Two-point Correlators in the Effective Field Theory of Large Scale Structures

Lorenzo Mercolli

Princeton University

COSMO 2013

In collaboration with E. Pajer

Outline

Standard perturbation theory and its problems

In SPT we have to deal with:

[Bernardeau et al. '02]

Standard perturbation theory and its problems

In SPT we have to deal with:

[Bernardeau et al. '02]

- Perturbative expansion not well defined, i.e. $\delta > 1$ locally.

Standard perturbation theory and its problems

In SPT we have to deal with:

[Bernardeau et al. '02]

- Perturbative expansion not well defined, i.e. $\delta > 1$ locally.
- Depending on the initial conditions, results are divergent.

Standard perturbation theory and its problems

In SPT we have to deal with:

[Bernardeau et al. '02]

- Perturbative expansion not well defined, i.e. $\delta > 1$ locally.
- Depending on the initial conditions, results are divergent.
- Limited to the perfect fluid approximation.

Standard perturbation theory and its problems

In SPT we have to deal with:

[Bernardeau et al. '02]

- Perturbative expansion not well defined, i.e. $\delta > 1$ locally.
- Depending on the initial conditions, results are divergent.
- Limited to the perfect fluid approximation.

EFToLSS addresses all these issues by consistently treating of the separation of short- and long-distance dynamics.

Formulation of the EFToLSS

General strategy of the EFToLSS:

- Setup: point particles in Newtonian cosmology.

Formulation of the EFToLSS

General strategy of the EFToLSS:

- Setup: point particles in Newtonian cosmology.
- Define a density and velocity field: $\rho(\vec{x},t) \equiv \sum_n \frac{m}{a^3}\delta_D(\vec{x}-\vec{x}_n)$.

Formulation of the EFToLSS

General strategy of the EFToLSS:

- Setup: point particles in Newtonian cosmology.
- Define a density and velocity field: $\rho(\vec{x}, t) \equiv \sum_n \frac{m}{a^3} \delta_D(\vec{x} - \vec{x}_n)$.
- Apply a smoothing over some scale Λ.

Formulation of the EFToLSS

General strategy of the EFToLSS:

- Setup: point particles in Newtonian cosmology.
- Define a density and velocity field: $\rho(\vec{x}, t) \equiv \sum_n \frac{m}{a^3} \delta_D(\vec{x} - \vec{x}_n)$.
- Apply a smoothing over some scale Λ.
- Study dynamics of smoothed fields while taking into account short-scale effects.

[Baumann et al. '10, Carrasco et al. '12, Hertzberg '12, Pietroni et al. '11]

Symmetries of the system

First possibility: equations of motion from the smoothed Boltzmann equation.

[Baumann et al. '10, Carrasco et al. '12, Hertzberg '12]

Symmetries of the system

First possibility: equations of motion from the smoothed Boltzmann equation.

[Baumann et al. '10, Carrasco et al. '12, Hertzberg '12]

EFT approach: write down the most general equations of motion compatible with the symmetries.

[LM & Pajer '13]

Power counting: expand in powers of smoothed fields and derivatives.

Equations of motion

In an Einstein-de Sitter universe:

$$\mathcal{H}a\partial_a\delta + \vec{\nabla}\left[(1+\delta)\vec{v}\right] = -\chi_1\frac{\triangle\delta}{\mathcal{H}} + \chi_2\frac{\triangle\vec{\nabla}\cdot\vec{v}}{\mathcal{H}^2} - \frac{J'}{\mathcal{H}} + \ldots$$

$$\mathcal{H}a\partial_a\vec{v} + \mathcal{H}\,\vec{v} + \vec{\nabla}\phi + (\vec{v}\cdot\vec{\nabla})\vec{v} = -c_s^2\,\vec{\nabla}\delta + \frac{3}{4}\frac{c_{sv}^2}{\mathcal{H}}\,\triangle\,\vec{v} + \frac{4c_{bv}^2 + c_{sv}^2}{4\mathcal{H}}\,\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) - \Delta\vec{J} + \ldots$$

PRINCETON UNIVERSITY

Equations of motion

In an Einstein-de Sitter universe:

$$\mathcal{H}a\partial_a\delta + \vec{\nabla}\left[(1+\delta)\vec{v}\right] \;=\; -\chi_1\frac{\triangle\delta}{\mathcal{H}} + \chi_2\frac{\triangle\vec{\nabla}\cdot\vec{v}}{\mathcal{H}^2} - \frac{J'}{\mathcal{H}} + \ldots$$

$$\mathcal{H}a\partial_a\vec{v} + \mathcal{H}\,\vec{v} + \vec{\nabla}\phi + (\vec{v}\cdot\vec{\nabla})\vec{v} \;=\; -c_s^2\,\vec{\nabla}\delta + \frac{3}{4}\frac{c_{sv}^2}{\mathcal{H}}\;\triangle\,\vec{v}$$

$$+\frac{4c_{bv}^2 + c_{sv}^2}{4\mathcal{H}}\,\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) - \Delta\vec{J} + \ldots$$

Note the extra (blue) terms! [LM & Pajer '13]

- $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$ parametrize the short scale dynamics (effective stress tensor).

- $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$ parametrize the short scale dynamics (effective stress tensor).
- All divergences are cancelled by the renormalization of $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$.

- $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$ parametrize the short scale dynamics (effective stress tensor).
- All divergences are cancelled by the renormalization of $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$.
- $\chi_{1,2}$ and J' are *needed* to cancel the divergences of the velocity correlators and are generated when renormalization group flow is considered.

- $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$ parametrize the short scale dynamics (effective stress tensor).

- All divergences are cancelled by the renormalization of $c_{s,sv,bv}$, $\chi_{1,2}$, J' and $\vec{J}$.

- $\chi_{1,2}$ and J' are *needed* to cancel the divergences of the velocity correlators and are generated when renormalization group flow is considered.

- In density correlators $\chi_{1,2}$ and J' are degenerate with $c_{s,sv,bv}$ and $\vec{J}$.

Self-similarity

In an EdS universe the equations of motion are self-similar, i.e. invariant under $t \to \lambda_t t$ and $\vec{x} \to \lambda_x \vec{x}$.

$\Longrightarrow$ For scale-free initial conditions $P_{in}(k) = A\, k^n$ also the solutions are self-similar.

PRINCETON UNIVERSITY

Self-similarity

In an EdS universe the equations of motion are self-similar, i.e. invariant under $t \to \lambda_t t$ and $\vec{x} \to \lambda_x \vec{x}$.

$\Longrightarrow$ For scale-free initial conditions $P_{in}(k) = A\, k^n$ also the solutions are self-similar.

$\Longrightarrow$ Correlators are *only* a function of $\frac{k}{k_{NL}}$ with $k_{NL} \equiv \left(\frac{2\pi^2}{Aa^2}\right)^{1/(3+n)}$

[Pajer & Zaldarriaga '13]

PRINCETON UNIVERSITY

General form of the two-point correlators

The two-point correlators $\Delta^2_{\delta\delta}$, $\Delta^2_{\delta\theta}$ and $\Delta^2_{\theta\theta}$:

$$\Delta^2 = \left(\frac{k}{k_{NL}}\right)^{n+3} + \left(\frac{k}{k_{NL}}\right)^{2(n+3)} \left[\alpha + \tilde{\alpha} \ln\left(\frac{k}{k_{NL}}\right)\right] + \beta \left(\frac{k}{k_{NL}}\right)^{n+5} + \gamma \left(\frac{k}{k_{NL}}\right)^{7} + \dots$$

α, $\tilde{\alpha}$ computable as in SPT and $\Delta^2 \equiv k^3 P(k)/2\pi^2$.

[Pajer & Zaldarriaga '13, LM & Pajer '13]

Bispectrum (work in progress)

Considering three-point correlators of δ and θ:

- How do the various contributions scale?

Bispectrum (work in progress)

Considering three-point correlators of δ and θ:

- How do the various contributions scale?
- Does the renormalization work for three-point functions?

Bispectrum (work in progress)

Considering three-point correlators of δ and θ:

- How do the various contributions scale?
- Does the renormalization work for three-point functions?
- Do we have to consider higher derivative terms?

Conclusions

- Separation of scales $\Longrightarrow$ effective field theories!
- EFToLSS solves conceptual issues of SPT.
- When considering the velocity field, new terms are necessary.
- Self-similarity $\Rightarrow$ simple form for the correlators.

PRINCETON UNIVERSITY

Linear equations of motion

$$\mathcal{H}^2\left\{-a^2\partial_a^2-\frac{3}{2}a\partial_a+\frac{3}{2}\right\}\delta = -d^2\,\triangle\,\delta_1 - J + \left(\frac{3}{2}+a\partial_a\right)\left(\tilde{d}^2\,\triangle\,\delta_1 + J'\right)\ ,$$

$$\mathcal{H}\left\{a^2\partial_a^2+\frac{5}{2}a\partial_a-1\right\}\theta = -\left(1+a\partial_a\right)\left(d^2\,\triangle\,\delta_1 + J\right) + \frac{3}{2}\left(\tilde{d}^2\,\triangle\,\delta_1 + J'\right)\ ,$$

$$\mathcal{H}\left\{a\partial_a+1\right\}\vec{\omega} = \frac{3}{4}\frac{c_{sv}^2}{\mathcal{H}}\,\triangle\,\vec{\omega} - \vec{\nabla}\times\Delta\vec{J}\ .$$

$$d^2\equiv c_s^2+c_{sv}^2+c_{bv}^2 \qquad \tilde{d}^2\equiv\chi_1+\chi_2 \qquad J\equiv\triangle\vec{\nabla}\cdot\vec{J}$$

PRINCETON UNIVERSITY

Vorticity

SPT: vorticity correlator decays as $\sim 1/a^2$.

EFToLSS: noise term is a constant source for vorticity.

Peebles argument: $P_{\omega\omega} \sim k^4 \quad \Longrightarrow \quad \mathcal{I}_{\omega\omega} = \gamma \left(\frac{k}{k_{NL}} \right)^7 .$

[Zel'dovich '65, Peebles '80, LM & Pajer '13]

PRINCETON UNIVERSITY

The non-linear scale

$$P_{in}(k) \;=\; Ak^n$$

$$\Delta^2(k) \;=\; \frac{k^3}{2\pi^2}\, P_{in}(k)$$

Definition of the non-linear scale k_{NL}:

$$\Delta^2_{lin}(k_{NL}) \equiv 1 \qquad \Longrightarrow \qquad k_{NL} \;=\; \left(\frac{2\pi^2}{Aa^2}\right)^{1/(3+n)}$$

[Pajer & Zaldarriaga '13]

PRINCETON UNIVERSITY

α_1 and $\tilde{\alpha}_1$

n	-2	$-3/2$	-1	$-1/2$	0	$1/2$	1	$3/2$	2	$5/2$	3
$\alpha_{1,\delta\delta}$	1.38	0.239	0.0489	0.537	0.336	0.257	0.00799	−0.0904	−0.0336	−0.0446	−0.0213
$\tilde{\alpha}_{1,\delta\delta}$	0	0	0.194	0	0	−0.0918	−0.0381	0.00188	0	−0.0134	0.0151

n	-2	$-3/2$	-1	$-1/2$	0	$1/2$	1	$3/2$	2	$5/2$	3
$\alpha_{1,\delta\theta}$	0.655	−0.442	−0.128	0.852	0.495	0.488	0.0445	−0.231	−0.0671	−0.0848	−0.0317
$\tilde{\alpha}_{1,\delta\theta}$	0	0	0.397	0	0	−0.0646	−0.125	0.0237	0	−0.0202	0.0248

n	-2	$-3/2$	-1	$-1/2$	0	$1/2$	1	$3/2$	2	$5/2$	3
$\alpha_{1,\theta\theta}$	0.0755	−1.03	−0.232	1.24	0.755	0.727	−0.0278	−0.394	−0.0755	−0.121	−0.0436
$\tilde{\alpha}_{1,\theta\theta}$	0	0	0.6	0	0	−0.124	−0.212	0.0845	0.	−0.0318	0.0345

For $\tilde{\alpha}_1 \neq 0$ α_1 is degenerate with fitting parameter and renormalization scheme dependent.

PRINCETON UNIVERSITY

Divergences

Non-linear corrections to two-point correlators are divergent:

$$P_{22}(k) \sim k^4 \int dq \, \frac{P_{lin}(q)^2}{q^2} \qquad P_{13} \sim k^2 P_{lin}(k) \int dq \, P_{lin}(q)$$

EFT contributions to the power spectra:

$$P_J \sim k^4 \qquad P_c(k) \sim k^2 P_{lin}(k)$$

$\Longrightarrow$ divergences cancel by renormalizing $c_{s,sv,bv}$, $\chi_{1,2}$, J' and J!

$\Longrightarrow$ physically meaningful results!

[Pajer & Zaldarriaga '13]

PRINCETON UNIVERSITY

Scaling