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Statistics of general functions of a Gaussian field-application to non-Gaussianity from preheating

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Massless preheating

$$V = \frac{\lambda}{4}\phi^4 + \frac{g^2}{2}\phi^2\chi^2$$

After inflation, ϕ oscillates and parametric resonance amplifies fluctuations of χ even on superhorizon scales (for particular parameter choice).

This invoked intensive studies of if it also results in the generation of the curvature perturbation on superhorizon scales. Evaluating amplitude of curvature perturbation and its correlation functions had been a longstanding issue.

Bassett and Viniegra (2000), etc. : standard perturbation analysis

Wands, Malik, Lyth and Liddle (2000) : suggestion of the use of deltaN formalism

Tanaka and Bassett (2003) : first application of deltaN formalism (only background)

The process is highly non-linear and requires numerical lattice simulations to compute mapping between field perturbation to curvature perturbation.

Chambers and Rajantie (2008) : first combinational use of lattice calculations and deltaN formalism (insufficient numerical accuracy)

Local type curvature perturbation

It is produced when the curvature perturbation is generated after inflation by fields other than inflaton.

(Models include curvaton model, modulated reheating scenario, and the massless preheating model)

$$\zeta(\vec{x}) = f(\chi(\vec{x}))$$

$\zeta(\vec{x})$: curvature perturbation

$\chi(\vec{x})$: sourcing field (assumed to be Gaussian)

Both variables depend on the same position.

In many models, Taylor expansion to the lowest order is sufficient (e.g., 2nd order for 3-point function) .

$$f(\chi) = a\chi + b\chi^2, \qquad a, b : \ \text{constants.}$$

Model dependent

$$\Rightarrow \quad \zeta = \zeta_g + \frac{3}{5} f_{\rm NL} \zeta_g^2,$$

However, things are more complicated in the case of massless preheating.

Mapping $f(\chi)$ in massless preheating

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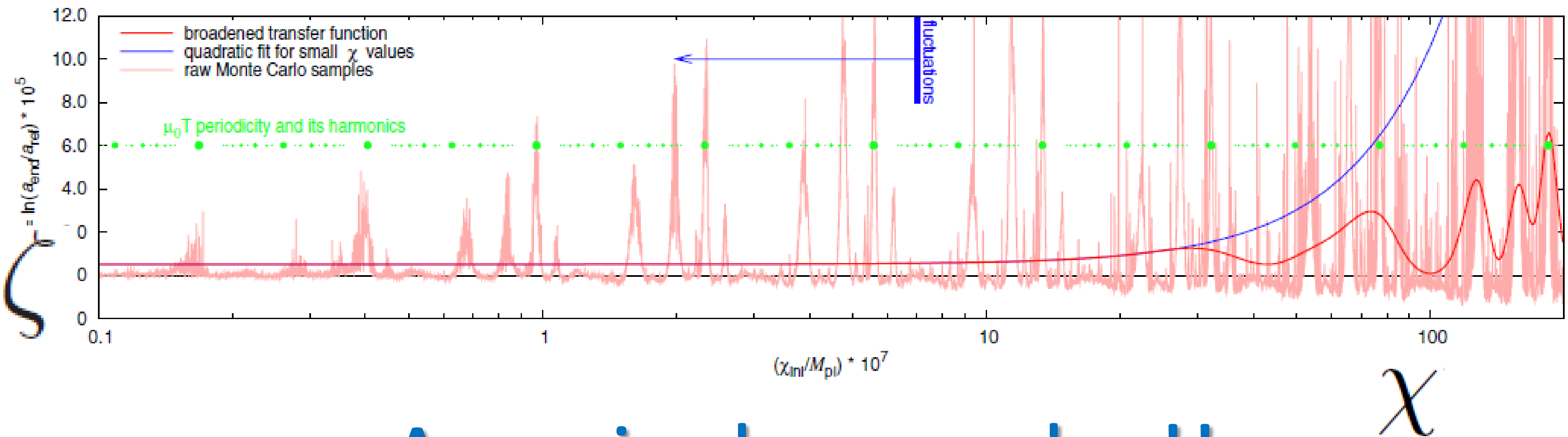


Amazingly complex!!

Motivation of my study is to compute the correlation functions of ζ produced in massless preheating model.

General formalism

$$f(\chi) = \int \frac{d\sigma}{2\pi}\, f_\sigma e^{i\chi\sigma},$$

Fourier transform in field space

Basic formula (for any function b(x))

$$\left\langle \exp\left(\int d^3x\, b(\vec{x})\chi(\vec{x})\right)\right\rangle = \exp\left[\frac{1}{2}\int d^3x_1 d^3x_2 \int \frac{d^3q}{(2\pi)^3} P_\chi(q) e^{i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)} b(\vec{x}_1)b(\vec{x}_2)\right]$$

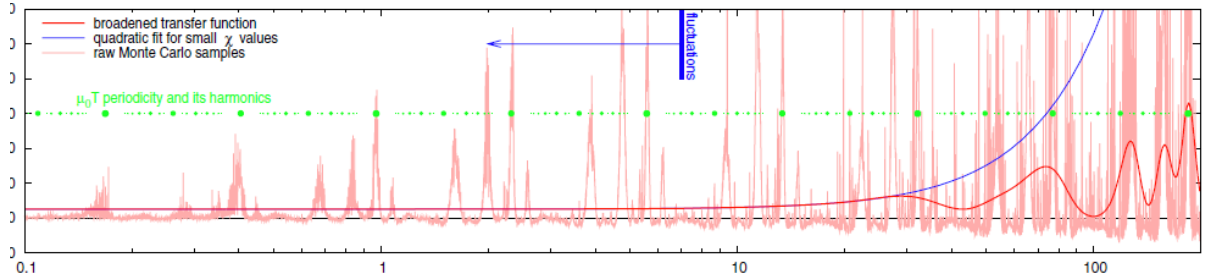
$$\langle f(\vec{x}_1)\cdots f(\vec{x}_N)\rangle = \int \frac{d\sigma_1\cdots d\sigma_N}{(2\pi)^N} f_{\sigma_1}\cdots f_{\sigma_N}\langle e^{i\chi(\vec{x}_1)\sigma_1+\cdots+i\chi(\vec{x}_N)\sigma_N}\rangle$$

$$= \int \left(\prod_{i=1}^{N}\frac{d\sigma_i}{2\pi} f_{\sigma_i} e^{-\frac{\langle\chi^2\rangle}{2}\sigma_i^2}\right)\exp\left(-\langle\chi^2\rangle\sum_{i<j}\sigma_i\sigma_j\xi_\chi(r_{ij})\right)$$

$$\xi_\chi(r_{ij}) = \langle\chi(\vec{x}_1)\chi(\vec{x}_2)\rangle/\langle\chi^2\rangle \text{ and } r_{ij} = |\vec{x}_i - \vec{x}_j|$$

Once we know f_σ , we can compute the N-point function.

Application to massless preheating



Fitting formula for the mapping (sum of normal distribution)

$$f(\chi) = \sum_p A_p \exp\left(-\frac{(\chi - \chi_p)^2}{2\kappa_p^2} \right)$$

position

amplitude

width

Use of the fitting formula enables us to obtain analytic f_σ and analytic integration.

2-point function

$$\langle f(\vec{x}) f(\vec{y}) \rangle = \sum_{p_1, p_2} \frac{A_{p_1} A_{p_2} \epsilon_{p_1} \epsilon_{p_2}}{\sqrt{(1+\epsilon_{p_1}^2)(1+\epsilon_{p_2}^2) - \xi_\chi^2(r)}} \times \exp\left(-\frac{1}{2} \frac{(1+\epsilon_{p_1}^2)\eta_{p_1}^2 + (1+\epsilon_{p_2}^2)\eta_{p_2}^2 - 2\xi_\chi(r)\eta_{p_1}\eta_{p_2}}{(1+\epsilon_{p_1}^2)(1+\epsilon_{p_2}^2) - \xi_\chi^2(r)} \right).$$

Once we know amplitude, width and position of each spike, we can immediately compute 2-point function by using this formula.

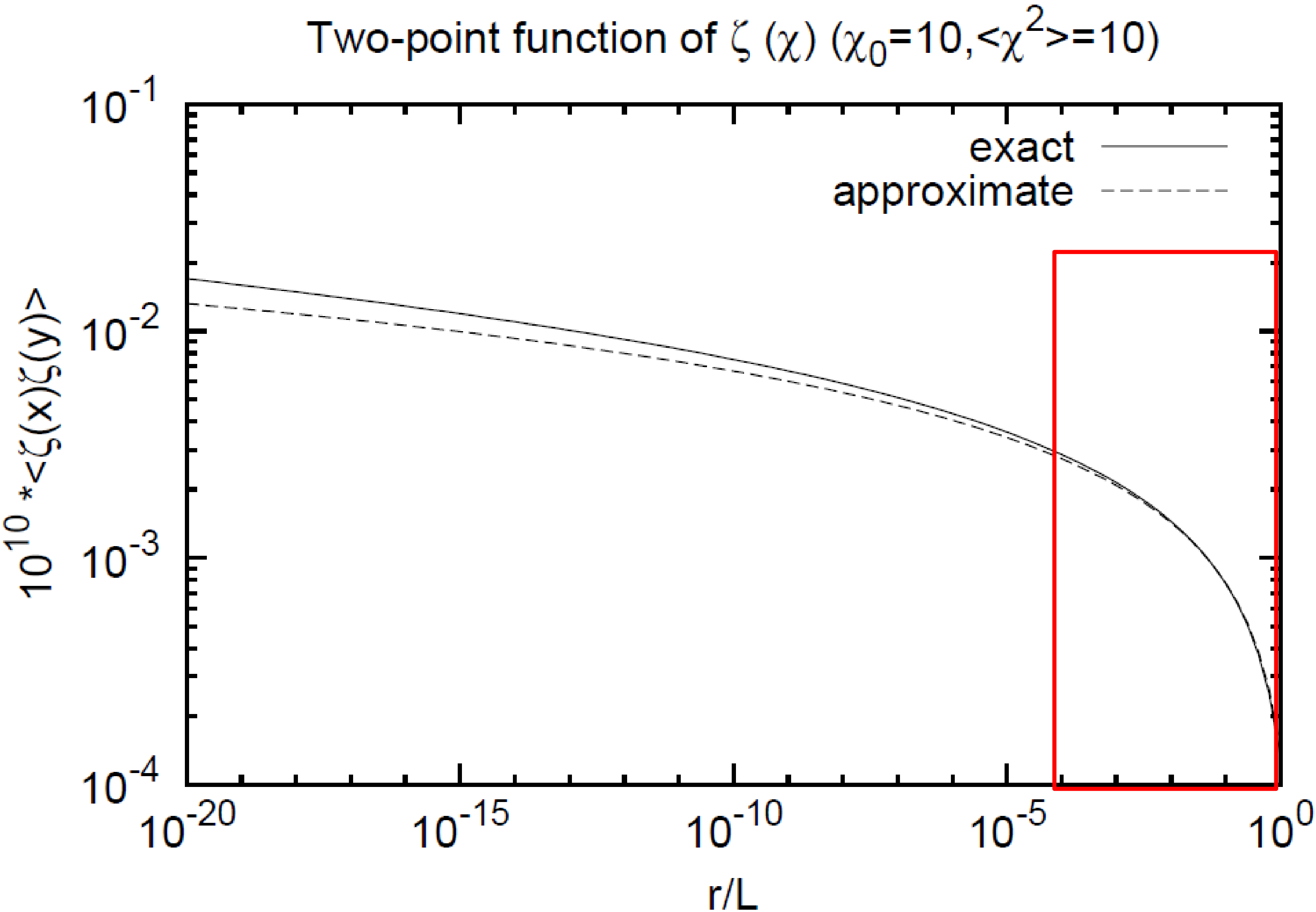
By making the approximation $\langle \chi^2 \rangle \gg \kappa_p^2$

$$\langle f(\vec{x}) f(\vec{y}) \rangle = I_0^2 + I_1^2 \xi_\chi(r) + \mathcal{O}(\xi_\chi^2)$$

$$I_0 = \sum_p A_p \epsilon_p e^{-\frac{\eta_p^2}{2}}, \qquad I_1 = \sum_p A_p \epsilon_p \eta_p e^{-\frac{\eta_p^2}{2}}$$

Curvature perturbation is approximately scale invariant!

2-point function



3, 4-point function (approximately standard local type)

$$\langle\zeta(\vec{x}_1)\zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle \approx \frac{I_2 - I_0}{I_1^2}\left(\langle\zeta(\vec{x}_1)\zeta(\vec{x}_2)\rangle\langle\zeta(\vec{x}_1)\zeta(\vec{x}_3)\rangle + 2\ \text{perms.}\right)$$

$$\langle\zeta(\vec{x}_1)\zeta(\vec{x}_2)\zeta(\vec{x}_3)\zeta(\vec{x}_4)\rangle \approx \frac{(I_2 - I_0)^2}{I_1^4}\left(\langle\zeta(\vec{x}_1)\zeta(\vec{x}_2)\rangle\langle\zeta(\vec{x}_2)\zeta(\vec{x}_4)\rangle\langle\zeta(\vec{x}_3)\zeta(\vec{x}_4)\rangle + 11\ \text{perms.}\right)$$
$$+\frac{(I_3 - 3I_1)}{I_1^3}\left(\langle\zeta(\vec{x}_1)\zeta(\vec{x}_2)\rangle\langle\zeta(\vec{x}_1)\zeta(\vec{x}_3)\rangle\langle\zeta(\vec{x}_1)\zeta(\vec{x}_4)\rangle + 3\ \text{perms.}\right) \quad (4.28)$$

$$I_2 = \sum_p A_p \epsilon_p \eta_p^2 e^{-\frac{\eta_p^2}{2}} \qquad I_3 = \sum_p A_p \epsilon_p \eta_p^3 e^{\frac{-\eta_p^2}{2}}$$

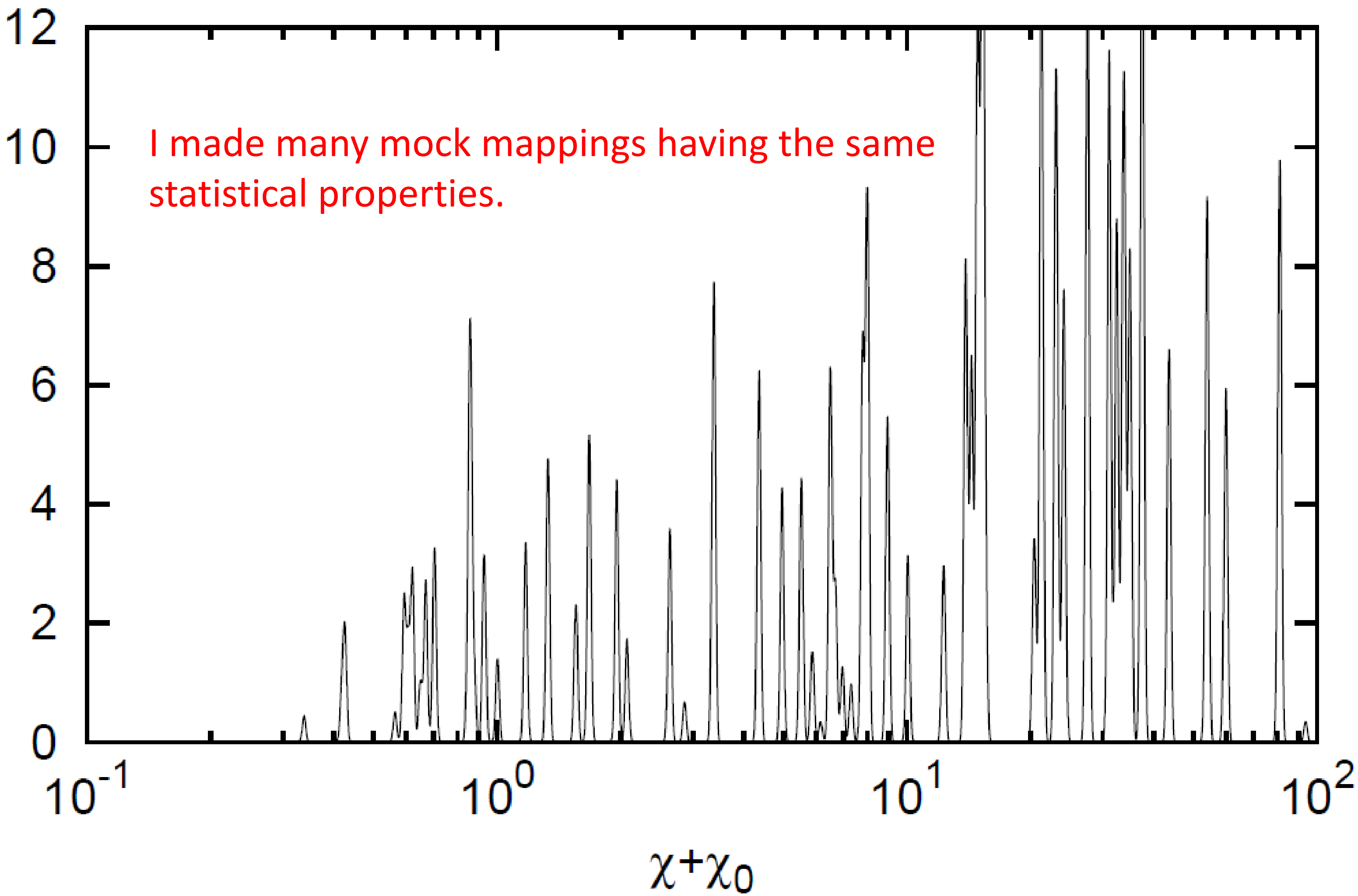
Approximate formula for the non-linearity parameters

$$\frac{6}{5} f_{\rm NL} = \frac{I_2 - I_0}{I_1^2}$$

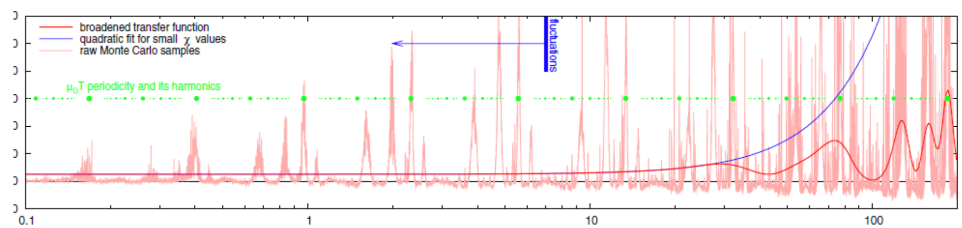
$$\tau_{\rm NL} = \frac{(I_2 - I_0)^2}{I_1^4} = \frac{36}{25} f_{\rm NL}^2, \qquad \frac{54}{25} g_{\rm NL} = \frac{(I_3 - 3I_1)}{I_1^3}$$

The non-linearity parameter is given by a simple expression in terms of the first three or four moments!

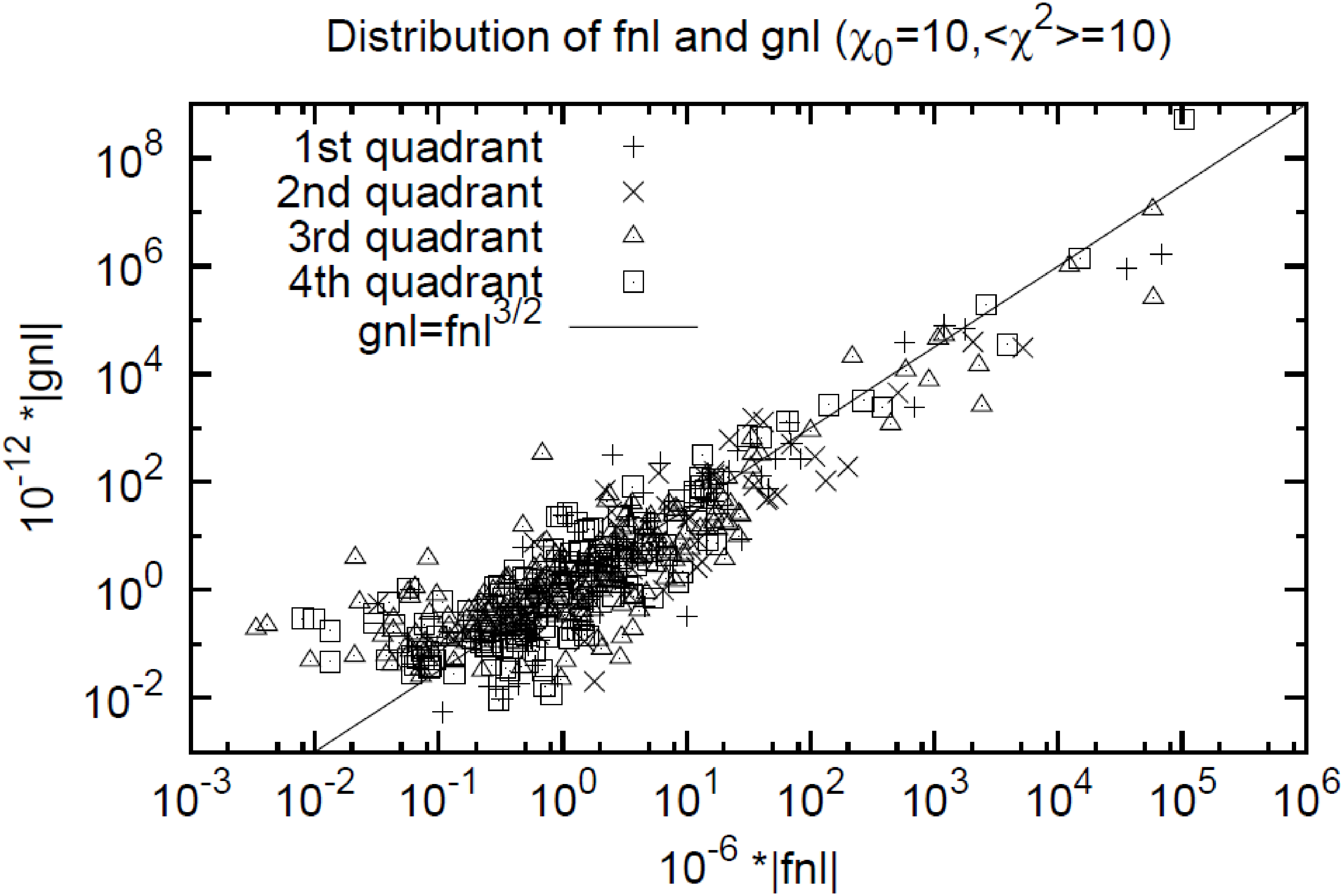
Mock mapping



1. To see general tendency
2. Non-availability of the raw data.



3, 4-point function



Observation implications

The curvature perturbation from preheating itself cannot explain the observed amplitude. (four orders of magnitude smaller)

Assuming $I_1 = 10^{-6}$ and $\xi_\chi = 0.04$ as typical values,

$$\langle \zeta(\vec{x})\zeta(\vec{y}) \rangle = 4 \times 10^{-14}$$

WMAP normalization

$$\langle \zeta(\vec{x})\zeta(\vec{y}) \rangle \approx 7 \times 10^{-10}$$

Massless preheating requires mixed scenario

$$\zeta = \zeta_{\rm inf} + \zeta_{\rm pre}$$

Mixed curvature perturbation

$$\zeta = \zeta_{\rm inf} + \zeta_{\rm pre}.$$

Inflaton perturbation dominates 2-point function,

$$\langle \zeta(\vec{x})\zeta(\vec{y})\rangle \approx \langle \zeta_{\rm inf}(\vec{x})\zeta_{\rm inf}(\vec{y})\rangle$$

But, higher order correlators have preheating origin,

$$\langle \zeta(\vec{x}_1)\zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle \approx \langle \zeta_{\rm pre}(\vec{x}_1)\zeta_{\rm pre}(\vec{x}_2)\zeta_{\rm pre}(\vec{x}_3)\rangle$$

Mixed curvature perturbation

$$s \equiv \frac{\langle \zeta_{\rm pre}(\vec{x})\zeta_{\rm pre}(\vec{y})\rangle}{\langle \zeta(\vec{x})\zeta(\vec{y})\rangle}$$

typically $s = 6 \times 10^{-5}$

$$\frac{6}{5} f_{\rm NL} = s^2 \frac{I_2 - I_0}{I_1^2}$$

$$\mathcal{O}(10^{-3}) \lesssim f_{\rm NL} \lesssim \mathcal{O}(0.1)$$

difficult to observe for the moment

$$\tau_{\rm NL} = \frac{(I_2 - I_0)^2}{I_1^4} s^3 = \frac{36}{25s} f_{\rm NL}^2,$$

$$\tau_{\rm NL} \lesssim \mathcal{O}(10^3)$$

may be observable in the near future

$$\frac{54}{25} g_{\rm NL} = \frac{(I_3 - 3I_1)}{I_1^3} s^3$$

$$g_{\rm NL} = \mathcal{O}(1)$$

difficult to observe for the moment

Summary

Massless preheating model generates (local type) curvature perturbation that cannot be simply Taylor-expanded in powers of the sourcing field.

We gave a general formalism to compute the correlation functions for such a case.

Correlation functions take approximately the same form as the standard case. Non-linearity parameters are expressed in a simple manner by using several moments.

The curvature perturbation from preheating has much smaller amplitude than observed, which means total curvature perturbation is mixture of inflationary origin and preheating one. The resultant degree of non-Gaussianity is not so significant. Maybe observable signal from trispectrum.

Comparison of different mappings

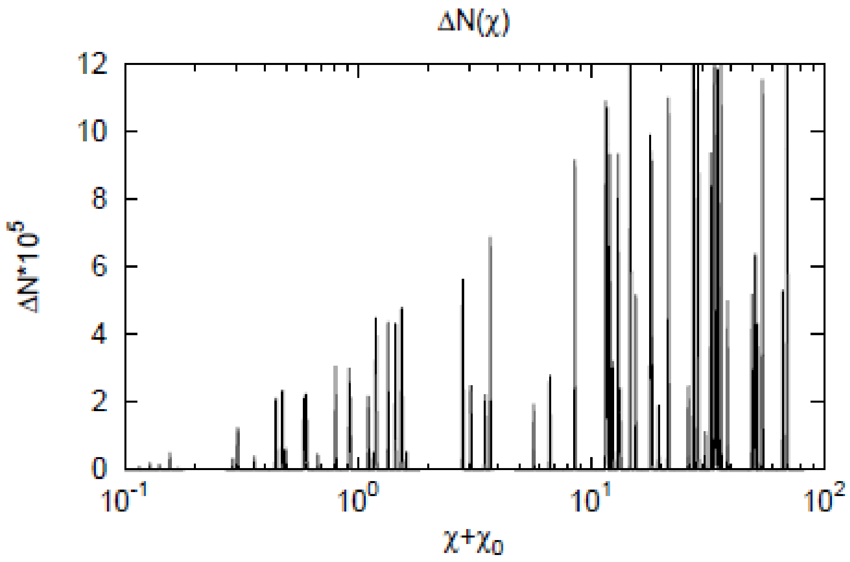


Figure 4. The function $f(\chi)$ for one realization that yields $10^{-6} \times f_{\rm NL} = 0.0055$.

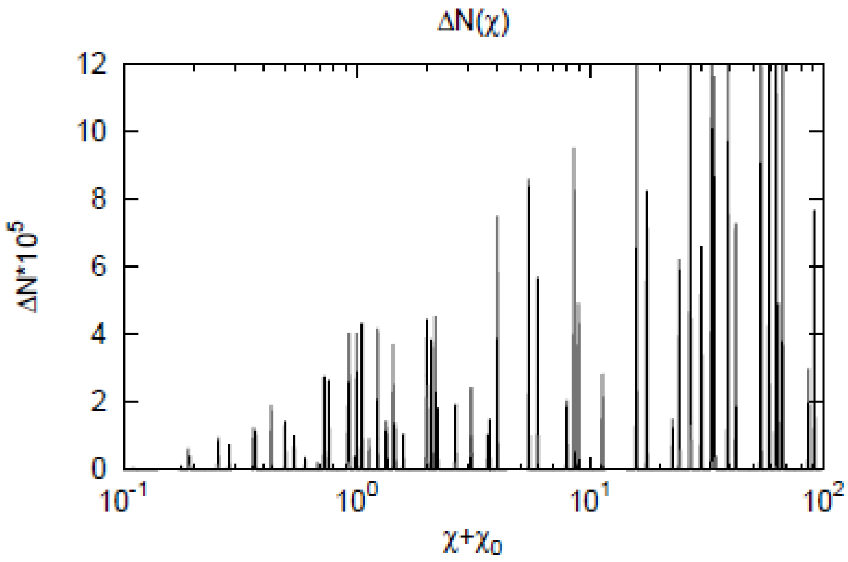


Figure 5. The function $f(\chi)$ for one realization that yields $10^{-6} \times f_{\rm NL} = 1.1 \times 10^5$.

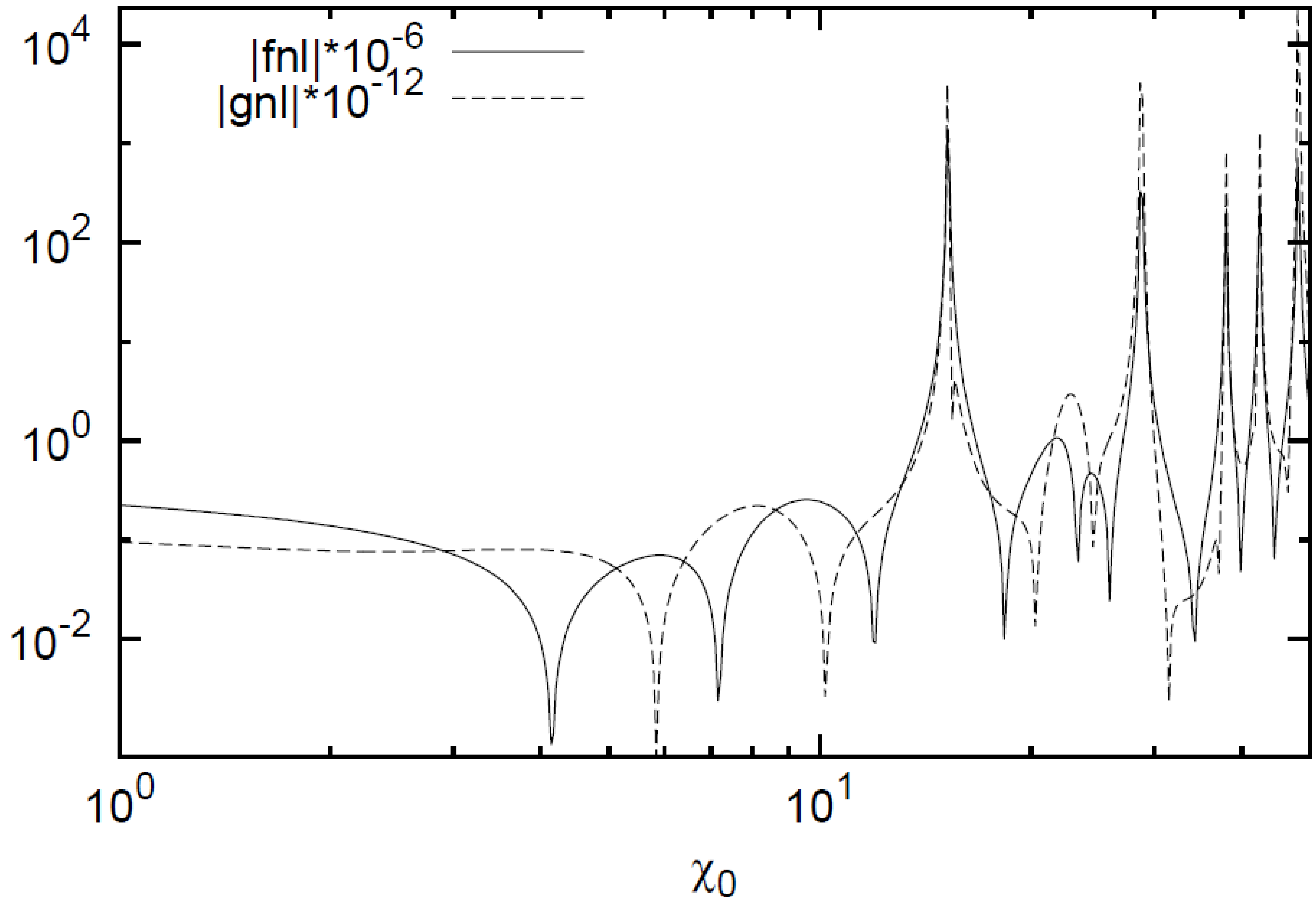
$$\frac{6}{5} f_{\rm NL} = \frac{I_2 - I_0}{I_1^2}$$

$$I_0 = \sum_p A_p \epsilon_p e^{-\frac{\eta_p^2}{2}}, \qquad I_1 = \sum_p A_p \epsilon_p \eta_p e^{-\frac{\eta_p^2}{2}}$$

$$I_2 = \sum_p A_p \epsilon_p \eta_p^2 e^{-\frac{\eta_p^2}{2}}$$

This result shows accidental cancellation among different moments yields extremely small or large value of fnl.

Dependence on the background value of χ_0



Indeed, non-linearity parameters take extreme values for some particular values of χ_0