Multi-field G-inflation

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Based on arXiv:1308.4798

Outline

- Most general scalar-tensor theory with 2nd-order EoM & candidate of its multi-scalar field extension:

  Generalized covariant multi-Galileon

- Cosmological perturbations

- Comparison with Multi-DBI inflation

  → Multi-DBI inflation $\not\subset$ Gen. Cov. Multi-Galileon

Generalized Covariant Galileon

- Generalized Galileon [Horndeski 1974]

Most general scalar-gravity theory with 2nd order EoMs

$$\mathcal{L}_2 = G_2(X, \phi)$$

$$\left(X \equiv -\frac{1}{2} (\nabla \phi)^2 \right)$$

$$\mathcal{L}_3 = -G_3(X, \phi) \Box \phi$$

$$\mathcal{L}_4 = G_4(X, \phi) R + \frac{\partial G_4}{\partial X} \left[(\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right]$$

$$\mathcal{L}_5 = G_5(X, \phi) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi - \frac{1}{6} \frac{\partial G_5}{\partial X} \left[(\Box \phi)^3 - 3 \Box \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right]$$

Generalized Covariant Multi-Galileon

- Single scalar ϕ → Multi scalar ϕ^I

[Padilla & Sivanesan 2012]

$$X \equiv -\frac{1}{2}(\nabla\phi)^2 \quad \rightarrow \quad X^{IJ} = -\frac{1}{2}\nabla_\mu\phi^I\nabla^\mu\phi^J$$

$$\mathcal{L}_2 = G_2(X,\phi) \quad \rightarrow \quad \mathcal{L}_2 = G_2(X^{IJ},\phi^K)$$

$$\mathcal{L}_3 = -G_3(X,\phi)\,\Box\phi \quad \rightarrow \quad \mathcal{L}_3 = -G_{3L}(X^{IJ},\phi^K)\,\Box\phi^L$$

$$\mathcal{L}_4 = G_4(X,\phi)\,R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \quad \rightarrow \quad \mathcal{L}_4 = G_4(X^{IJ},\phi^K)\,R + \frac{\partial G_4}{\partial X^{IJ}}\left[\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J\right]$$

$$\mathcal{L}_5 = G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right] \quad \rightarrow \quad \mathcal{L}_5 = G_{5L}(X^{IJ},\phi^K)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi^L - \frac{1}{6}\frac{\partial G_{5I}}{\partial X^{JK}}\left[\Box\phi^I\Box\phi^J\Box\phi^K - \ \cdots\right.$$

Generalized Covariant Multi-Galileon

- Single scalar ϕ → Multi scalar ϕ^I

[Padilla & Sivanesan 2012]

To keep EoMs 2nd order,

$$\frac{\partial G_{3I}}{\partial X^{JK}}, \quad \frac{\partial^2 G_4}{\partial X^{IJ}\partial X^{KL}}, \quad \frac{\partial G_{5I}}{\partial X^{JK}}, \quad \frac{\partial^2 G_{5I}}{\partial X^{JK}\partial X^{LM}}$$

must be symmetric w.r.t. I, J, K, L, M

$$\mathcal{L}_3 = -G_3(X,\phi)\,\Box\phi \quad \rightarrow \quad \mathcal{L}_3 = -G_{3L}(X^{IJ},\phi^K)\,\Box\phi^L$$

$$\mathcal{L}_4 = G_4(X,\phi)\,R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \quad \rightarrow \quad \mathcal{L}_4 = G_4(X^{IJ},\phi^K)\,R + \frac{\partial G_4}{\partial X^{IJ}}\left[\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J\right]$$

$$\mathcal{L}_5 = G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right] \quad \rightarrow \quad \mathcal{L}_5 = G_{5L}(X^{IJ},\phi^K)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi^L - \frac{1}{6}\frac{\partial G_{5I}}{\partial X^{JK}}\left[\Box\phi^I\Box\phi^J\Box\phi^K - \cdots\right]$$

Generalized Covariant Multi-Galileon

- Flat Friedmann background $ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$
  - Friedmann eq: $\mathcal{E}(\phi^I, \dot\phi^J, H) = 0$, $\mathcal{P}(\phi^I, \dot\phi^J, \ddot\phi^K, H, \dot H) = 0$
  - ϕ^I EoMs: $\dfrac{1}{a^3}\dfrac{\mathrm{d}}{\mathrm{d}t}\left(a^3 \mathcal{J}_I\right) = \dfrac{\partial \mathcal{P}}{\partial \phi^I}$

$$
\begin{aligned}
\mathcal{E}(\phi^I, \dot\phi^J, H) \;=\;& 2X^{IJ}G_{2,\langle IJ\rangle} - G_2 + 6H\dot\phi^I X^{JK} G_{3IJK} - 2X^{IJ}G_{3I,J} \\
& -6H^2 G_4 + 24H^2 X^{IJ}\left(G_{4IJ} + X^{KL}G_{4IJKL}\right) - 12H\dot\phi^I X^{JK} G_{4IJ,K} - 6H\dot\phi^I G_{4,I} \\
& +2H^3\dot\phi^I X^{JK}\left(5G_{5IJK} + 2X^{LM}G_{5IJKLM}\right) - 6H^2 X^{IJ}\left(3G_{5I,J} + 2X^{KL}G_{5IJK,L}\right)
\end{aligned}
$$

$$
\mathcal{P}(\phi^I, \dot\phi^J, \ddot\phi^K, H, \dot H) \;=\; \tilde{\mathcal{P}}(\phi^I, \dot\phi^J, H) + \ddot\phi^K \mathcal{B}_K(\phi^I, \dot\phi^J, H) + 2\dot H \mathcal{G}_{\mathrm{T}}(\phi^I, \dot\phi^J, H)
$$

$$
\begin{aligned}
\tilde{\mathcal{P}} \;:=\;& G_2 - 2X^{IJ}G_{3I,J} + 6H^2 G_4 - 12H^2 X^{IJ} G_{4IJ} + 4H\dot\phi^I G_{4,I} + 4X^{IJ}G_{4,I,J} - 8HX^{IJ}\dot\phi^K G_{4IJ,K} \\
& -4H^3 X^{IJ}\dot\phi^K G_{5IJK} - 4H^2 X^{IJ}X^{KL}G_{5IJK,L} + 6H^2 X^{IJ}G_{5I,J} + 4HX^{IJ}\dot\phi^K G_{5I,J,K}, \\
\mathcal{B}_I \;:=\;& -2X^{JK}G_{3IJK} - 4H\dot\phi^J G_{4IJ} - 8HX^{JK}\dot\phi^L G_{4IJKL} + 2G_{4,I} + 4X^{JK}G_{4IJ,K} \\
& -6H^2 X^{JK}G_{5IJK} - 4H^2 X^{JK}X^{LM}G_{5IJKLM} + 4HX^{JK}\dot\phi^L G_{5IJK,L} + 4H\dot\phi^J G_{5(I,J)},
\end{aligned}
$$

$$
\mathcal{G}_{\mathrm{T}} := 2\left[G_4 - 2X^{IJ}G_{4IJ} - X^{IJ}\left(H\dot\phi^K G_{5IJK} - G_{5I,J}\right)\right]
$$

$$
\begin{aligned}
\mathcal{J}_I \;:=\;& \dot\phi^J G_{2,\langle IJ\rangle} + 6HX^{JK}G_{3IJK} - 2\dot\phi^J G_{3(I,J)} \\
& -2HG_{4,I} + 2\dot\phi^J G_{4,I,J} + 6H^2\dot\phi^J G_{4IJ} + 12H^2\dot\phi^J X^{KL}G_{4IJKL} - 8HX^{JK}G_{4JK,I} - 12HX^{JK}G_{4IJ,K} \\
& -6H^2\dot\phi^J G_{5(I,J)} + 4HX^{JK}G_{5J,K,I} + 6H^3 X^{JK}G_{5IJK} - 6H^2 G_{5IJK,L}\dot\phi^J X^{KL} - 2H^2\dot\phi^J X^{KL}G_{5JKL,I} \\
& +4H^3 X^{JK}X^{KL}G_{5IJKLM}.
\end{aligned}
$$

Generalized Covariant Multi-Galileon

- Cosmological perturbations

$$ds^2 = -(1+\alpha)^2 dt^2 + \gamma_{ij}\left(dx^i + \partial^i \beta dt\right)\left(dx^j + \partial^j \beta dt\right)$$

$$\phi^I = \phi_0^I(t) + Q^I(t, \vec{x}) \qquad \gamma_{ij} = a^2\left(\delta_{ij} + h_{ij} + \frac{1}{2}h_{ik}h_{kj}\right)$$

- Tensor perturbations

$$S_{\mathrm{T}}^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\mathcal{G}_{\mathrm{T}}\dot{h}_{ij}^2 - \frac{\mathcal{F}_{\mathrm{T}}}{a^2}\partial_k h_{ij}\partial_k h_{ij}\right]$$

$$\begin{cases} \mathcal{F}_{\mathrm{T}} = 2\left[G_4 - X^{IJ}\left(\ddot{\phi}^K G_{5IJK} + G_{5I,J}\right)\right] \\ \mathcal{G}_{\mathrm{T}} = 2\left[G_4 - 2X^{IJ}G_{4IJ} - X^{IJ}\left(H\dot{\phi}^K G_{5IJK} - G_{5I,J}\right)\right] \end{cases}$$

Generalized Covariant Multi-Galileon

- Scalar perturbations

$$\begin{cases} ds^2 = -(1+\alpha)^2 dt^2 + a^2 \left(dx_i + \partial_i \beta dt\right)\left(dx^i + \partial^i \beta dt\right) \\ \phi^I = \phi_0^I(t) + Q^I(t, \vec{x}) \end{cases}$$

$$S_{\rm S}^{(2)} = \int {\rm dtd}^3x\, a^3 \Big[\Sigma\alpha^2 - \frac{\partial\mathcal{E}}{\partial\dot\phi^I}\dot Q^I\alpha - \frac{\partial\mathcal{E}}{\partial\phi^I}Q^I\alpha - \frac{\mathcal{B}_I}{a^2}\partial^2 Q^I\alpha + \left(\mathcal{J}_I Q^I + \mathcal{B}_I\dot Q^I - 2\Theta\alpha\right)\frac{\partial^2\beta}{a^2} + \frac{1}{2}\left(\mathcal{A}_{IJ}\dot Q^I\dot Q^J + \frac{\partial^2\mathcal{P}}{\partial\phi^I\partial\phi^J}Q^I Q^J\right) + \frac{\partial\mathcal{J}_J}{\partial\phi^I}Q^I\dot Q^J - \frac{1}{2a^2}\mathcal{C}_{IJ}\partial_i Q^I\partial^i Q^J\Big]$$

$$\mathcal{A}_{IJ} = \frac{1}{2}\left(\frac{\partial\mathcal{J}_I}{\partial\dot\phi^J} + \frac{\partial\mathcal{J}_J}{\partial\dot\phi^I}\right) - \frac{1}{2}\left(\frac{\partial\mathcal{B}_I}{\partial\phi^J} + \frac{\partial\mathcal{B}_J}{\partial\phi^I}\right) \qquad \Theta = -\frac{1}{6}\frac{\partial\mathcal{E}}{\partial H} \qquad \Sigma = \frac{1}{2}\left(\dot\phi^I\frac{\partial\mathcal{E}}{\partial\dot\phi^I} + H\frac{\partial\mathcal{E}}{\partial H}\right)$$

$$\begin{aligned} \mathcal{C}_{IJ} = {} & G_{2,\langle IJ\rangle} + \left(\ddot\phi^K + 3H\dot\phi^K\right)G_{3IJK} - 2G_{3(I,J)} + a^{-1}\partial_t\left[a\dot\phi^K G_{3IJK}\right] + \left(8H^2 + 6\dot H\right)G_{4IJ} + 6H\left(\dot X^{KL} + 2HX^{KL}\right)G_{4IJKL} - 4\left(\ddot\phi^K + 2H\dot\phi^K\right)G_{4K(I,J)} \\ & + 2a^{-1}\partial_t\left[-aHG_{4IJ} + 4aHX^{KL}G_{4IJKL} - a\dot\phi^K G_{4IJ,K}\right] - 2\left(3H^2 + 2\dot H\right)G_{5(I,J)} + \left(3H^2\ddot\phi^K + 5H^3\dot\phi^K + 6H\dot H\dot\phi^K\right)G_{5IJK} \\ & - 4H\left(\dot X^{KL} + HX^{KL}\right)G_{5KL(I,J)} + \left(3H^2\dot X^{KL}\dot\phi^M + 2H^3X^{KL}\dot\phi^M\right)G_{5IJKLM} + a^{-1}\partial_t\left[-aH^2\dot\phi^K G_{5IJK} - 4aHX^{KL}G_{5IJK,L} + 2aH^2X^{KL}\dot\phi^M G_{5IJKLM}\right] \end{aligned}$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Single-field DBI inflation

$$S = \int d^4x \sqrt{-\gamma} \left[-\frac{1}{f} + \frac{M^2}{2} R[\gamma] \right]$$

with $\gamma_{\mu\nu} = g_{\mu\nu} + f \partial_\mu \phi \partial_\nu \phi$

$$\sqrt{-\gamma} = \sqrt{-g}\sqrt{1-2fX} \qquad \left(X \equiv -\frac{1}{2}(\nabla\phi)^2 \right)$$

$$\sqrt{-\gamma} R[\gamma] = \sqrt{-g} \left\{ \sqrt{1-2fX} R[g] - \frac{f}{\sqrt{1-2fX}} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \right\}$$

$$G_2 = -f^{-1}\sqrt{1-2fX} \qquad G_4 = \sqrt{1-2fX}$$

[de Rham, Tolley 2010]

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- Multi-field DBI inflation

$$S = \int d^4x \sqrt{-\gamma}\left[-\frac{1}{f} + \frac{M^2}{2} R[\gamma]\right]$$

with $\gamma_{\mu\nu} = g_{\mu\nu} + f\delta_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J$

$$\sqrt{-\gamma} = \sqrt{-g}\sqrt{\det\left(\delta_I{}^J - 2fX_I{}^J\right)} \qquad \left(X^{IJ} = -\frac{1}{2}\nabla_\mu\phi^I\nabla^\mu\phi^J\right)$$

$$\sqrt{-\gamma}R[\gamma] = \sqrt{-g}\left\{ \quad * \quad * \quad * \quad * \quad * \quad \right\}$$

$$G_2 = -f^{-1}\sqrt{\det\left(\delta_I{}^J - 2fX_I{}^J\right)} \qquad G_4 = \cdots$$

[Hinterbichler+, 2010]

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Multi-field DBI inflation

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with $\gamma_{\mu\nu} = g_{\mu\nu} + f \delta_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J$

$$\sqrt{-\gamma} R[\gamma] = \sqrt{-g} \left[R[g] + f \mathcal{L}^{(1)} + f^2 \mathcal{L}^{(2)} + \mathcal{O}(f^3) \right]$$

$$\mathcal{L}^{(1)} = -X R[g] - \delta_{IJ} \left(\Box\phi^I \Box\phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla^\mu \nabla^\nu \phi^J \right)$$

$$\mathcal{L}^{(2)} = -\frac{1}{6} \left(X^2 + 2X_{IJ} X^{IJ} \right) R - \frac{1}{3} \left(X \delta_{IJ} + 2X_{IJ} \right) \left(\Box\phi^I \Box\phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla^\mu \nabla^\nu \phi^J \right)$$
$$+ \frac{1}{3} L^{\mu\alpha\nu\beta} \partial_\mu \phi_I \partial_\nu \phi^I \partial_\alpha \phi_J \partial_\beta \phi^J$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Multi-field DBI inflation

$$S = \int \mathrm{d}^4x\sqrt{-\gamma}\left[-\frac{1}{f} + \frac{M^2}{2}R[\gamma]\right]$$

$$L^{\mu\alpha\nu\beta} = \frac{1}{4}\mathcal{E}^{\mu\alpha\rho\sigma}R_{\rho\sigma\xi\zeta}\mathcal{E}^{\xi\zeta\nu\beta}$$

$$= R^{\mu\alpha\nu\beta} + \left(R^{\mu\beta}g^{\nu\alpha} + R^{\nu\alpha}g^{\mu\beta} - R^{\mu\nu}g^{\alpha\beta} - R^{\alpha\beta}g^{\mu\nu}\right)$$

$$+ \frac{1}{2}R\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}\right)$$

$$\mathcal{L}^{(2)} = -\frac{1}{6}\left(X^2 + 2X_{IJ}X^{IJ}\right)R - \frac{1}{3}\left(X\delta_{IJ} + 2X_{IJ}\right)\left(\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J\right)$$

$$+ \frac{1}{3}L^{\mu\alpha\nu\beta}\partial_\mu\phi_I\partial_\nu\phi^I\partial_\alpha\phi_J\partial_\beta\phi^J$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Multi-field DBI inflation

$$S = \int \mathrm{d}^4x \sqrt{-\gamma} \left[-\frac{1}{f} + \frac{M^2}{2} R[\gamma] \right]$$

$$\therefore \frac{\sqrt{-\gamma} R[\gamma]}{\sqrt{-g}} = G_4(X^{IJ}, \phi^K)\, R + \frac{\partial G_4}{\partial X^{IJ}} \left[\Box\phi^I \Box\phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla^\mu \nabla^\nu \phi^J \right] + \mathcal{L}_*$$

with

$$G_4 = 1 - f \delta_{IJ} X^{IJ} - \frac{f^2}{6} \left(\delta_{IJ}\delta_{KL} + \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK} \right) X^{IJ} X^{KL} + \mathcal{O}(f^3)$$

$$\mathcal{L}_* = \frac{f^2}{3} \delta_{IJ} \delta_{KL} L^{\mu\alpha\nu\beta} \partial_\mu \phi^I \partial_\nu \phi^J \partial_\alpha \phi^K \partial_\beta \phi^L + \mathcal{O}(f^3)$$

Generalized Covariant Multi-Galileon

- Single scalar ϕ → Multi scalar ϕ^I

[Padilla & Sivanesan 2012]

$$X \equiv -\frac{1}{2}(\nabla\phi)^2 \quad \rightarrow \quad X^{IJ} = -\frac{1}{2}\nabla_\mu\phi^I\nabla^\mu\phi^J$$

$$\mathcal{L}_2 = G_2(X,\phi) \quad \rightarrow \quad \mathcal{L}_2 = G_2(X^{IJ},\phi^K)$$

$$\mathcal{L}_3 = -G_3(X,\phi)\,\Box\phi \quad \rightarrow \quad \mathcal{L}_3 = -G_{3L}(X^{IJ},\phi^K)\,\Box\phi^L$$

$$\mathcal{L}_4 = G_4(X,\phi)\,R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \quad \rightarrow \quad \mathcal{L}_4 = G_4(X^{IJ},\phi^K)\,R + \frac{\partial G_4}{\partial X^{IJ}}\left[\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J\right]$$

$$\mathcal{L}_5 = G_5(X,\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right] \quad \rightarrow \quad \mathcal{L}_5 = G_{5L}(X^{IJ},\phi^K)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi^L - \frac{1}{6}\frac{\partial G_{5I}}{\partial X^{JK}}\left[\Box\phi^I\Box\phi^J\Box\phi^K - \cdots\right]$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Multi-field DBI $\subset$ Gen. Covariant Multi-Galileon ?

$$\mathcal{L}_* = \frac{f^2}{3}\delta_{IJ}\delta_{KL}L^{\mu\alpha\nu\beta}\partial_\mu\phi^I\partial_\nu\phi^J\partial_\alpha\phi^K\partial_\beta\phi^L + \mathcal{O}(f^3)$$

$$\rightarrow \frac{4}{3}f^2H^2(X_{IJ} - \delta_{IJ}X)\partial_i Q^I\partial^i Q^J + \mathcal{O}(f^3)$$

$$S_{\rm S}^{(2)} = \int {\rm dtd}^3x\, a^3\Big[\Sigma\alpha^2 - \frac{\partial\mathcal{E}}{\partial\dot{\phi}^I}\dot{Q}^I\alpha - \frac{\partial\mathcal{E}}{\partial\phi^I}Q^I\alpha - \frac{\mathcal{B}_I}{a^2}\partial^2Q^I\alpha + \left(\mathcal{J}_IQ^I + \mathcal{B}_I\dot{Q}^I - 2\Theta\alpha\right)\frac{\partial^2\beta}{a^2}$$
$$+\frac{1}{2}\left(\mathcal{A}_{IJ}\dot{Q}^I\dot{Q}^J + \frac{\partial^2\mathcal{P}}{\partial\phi^I\partial\phi^J}Q^IQ^J\right) + \frac{\partial\mathcal{J}_J}{\partial\phi^I}Q^I\dot{Q}^J - \frac{1}{2a^2}\mathcal{C}_{IJ}\partial_iQ^I\partial^iQ^J\Big]$$

$$\mathcal{C}_{IJ} \supset H^2\big(6G_{4IJ} + 20X^{KL}G_{4IJKL} - 6G_{5(I,J)}$$
$$-4X^{KL}G_{5KL(I,J)} - 6X^{KL}G_{5IJK,L} + 4X^{KL}X^{MN}G_{5IJKLM,N}\big)$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Multi-field DBI $\subset$ Gen. Covariant Multi-Galileon ?

To keep EoMs 2nd order,

$$\frac{\partial G_{3I}}{\partial X^{JK}}, \quad \frac{\partial^2 G_4}{\partial X^{IJ}\partial X^{KL}}, \quad \frac{\partial G_{5I}}{\partial X^{JK}}, \quad \frac{\partial^2 G_{5I}}{\partial X^{JK}\partial X^{LM}}$$

must be symmetric w.r.t. I, J, K, L, M

$$\Rightarrow G_{4IJ},\ G_{5(I,J)} \supset (\delta_{IJ}\delta_{KL} + \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK})\, X^{KL}$$

$$S_{\rm S}^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \Big[\Sigma\alpha^2 - \frac{\partial\mathcal{E}}{\partial\dot\phi^I}\dot Q^I\alpha - \frac{\partial\mathcal{E}}{\partial\phi^I}Q^I\alpha - \frac{\mathcal{B}_I}{a^2}\partial^2 Q^I\alpha + \left(\mathcal{J}_I Q^I + \mathcal{B}_I\dot Q^I - 2\Theta\alpha\right)\frac{\partial^2\beta}{a^2} + \frac{1}{2}\left(\mathcal{A}_{IJ}\dot Q^I\dot Q^J + \frac{\partial^2\mathcal{P}}{\partial\phi^I\partial\phi^J}Q^IQ^J\right) + \frac{\partial\mathcal{J}_J}{\partial\phi^I}Q^I\dot Q^J - \frac{1}{2a^2}\mathcal{C}_{IJ}\partial_i Q^I\partial^i Q^J\Big]$$

$$\mathcal{C}_{IJ} \supset H^2\big(6G_{4IJ} + 20X^{KL}G_{4IJKL} - 6G_{5(I,J)} - 4X^{KL}G_{5KL(I,J)} - 6X^{KL}G_{5IJK,L} + 4X^{KL}X^{MN}G_{5IJKLM,N}\big)$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Multi-field DBI $\not\subset$ Gen. Covariant Multi-Galileon

$$\mathcal{L}_* = \frac{f^2}{3}\delta_{IJ}\delta_{KL}L^{\mu\alpha\nu\beta}\partial_\mu\phi^I\partial_\nu\phi^J\partial_\alpha\phi^K\partial_\beta\phi^L + \mathcal{O}(f^3)$$

$$\rightarrow \frac{4}{3}f^2H^2(X_{IJ} - \delta_{IJ}X)\partial_i Q^I\partial^i Q^J + \mathcal{O}(f^3)$$

$\neq$

$$S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\Sigma\alpha^2 - \frac{\partial\mathcal{E}}{\partial\dot{\phi}^I}\dot{Q}^I\alpha - \frac{\partial\mathcal{E}}{\partial\phi^I}Q^I\alpha - \frac{\mathcal{B}_I}{a^2}\partial^2Q^I\alpha + \left(\mathcal{J}_IQ^I + \mathcal{B}_I\dot{Q}^I - 2\Theta\alpha\right)\frac{\partial^2\beta}{a^2}\right.$$

$$G_{4IJ},\ G_{5(I,J)} \supset (\delta_{IJ}\delta_{KL} + \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK})\,X^{KL}$$

$$\mathcal{C}_{IJ} \supset H^2\left(6G_{4IJ} + 20X^{KL}G_{4IJKL} - 6G_{5(I,J)}\right.$$
$$\left. -4X^{KL}G_{5KL(I,J)} - 6X^{KL}G_{5IJK,L} + 4X^{KL}X^{MN}G_{5IJKLM,N}\right)$$

Multi-field DBI $\subset$ Gen. Cov. Multi-Galileon?

- Most-general vector-gravity theory w/ 2[nd]-order EoM

[Horndeski 1976]

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_{Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4M^2} L^{\mu\alpha\nu\beta} F_{\mu\nu} F_{\alpha\beta} \right]$$

- Multi-field DBI inflation

$$\mathcal{L} = G_4(X^{IJ}, \phi^K)\, R + \frac{\partial G_4}{\partial X^{IJ}} \left[\Box\phi^I \Box\phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla^\mu \nabla^\nu \phi^J \right] + \mathcal{L}_*$$

with

$$G_4 = 1 - f \delta_{IJ} X^{IJ} - \frac{f^2}{6} \left(\delta_{IJ}\delta_{KL} + \delta_{IK}\delta_{JL} + \delta_{IL}\delta_{JK} \right) X^{IJ} X^{KL} + \mathcal{O}(f^3)$$

$$\mathcal{L}_* = \frac{f^2}{3} \delta_{IJ} \delta_{KL} L^{\mu\alpha\nu\beta} \partial_\mu \phi^I \partial_\nu \phi^J \partial_\alpha \phi^K \partial_\beta \phi^L + \mathcal{O}(f^3)$$

Summary

- Most general multi scalar-tensor theory w/ 2nd-order EoM:
  Generalized Covariant multi-Galileon
- Cosmological perturbations
- Comparison with Multi-DBI inflation
  → Multi-DBI inflation $\not\subset$ Gen. Cov. multi-galleon

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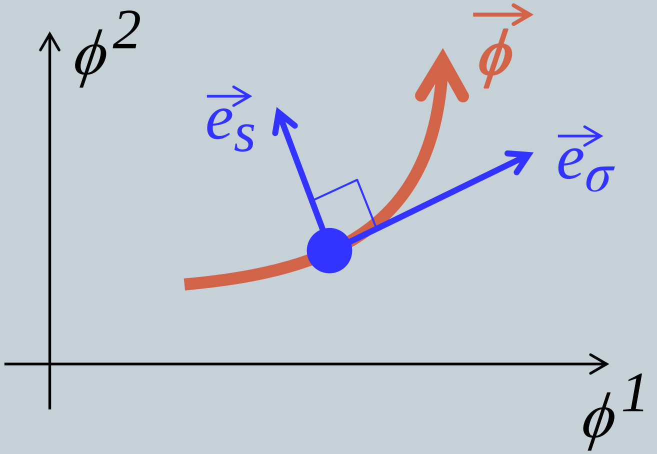
- Needs systematic approach to get most-general theory
- Similarity with Vector Horndeski theory?
- Application to cosmology

Two-field example ($I = 1, 2$)

- Adiabatic & Entropy modes

$$Q^I = Q_\sigma e_\sigma^I + Q_s e_s^I$$

$$\mathcal{R} \equiv \frac{H}{|\dot{\phi}^I|} Q_\sigma \qquad \mathcal{S} \equiv \frac{H}{|\dot{\phi}^I|} Q_s$$



$$S = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_\mathrm{S}\dot{\mathcal{R}}^2 - \frac{\mathcal{F}_\mathrm{S}}{a^2}(\partial\mathcal{R})^2 + \mathcal{L}_{\mathcal{S}\mathcal{S}} + \mathcal{L}_{\mathcal{R}\mathcal{S}}\right]$$

- $\mathcal{S}$ is independent of $\mathcal{R}$ ($k \to 0$)

$$\frac{\Theta\mathcal{G}_\mathrm{S}}{\mathcal{G}_\mathrm{T}}\dot{\mathcal{R}} \simeq I(\mathcal{S}, \dot{\mathcal{S}}) \qquad (k \to 0)$$

Generalized Multi-Galileon

- Scalar perturbations

$$S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\Sigma\alpha^2 - \frac{\partial\mathcal{E}}{\partial\dot{\phi}^I}\dot{Q}^I\alpha - \frac{\partial\mathcal{E}}{\partial\phi^I}Q^I\alpha - \frac{\mathcal{B}_I}{a^2}\partial^2 Q^I\alpha + \left(\mathcal{J}_I Q^I + \mathcal{B}_I\dot{Q}^I - 2\Theta\alpha\right)\frac{\partial^2\beta}{a^2} + \frac{1}{2}\left(\mathcal{A}_{IJ}\dot{Q}^I\dot{Q}^J + \frac{\partial^2\mathcal{P}}{\partial\phi^I\partial\phi^J}Q^IQ^J\right) + \frac{\partial\mathcal{J}_J}{\partial\phi^I}Q^I\dot{Q}^J - \frac{1}{2a^2}\mathcal{C}_{IJ}\partial_iQ^I\partial^iQ^J\right]$$

α, β

$$0 = 2\Sigma\alpha - \frac{\partial\mathcal{E}}{\partial\dot{\phi}^I}\dot{Q}^I - \frac{\partial\mathcal{E}}{\partial\phi^I}Q^I - \mathcal{B}_I\frac{\partial^2 Q^I}{a^2} - 2\Theta\frac{\partial^2\beta}{a^2}$$

$$0 = \mathcal{J}_I Q^I + \mathcal{B}_I\dot{Q}^I - 2\Theta\alpha$$

$$S_S^{(2)} = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\mathcal{K}_{IJ}\dot{Q}^I\dot{Q}^J - \frac{1}{a^2}\mathcal{D}_{IJ}\partial_iQ^I\partial^iQ^J - \mathcal{M}_{IJ}Q^IQ^J + 2\Omega_{IJ}Q^I\dot{Q}^J\right]$$