Dark Matter from Decaying Topological Defects

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MH, Kirk, West (in prep.)

Outline

Dark Matter

- Strong evidence from multiple sources for dark matter (DM)
- Planck + ΛCDM:[1] $\Omega_{\rm DM} h^2 = 0.1186 \pm 0.0031$
- A leading candidate: weakly interacting massive particle (WIMP)
- Standard thermal freeze-out:[2]
  relic abundance $\sim$ (total annihilation cross-section)$^{-1}$
- Refinements and other production mechanisms:
  - co-annihilation, near-threshold/resonant annihilation,[3]
- Other production mechanisms
  - freeze-in[4]
  - gravitino decay
  - and ...

[1] Ade et al 2013
[2] Zel'dovich 1965; Lee, Weinberg 1977
[3] Greist, Seckel 1991
[4] Hall et al 2010

“Top-Down" production of particles

- BSM physics often also predicts extra symmetries
- spontaneous breaking at scale $v_{\rm d} \rightarrow$
  extra phase transitions at temperature $T \simeq v_{\rm d}$
- phase transitions can produce topological defects:[5]
  - cosmic strings
  - textures, semilocal strings, monopoles, necklaces
- Decay of topological defects produces particles
  - SM states (γ, $e^{\pm}$, p, $\bar{p}$, ν, $\bar{\nu}$) $\rightarrow$ cosmic rays, γ-ray background[6]
  - ... and dark matter[7]

[5] Kibble 1976
[6] Review: Bhattacharjee, Sigl 2000
[7] Jeannerot, Zhang, Brandenberger 1999

TD = Topological Defect and Top-Down

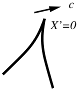
- TDs decay into a new sector of particles X (branch. frac. f_X)
- X particles decay into stable states including DM
- Energy injection rate per unit volume $Q(t) \sim t^{4-p}$
- Parameters of a TD model
  - mass of DM particle m_χ
  - energy density injection rate at $T = T_\alpha = m_\chi$: Q_α
  - exponent of power law p
  - average energy of X particles $\bar{E}_X$
  - average multiplicity of X decays N_χ
- DM number injection rate per unit volume:

$$j_\chi^{\text{inj}} = \frac{f_X N_\chi}{\bar{E}_X} Q$$

- Important combination: $q_X = Q_\alpha f_X / \rho_\alpha H_\alpha$
  (ρ - energy density, H - Hubble rate, evaluated at T_α)

Cosmic string TD models

- Strings decay into particles and gravitational radiation
- Branching fractions uncertain and model-dependent
- Strings parametrised by mass per unit length $\mu \simeq 2\pi v_{\mathrm{d}}^2$
- Consider two string decay scenarios:
  - A) Strings decay entirely into X particles
  - B) Strings decay mostly into g-radiation, small fraction X particles from string-antistring annihilation at cusps



- X-particle decay scenarios:
  - X1) $\bar{E}_X \sim v_{\mathrm{d}}$ (X particle masses at symmetry-breaking scale)
  - X2) $\bar{E}_X \sim m_\chi$ (X particle masses at DM scale - e.g. M_{susy})
- NB Third string scenario: particles from final string loop collapse[8] - subdominant contribution to particle production.

[8] Jeannerot, Zhang, Brandenberger 1999

Boltmann equation with source

Number density of dark matter states n_χ obeys

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma_\chi v \rangle \left(n_\chi^2 - n_{\chi,\text{eq}}^2 \right) + \frac{N_\chi f_X Q(t)}{\bar{E}_X},$$

- $\langle \sigma_\chi v \rangle$: thermally-averaged dark matter annihilation cross section
- ... weighted by v, relative speed of annihilating particles
- $n_{\chi,\text{eq}}$: equilibrium dark matter number density

Yield equation

- Definitions:
  - $x = m_\chi/T$ (proportional to scale factor)
  - $\langle \sigma_\chi v \rangle = \sigma_0 x^{-n}$ (s-wave: $n = 0$; p-wave: $n = 1$)
  - Dark matter yield $Y_\chi = n_\chi/s$ (where s is entropy density)
- Equation for yield:

$$\frac{dY_\chi}{dx} = -\frac{A}{x^{n+2}}\left(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2\right) + \frac{B}{x^{4-2p}},$$

where

$$A = \sqrt{\frac{\pi}{45}} M_{\mathrm{Pl}} m_\chi \sigma_0, \quad B = \frac{3}{4} x_\alpha^{2-2p} \left(\frac{N_\chi m_\chi}{\bar{E}_X}\right)\left(\frac{Q_\alpha f_X}{\rho_\alpha H_\alpha}\right).$$

Planck mass $M_{\mathrm{Pl}} = 1/\sqrt{G} \simeq 1.22 \times 10^{19}$ GeV

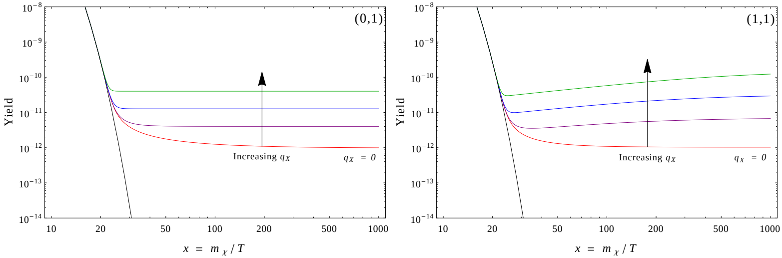
Model parameters

- Recall that yield equation depends on two parameters

$$A = \sqrt{\frac{\pi}{45}} M_{\mathrm{Pl}} m_\chi \sigma_0, \quad B = \frac{3}{4} x_\alpha^{2-2p} \left(\frac{N_\chi m_\chi}{\bar{E}_X} \right) \left(\frac{Q_\alpha f_X}{\rho_\alpha H_\alpha} \right).$$

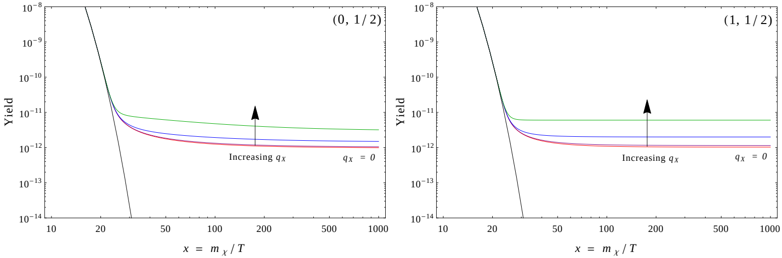
- Define:
  - χ multiplicity parameter: $\nu_\chi = \frac{N_\chi m_\chi}{\bar{E}_X}$
  - X injection rate parameter: $q_X = \frac{Q_\alpha f_X}{\rho_\alpha H_\alpha}$
- Scenario A: $p = 1$; Scenario B: $p = \frac{1}{2}$;
- Take $\nu_\chi \simeq 1$ (X particle decay scenario X2)
- Derive constraints on q_X for s-wave and p-wave annihilation
- Gives 4 models: $(n, p) = (0, 1)$, $(1, 1)$, $(0, \frac{1}{2})$, $(1, \frac{1}{2})$.

Numerical solutions: $(n,p) = (0,1), (1,1)$



$m_\chi = 500$ GeV, $\bar{E}_X = 1$ TeV, $N_\chi = 1$ GeV ($\nu_\chi = 0.5$),
$(n,p) = (0,1)$: $\sigma_0 = 1.6 \times 10^{-26}$ cm^{3}s^{-1}
$(n,p) = (1,1)$: $\sigma_0 = 7.0 \times 10^{-25}$ cm^{3}s^{-1}
Coloured lines: $q_X = 0, 10^{-9}, 10^{-8}, 10^{-7}$
Solid black line: equilibrium yield

Numerical solutions: $(n, p) = (0, 1/2), (1, 1/2)$



$m_\chi = 500$ GeV, $\nu_\chi = 0.5$,
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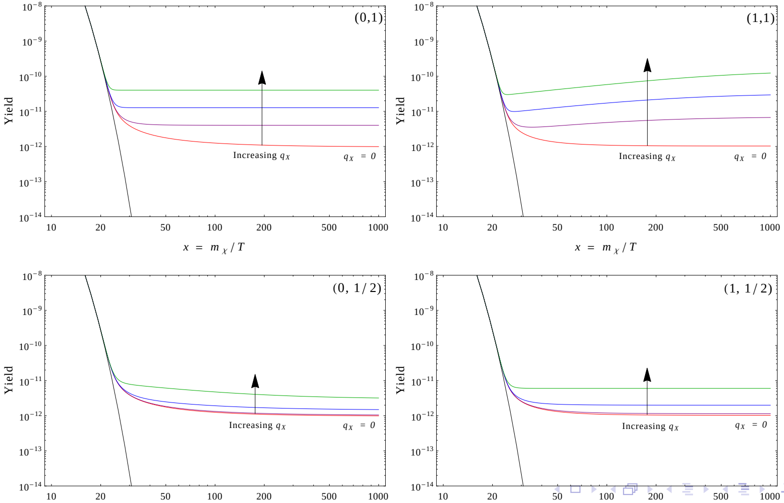
$(n, p) = (1, \frac{1}{2})$: $\sigma_0 = 7.0 \times 10^{-25}$ cm^{3}s^{-1}
$q_X = 0, 10^{-9}, 10^{-8}, 10^{-7}$ are plotted.
Solid black line: equilibrium yield

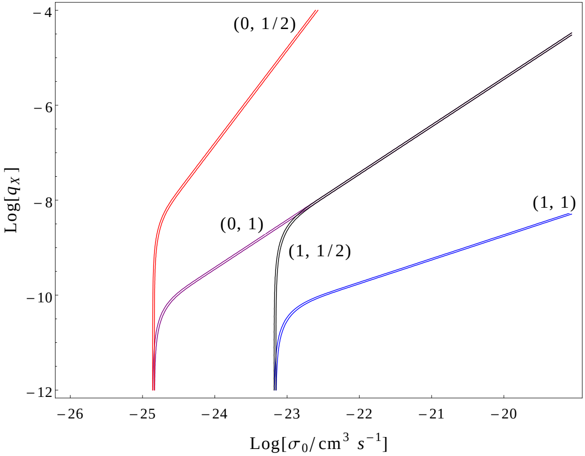
Numerical solutions: summary



Comments on numerical solutions

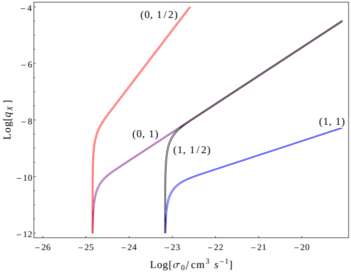
- Increasing in asymptotic yield with increasing q_X (expected)
- Recall yield eqn: $\frac{dY_\chi}{dx} = -\frac{A}{x^{n+2}}\left(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2\right) + \frac{B}{x^{4-2p}}$,
  post freeze-out behaviour depends on sign of $(n+2)-(4-2p)$
  - ($n+2 > 4-2p$) source drops less quickly than annihilation term
    – relic density dominated by source decays after freeze-out
    e.g. $(n,p) = (1,1)$
  - ($n+2 < 4-2p$) source drops more quickly than annihilation term
    – relic density close to ordinary freeze-out
    e.g. $(n,p) = (0,\frac{1}{2})$
  - ($n+2 = 4-2p$) source and annihilation terms drop at same rate
    – rapid asymptote to $Y_\chi(\infty) = \sqrt{B/A}$
    e.g. $(n,p) = (0,1),(1,\frac{1}{2})$

Fitting to Planck dark matter abundance



Comments on fit to Planck dark matter abundance

- Large q_X: power-law relationship – final yield still depends on DM annihilation cross-section[a]
- Small q_X: yield asymptotes to ordinary freeze-out value and becomes independent of source
- Slope of curve depends on (n, p)

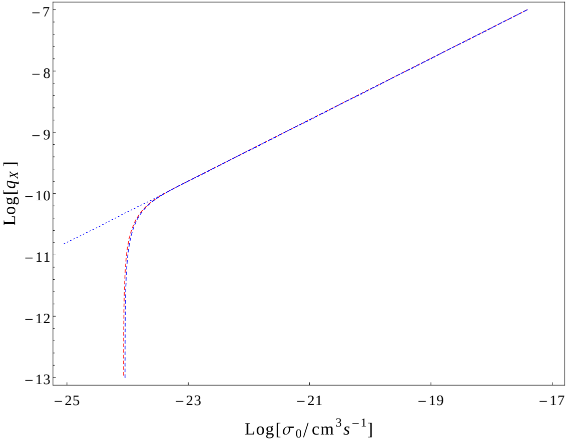


[a] Incorrect to integrate source from freeze-out

Analytic solution: Ricatti equation

- As x gets large, $Y_{\chi,\text{eq}} \to 0$ and Boltzmann equation can be approximated as
$$\frac{dY_\chi}{dx} = -\frac{A}{x^{n+2}} Y_\chi^2 + \frac{B}{x^{4-2p}}.$$
- Ricatti equation form - exact solution available.
- In large q_X limit: $Y_\chi(\infty) \approx (\alpha+\beta)^{\frac{\beta-\alpha}{\alpha+\beta}} \dfrac{B^{\frac{\alpha}{\alpha+\beta}} \Gamma\left(\frac{\beta}{\alpha+\beta}\right)}{A^{\frac{\beta}{\alpha+\beta}} \Gamma\left(\frac{\alpha}{\alpha+\beta}\right)}$
  where $\alpha = n+1$ and $\beta = 3-2p$.
- e.g. $n+2 = 4-2p$ gives $Y_\chi(\infty) \simeq \sqrt{B/A}$ as above

Comparison: analytic and numerical $(n, p) = (1, 1)$



Unitarity limit

- Annihilation cross-section constrained:[9]

$$\langle \sigma v_{rel} \rangle \leq \frac{4(2n+1)\sqrt{\pi x_d}}{m_\chi^2}$$

- Sourced freeze-out temperature x_d defined by $Y_\chi(x_d) - Y_{\chi,\mathrm{eq}}(x_d) \approx c Y_{\chi,\mathrm{eq}}(x_d)$ with $c = \mathrm{O}(1)$.

[9] Griest, Kamionkowski 1990

Indirect Fermi-LAT limit (model-dependent)

- Searches for γ continuum in dwarf galaxies give model-dependent limits to DM density[10]
- Assumptions in representative model:
  - s-wave annihilation ($n = 0$)[11]
  - $\chi\chi \to WW$

[10] Fermi-LAT 2011, Drlica-Wagner (talk) 2012
[11] Constraints on p-wave annihilation ($n = 1$) much weaker due to v-dependence of annihilation

Diffuse γ-ray background (model-dependent)

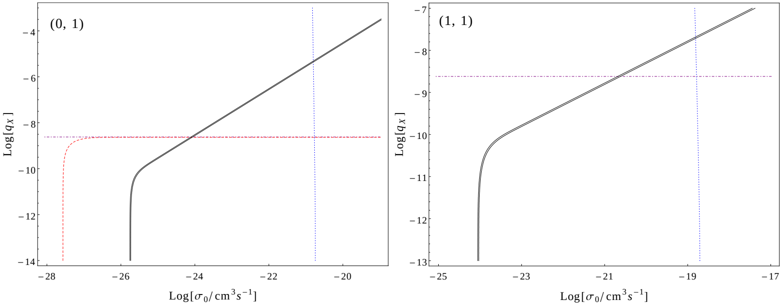
- X particles may also decay into SM particles
- Cascade decays to γ, $e^{\pm}$, p, $\bar{p}$, ν, $\bar{\nu}$
- Interaction with cosmic backgrounds, magnetic fields
- Result: cosmic rays, γ-ray background (GRB)[12]
- Observed GRB limits energy injection rate into EM cascade today[13]

$$Q_0 < 2.2 \times 10^{-23}(3p - 1)h \,\mathrm{eV\,cm^{-3}s^{-1}}$$

- No significant constraints for $p < 1$ (Q decays too quickly)

[12] Review: Bhattacharjee, Sigl 2000
[13] Sigl, Lee, Bhattacharjee, Yoshida 1998, using EGRET data

Constraints



(n,p)	Unitarity	Fermi-LAT	EGRET
$(0,1)$	$q_X \lesssim 4.6 \times 10^{-6}$	$q_X \lesssim 2.3 \times 10^{-9}$	$q_X \lesssim 2.4 \times 10^{-9}$
$(1,1)$	$q_X \lesssim 2.0 \times 10^{-8}$	-	$q_X \lesssim 2.4 \times 10^{-9}$
$(0,1/2)$	$q_X \lesssim 19$	$q_X \lesssim 6.1 \times 10^{-6}$	
$(1,1/2)$	$q_X \lesssim 3.8 \times 10^{-4}$	-	

Cosmic string models

- String mass per unit length $\mu \simeq 2\pi v_{\rm d}^2$.
- String density $\rho_{\rm d}$, average equation of state $w_{\rm d}$, density parameter $\Omega_{\rm d} = \rho_{\rm d}/\rho$.
- Numerical simulations: $w_{\rm d} \simeq 0$
- Total energy injection rate into (particles) + (gravitational radiation): Q
- Conservation of energy: $\frac{Q}{\rho H} = 3(w - w_{\rm d})\Omega_{\rm d} \simeq \frac{3}{2}\Omega_{\rm d}$

Constraints on cosmic string scenarios

- A: constant fraction $f_X \sim 1$ into X particles, $p = 1$
  - Numerical simulations: $\Omega_\mathrm{d} \simeq 5.3(8\pi G\mu)$
  - $q_X \simeq \left(\frac{v_\mathrm{d}}{10^{16}\,\mathrm{GeV}}\right)^2 10^{-3}$

(n,p)	Unitarity	Fermi-LAT	EGRET
$(0,1)$	$v_\mathrm{d} < 7.1 \cdot 10^{14}$ GeV	$v_\mathrm{d} < 1.6 \cdot 10^{13}$ GeV	$v_\mathrm{d} < 1.6 \cdot 10^{13}$ GeV
$(1,1)$	$v_\mathrm{d} < 4.7 \cdot 10^{13}$ GeV	-	$v_\mathrm{d} < 1.6 \cdot 10^{13}$ GeV

- B: subdominant X emission from cusps on string loops, $p = \frac{1}{2}$
  - Main loop decay channel gravitational waves, power $P_g = \Gamma G\mu^2$
  - Lower $\mu \to$ higher loop density $\to$ more cusps $\to$ more particles
  - $q_X = E\left(\frac{10^{16}\,\mathrm{GeV}}{v_\mathrm{d}}\right)^{\frac{5}{2}}\left(\frac{m_\chi}{\mathrm{TeV}}\right) 10^{-11}$ ($E = O(1)$ parameter combination)

(n,p)	Unitarity	Fermi-LAT	EGRET
$(0,1/2)$	$v_\mathrm{d} > 2.1 \cdot 10^{10} E^{\frac{2}{5}}$ GeV	$v_\mathrm{d} > 8.3 \cdot 10^{12} E^{\frac{2}{5}}$ GeV	
$(1,1/2)$	$v_\mathrm{d} > 1.6 \cdot 10^{12} E^{\frac{2}{5}}$ GeV	-	

Summary

- Dark matter produced "top-down" by decaying topological defects
- Analytic formula for DM yield in TD scenarios
- Depends on
  - DM particle mass m_χ, annihilation cross-section parameter σ_0
  - DM multiplicity parameter: $\nu_\chi = N_\chi m_\chi / \bar{E}_X$
  - X injection rate parameter: $q_X = Q_\alpha f_X / \rho_\alpha H_\alpha$
- (q_X, σ_0) parameter space consistent with Planck relic density
- Constraints on cosmic strings from unitarity, indirect detection (c.f. GRB)
  - Scenario A: upper bounds on $v_{\rm d}$
  - Scenario B: lower bounds on $v_{\rm d}$
- Outlook: specific models
  - Combine direct detection, collider limits, cosmic rays, g-waves
  - New predictions for indirect detection
  - New limits for TDs

Back-up slide A.1

- Ricatti equation $\frac{dY_\chi}{dx} = -\frac{A}{x^{n+2}} Y_\chi^2 + \frac{B}{x^{4-2p}}$,
- Exact asymptotic solution

$$Y_\chi(\infty) \approx (\alpha+\beta)^{\frac{\beta-\alpha}{\alpha+\beta}} \frac{B^{\frac{\alpha}{\alpha+\beta}} \Gamma\left(\frac{\beta}{\alpha+\beta}\right) I_{\frac{-\alpha}{\alpha+\beta}}\left(\frac{2\sqrt{AB}}{(\alpha+\beta)x_d^{(\alpha+\beta)/2}}\right)}{A^{\frac{\beta}{\alpha+\beta}} \Gamma\left(\frac{\alpha}{\alpha+\beta}\right) I_{\frac{\alpha}{\alpha+\beta}}\left(\frac{2\sqrt{AB}}{(\alpha+\beta)x_d^{(\alpha+\beta)/2}}\right)},$$

where $\alpha = n+1$ and $\beta = 3-2p$,

  - x_d defined as sourced freeze-out temperature: $Y_\chi(x_d) - Y_{\chi,\mathrm{eq}}(x_d) = cY_{\chi,\mathrm{eq}}(x_d)$, with $c = O(1)$ chosen to fit numerical solutions.
  - Iterative solution: $x_d \approx \log[Ac(c+2)k] - \left(n+\frac{1}{2}\right)\log[Ac(c+2)k] - \log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4Ac(c+2)B}{(\log[Ac(c+2)k])^{6+n-2p}}}\right)\right]$.

Back-up slide A.2

- Loop number density distribution: $n(\ell, t) = \frac{\nu}{t^{\frac{3}{2}}(\ell+\beta t)^{\frac{5}{2}}}$
  - $\nu = \mathrm{O}(1)$ constant
  - $\beta = \Gamma G\mu$, with $\Gamma \sim 10^2$ (gravitational radiation efficiency)
- Cusp emission power: $P_c = \beta_c \mu / \sqrt{v_\mathrm{d}\ell}$
- Energy injection rate: $Q_\mathrm{c} = \int_0^\infty d\ell \beta_c \mu \sqrt{\frac{1}{v_\mathrm{d}\ell}} n(\ell, t)$
- $q_X = \left.\frac{Q_\mathrm{c}}{\rho H}\right|_{T_\alpha} \simeq \frac{\beta_c \nu}{\beta^2} \frac{\mu}{m_\mathrm{P}^2} \left(\frac{\pi^2 g}{90}\right)^{\frac{1}{4}} \left(\frac{T_\alpha^2}{m_\mathrm{P} v_\mathrm{d}}\right)^{\frac{1}{2}}.$
- $q_X \sim \frac{\beta_c \nu}{\Gamma_{100}^2} \left(\frac{10^{16}\,\mathrm{GeV}}{v_\mathrm{d}}\right)^{\frac{5}{2}} \left(\frac{T_\alpha}{\mathrm{TeV}}\right) 10^{-11},$