Primordial trispectrum and orthogonal non-Gaussianities

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Motivations / message

- Data analysis of the trispectrum has received much less attention than the bispectrum: only $\tau_{NL}, \quad g_{NL}, \quad t_{NL}^{\mathrm{eq}}$
- Until recently, no counterpart to orthogonal bispectrum

⇨

- Generation of 'orthogonal-type' trispectra in a significant fraction of parameter space in the simplest theoretical context
- Use as a diagnostic for the appearance of a large orthogonal bispectrum (pre-Planck)

Renaux-Petel, 1302.6978 and 1303.2618

EFT of single-clock inflation

- Unitary gauge action: $(\delta\phi = 0)$

Cheung et al, 0709.0293

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2(3H^2 + \dot{H}) + F(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \nabla_\mu; t)\right]$$

- Lowest order in derivatives, up to 4th order in fluctuations:

$$F = \frac{1}{2}M_2(t)^4(\delta g^{00})^2 + \frac{1}{3!}M_3(t)^4(\delta g^{00})^3 + \frac{1}{4!}M_4(t)^4(\delta g^{00})^4 .$$

- Stückelberg time diffeomorphism $t \to t + \pi(x)$ in the decoupling regime (neglect the mixing with gravity)

$$\Rightarrow \quad \delta g^{00} \to -2\dot{\pi} - \dot{\pi}^2 + \frac{(\partial_i \pi)^2}{a^2}$$

with approximate shift-symmetry: constant $M_n(t)$

EFT of single-clock inflation

$$S_{\rm DL} = \int d^4x\sqrt{-g}\left[M_{\rm Pl}^2\dot{H}(\partial_\mu\pi)^2 + 2M_2^4\left(\dot{\pi}^2 - \dot{\pi}(\partial_\mu\pi)^2 + \frac{1}{4}((\partial_\mu\pi)^2)^2\right) + \frac{2M_3^4}{3}\left(-2\dot{\pi}^3 + 3\dot{\pi}^2(\partial_\mu\pi)^2\right) + \frac{2M_4^4}{3}\dot{\pi}^4\right]$$

where $(\partial_\mu\pi)^2 \equiv -\dot{\pi}^2 + (\partial_i\pi)^2/a^2$ and $\zeta = -H\pi$

Let us introduce:

$$\frac{1}{c_s^2} - 1 \equiv -\frac{2M_2^4}{M_p^2\dot{H}}$$

$$A/c_s^2 \equiv -1 + \frac{2}{3}\left(\frac{M_3}{M_2}\right)^4$$

$$\frac{B}{2c_s^4} \equiv 1 - \left(\frac{M_3}{M_2}\right)^4 + \frac{1}{2}\left(\frac{M_3}{M_2}\right)^8 - \frac{1}{6}\left(\frac{M_4}{M_2}\right)^4$$

EFT of single-clock inflation

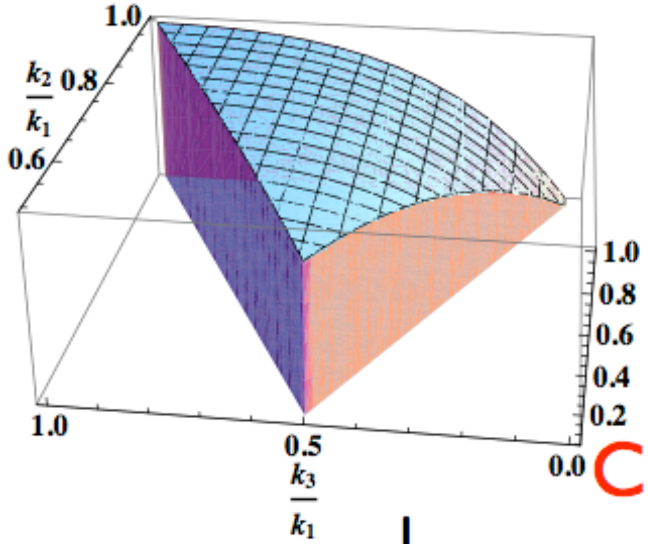
$$S_{\rm DL} = \int d^4x\sqrt{-g}\left[-\frac{M_P^2\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + \frac{M_P^2\dot{H}}{c_s^2}\left(\frac{\dot{\pi}(\partial_i\pi)^2}{a^2} + \frac{A}{c_s^2}\dot{\pi}^3\right) - \frac{M_P^2\dot{H}}{c_s^2}\left(\frac{((\partial_i\pi)^2)^2}{4a^4} + \frac{3A}{2c_s^2}\dot{\pi}^2\frac{(\partial_i\pi)^2}{a^2} + \frac{(9A^2/4 - B)}{c_s^4}\dot{\pi}^4\right)\right]$$

(leading-order terms in the low sound speed limit)

For A and B of order one, all operators introduce the same strong coupling scale ⇨ technically natural

Well known bispectrum: one-parameter family of shapes; equilateral, and orthogonal for $3.1 < A < 4.2$

Orthogonal bispectrum



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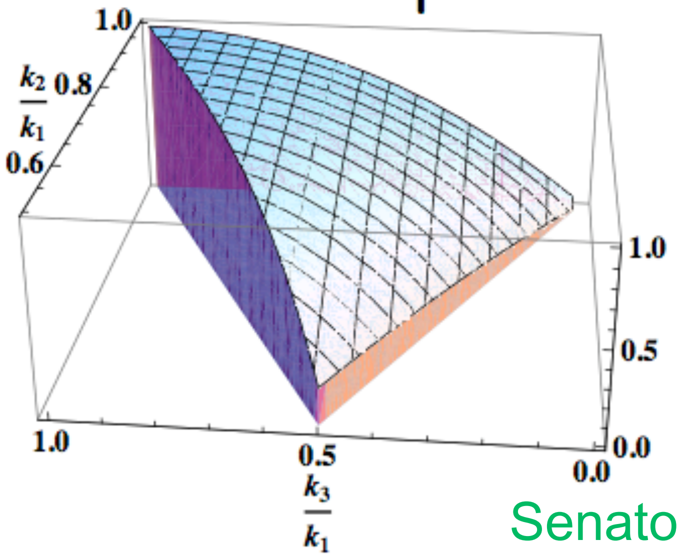

Change of basis

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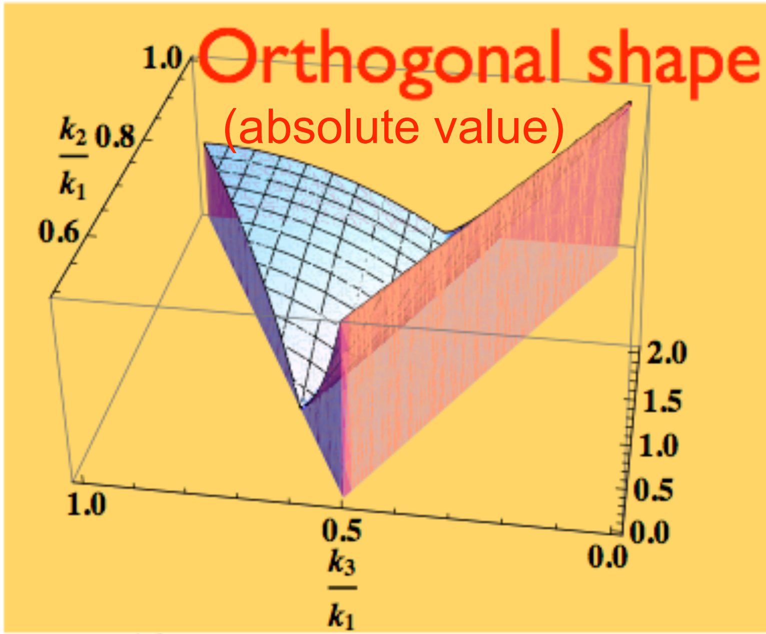
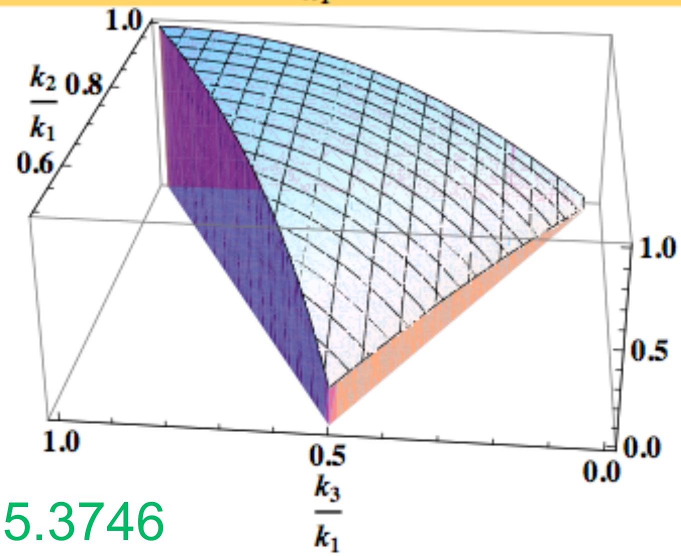




Senatore et al, 0905.3746

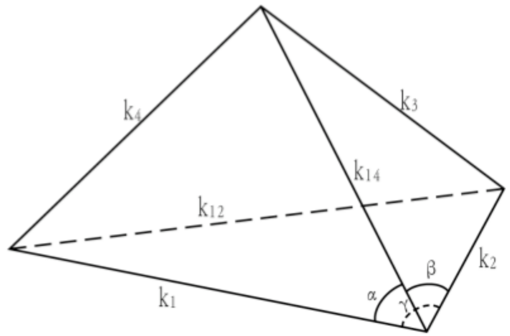
Trispectrum

Set-up computationaly equivalent to k-inflation

Chen et al, 0905.3494

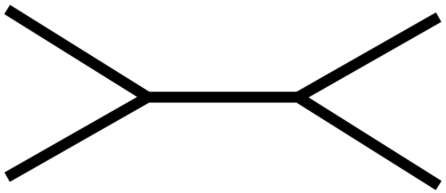
$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle_c = (2\pi)^9\mathcal{P}_\zeta^3\delta(\sum_{\mathbf{i}}\mathbf{k}_{\mathbf{i}})\prod_{i=1}^{4}\frac{1}{k_i^3}\mathcal{T}(k_1,k_2,k_3,k_4,k_{12},k_{14})$$

$$\mathcal{T}(A,B) = \frac{1}{c_s^4}\left(\frac{A^2}{4}T_{s1} - \frac{A}{2}T_{s2} + T_{s3} - BT_{c1}\right)$$

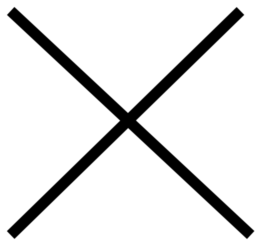


$$k_{ij} = |\boldsymbol{k}_i + \boldsymbol{k}_j|$$

s=scalar exchange



c=contact interactions



‘Equilateral’ trispectrum

All 4 shapes are similar, and peak near the regular tetrahedron (RT) limit $k_1 = k_2 = k_3 = k_4 = k_{12} = k_{14}$

Simple representative ansatz: $T_{c1} = 36 \frac{(k_1 k_2 k_3 k_4)^2}{(k_1 + k_2 + k_3 + k_4)^5}$

Simple measure of the amplitude:

$$\frac{1}{k^3} \mathcal{T}(k_1, k_2, k_3, k_4, k_{12}, k_{14}) \xrightarrow[\text{limit}]{\text{RT}} t_{NL}$$

Only constraint to date on a trispectrum from quantum origin (non-local type) :

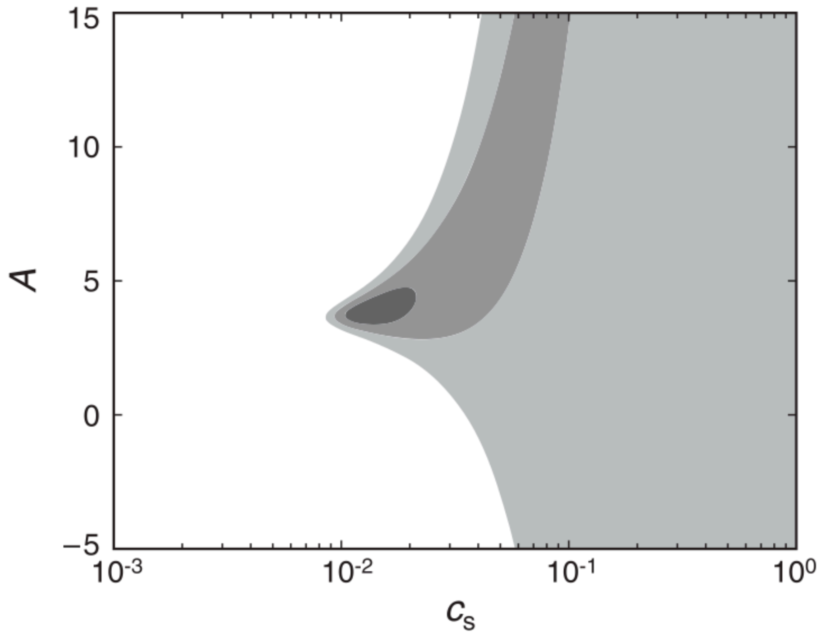
Assuming $\mathcal{T} = t_{NL}^{\text{eq}} / t_{NL}(T_{c1}) \times T_{c1}$

⇨

$$t_{NL}^{\text{eq}} = (-3.11 \pm 7.5) \times 10^6$$

Fergusson, Regan, Shellard (10)

A diagnostic of a large orthogonal bispectrum (pre-Planck)



WMAP9

WMAP9 (weakly) favored values of c_s of order 10^{-2}

For such values, the trispectrum, scaling like $1/c_s^4$

is of order 10^8 and is greater than the observational bound

$$t_{NL}^{\mathrm{eq}} = (-3.11 \pm 7.5) \times 10^6$$

Strong constraints on models that can generate the large orthogonal bispectrum suggested by WMAP9 … unless the observational bound is not applicable!

Trispectrum

$$\mathcal{T}(A,B) = \frac{1}{c_s^4}\left(\frac{A^2}{4}T_{s1} - \frac{A}{2}T_{s2} + T_{s3} - BT_{c1}\right)$$

Two-parameter family of shapes. For any A, T interpolates between highly correlated and anti-correlated with Tc1 as B goes from large negative to positive values. A shape qualitatively different from Tc1 necessarily arises in the neighborhood of a particular value of B (function of A).

$\Rightarrow$

Around which value? How narrow is this region?
What does the shape look like? How does it vary with A?

Orthogonal trispectrum

In 1302.6978, we concentrated on the region 3.1< A < 4.2 where the orthogonal bispectrum arises, motivated by the hints of a large orthogonal bispectrum in WMAP9 analysis.

Visual inspection of shapes in various representative limits: Tc1 is representative only for

- $B \lesssim 5$ and $B \gtrsim 11$ $(A = 3.2)$
- $B \lesssim 5$ and $B \gtrsim 14$ $(A = 3.6)$
- $B \lesssim 10$ and $B \gtrsim 19$ $(A = 4)$

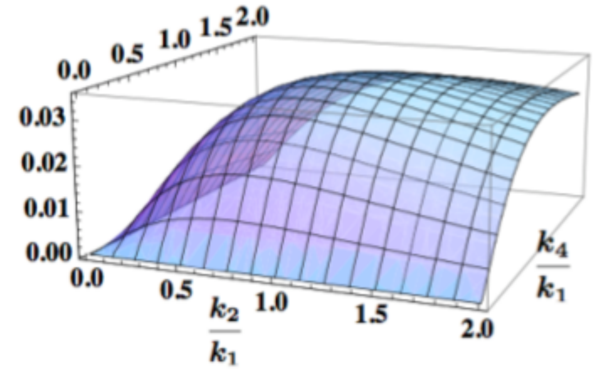
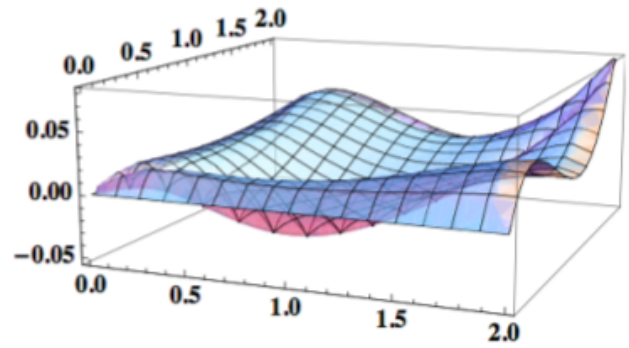
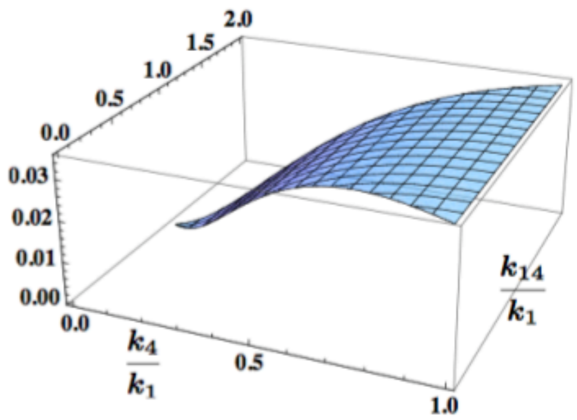
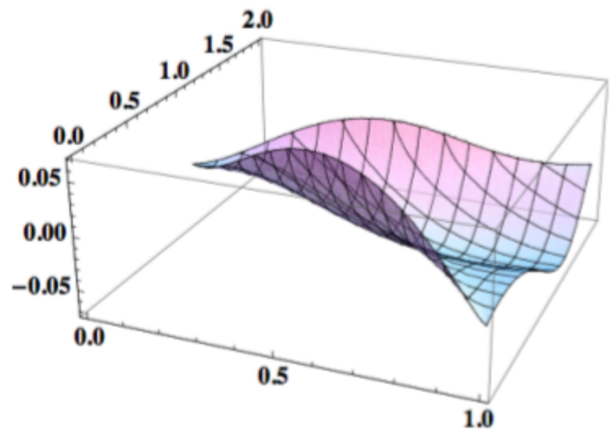
A qualitatively different shape arises in the complementary region, centered around:

- $B \simeq 8.5$ $(A = 3.2)$
- $B \simeq 11.5$ $(A = 3.6)$
- $B \simeq 14.5$ $(A = 4)$

The shapes of these various orthogonal trispectra depend very weakly on A

Orthogonal trispectrum

Plots of the dimensionless quantities $\tilde{\mathcal{T}} = \mathcal{T}/(k_1k_2k_3k_4)^{3/4}$

	$\tilde{T}_{c1}$	$\tilde{\mathcal{T}}(3.6, 11.5)$
Specialized planar limit		
Folded limit		

More in 1302.6978

Family of orthogonal trispectra

More generally, the cancellation of tNL gives a good estimate of when Tc1 is not representative of the total shape.

⇨

One-parameter family of ‘orthogonal-type’ trispectra:

$$\frac{A^2}{4}T_{s1} - \frac{A}{2}T_{s2} + T_{s3} - 8.66\left(1 - 0.69A + 0.20A^2\right)T_{c1}$$

This is in full agreement with the visual inspection of shapes

Conclusion

- Data analysis of the trispectrum has received much less attention than the bispectrum. Until recently, no counterpart to orthogonal bispectrum.
- We have shown that the simple ‘equilateral’ ansatz used to represent trispectra from derivative operators is not enough: qualitatively new ‘orthogonal-type’ trispectra are generated in a substantial fraction of parameter space in the simplest theoretical context. Current constraints are mostly blind to them. Need for dedicated data analysis.
- See 1303.2618 for a discussion of the trispectrum in the case of DBI-Galileon inflation, which can generate a bispectrum of orthogonal shape.