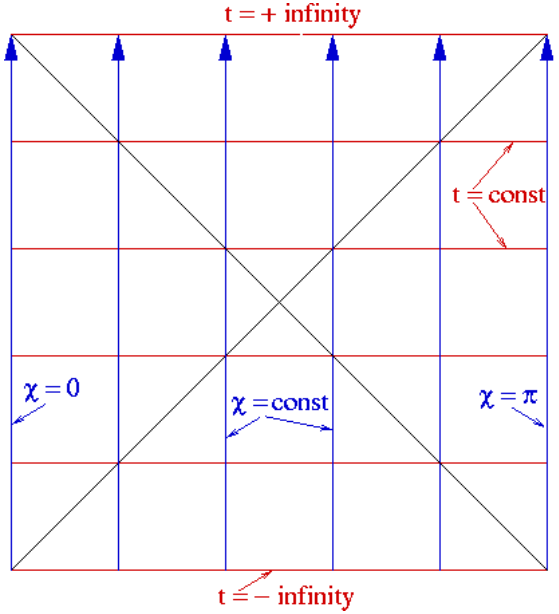
DE SITTER SPACE: CARTER-PENROSE

- CUTTING VERTICALLY GLOBAL dS in global coordinates:

$\Rightarrow$ suppress the angles (θ,ϕ): $ds^2_{conf} = -d\eta^2 + d\chi^2 + H^2 \sin^2(H\chi) d\Omega^2 \Rightarrow -d\eta^2 + d\chi^2$



CONFORMAL DIAGRAM



QUANTUM EFFECTS ON DE SITTER: SCALARS

QUANTUM FIELDS ON DE SITTER

ONE CAN QUANTIZE: SCALAR, FERMIONIC, VECTOR AND TENSOR (GRAVITON) FIELDS ON DE SITTER, AND CONSIDER THEIR (NON-)PERTURBATIVE EVOLUTION AND BACKREACTION ON THE dS BACKGROUND

EXAMPLE 1: MASSLESS SCALAR ON DE SITTER (D=4):

ACTION: $$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu}\right], \quad \sqrt{-g} = a^4, \quad g^{\mu\nu} = a^{-2}\eta^{\mu\nu}, \quad \eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1)$$

- **EOM for ϕ:** $$\frac{1}{a^2}(-\partial_\eta^2 - 2aH\partial_\eta + \partial_i^2)\hat{\phi}(x) = 0$$

- **In a <u>dS invariant state</u>, the propagator must be a function of dS inv distance l:**

$$y(x;x') = 4\sin^2\left[Hl(x;x')/2\right], \quad y(x;x') = H^2 a(\eta)a(\eta')\left[(\eta-\eta'|-\iota\varepsilon)^2 + \|\vec{x}-\vec{x}'\|^2\right]$$

$\Rightarrow$ **then the propagator obeys a dS inv equation:**

$$\left(y(4-y)\frac{d^2}{dy^2} + D(2-y)\frac{d}{dy}\right)i\Delta(x;x') = i\frac{\delta^D(x-x')}{H^2\sqrt{-g}}, \quad (D=4)$$

$\Rightarrow$ **This equation has no solution!**

Allen, Folacci, PRD35 (1987)

MASSLESS SCALAR ON DE SITTER

MASSLESS MINIMALLY COUPLED SCALAR (MMCS) D=4:

$$\left(y(4-y)\frac{d}{dy} + 4(2-y) \right)\frac{d}{dy} i\Delta(x;x') = i\frac{\delta^4(x-x')}{H^2\sqrt{-g}} \qquad (*)$$

THE NAIVE SOLUTION (in D=4) is (up to an irrelevant constant):

$$i\Delta(x;x') \to F(y) = \frac{H^2}{4\pi^2}\left\{ \left[\frac{1}{y} - \frac{1}{2}\log\left(\frac{y}{4}\right) \right] - \left[\frac{1}{4-y} - \frac{1}{2}\log(4-y) \right] \right\}$$

- **This UNIQUE dS inv solution solves a wrong equation:**

$$\left(y(4-y)\frac{d}{dy} + 4(2-y) \right)\frac{d}{dy} i\Delta(x;x') = i\frac{\delta^4(x-x')}{H^2\sqrt{-g}} + i\frac{\delta^4(x-\tilde{x}')}{H^2\sqrt{-g}} \qquad (**)$$

$\Rightarrow$ **Here** $\tilde{x}^\mu = (-\eta, x^i)$ **denotes the antipodal point of** $x^\mu = (\eta, x^i)$**:**

$\Rightarrow$ **From** $y(x;\tilde{x}') = -[4 - y(x;x')]$ **it follows that the part in second square brackets [.] sources the second δ-function in Eq. (**).**

$\Rightarrow$ **The solution (**) follows from:**

$$\partial^2\left(\frac{1}{4\pi^2}\frac{1}{\Delta x^2(x;x')} \right) = i\delta^4(x-x'), \quad \partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu, \quad \Delta x^2(x;x') = -(|t-t'|-i\varepsilon)^2 + \|\vec{x}-\vec{x}'\|^2$$

MASSLESS SCALAR ON DE SITTER 2

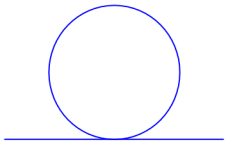
CONCLUSION: MMCS cannot be quatized in a dS inv way.

ONE POSSIBLE SOLUTION (that breaks dS sym, but solves the right equation) is:

$$i\Delta(x;x') = \frac{H^2}{4\pi^2}\left\{\left[\frac{1}{y} - \frac{1}{2}\log\left(\frac{y}{4}\right)\right] + \frac{1}{2}\ln\left(a(\eta)a(\eta')\right)\right\}$$

- **This dS breaking solution solves the right equation, but (when dim reg is applied) the coincident propagator grows with time.**

- **This has physical consequences. For example, when one introduces a quartic interaction** $\mathcal{L}_{\text{quartic}} = -\frac{\lambda}{4!}\phi^4$ **at one loop one generates a Hartree mass:**

$(\delta m^2)_{\text{Hartree}} =$  $= \frac{\lambda}{2} i\Delta(x;x) = \frac{\lambda H^2}{4\pi^2}\ln(a), \quad \ln(a) = Ht$

- In a self-interacting scalar theory, the mass squared thus grows linearly in time. This is a consequence of abundant particle creation in dS. Precisely this type of particle creation generates scalar cosmological perturbations, which in turn induce CMB temperature fluctuations and seed the Universe's large scale structure.

☺ **QUESTION: IS THERE A dS INV SCALAR MASS?**

MASSIVE SCALAR FIELD PROPAGATOR

dS INVARIANT SCALAR FIELD PROPAGATOR

Chernikov, Tagirov, Annales Poincaré Phys.Theor. A9 (1968)

$$\imath\Delta(x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma\left(\frac{D}{2}\right)} \times {}_2F_1\left(\frac{D-1}{2}+\nu_D, \frac{D-1}{2}-\nu_D; \frac{D}{2}; 1-\frac{y(x;x')}{4}\right)$$

$$\nu_D = \left(\frac{D-1}{2}\right)^2 - \frac{m^2}{H^2}, \qquad y(x;x') = -\left(|\eta-\eta'| - \imath\varepsilon\right)^2 + \left\|\vec{x}-\vec{x}'\right\|^2$$

COINCIDENT SCALAR PROPAGATOR

$$\imath\Delta(x;x) = \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma\left(\frac{1}{2}+\nu_D\right)\Gamma\left(\frac{1}{2}-\nu_D\right)} \Gamma\left(-\frac{D}{2}\right) = \imath\Delta(x;x)_{\rm div} + \imath\Delta(x;x)_{\rm fin}$$

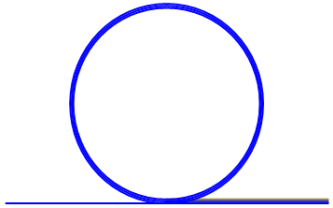
$$\imath\Delta(x;x)_{\rm div} = \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}} \left[\psi(D/2) - \psi(D-1) - \psi(-D/2) - \gamma_E + \frac{1}{D-1}\right]$$

$$\imath\Delta(x;x)_{\rm fin} = \frac{\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{(D+1)/2}} \frac{H^D}{m^2}$$

$\Rightarrow$The m$\rightarrow$0 LIMIT IS SINGULAR;
EXPLAINS WHY THERE IS NO dS INV SOLUTION.

SCALAR FIELD: MASS GENERATION

in a self-interacting scalar theory, scalar has a growing mass.
Is there a dS inv late time limit?



RESUMMATION: SELF-CONSISTENT HARTREE:

ACTION

$$S = \int d^D x \sqrt{-g}\left[-\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m_0^2 \phi^2 - \frac{\lambda}{4!}\phi^4\right]$$

GAP EQUATION

$$m_{\mathrm{MF}}^2 = m^2 + \frac{\lambda}{2}\imath\Delta(x;x)_{\mathrm{fin}}, \text{ if } \phi^2 = 0, \quad m^2 = m_0^2 + \frac{\lambda}{2}\imath\Delta(x;x)_{\mathrm{div}} > 0$$

SOLUTION

Ford, Vilenkin, PRD 26 (1982); Serreau; Garbrecht, Rigopoulos (2011)

$$m_{\mathrm{MF}}^2 = \frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{m_{\mathrm{cr}}^4}{2}} > 0, \ (\phi^2 = 0), \quad |m^2| < m_{\mathrm{cr}}^2, \qquad m_{\mathrm{cr}}^2 = \sqrt{\frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}} \xrightarrow{D=4} \sqrt{\frac{3\lambda}{2}}\frac{H^2}{2\pi}$$

THERE ARE STUDIES OF 2 and 3 LOOPS
(primarily to check Stochastic inflation; also w<-1 found)

Kahya, Onemli, Woodard (2010);
Onemli, Woodard (2004)

NONPERTURBATIVE METHODS: STOCHASTIC INFLATION

Starobinsky (1986); Starobinsky, Yokoyama (1996)

STOCHASTICIZING SCALAR THEORY

Prokopec, Tsamis, Woodard, Ann.Phys.323 (2008); Tsamis, Woodard, Nucl. Phys B724 (2005)

EXACT EQUATION OF MOTION:

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\hat{\phi}(x)=\left(-\partial_t^2-(D-1)H\partial_t+\frac{\nabla^2}{a^2}\right)\hat{\phi}(x)=-V'_{\rm ren}(\hat{\phi}),\quad V'_{\rm ren}\equiv\frac{1}{1+\delta Z}\frac{dV}{d\phi}$$

$\Rightarrow$ **THE YANG-FELDMAN EQ. (also exact):**

$$\hat{\phi}(x)=\hat{\phi}_0(x)-\int d^{D-1}x'dt'a^{D-1}(t')G_r(x;x')V'_{\rm ren}(\hat{\phi}(x')),\quad (*)$$

- **G_r: RETARDED GREEN'S FUNCTION**

$$\left(-\partial_t^2-(D-1)H\partial_t+\frac{\nabla^2}{a^2}\right)G_r(x;x')=\frac{\delta^D(x-x')}{\sqrt{-g}},\quad G_r(x;x')=i\theta(\Delta t)\langle\Omega|\left[\hat{\phi}_0(x),\hat{\phi}_0(x')\right]|\Omega\rangle$$

$\Rightarrow$ One can <u>prove</u> that – to recover the leading order late time n-point functions – it suffices to approximate G_r by:

$$G_r(x;x')\approx\frac{\theta(\Delta t)\delta^{D-1}(\vec{x}-\vec{x}')}{(D-1)Ha(t')^{D-1}}$$

See e.g. Prokopec, Tsamis, Woodard, Ann.Phys.323 (2008)

$\Rightarrow$ the free field ϕ_0 can be approximated by modes aH>k>H:

$$\hat{\phi}_0(x)\to\hat{\Phi}_0(x)=\int\frac{d^3k}{(2\pi)^3}\theta(k-H)\theta(aH-k)\frac{H}{\sqrt{2k^3}}\left\{e^{i\vec{k}\cdot\vec{x}}\hat{\alpha}(\vec{k})+e^{-i\vec{k}\cdot\vec{x}}\hat{\alpha}^+(\vec{k})\right\},\quad\left[\hat{\alpha}(\vec{k}),\hat{\alpha}^+(\vec{k}')\right]=(2\pi)^3\delta^3(\vec{k}-\vec{k}')$$

Note that: $\left[\hat{\Phi}(x),\hat{\Phi}(x')\right]=0$ $\Rightarrow$ CLASS. STOCHASTIC THEORY w/ MARKOWIAN NOISE!

STOCHASTIC SCALAR THEORY 2

Prokopec, Tsamis, Woodard, Ann.Phys.323 (2008)

⇒ THE YANG-FELDMAN EQ. (*) ON p.20 REDUCES TO

$$\hat{\Phi}(x) = \hat{\Phi}_0(x) - \frac{1}{3H}\int_0^t dt' V'_{\rm ren}(\hat{\Phi}(\vec{x},t')) \quad \Rightarrow \quad \partial_t \hat{\Phi}(x) + \frac{1}{3H} V'_{\rm ren}(\hat{\Phi}(x')) = \partial_t \hat{\Phi}_0(x)$$

- WHERE THE FREE FIELD Φ_0 IS A CLASSICAL STOCHASTIC FIELD WITH A WHITE (MARKOWIAN) NOISE

$$\langle\Omega|\partial_t \hat{\Phi}_0(t,\vec{x})\partial_{t'}\hat{\Phi}_0(t',\vec{x})|\Omega\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$$

- Field Φ obeys a Langevin equation with damping γ=3H and noise: $\xi(x) = 3H\partial_t \hat{\Phi}_0(x)$

$$\gamma\partial_t \hat{\Phi}(x) + V'_{\rm ren}(\hat{\Phi}(x')) = \xi(x)$$

Rigopoulos, 1305.0229 [astro-ph.CO] (2013)

- FLUCT.-DISSIPATION RELATION WITH A **HAWKING-GIBBONS** TEMP.:

$$\langle\Omega|\xi(t,\vec{x})\xi(t',\vec{x})|\Omega\rangle = \frac{\gamma T_{GH}}{(4\pi/3)H^{-3}}\delta(t-t'), \quad T_{GH} = \frac{H}{2\pi}$$

- UPON INCLUDING $d^2\Phi/dt^2$, ONE CAN PROVE AN EQUIPARTITION RELATION

Rigopoulos, 1305.0229 [astro-ph.CO] (2013)

$$\frac{1}{2}\langle\Omega|(\partial_t\hat{\Phi})^2|\Omega\rangle = 2\langle\Omega|V(\hat{\Phi})|\Omega\rangle = \frac{3H^4}{16\pi^2} = \frac{1}{2}\frac{T_{GH}}{(4\pi/3)H^{-3}}$$

- (OLD) LANGEVIN DESCRIPTION IS EQUIVALENT TO A FOKKER-PLANCK EQ.:

$$\partial_t \rho(t,\phi(\vec{x})) = \frac{1}{3H}\partial_\phi(V'\rho) + \frac{1}{2}\partial_\phi^2\left(\frac{H^3}{4\pi^2}\rho\right) \to \rho_\infty = \rho_0 \exp\left(-\frac{8\pi^2 V(\phi)}{3H^4}\right)$$

Starobinsky, Yokoyama, PRD50 (1994)

STOCHASTIC THEORIES OF INFLATION

SQED: Prokopec, Tsamis, Woodard, Ann.Phys.323 (2008)

Yukawa: Miao, Woodard, PRD74 (2006)

APART FROM SCALAR THEORIES WITH ARBITRARY POTENTIAL $V(\phi)$, ONE CAN STOCHASTICIZE SCALAR ELECTRODYNAMICS & YUKAWA TH. SO FAR ATTEMPTS TO STOCHASTICIZE GRAVITY HAVE FAILED.

- STRATEGY: identify active and passive fields.
- Active fields are amplified in IR and can produce leading log(a).
- Passive fields (gauge fields, fermions): NEED TO BE INTEGRATED OUT
- Stochasticize the resulting EFFECTIVE SCALAR THEORY, with Veff(Φ).

RESULTS FOR **SQED**:

the photon and scalar acquire a (nonpert) mass: $m_\gamma^2 = 3.3H^2; \quad m_\varphi^2 = 0.034e^2H^2$

fluctuations give constant contributions to density invariants & Λ:

$$\left\langle F_{\mu\nu}(x)F_{\rho\sigma}(x)\right\rangle = -0.12H^4(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}); \quad \left\langle V_{\text{eff}}(\phi)\right\rangle = 0.027H^4; \quad \delta\Lambda = -0.2083GH^4$$

NB: These results are fully nonperturbative (not suppressed by a coupling constant e)

RESULTS FOR **YUKAWA**: EFFECTIVE SCALAR THEORY UNSTABLE AND UNBOUNDED FROM BELOW (vacuum stability?)!

HIGGS FOUND VERY NEAR THE STABILITY BOUND! MESSAGE??

SYMMETRY RESTORATION ON DE SITTER

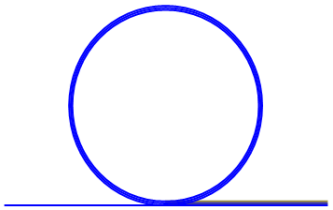
Prokopec, JCAP 1212 (2012) [arXiv:1110.3187 [gr-qc]]

Lazzari, Prokopec, arXiv:1304.0404 [hep-th] (2013)

MEAN FIELD MASS

BROKEN SYMMETRY CASE: m²<0:

MEAN FIELD MASS (self-consistent Hartree approximation)



$$\imath\Delta(x;x)_{\text{fin}} = \frac{\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{(D+1)/2}}\frac{H^D}{m_{\text{MF}}^2}$$

$$m_{\text{MF}}^2 = \underbrace{m^2}_{<0} + \frac{\lambda}{2}\left(\phi^2 + \imath\Delta(x;x)_{\text{fin}}\right) = -2m^2 - \lambda\imath\Delta(x;x)_{\text{fin}}, \text{ if } \phi^2 > 0$$

SOLVING THIS YIELDS (BROKEN SYMMETRY CASE, m²<0):

$$m_{\text{MF}}^2 = -m^2 + \sqrt{m^4 - m_{\text{cr}}^4} > 0,\ (\phi^2 > 0), \quad |m^2| > m_{\text{cr}}^2 = \sqrt{\frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}} \xrightarrow{D=4} \sqrt{\frac{3\lambda}{2}}\frac{H^2}{2\pi}$$

NB: ANALOGOUS ANALYSIS CAN BE DONE FOR O(N) MODEL
$\Rightarrow$MASSIVE WOULD-BE GOLDSTONE BOSONS (??)

QUESTION: HOW ACCURATE IS THIS RESULT/METHOD?

ANSWER: ONE CAN CHECK IT BY USING STOCHASTIC THEORY.

EFFECTIVE ACTION APPROACH

Lazzari, Prokopec, arXiv:1304.0404 [hep-th] (2013)

USE THE FOKKER PLANCK EQUATION:

$$\partial_t \rho(t,\phi(\vec{x})) = \frac{1}{3H}\partial_\phi \left(V'\rho\right) + \frac{1}{2}\partial_\phi^2\left(\frac{H^3}{4\pi^2}\rho\right)$$

ASSUME FOR SIMPLICITY AN INITIAL STATE

$$\rho = \rho_0 \exp\left(-\frac{8\pi^2 V(\phi)}{3H^4}\right) \equiv \rho_0 \exp\left(-v(\phi)\right)$$

USE A LEGENDRE TRANSFORM $\Rightarrow$ PARTITION FUNCTION

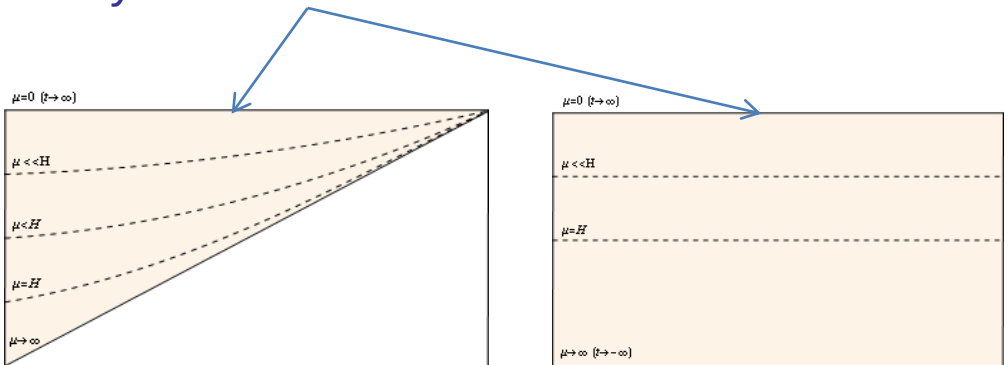
$$Z(J) \equiv \mathrm{e}^{-\mathrm{W(J)}} = \int d\phi \rho_0 \mathrm{e}^{-\mathrm{v}(\phi)+\mathrm{J}\phi} = \left\langle \mathrm{e}^{\mathrm{J}\phi}\right\rangle = \sum_{n=0}^{\infty}\frac{\left\langle \phi^{2n}\right\rangle}{(2n)!}J^{2n}$$

$$v_{\mathrm{eff}}(\bar{\phi}) \equiv \frac{8\pi^2}{3\mathrm{H}^4}V_{\mathrm{eff}}(\bar{\phi}) = W(J) + J\bar{\phi}, \quad \frac{d\ln[Z(J)]}{dJ} = \bar{\phi}(J)$$

<u>NB</u>: This method integrates out all super-Hubble fluctuations and yields a PDF for deep infrared fields at asymptotically late times.

$$\rho_{\mathrm{eff}}(\bar{\phi}) = \rho_{0\mathrm{eff}} \exp\left(-\frac{8\pi^2 V_{\mathrm{eff}}(\bar{\phi})}{3H^4}\right)$$

CONFORMAL DIAGRAM OF dS:
<u>flat</u> and <u>global</u> coordinates

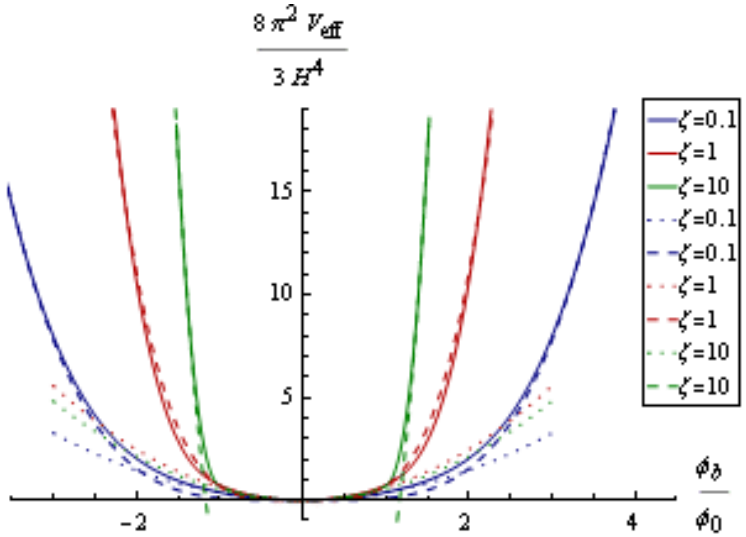


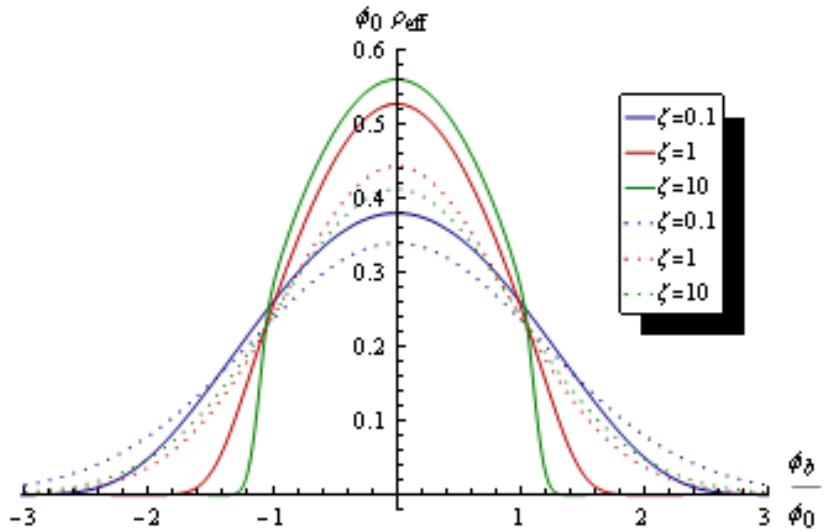
ASYMPTOTIC STATE FOR REAL SCALAR

EFFECTIVE ACTION AND ITS PDF: SYM BREAKING CASE

$$\rho_{\text{eff}}(\bar{\phi}) = \rho_{0\text{eff}} \exp\left(-\frac{8\pi^2 V_{\text{eff}}(\bar{\phi})}{3H^4}\right)$$

$$\zeta = \frac{4\pi^2\mu^4}{\lambda H^4}, \quad \mu^2 = m^2 < 0, \quad \phi_0^2 = \frac{6\mu^2}{\lambda}$$





CONCLUSION: SYMMETRY GETS RESTORED FOR ARBITRARY TREE POTENTIAL! SAME CONCLUSION REACHED IN THE O(N) CASE: GOLDSTONE THM RESPECTED (GOLDSTONE BOSONS REMAIN MASSLESS)

HENCE: quantum fluctuations in dS strong enough to restore symmetry!

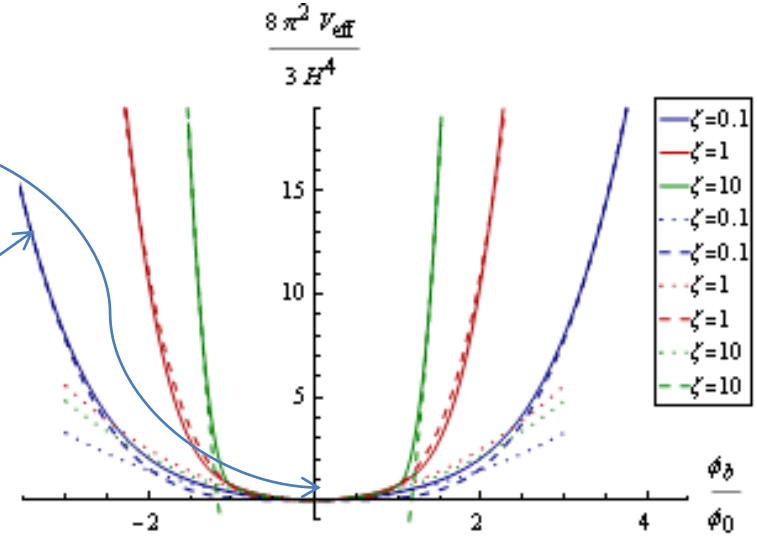
For large ζ (weak λ) at $\phi \sim \phi_0$: a sudden turn in V_{eff} (a Maxwell's construction)

ASYMPTOTIC STATE REAL SCALAR 2

ANALYTIC APPROXIMATIONS FOR EFFECTIVE ACTION

FOR SMALL FIELD VALUES:

$$m^2(\bar{\phi} \approx 0) = \frac{3H^4}{8\pi^2 \langle \phi^2 \rangle} \Rightarrow \quad \rho_{\text{eff}}(\bar{\phi} \approx 0) \propto \exp\left(-\frac{\bar{\phi}^2}{2\langle \phi^2 \rangle}\right)$$



FOR LARGE FIELD VALUES:

$$V_{\text{eff}}(\bar{\phi}) = -\frac{\mu^2}{2}\bar{\phi}^2 + \frac{\lambda}{4!}\bar{\phi}^4 + \Delta V_{\text{eff}}(\bar{\phi});$$

$$\Delta V_{\text{eff}}(\bar{\phi}) \sim \frac{3\mu^4}{4\lambda} + \frac{3H^4}{16\pi^2}\ln\left(\frac{\bar{\phi}^2}{H^2}\right) + \frac{3H^4}{8\pi^2}\ln\left(\frac{\pi^{3/2}\mu}{H}\left[\mathrm{I}_{1/4}\left(\frac{\varsigma}{2}\right) + \mathrm{I}_{-1/4}\left(\frac{\varsigma}{2}\right)\right]\right)$$

FLUCTUATIONS GENERATE LARGE AMOUNT OF ENERGY

FLUCTUATIONS ARE LIGHT FOR SMALL BACKGROUND FIELD; HEAVY FOR LARGE VALUES OF BACKGROUND FIELD

SAME CONCLUSIONS ARE REACHED FOR A GENERAL O(N) MODEL

DISCUSSION

- PHYSICS OF DE SITTER IS ESSENTIALLY NONPERTURBATIVE (for massless and light scalars, gravitons, and other fields that couple to them).

- WHILE IT IS NOT KNOWN HOW TO STUDY EFFECTS OF QUANTUM GRAVITON FLUCTUATIONS, PROGRESS HAS BEEN MADE FOR SCALAR, VECTOR AND FERMIONIC FIELDS (both using perturbative methods and stochastic inflation).

- SCALAR AND VECTOR FIELDS ACQUIRE/YIELD COMPUTABLE CORRECTIONS DURING INFLATION; FERMIONIC FIELDS TEND TO DESTABILIZE DE SITTER.

- QUANTUM FLUCTUATIONS OF SCALARS ARE STRONG ENOUGH TO RESTORE A BROKEN SYMMETRY IN THE CASE OF REAL AND N-COMPONENT SCALAR FIELD WITH AN O(N) SYMMETRY. LATE TIME STATE CAN BE COMPUTED: FLUCTUATIONS ARE LIGHT AROUND THE ORIGIN, HEAVY AT LARGE FIELD VALUES.