Cosmological Perturbations:

Predictions and Observations

Inflation:

$$\frac{\varepsilon + P}{\varepsilon} \ll 1 \Longrightarrow \frac{\varepsilon + P}{\varepsilon} \simeq O(1)$$

$$a = a_f \exp(-N) \quad ; \quad 1 + \frac{P}{\varepsilon} = \frac{\beta}{(N+1)^\alpha}$$

where $\alpha, \beta \sim O(1)$

Predictions:

- $\Omega_0 = 1 \pm 10^{-5}$

The generated perturbations are:

a) adiabatic

b) Gaussian: $\Phi = \Phi_g + f_{NL}\Phi_g^2$

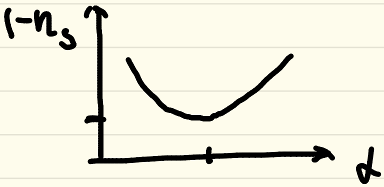
$f_{NL} \sim O(1)$

c) $\Phi^2 \propto \lambda^{1-n_s}$

$$n_s - 1 = -3\left(1 + \frac{p}{\varepsilon}\right) + \frac{d}{dN}\ln\left(1 + \frac{p}{\varepsilon}\right)$$

d) $r = \frac{T}{S} = 24\left(1 + \frac{p}{\varepsilon}\right)$

$$n_s - 1 = -\frac{3\beta}{N^\alpha} - \frac{\alpha}{N}$$



$(1-n_s)_{min} = 0.032$ for $N=50$

$$\boxed{n_s < 0.968}$$

$r = \frac{24\beta}{N^\alpha} \Rightarrow$ for $\alpha=2$, r in 50 times less than for $\alpha=1$

But for $\alpha>1$, $n_s \simeq -\frac{\alpha}{N} \gtrsim 0.96 \Rightarrow$

$\alpha \lesssim 2$ $\quad r_{min} \gtrsim \frac{24\beta}{N^2} \sim 10^{-2}\beta$

Scenarios:

$$\dot{\varepsilon} = -3H(\varepsilon + p)$$

$$\frac{d\ln\varepsilon}{dN} = 3\left(1+\frac{p}{\varepsilon}\right) = \frac{3\beta}{N^{\alpha}}$$

$$V(\varphi) \simeq \varepsilon(N) = \begin{cases} \varepsilon_f N^{3\beta} & , \alpha = 1 \\ \varepsilon_0 \exp\left(-\frac{3\beta}{\alpha-1}\frac{1}{N^{\alpha-1}}\right. & , \alpha \neq 1 \end{cases}$$

$N(\varphi)$ - ?

$$\frac{d\varphi}{dN} = \frac{\dot{\varphi}}{-H} = \sqrt{\frac{3}{8\pi}\left(1+\frac{p}{\varepsilon}\right)} \propto \pm\frac{1}{N^{\alpha/2}}$$

$$N = \begin{cases} \exp\left(\pm\sqrt{\frac{8\pi}{3\beta}}\,\varphi\right) & , \alpha = 2 \\ \dots\ (\pm\varphi + C)^{\frac{2}{2-\alpha}} & , \alpha \neq 2 \end{cases}$$

$\alpha = 1$, $\alpha = 2$, $\alpha \neq 1,2$

$\alpha = 1$

$$V = V(\varphi_f)\left(\frac{\varphi}{\varphi_f}\right)^{6\beta}$$

$$n_s - 1 = -\frac{3\beta+1}{N}, \qquad r = \frac{24\beta}{N} \quad \Rightarrow$$

$n_s = 0.96$, $r = 0.16$ for $3\beta = 1 \equiv \varphi^2$

$n_s = 0.94$, $r = 0.32$ for $3\beta = 2 \equiv \varphi^4$

$\alpha = 2$

$$n_s - 1 \simeq -\frac{2}{N}, \qquad \underline{n_s = 0.96}$$

$$r = \frac{24\beta}{N^{2}} = 10^{-2}\beta$$

$$V = V_0 \exp\left(-3\beta \exp(-\ldots\varphi)\right) \simeq V_0\left(1 - e^{-\ldots\varphi}\right)^2$$

$\beta = \frac{1}{2} \Leftrightarrow R^2$, Higgs

$\alpha \neq 1, 2$

$\alpha < 1$

$\frac{1}{2}$

$V \propto \exp(\dots \varphi^{2/3})$

$n_s = 0.85$

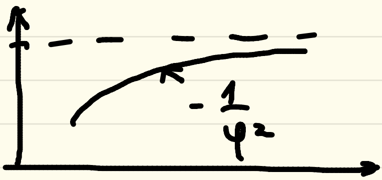
$r \simeq 1.13$

$1 < \alpha < 2$

$\frac{3}{2}$

$$V \simeq V_0 \exp\left(-\dots \frac{1}{\varphi^2}\right)$$

$$\simeq V_0 \left(1 - \dots \frac{1}{\varphi^2}\right)$$



$n_s \simeq 0.967$

$r \simeq 2 \cdot 10^{-2}$

$\alpha > 2$

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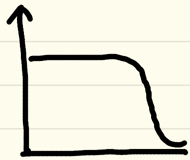
$V \propto e^{-\dots \varphi^4}$

$\simeq (1 - \dots \varphi^4)$



$n_s \simeq 0.94$

$r \simeq \frac{24\beta}{N^3}$

$\sim 6 \cdot 10^{-5} \beta$

K-Inflation

$$c_s \neq 1$$

$$n_s - 1 = -3\left(1+\frac{p}{\varepsilon}\right) + \frac{d}{dN}\ln\left(c_s\left(1+\frac{p}{\varepsilon}\right)\right)$$

$$r = \frac{T}{S} = 24\, c_s \left(1+\frac{p}{\varepsilon}\right)$$

$$f_{NL} \sim \frac{O(1)}{c_s^2}$$