

Cosmology of dRGT Massive Bigravity

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Massive Gravity Vs Massive Bigravity

Massive deformation of General Relativity

$$\sqrt{g} R + V \xrightarrow{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}} (\partial h)^2 + h_{\mu\nu} h^{\mu\nu} - h^2 + \dots$$

- Non linear MG **needs** an extra rank-2 tensor $f_{\mu\nu}$
(no non-trivial self coupling with only $g_{\mu\nu}$, i.e. $g_{\mu\alpha} g^{\alpha\nu} = \delta_{\mu}^{\nu}$)
- Non-derivative coupling $X_{\nu}^{\mu} = g^{\mu\alpha} f_{\alpha\nu}$ gives scalars $\tau_n = \text{Tr}(X^n)$
(e.g. for $f_{\mu\nu} = \eta_{\mu\nu}$, $\tau_1 = 4 - h + h^{\mu\nu} h_{\mu\nu} + \dots$)

$f_{\mu\nu}$ can be fixed or dynamical

Massive Gravity

Stuckelberg approach restores diff invariance

$$f_{\mu\nu} = \frac{\partial\phi^A}{\partial x^{\mu}} \frac{\partial\phi^B}{\partial x^{\nu}} \eta_{AB}$$

Unitary gauge: $\frac{\partial\phi^A}{\partial x^{\mu}} = \delta_{\mu}^A \Rightarrow f_{\mu\nu} = \eta_{\mu\nu}$

Massive Bigravity

$$M_P^2 \left(\sqrt{g} R + V + \kappa \sqrt{f} \tilde{R} \right)$$

- $f_{\mu\nu}$ dynamically determined by the eom
- two gravitons, one massive and one massless, propagate
- $\kappa \rightarrow \infty$ freezes the dynamic of $f_{\mu\nu}$

$$X_{\nu}^{\mu} = g^{\mu\alpha} f_{\alpha\nu} \quad Y_{\nu}^{\mu} = \left(\sqrt{X}\right)_{\nu}^{\mu} \quad \tau_n = \text{Tr}(Y^n)$$

$$V = \sum_{n=0}^4 a_n V_n$$

$$V_0 = 1, \quad V_1 = \tau_1, \quad V_2 = \tau_1^2 - \tau_2, \quad V_3 = \tau_1^3 - 3\tau_1\tau_2 + 2\tau_3,$$

$$V_4 = \tau_1^4 - 6\tau_1^2\tau_2 + 8\tau_1\tau_3 + 3\tau_2^2 - 6\tau_4.$$

- 5 DoF propagate to all orders (no BD instability)
- Valid also for general $f_{\mu\nu}$ and for Bigravity (7 DoF)

Hassan – Rosen (2011)

Theoretically consistent → study the phenomenology

- Issue: vDVZ discontinuity at linear level → spherically symmetric solutions & Vainshtein mechanism
- Cosmology
- Cosmological perturbations

Static Spherically Symmetric Solutions & Vainshtein

No Birkhoff theorem \rightarrow no unicity of the solution: 2 classes of solution

- Type I: non bi-diagonal metrics \rightarrow Schwarzschild de Sitter

Comelli, Nesti, Pilo, MC

$$ds^2 = -J(r) dt^2 + K(r) dr^2 + r^2 d\Omega^2$$

$$d\tilde{s}^2 = -C(r) dt^2 + A(r) dr^2 + 2 D(r) dt dr + B(r) d\Omega^2$$

$$J = 1 - \frac{2M_1}{r} + \Lambda_1 r^2, \quad K = \frac{1}{J}, \quad B = \omega^2 r^2, \quad D^2 + AC = \omega^4,$$

$$C = \omega^2 \left(1 - \frac{2M_2}{\kappa r} + \frac{\Lambda_2 r^2}{\kappa} \right), \quad A = \frac{2\omega^2 J - C}{J^2}$$

- $JK = 1 \rightarrow$ no vDVZ discontinuity
- for asymptotically flat solutions, the graviton mass is zero
- no Yukawa modifications

Take the limit $\kappa \rightarrow \infty$ and change coordinates to bring the second metric in a diagonal Minkowski form

$$f_{\mu\nu} = \eta_{\mu\nu}, \quad ds^2 = -J(r) dt^2 + K(r) dr^2 + 2 D(r) dt dr + B(r) d\Omega^2$$

$$B(r) = \omega^2 r^2, \quad J = \omega^2 - \frac{2M}{r} + \Lambda r^2, \quad J + K = 2\omega^2, \quad D^2 + JK = \omega^4$$

Koyama, Niz, Tasinato

Static Spherically Symmetric Solutions & Vainshtein

No Birkhoff theorem \rightarrow no unicity of the solution: 2 classes of solution

- Type II: bi-diagonal metrics (for small density source ρ)

Babichev, MC

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2$$

$$d\tilde{s}^2 = -e^n dt^2 + e^l (r + r\mu)^{l/2} dr^2 + (r + r\mu)^2 d\Omega^2$$

$\rho \ll 1 \quad \Rightarrow \quad \{\lambda, \nu, l, n\} \ll 1$, retain all the non-linearities in μ

r	$R_\odot < r \ll r_V$	$r_V \ll r \ll 1/m$	$r > 1/m$
μ	$\text{const} + \frac{m^2 r^3}{r_S}$	$-\frac{r_S \kappa}{3 m^2 r^3 (1 + \kappa)}$	$-\frac{r_S}{r} e^{-m r \sqrt{1+1/\kappa}}$
λ	$\frac{r_S}{r} - m^2 r^2$	$\frac{r_S (3 + 2\kappa)}{3 r (1 + \kappa)}$	$\frac{r_S}{r} \left(\frac{1}{\kappa} + e^{-m r \sqrt{1+1/\kappa}} \right)$
ν	$-\frac{r_S}{r} + m^2 r^2$	$-\frac{r_S (3 + 4\kappa)}{3 r (1 + \kappa)}$	$-\frac{r_S}{r} \left(\frac{1}{\kappa} + e^{-m r \sqrt{1+1/\kappa}} \right)$
l	$-\frac{m^2 r^2}{\kappa}$	$\frac{r_S}{3 r (1 + \kappa)}$	$\frac{r_S}{\kappa r} \left(1 - e^{-m r \sqrt{1+1/\kappa}} \right)$
n	$\frac{m^2 r^2}{\kappa}$	$\frac{r_S}{3 r (1 + \kappa)}$	$-\frac{r_S}{\kappa r} \left(1 - e^{-m r \sqrt{1+1/\kappa}} \right)$

Vainshtein mechanism works well also in bigravity

$\kappa \rightarrow \infty$ reproduces solutions found with fixed $f_{\mu\nu} = \eta_{\mu\nu}$

Koyama, Niz, Tasinato
Chkareuli, Pirtskhalava

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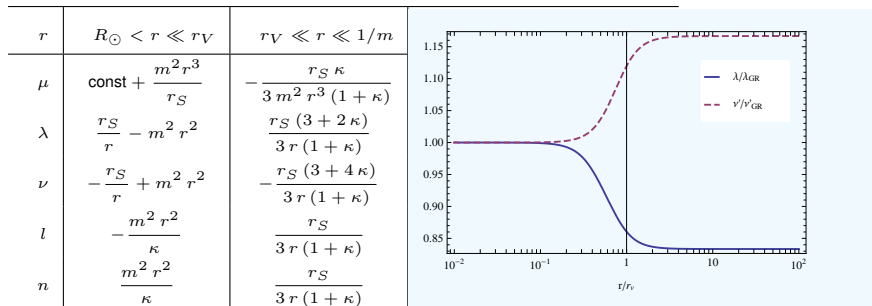
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Cosmological Solutions & Perturbations

Consider both metrics homogeneous and isotropic

Comelli, Nesti, Pilo, MC

Volkov

von Strauss et al.

$$ds^2 = a^2(t) \left(-dt^2 + \frac{dr^2}{1 - k_1 r^2} + r^2 d\Omega^2 \right)$$

$$\tilde{d}s^2 = \omega^2(t) \left[-c^2(t) dt^2 + \frac{dr^2}{1 - k_2 r^2} + r^2 d\Omega^2 \right]$$

$k_1 \neq k_2 \Rightarrow H = 0$, so take $k_1 = k_2 = k$

Bianchi gives **two branches of solution**:

- **Branch I:** $\omega/a = \text{const.}$ \rightarrow **FRW + CC**

$$H_\omega / H = \text{const.}^{-1}$$

$$H^2 = \frac{8\pi G}{3} \rho_m - \frac{k}{a^2} + m^2 \Lambda_1 \quad \frac{H_\omega^2}{c^2} = -\frac{k}{\omega^2} + \frac{m^2 \Lambda_2}{\kappa}$$

- the condition $\omega/a = \text{const.}$ at quadratic level around bi-flat solution gives zero graviton mass
- linear perturbations for scalars and vectors are stuck to GR, and both the tensors propagate.

$8 - (4 + 1) = 3$ DoF (1S+1V) are probably strongly coupled

Comelli, Pilo, MC

$\kappa \rightarrow \infty$ forces $k < 0 \Rightarrow$ No spatially flat Universe

Non-linear perturbations are unstable

Possible solutions: anisotropy, quasi-dilaton, mass-varying...

D'Amico et al.

Gumrukcuoglu, Lin, Mukohyama

De Felice, Gumrukcuoglu, Mukohyama

Cosmological Solutions & Perturbations

Consider both metrics homogeneous, isotropic and flat

Comelli, Nesti, Pilo, MC
Volkov
von Strauss et al.

$$ds^2 = a^2(t) (-dt^2 + dr^2 + r^2 d\Omega^2)$$
$$\tilde{d}s^2 = \omega^2(t) [-c^2(t) dt^2 + dr^2 + r^2 d\Omega^2]$$
$$\xi \equiv \frac{\omega}{a}$$

Bianchi gives **two branches of solution**:

● **Branch II**: $c = \frac{H\omega}{H} \xi \quad \longrightarrow \quad H^2 = \frac{m^2}{\kappa} \rho_{\text{eff}}(\xi) \quad F(\xi) = \frac{G \rho_m}{m^2}$

Fixed point for $\rho_m \rightarrow 0 \Rightarrow \xi = \text{const.} \quad c = 1$ **late time de Sitter phase**

At early time $\frac{G \rho_m}{m^2} \gg 1 \longrightarrow$ **viable cosmology only** for small $\xi \sim \frac{m^2}{G \rho_m}$

$$H^2 = \frac{8\pi G \rho_m}{3} [1 + f(\xi)] \quad f(\xi) \sim \xi \ll 1 \quad c = 4 + 3\omega$$

- all the 7 + 1 DoF propagate at linear level
- perturbations around the de Sitter phase are fine
- early time perturbations are problematic in the scalar sector inside the horizon:
gradient instability in the second metric scalar

Comelli, Pilo, MC

$$\kappa \rightarrow \infty \Rightarrow H = 0$$

since $\kappa \rightarrow \infty$ fixes $c = \frac{\omega H\omega}{\sqrt{-k}}$, **this branch does not exist with fixed $f_{\mu\nu}$**

Conclusions

- Spherically symmetric solutions are quite similar in Massive Gravity and Bigravity
 - both have a branch where the effective modification reduces to a cosmological constant term, so we have Schwarzschild de Sitter solutions
 - both have a branch where inside the Vainshtein radius we recover GR solutions (so it is ok for Solar System tests) and asymptotically the Yukawa decay
- Cosmological solutions instead are very different in Massive Gravity and Bigravity
 - massive gravity has only one branch of solution of the type FRW + CC that allows just open Universe
 - this solution is unstable (ghost) at non-linear level
 - one has to rely on anisotropy or extra-fields to avoid instabilities
 - massive bigravity has also another branch with viable cosmological background solutions
 - linear perturbations start growing exponentially in the scalar sector already in the radiation dominated era
 - it should require a cosmological version of the Vainshtein mechanism
- Both Massive gravity and Bigravity are problematic in the cosmological setup, even if the bigravity formulation seems to work a little bit better

Thank you

Cosmological perturbations: Equations

$$GR(\Phi_1) + a^2 m^2 f(\xi) F(\Phi_1, \Phi_2) = G a^2 \delta \rho$$

$$a^2 m^2 f(\xi) \approx a^2 m^2 \xi \approx a^2 \frac{m^4}{G \rho} \xrightarrow{m \rightarrow 0} 0 \quad \text{GR}$$

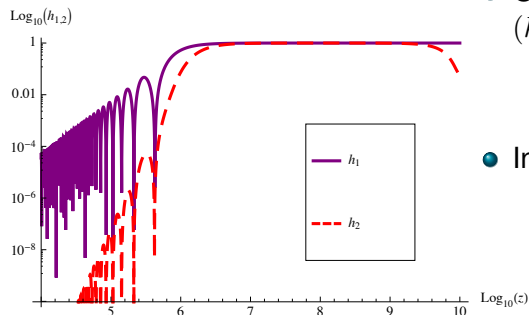
$$GR(\Phi_2) + \frac{m^2 a^2 f(\xi)}{\kappa \xi^2} F(\Phi_1, \Phi_2) = 0$$

$$\frac{m^2 a^2 f(\xi)}{\kappa \xi^2} \approx \frac{m^2 a^2}{\xi} \approx a^2 G \rho \xrightarrow{m \rightarrow 0} \text{finite}$$

Nontrivial structure in k

Cosmological perturbations: Tensor Sector

RD era:



- Outside horizon ($kt \ll 1$)

$$h_2 = h_1$$

- Inside horizon ($kt \gg 1$)

$$h_1 \sim \frac{1}{t}$$

$$h_2 \sim \frac{1}{t^5}$$

Cosmological perturbations: Scalar Sector

RD era:

- Outside horizon ($kt \ll 1$)

$$\Phi_1 \sim \text{const.}$$

$$\Phi_2'' + \frac{14}{t} \Phi_2' + \frac{37 \Phi_2 - 39 \Phi_1}{t^2} = 0 \Rightarrow \Phi_2 \sim \frac{39}{37} \Phi_1 = \text{const.}$$

Both scalar perturbations are frozen

- Inside horizon ($kt \gg 1$)

$$\Phi_1 \sim \frac{1}{k^2 t^2} \cos(kt)$$

$$\Phi_2'' + \frac{10}{t} \Phi_2' - \frac{5k^2}{3} \Phi_2 - \frac{36}{k^2 t^3} \Phi_1' - \frac{15}{t^2} \Phi_1 = 0 \Rightarrow \Phi_2 \sim \frac{1}{(kt)^{1/2}} e^{+(\frac{5}{3})^{1/2} kt} + \mathcal{O}(\Phi_1)$$

Gradient instability



- Scalar sector

$$\Phi'' + 2\mathcal{H}\Phi' \left[\frac{2k^4}{9a^2\mathcal{H}^2 m_\Phi^2 + k^4 - 18\mathcal{H}^4} - 1 \right] + \frac{1}{3}\Phi \left[\frac{4(k^6 - 3k^4\mathcal{H}^2)}{9a^2\mathcal{H}^2 m_\Phi^2 + k^4 - 18\mathcal{H}^4} + 3a^2 m_\Phi^2 - k^2 - 6\mathcal{H}^2 \right] = 0$$

- Vector sector

$$\mathcal{V}_i'' + \frac{2\mathcal{H}\mathcal{V}_i'(2k^2 + a^2 m_\Phi^2)}{k^2 + a^2 m_\Phi^2} + \mathcal{V}_i(k^2 + a^2 m_\Phi^2) = 0$$

- Tensor sector

$$h_{+ij}^{TT}'' + 2\mathcal{H}h_{+ij}^{TT}' + k^2 h_{+ij}^{TT} = 0$$

$$h_{-ij}^{TT}'' + 2\mathcal{H}h_{-ij}^{TT}' + (k^2 + a^2 m_\Phi^2) h_{-ij}^{TT} + a^2 m_\Phi^2 \frac{(\xi^2 \kappa - 1)}{(\xi^2 \kappa + 1)} h_{+ij}^{TT} = 0$$

Perturbations in RD era at leading order

- Scalar sector

$$\Phi_1'' + \frac{4}{t} \Phi_1' + \frac{k^2}{3} \Phi_1 = 0$$

$$\Phi_2'' + \frac{10x^2 + 42}{t(x^2 + 3)} \Phi_2' + \frac{-5x^6 - 15x^4 + 333x^2 + 999}{3t^2(x^2 + 3)^2} \Phi_2 - \frac{36}{t(x^2 + 3)} \Phi_1' - \frac{3(5x^2 + 39)}{t^2(x^2 + 3)} \Phi_1 = 0$$

$$x = kt$$

- Vector sector

$$\mathcal{V}'' + \frac{8k^2 t^2 + 50}{t(k^2 t^2 + 5)} \mathcal{V}' + \frac{3}{t^2} (k^2 t^2 + 5) \mathcal{V} - \frac{48k^2 t^2 + 320}{k^2 t^3 (k^2 t^2 + 5)} \delta v = 0$$

- Tensor sector

$$h_1'' + \frac{2}{t} h_1' + k^2 h_1 = 0$$

$$h_2'' + \frac{10}{t} h_2' + 25k^2 h_2 + \frac{15}{t^2} (h_1 - h_2) = 0$$