Cosmological and Particle Physiccal Implications of Nonlinear-Supersymmetric General Relativity Theory

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OUTLINE

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1. Motivation

@ How to unify Two SMs for space-time and matter, i.e. GRT and GWS model..

@ SUSY may be an essential notion beyond SMs, $\rightarrow$ MSSM, SUSYGUT, SUGRA
• SUSY stabilizes the low mass Higgs particle!?

@ Many unsolved problems in SMs:
• Origin of SUSY breaking,
• Proton decay,
• Three generations of quarks and leptons,
• ν oscillations,
• Dark Matter, Dark enegy density; $\rho_D \sim (M_\nu)^4 \Leftrightarrow \Lambda$(cosmological term)

@ SUSY constitutes space-time symmetry and describes geometry of space-time.

@Geometry and symmetry of specific space-time
• SUGRA $\Longleftrightarrow$ Geometry of superspace (Mathematical:$[x^\mu, \theta_\alpha]$, sPoicaré)
While,
• General Relativity(GRT) $\Longleftrightarrow$ Geometry of Riemann space(Physical:$[x^\mu]$, GL(4,R))

$\Longrightarrow$ **New SUSY paradigm on particular physical space-time**.

@ SUSY and its spontaneous breakdown are profound notions essentially related to the space-time symmetry, therefore, to be studied in particle physics, cosmology(gravitation) and their relations.

$\Longrightarrow$ SO(N) superPoincaré(sP) symmetry gives a natural framework.

@ We found group theoretically *(Z.P,1983.E.P.J.,1999)*:

• SM with just three generations equipped with ν_R emerges from one irrep representation of SO(10) sP with the decomposition $\underline{10} = \underline{5} + \underline{5}^*$ corresponding to $SO(10) \supset SU(5)$, where $\underline{5}_{SU(5)GUT}$ quantum numbers are assigned to $\underline{5}$.

• Proton is stable due to the selection rule despite $SU(5)$, provided all particles are regarded as composites of fundamental spin $\frac{1}{2}$ objects $\underline{5} = \underline{5}_{SU(5)GUT}$ (Superon Quintet Model)(**SQM, spin** $\frac{1}{2}$).

SO(N>8) Linear(L) SUSY $\Longrightarrow$ **NO-GO** theorem in S-matrix !

SUSY indicates gravitational compositeness of matter before BB?

$SU(3)$	Q_e	$SU(2) \otimes U(1)$
$\underline{\mathbf{1}}$	0 -1 -2	$\begin{pmatrix} \nu_1 \\ l_1 \end{pmatrix} \begin{pmatrix} \nu_2 \\ l_2 \end{pmatrix} \begin{pmatrix} \nu_3 \\ l_3 \end{pmatrix}$ E
$\underline{\mathbf{3}}$	$\frac{2}{3}$ $-\frac{1}{3}$ $-\frac{4}{3}$	$\begin{pmatrix} u_1 \\ d_1 \end{pmatrix} \begin{pmatrix} u_2 \\ d_2 \end{pmatrix} \begin{pmatrix} u_3 \\ d_3 \end{pmatrix}$ $\begin{pmatrix} h \\ o \end{pmatrix}$
$\underline{\mathbf{6}}$	$\frac{4}{3}$ $\frac{1}{3}$ $-\frac{2}{3}$	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
$\underline{\mathbf{8}}$	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

@ A way to field theoretical breakthrough:

We show in this talk:

- The nonlinear(NL) SUSY invariant coupling of spin $\frac{1}{2}$ fermion with spin 2 graviton is crucial to circumvent the no-go theorem of S-matrix arguments for SO(N>8) Linear SUSY.

- This is attributed to the geometrical description of particular (empty) unstable space-time unifying:

the fundamental object(spin $\frac{1}{2}$ NLSUSY) and the background space-time manifold(general relativity).

- There may be a certain composite (SQM) structure and/or a fundamental fermionic structure beyond the SM.

A brief review of NLSUSY:

- Take flat space-time specified by x^a and ψ_α.
- Consider one form $\omega^a = dx^a - \frac{\kappa^2}{2i}(\bar{\psi}\gamma^a d\psi - d\bar{\psi}\gamma^a\psi)$,

κ is an arbitrary constant with the dimension l^{+2}.

- $\delta\omega^a = 0$ under $\delta x^a = \frac{i\kappa^2}{2}(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)$ and $\delta\psi = \zeta$

with a global spinor parameter ζ.

- An invariant acction($\sim$ invariant volume) is obtained:

$S = -\frac{1}{2\kappa^2}\int \omega^0 \wedge \omega^1 \wedge \omega^2 \wedge \omega^3 = \int d^4x L_{VA}$,

L_{VA} is N=1 Volkov-Akulov model of NLSUSY given by

$L_{\mathrm{VA}} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right]$,

$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,

$t^a{}_b = -i\kappa^2(\bar{\psi}\gamma^a\partial_b\psi - \bar{\psi}\gamma^a\partial_b\psi)$,

which is invariant under N=1 NLSUSY transformation:

$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - i\kappa(\bar{\zeta}\gamma^a\psi - \bar{\zeta}\gamma^a\psi)\partial_a\psi$. $\longleftrightarrow$ NG fermioon for SB SUSY

- ψ is NG fermion (the coset space coordinate) of $\frac{Super-Poincare}{Poincare}$.
- ψ is quantized canonically in compatible with SUSY algebra.

2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

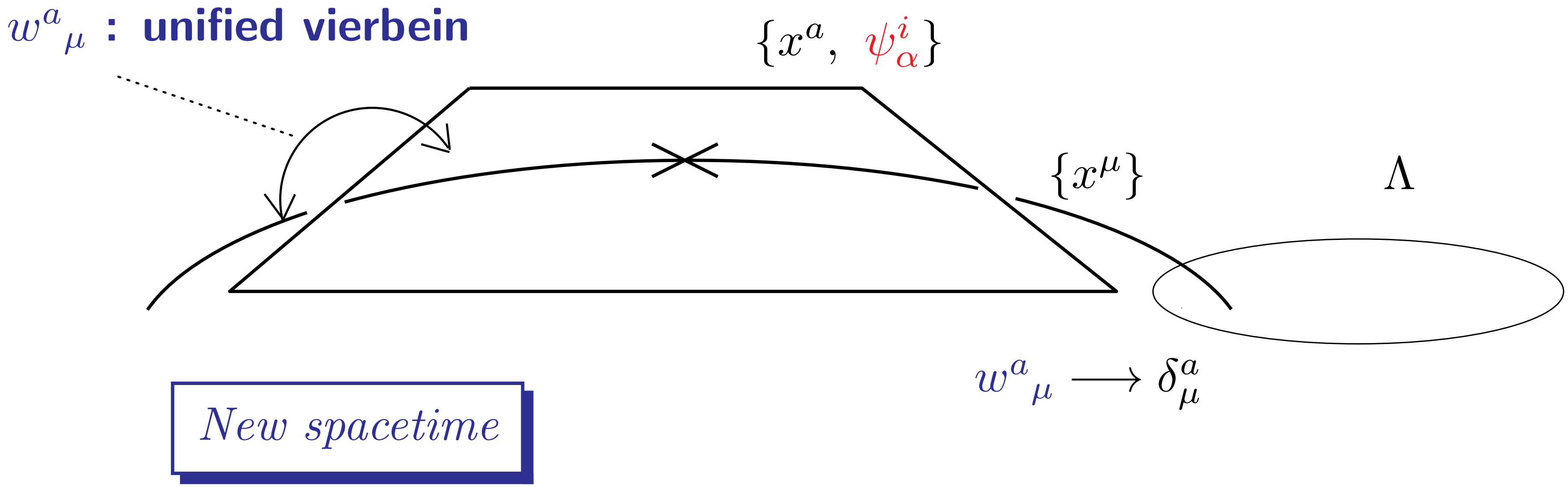
2.1. New Space-time as Ultimate Shape of Nature

We consider the following **new (unstable) space-time inspired by nonlinear(NL) SUSY :**

The tangent space of new space-time is specified by
SL(2,C) Grassmann coordinates ψ_α for NLSUSY
besides the ordinary SO(1,3) Minkowski coordinates x^a,

i.e ψ_α the coordinates of the the coset space $\frac{superGL(4,R)}{GL(4,R)}$ turning to the NLSUSY NG fermion (called $superon$ hereafter) and x^a are attached at every curved space-time point.

• **Ultimate shape of nature** $\Longleftrightarrow$ **(empy) unstable space-time:**



(Locally homomorphic non-compact groups SO(1,3) and SL(2,C) for space-time symmetry are analogous to compact groups SO(3) and SU(2) for gauge symmetry of 't Hooft-Polyakov monopole, though SL(2,C) is realized nonlinearly.)

• Note that $SO(1,3) \cong SL(2,C)$ is crucial for NLSUSYGR scenario.

4 dimensional space-time is singled out.

2.2. Nonlinear-Supersymmetric General Relativity (NLSUSYGR)

We have found that geometrical arguments of Einstein general relativity(EGR) can be extended to **new (unstable) space-time :**

- Unified vierbein of new space-time:

$$w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi),$$

$$w^\mu{}_a(x) = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a,$$

$$w^a{}_\mu(x) w^\mu{}_b(x) = \delta^a{}_b$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I \gamma^a \partial_\mu \psi^I - \partial_\mu \bar{\psi}^I \gamma^a \psi^I), (I = 1, 2, .., N)$$

(Note: The first and the second indices of t represent those of γ-matrix and the covariant derivative, respectively.)

- N-extended NLSUSYGR action of EH-type in new (empty) space-time:

N-extended NLSUSY GR action:

$$L_{NLSUSYGR}(w) = -\frac{c^4}{16\pi G}|w|(\Omega(w) + \Lambda), \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \tag{2}$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar{\psi}^I\gamma^a\partial_\mu\psi^I - \partial_\mu\bar{\psi}^I\gamma^a\psi^I), (I = 1, 2, .., N) \tag{3}$$

- $w^a{}_\mu(x)(= e^a{}_\mu + t^a{}_\mu(\psi))$: the unified vierbein of new space-time,
- $e^a{}_\mu(x)$: the ordinary vierbein for the local SO(1,3) of EGR,
- $t^a{}_\mu(\psi(x))$: the mimic vierbein for the local SL(2,C) composed of the stress-energy-momentum of NG fermion $\psi(x)^I$(called *superons*),
- $\Omega(w)$: the unified Ricci scalar curvature of new space-time in terms of $w^a{}_\mu$,
- $s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$, $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) w^{\nu a}(x)$: unified metric tensors of new space-time.
- G : the Newton gravitational constant.
- Λ : cosmological constant in new space-time indicating NLSUSY of tangent space.

• No-go theorem for has been circumvented in a sense that
SO(N>8) SUSY with the non-trivial gravitational interaction has been constructed
by using NLSUSY, i.e. the vacuum degeneracy.

• Note that $SO(1, D-1) \cong SL(d, C)$, i.e.

$$\frac{D(D-1)}{2} = 2(d^2 - 1)$$

holds only for $D = 4, d = 2$.

NLSUSYGR scenario predicts 4 dimensional space-time.

- Remarkably NLSUSYGR scenario fixes the arbitrary constatnt κ^2 to

$$\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1},$$

with the dimension $(length)^4 \sim (enegy)^{-4}$.

- Also $\Lambda > 0$ in the action is now fixed uniuely

to give the correct sign to the kinetic term of $\psi(x)$

and indicates

(i) the positive potential minimum $V_{P.E.}(w) = \Lambda > 0$ for ${w^a}_\mu(x)$

and

(ii) the negative dark energy density interpretation for Λ ($\rightarrow$ Sec.4).

2.3. Symmetries of NLSUSY GR(N-extended action)

• NLSUSY GR action is invariant at least under the following space-time symmetries which is homomorphic to sP:

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4,\text{R})] \otimes [\text{local Lorentz}] \otimes [\text{local spinor translation}] \tag{4}$$

and

• the following internal symmetries for N-extended NLSUSY GR
(with N-superons ψ^I $(I = 1, 2, ..N)$) :

$$[\text{global SO(N)}] \otimes [\text{local U}(1)^{\text{N}}] \otimes [\text{chiral}]. \tag{5}$$

For Example:

- Invariance under the new NLSUSY transformation;

$$\delta_\zeta \psi^I = \frac{1}{\kappa}\zeta^I - i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_\rho\psi^I, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}^J\gamma^\rho\psi^J\partial_{[\mu}e^a{}_{\rho]}, \tag{6}$$

Because (6) induce GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{7}$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and $\xi^\rho = -i\kappa\bar{\zeta}^I\gamma^\rho\psi^I$).

Commutators of two new NLSUSY transformations (6) on ψ^I and $e^a{}_\mu$ close to GL(4,R),

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi^I = \Xi^\mu\partial_\mu\psi^I, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \tag{8}$$

where $\Xi^\mu = 2i\bar{\zeta}^I{}_1\gamma^\mu\zeta^I{}_2 - \xi_1^\rho\xi_2^\sigma e_a{}^\mu\partial_{[\rho}e^a{}_{\sigma]}$. *Q.E.D.*

- New NLSUSY (6) is the square-root of GL(4,R);

$$[\delta_1, \delta_2] = \delta_{GL(4,R)}, \quad i.e. \quad \delta \sim \sqrt{\delta_{GL(4,R)}}.$$

c.f. SUGRA

$$[\delta_1, \delta_2] = \delta_P \underline{+\delta_L + \delta_g}$$

- The ordinary local GL(4,R) invariance is manifest by the construction.

- Invariance under the local Lorentz transformation;

$$\delta_L \psi^I = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi^I, \quad \delta_L e^a{}_\mu = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^I)\gamma_5\gamma_d\psi^I(\partial_\mu \epsilon_{bc}) \tag{9}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$.

Because (9) induce the familiar local Lorentz transformation on $w^a{}_\mu$:

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{10}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

The local Lorentz transformation forms a closed algebra, for example, on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu \beta_{bc}), \tag{11}$$

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$. $Q.E.D.$

2.4. Big Decay of New Space-Time:

The supercurrent obtained by the Noether theorem

$$S^{I\mu} = i\frac{c^4\Lambda}{16\pi G}|e|e_a{}^{\mu}\gamma^a\psi^I + i\frac{c^4}{16\pi G}|e|R^{\mu\nu}e^a{}_{\nu}\gamma_a\psi^I + \cdots, \tag{12}$$

shows that New space-time described by $L_{NLSUSYGR}(w)$ is unstable and would break down spontaneously to

***ordinary Riemann space-time*(EH action) and**
***massless superons*(NG fermion)**,

called Superon-Graviton Model(SGM) and induces quantum mechanical rapid expansion

$$L_{NLSUSYGR}(w) = L_{SGM}(e,\psi) = -\frac{c^4}{16\pi G}|e|\{R(e) + |w_{VA}(\psi^I)|\Lambda + \tilde{T}(e,\psi^I)\}. \tag{13}$$

- $R(e)$: the ordinary Ricci scalar curvature of EH action
- Λ : the cosmological term; $V_{P.E} = \Lambda > 0$
- $\tilde{T}(e,\psi^I)$: the gravitational interaction of superon.
- $|w_{VA}(\psi^I)| = \det w^a{}_b = \det(\delta^a{}_b + t^a{}_b(\psi^I))$

Note that

- $L_{SGM}(e, \psi^I)$ (with N-superons ψ^I $(I = 1, 2, ..N)$) is invariant under under the following space-time symmetries which is homomorphic to sP:

$$[\text{new NLSUSY}] \otimes [\text{local GL}(4, \text{R})] \otimes [\text{local Lorentz}] \otimes [\text{local spinor translation}] \quad (14)$$

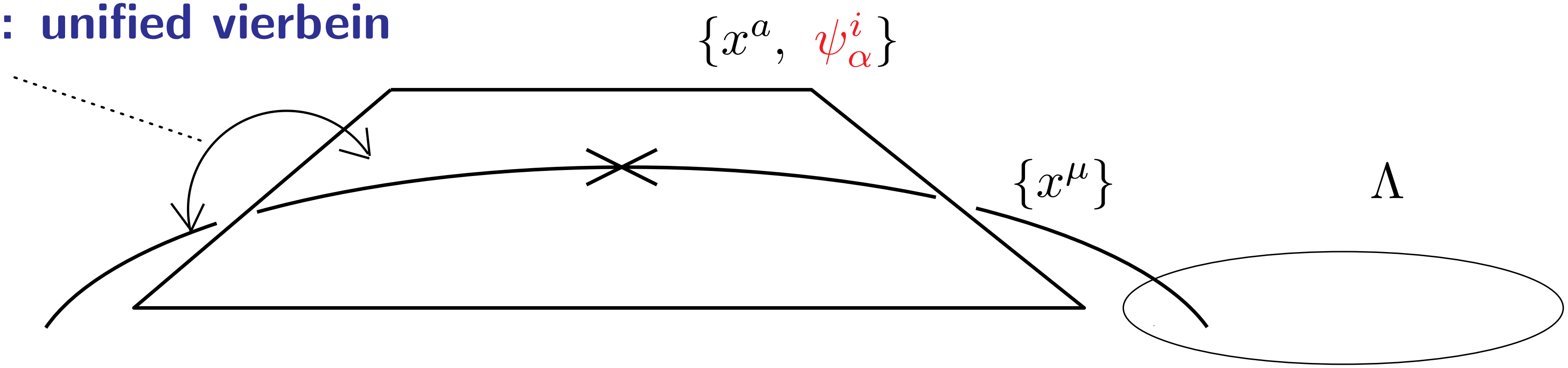
and the following internal symmetries for N-extended NLSUSY GR:

$$[\text{global SO(N)}] \otimes [\text{local U}(1)^{\text{N}}] \otimes [\text{chiral}]. \quad (15)$$

- $L_{SGM}(e, \psi^I)$ is expected to form **gravitational composite massless-eigenstates of SO(N)sP** continuing to Big Bang SMs.

The ignition of Big Bang proceeding to the true vacuum.

$w^a{}_\mu$ **: unified vierbein**

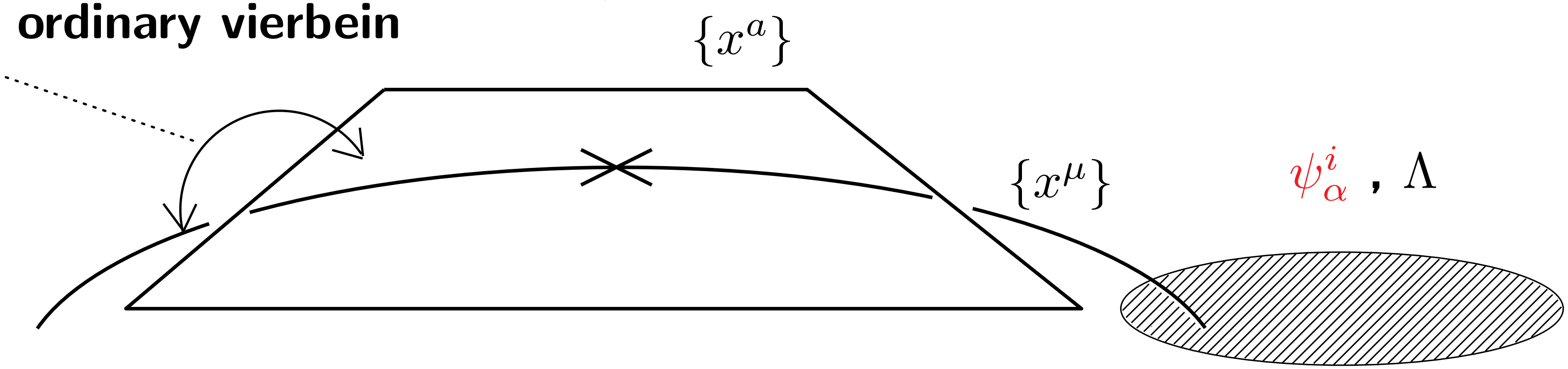


$w^a{}_\mu \longrightarrow \delta^a_\mu$

New spacetime

$\Downarrow$ **Big Decay**

$e^a{}_\mu$ **: ordinary vierbein**



$e^a{}_\mu \longrightarrow \delta^a_\mu$

Riemann spacetime $\oplus$ **matter**

Ignition of Big Bang towards the true vacuum

3. Phase Transition of $L_{SGM}(e, \psi)$

We expect

SUSY (algebra) dictates the vacuum configuration of $L_{SGM}(e, \psi)$.

By respecting SUSY algebra throughout we show in *flat space* :

- N-LSUSY theory

emerges in the true vacuum of N-NLSUSY theory $L_{SGM}(e, \psi)$. expressed uniquely as massless composites of NG fermions

$\Longleftrightarrow$ NL/L SUSY relations $\longleftrightarrow$ **BCS/LG**

- The systematics for NL/L SUSY relation are simple so far and

carried out for $N = 1$(toy model), 2(SUSY QED), 3(SUSY QCD) in flat space-time.

- These phenomena are the phase transition of NLSUSY $L_{SGM}(e, \psi)$

from the false vacuum with $V_{P.E.} = \Lambda > 0$

towards the true vacuum with $V_{P.E.} = 0$

achieved by forming massless composite states of LSUSY.

3.1. NL/L Relation for N=2 SUSY :

We demonstrate NL/L relation for N=2 SUSY in *flat space* as Low Energy Theory of N=2 SGM.

($N \geq 2$ SUSY can give a realistic model in SGM scenario.)

- N=2 SGM in Riemann-flat ($e^a{}_\mu(x) \to \delta^a{}_\mu$) space-time produces $N = 2$ NLSUSY:

$$\underline{L_{N=2SGM}(e,\psi) \longrightarrow L_{N=2NLSUSY}(\psi)} \leftrightarrow \text{cosmological constant of SGM.}$$

$N=2$ NL/L SUSY relation (two dimensional space-time for simplicity):

N=2, d=2 NLSUSY model:

$$L_{\rm VA} = -\frac{1}{2\kappa^2}|w_{VA}| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \qquad (16)$$

where,
$|w_{VA}| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$,
$t^a{}_b = -i\kappa^2(\bar{\psi}_j\gamma^a\partial_b\psi^j - \bar{\psi}_j\gamma^a\partial_b\psi^j)$, $(j = 1, 2)$,

which is invariant under N=2 NLSUSY transformation,

$\delta_\zeta\psi^j = \frac{1}{\kappa}\zeta^j - i\kappa(\bar{\zeta}_k\gamma^a\psi^k - \bar{\zeta}_k\gamma^a\psi^k)\partial_a\psi^j, (j = 1, 2)$.

N=2, d=2 LSUSY Theory (SUSY QED):

- Helicity states of N=2 vector supermultiplet:

$$\left(\begin{array}{c} +1 \\ +\frac{1}{2}, \ +\frac{1}{2} \\ 0 \end{array}\right) + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY off-shell vector supermultiplet: $(v^a, \lambda^i, A, \phi, D;$i=1,2$)$. in *WZ gauge*. ($A$ and ϕ are two singlets, 0^+ and 0^-, scalar fields.)

- Helicity states of N=2 scalar supermultiplet:

$$\left(\begin{array}{c} +\frac{1}{2} \\ 0, \ 0 \\ -\frac{1}{2} \end{array}\right) + [\mathrm{CPTconjugate}]$$

corresponds to N=2, d=2 LSUSY two scalar supermultiplets:
$(\chi,\ B^i,\ \nu,\ F^i;\ i=1,2)$.

- **The most genaral $N=2, d=2$ SUSYQED action ($m=0$ case)** :

$$L_{N=2SUSYQED} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf}, \tag{17}$$

$$\begin{aligned}
L_{V0} &= -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D, \\
L'_{\Phi 0} &= \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}(F^i)^2, \\
L_e &= e\Big\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5 \nu \\
&\quad + B^i(\bar{\lambda}^i \chi - \epsilon^{ij}\bar{\lambda}^j \nu) - \frac{1}{2}(B^i)^2 D \Big\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2, \\
L_{Vf} &= f\{A\bar{\lambda}^i \lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i \gamma_5 \lambda^j + (A^2 - \phi^2)D - \epsilon^{ab}A\phi F_{ab}\}.
\end{aligned} \tag{18}$$

- Note that
$J=0$ states in the vector multiplet for $N \geq 2$ SUSY induce Yukawa coupling.

$L_{\mathrm{N=2SUSYQED}}$ is invariant under $N=2$ LSUSY transformation:

- For the vector off-shell supermultiplet:

$$
\begin{aligned}
\delta_\zeta v^a &= -i\epsilon^{ij}\bar{\zeta}^i\gamma^a\lambda^j, \\
\delta_\zeta \lambda^i &= (D - i\partial\!\!\!/ A)\zeta^i + \frac{1}{2}\epsilon^{ab}\epsilon^{ij}F_{ab}\gamma_5\zeta^j - i\epsilon^{ij}\gamma_5\partial\!\!\!/\phi\zeta^j, \\
\delta_\zeta A &= \bar{\zeta}^i\lambda^i, \\
\delta_\zeta \phi &= -\epsilon^{ij}\bar{\zeta}^i\gamma_5\lambda^j, \\
\delta_\zeta D &= -i\bar{\zeta}^i\partial\!\!\!/\lambda^i.
\end{aligned}
\tag{19}
$$

$$
[\delta_{Q1}, \delta_{Q2}] = \delta_P(\Xi^a) + \delta_g(\theta), \tag{20}
$$

where $\zeta^i, i=1,2$ are constant spinors and $\delta_g(\theta)$ is the $U(1)$ gauge transformation only for v^a with $\theta = -2(i\bar{\zeta}_1^i\gamma^a\zeta_2^i\ v_a - \epsilon^{ij}\bar{\zeta}_1^i\zeta_2^j A - \bar{\zeta}_1^i\gamma_5\zeta_2^i\phi)$.

- For the two scalar off-shell supermultiplets:

$$\begin{aligned}
\delta_\zeta \chi &= (F^i - i\partial\!\!\!/ B^i)\zeta^i - e\epsilon^{ij}V^i B^j, \\
\delta_\zeta B^i &= \bar{\zeta}^i \chi - \epsilon^{ij}\bar{\zeta}^j \nu, \\
\delta_\zeta \nu &= \epsilon^{ij}(F^i + i\partial\!\!\!/ B^i)\zeta^j + eV^i B^i, \\
\delta_\zeta F^i &= -i\bar{\zeta}^i \partial\!\!\!/ \chi - i\epsilon^{ij}\bar{\zeta}^j \partial\!\!\!/ \nu \\
&\quad -e\{\epsilon^{ij}\bar{V}^j \chi - \bar{V}^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i)B^j - \bar{\zeta}^j \lambda^j B^i\}, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\chi = \Xi^a \partial_a \chi - e\theta\nu, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]B^i = \Xi^a \partial_a B^i - e\epsilon^{ij}\theta B^j, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]\nu = \Xi^a \partial_a \nu + e\theta\chi, \\
&\quad [\delta_{\zeta_1}, \delta_{\zeta_2}]F^i = \Xi^a \partial_a F^i + e\epsilon^{ij}\theta F^j,
\end{aligned} \tag{21}$$

with $V^i = iv_a\gamma^a\zeta^i - \epsilon^{ij}A\zeta^j - \phi\gamma_5\zeta^i$ and the U(1) gauge parameter θ.

$N=2$ NL/L SUSY relation:

$$L_{\mathrm{N=2SUSYQED}} = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} = L_{\mathrm{N=2NLSUSY}} + [\text{surface terms}], \quad (22)$$

achieved by the followings:

(i) Construct SUSY invariant relations which express component fields of LSUSY supermultiplet as the composites of superons ψ_j of NLSUSY.

(ii) Show that performing NLSUSY transformations of constituent superons ψ^j in SUSY invariant relations
reproduces familiar LSUSY transformations among the LSUSY supermultiplet recasted by SUSY invariant relations.

(iii) Substituting SUSY invariant relations into $L_{\mathrm{N=2LSUSYQED}}$,
the NL/L SUSY relation is established.

- SUSY invariant relationsns for the vector off-shell supermultiplet:

$$
\begin{aligned}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\
\lambda^i &= \xi\psi^i|w|, \\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\
D &= \frac{\xi}{\kappa}|w|.
\end{aligned}
\tag{23}
$$

- Note that the global SU(2) emerges for N=2, d=4 SGM.

• SUSY invariant relations for scalar off-shell supermultiplets:

$$
\begin{aligned}
&\chi = \xi^i \left[\psi^i |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j |w| \} \right] \\
&B^i = -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|, \\
&\nu = \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k |w| \} \right], \\
&F^i = \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8}\kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|) \right\} - i\kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|) \\
&\underline{-\frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k |w|}.
\end{aligned}
\tag{24}
$$

The quartic fermion self-interaction term in F^i is the origin of the local $U(1)$ gauge symmetry of LSUSY.

• SUSY invariant relations produce a new off-shell commutator algebra which closes on only a translation:

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \tag{25}$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}\gamma^a\zeta_{2L} - \bar{\zeta}_{1R}\gamma^a\zeta_{2R}) \tag{26}$$

• Note that the commutator does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.

• Substituting these SUSY invariant relations into $L_{N=2LSUSYQED}$,
we find NL/L SUSY relation:

$$L_{N=2LSUSYQED} = f(\xi, \xi^i) L_{\rm N=2NLSUSY} + [\text{suface terms}], \tag{27}$$

$$f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1. \tag{28}$$

⇒ composite eigenstates of global space-time (bulk) symmetries !?

• NL/L SUSY relation gives the relation between the cosmology and the low energy particle physics in NLSUSY GR. (⇒ Sec. 4).

• The direct linearization of highly nonlinear SGM action (13) in curved space remains to be carried out.

In Riemann flat space-time of SGM,
ordinary LSUSY gauge theory with the spontaneous SUSY breaking
emerges
as massless composites of NG fermion
from
the NLSUSY cosmological constant of SGM.

♣ Systematics of NL/L SUSY relation and $N = 2$ SUSY QED

SUSY invariant relations: in the superfield formulation.

Linearization of NLSUSY in the $d = 2$ superfield formulation

- General superfields are given for the $N = 2$ vector supermultiplet by

$$\begin{aligned}\mathcal{V}(x,\theta^i) \ =\ & C(x) + \bar\theta^i\Lambda^i(x) + \frac{1}{2}\bar\theta^i\theta^j M^{ij}(x) - \frac{1}{2}\bar\theta^i\theta^i M^{jj}(x) + \frac{1}{4}\epsilon^{ij}\bar\theta^i\gamma_5\theta^j\phi(x) \\ & -\frac{i}{4}\epsilon^{ij}\bar\theta^i\gamma_a\theta^j v^a(x) - \frac{1}{2}\bar\theta^i\theta^i\bar\theta^j\lambda^j(x) - \frac{1}{8}\bar\theta^i\theta^i\bar\theta^j\theta^j D(x), \end{aligned} \tag{29}$$

and for the $N = 2$ scalar supermultiplet by

$$\begin{aligned}\Phi^i(x,\theta^i) \ =\ & B^i(x) + \bar\theta^i\chi(x) - \epsilon^{ij}\bar\theta^j\nu(x) - \frac{1}{2}\bar\theta^j\theta^j F^i(x) + \bar\theta^i\theta^j F^j(x) - i\bar\theta^i \partial\!\!\!/ B^j(x)\theta^j \\ & +\frac{i}{2}\bar\theta^j\theta^j(\bar\theta^i\partial\!\!\!/\chi(x) - \epsilon^{ik}\bar\theta^k\partial\!\!\!/\nu(x)) + \frac{1}{8}\bar\theta^j\theta^j\bar\theta^k\theta^k\partial_a\partial^a B^i(x). \end{aligned} \tag{30}$$

• Take the following ψ^i-dependent specific supertranslations with $-\kappa\psi(x)$,

$$x'^a = x^a + i\kappa\bar{\theta}^i\gamma^a\psi^i, \quad \theta'^i = \theta^i - \kappa\psi^i, \tag{31}$$

and denote the resulting superfields on (x'^a, θ'^i) and their θ-epansions as

$$\mathcal{V}(x'^a, \theta'^i) = \tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \quad \Phi(x'^a, \theta'^i) = \tilde{\Phi}(x^a, \theta^i; \psi^i(x)). \tag{32}$$

• Hybrid global SUSY transformations $\delta^h = \delta^L(x.\theta) + \delta^{NL}(\psi)$ on (x'^a, θ'^i) give:

$$\delta^h\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\mathcal{V}}(x^a, \theta^i; \psi^i(x)), \delta^h\tilde{\Phi}(x^a, \theta^i; \psi^i(x)) = \xi_\mu\partial^\mu\tilde{\Phi}(x^a, \theta^i; \psi^i(x)), \tag{33}$$

• Therefore, the following conditions, i.e. SUSY invariant constraints

$$\tilde{\varphi}^I_{\mathcal{V}}(x) = \xi^I_{\mathcal{V}}(\mathrm{constant}) \quad \tilde{\varphi}^I_{\Phi}(x) = \xi^I_{\Phi}(\mathrm{constant}), \tag{34}$$

are invariant (conserved quantities) under hybrid supertrasformations,
which provide SUSY invariant relations.

• Putting in general constants as follows:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \xi^i_\Lambda, \quad \tilde{M}^{ij} = \xi^{ij}_M, \quad \tilde{\phi} = \xi_\phi, \quad \tilde{v}^a = \xi^a_v, \quad \tilde{\lambda}^i = \xi^i_\lambda, \quad \tilde{D} = \frac{\xi}{\kappa}, \tag{35}$$

$$\tilde{B}^i = \xi^i_B, \quad \tilde{\chi} = \xi_\chi, \quad \tilde{\nu} = \xi_\nu, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{36}$$

where mass dimensions of constants (or constant spinors) in $d = 2$ are defined by (-1, $\frac{1}{2}$, 0, 0, 0, $-\frac{1}{2}$) for (ξ_c, ξ^i_Λ, ξ^{ij}_M, ξ_ϕ, ξ^a_v, ξ^i_λ), (0, $-\frac{1}{2}$, $-\frac{1}{2}$) for (ξ^i_B, ξ_χ, ξ_ν) and 0 for ξ^i for convenience.

• we obtain straightforwardly the following SUSY invariant relations $\varphi^I_{\mathcal{V}} = \varphi^I_{\mathcal{V}}(\psi)$ for the vector supermultiplet

$$\begin{aligned} C \ &= \ \xi_c + \kappa\bar{\psi}^i\xi^i_\Lambda + \frac{1}{2}\kappa^2(\xi^{ij}_M\bar{\psi}^i\psi^j - \xi^{ii}_M\bar{\psi}^j\psi^j) + \frac{1}{4}\xi_\phi\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j - \frac{i}{4}\xi^a_v\kappa^2\epsilon^{ij}\bar{\psi}^i\gamma_a\psi^j \\ &\quad -\frac{1}{2}\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\xi^j_\lambda - \frac{1}{8}\xi\kappa^3\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\ \Lambda^i \ &= \ \xi^i_\Lambda + \kappa(\xi^{ij}_M\psi^j - \xi^{jj}_M\psi^i) + \frac{1}{2}\xi_\phi\kappa\epsilon^{ij}\gamma_5\psi^j - \frac{i}{2}\xi^a_v\kappa\epsilon^{ij}\gamma_a\psi^j \end{aligned}$$

$$
\begin{aligned}
& -\frac{1}{2}\xi^i_\lambda \kappa^2 \bar{\psi}^j \psi^j + \frac{1}{2}\kappa^2(\psi^j \bar{\psi}^i \xi^j_\lambda - \gamma_5 \psi^j \bar{\psi}^i \gamma_5 \xi^j_\lambda - \gamma_a \psi^j \bar{\psi}^i \gamma^a \xi^j_\lambda) \\
& -\frac{1}{2}\xi \kappa^2 \psi^i \bar{\psi}^j \psi^j - i\kappa \partial\!\!\!/ C(\psi)\psi^i, \\
M^{ij} &= \xi^{ij}_M + \kappa \bar{\psi}^{(i} \xi^{j)}_\lambda + \frac{1}{2}\xi \kappa \bar{\psi}^i \psi^j + i\kappa \epsilon^{(i|k|}\epsilon^{j)l} \bar{\psi}^k \partial\!\!\!/ \Lambda^l(\psi) - \frac{1}{2}\kappa^2 \epsilon^{ik}\epsilon^{jl} \bar{\psi}^k \psi^l \partial^2 C(\psi), \\
\phi &= \xi_\phi - \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \xi^j_\lambda - \frac{1}{2}\xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \partial\!\!\!/ \Lambda^j(\psi) + \frac{1}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j \partial^2 C(\psi), \\
v^a &= \xi^a_v - i\kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \xi^j_\lambda - \frac{i}{2}\xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j - \kappa \epsilon^{ij} \bar{\psi}^i \partial\!\!\!/ \gamma^a \Lambda^j(\psi) + \frac{i}{2}\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j \partial^2 C(\psi) \\
& -i\kappa^2 \epsilon^{ij} \bar{\psi}^i \gamma^b \psi^j \partial^a \partial_b C(\psi), \\
\lambda^i &= \xi^i_\lambda + \xi \psi^i - i\kappa \partial\!\!\!/ M^{ij}(\psi)\psi^j + \frac{i}{2}\kappa \epsilon^{ab}\epsilon^{ij} \gamma_a \psi^j \partial_b \phi(\psi) \\
& -\frac{1}{2}\kappa \epsilon^{ij} \left\{ \psi^j \partial_a v^a(\psi) - \frac{1}{2}\epsilon^{ab} \gamma_5 \psi^j F_{ab}(\psi) \right\} \\
& -\frac{1}{2}\kappa^2 \{ \partial^2 \Lambda^i(\psi) \bar{\psi}^j \psi^j - \partial^2 \Lambda^j(\psi) \bar{\psi}^i \psi^j - \gamma_5 \partial^2 \Lambda^j(\psi) \bar{\psi}^i \gamma_5 \psi^j
\end{aligned}
$$

$$
\begin{aligned}
& -\gamma_a \partial^2 \Lambda^j(\psi)\bar{\psi}^i \gamma^a \psi^j + 2\partial\!\!\!/ \partial_a \Lambda^j(\psi)\bar{\psi}^i \gamma^a \psi^j\} - \frac{i}{2}\kappa^3 \partial\!\!\!/ \partial^2 C(\psi)\psi^i \bar{\psi}^j \psi^j, \\
D \ = \ & \frac{\xi}{\kappa} - i\kappa \bar{\psi}^i \partial\!\!\!/ \lambda^i(\psi) \\
& +\frac{1}{2}\kappa^2 \left\{ \bar{\psi}^i \psi^j \partial^2 M^{ij}(\psi) - \frac{1}{2}\epsilon^{ij}\bar{\psi}^i \gamma_5 \psi^j \partial^2 \phi(\psi) \right. \\
& \left. +\frac{i}{2}\epsilon^{ij}\bar{\psi}^i \gamma_a \psi^j \partial^2 v^a(\psi) - i\epsilon^{ij}\bar{\psi}^i \gamma_a \psi^j \partial_a \partial_b v^b(\psi) \right\} \\
& -\frac{i}{2}\kappa^3 \bar{\psi}^i \psi^i \bar{\psi}^j \partial\!\!\!/ \partial^2 \Lambda^j(\psi) + \frac{1}{8}\kappa^4 \bar{\psi}^i \psi^i \bar{\psi}^j \psi^j \partial^4 C(\psi), \qquad (37)
\end{aligned}
$$

and the following SUSY invariant relations for the vector multiplet $\varphi_\Phi^I = \varphi_\Phi^I(\psi)$:

$$
\begin{aligned}
B^i \ = \ & \xi_B^i + \kappa(\bar{\psi}^i \xi_\chi - \epsilon^{ij}\bar{\psi}^j \xi_\nu) - \frac{1}{2}\kappa^2 \{\bar{\psi}^j \psi^j F^i(\psi) - 2\bar{\psi}^i \psi^j F^j(\psi) + 2i\bar{\psi}^i \partial\!\!\!/ B^j(\psi)\psi^j\} \\
& -i\kappa^3 \bar{\psi}^j \psi^j \{\bar{\psi}^i \partial\!\!\!/ \chi(\psi) - \epsilon^{ik}\bar{\psi}^k \partial\!\!\!/ \nu(\psi)\} + \frac{3}{8}\kappa^4 \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k \partial^2 B^i(\psi), \\
\chi \ = \ & \xi_\chi + \kappa\{\psi^i F^i(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^i\}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\chi(\psi)\bar{\psi}^i\psi^i - \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\nu(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\nu(\psi)\}] \\
& +\frac{1}{2}\kappa^3\psi^i\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \frac{i}{2}\kappa^3\partial\!\!\!/ F^i(\psi)\psi^i\bar{\psi}^j\psi^j + \frac{1}{8}\kappa^4\partial^2\chi(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
\nu \ = \ & \xi_\nu - \kappa\epsilon^{ij}\{\psi^i F^j(\psi) - i\partial\!\!\!/ B^i(\psi)\psi^j\} \\
& -\frac{i}{2}\kappa^2[\partial\!\!\!/\nu(\psi)\bar{\psi}^i\psi^i + \epsilon^{ij}\{\psi^i\bar{\psi}^j\partial\!\!\!/\chi(\psi) - \gamma^a\psi^i\bar{\psi}^j\partial_a\chi(\psi)\}] \\
& +\frac{1}{2}\kappa^3\epsilon^{ij}\psi^i\bar{\psi}^k\psi^k\partial^2 B^j(\psi) + \frac{i}{2}\kappa^3\epsilon^{ij}\partial\!\!\!/ F^i(\psi)\psi^j\bar{\psi}^k\psi^k + \frac{1}{8}\kappa^4\partial^2\nu(\psi)\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j, \\
F^i \ = \ & \frac{\xi^i}{\kappa} - i\kappa\{\bar{\psi}^i\partial\!\!\!/\chi(\psi) + \epsilon^{ij}\bar{\psi}^j\partial\!\!\!/\nu(\psi)\} \\
& -\frac{1}{2}\kappa^2\bar{\psi}^j\psi^j\partial^2 B^i(\psi) + \kappa^2\bar{\psi}^i\psi^j\partial^2 B^j(\psi) + i\kappa^2\bar{\psi}^i\partial\!\!\!/ F^j(\psi)\psi^j \\
& +\frac{1}{2}\kappa^3\bar{\psi}^j\psi^j\{\bar{\psi}^i\partial^2\chi(\psi) + \epsilon^{ik}\bar{\psi}^k\partial^2\nu(\psi)\} - \frac{1}{8}\kappa^4\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k\partial^2 F^i(\psi). \qquad (38)
\end{aligned}
$$

• Choosing the following simple SUSY invariant constraints of the component fields in $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$,

$$\tilde{C} = \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \tilde{D} = \frac{\xi}{\kappa}, \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}, \tag{39}$$

give previous simple SUSY invariant relations.

Actions in the $d = 2$, $N = 2$ NL/L SUSY relation

By changing the integration variables $(x^a, \theta^i) \to (x'^a, \theta'^i)$, we can confirm systematically that LSUSY actions reduce to NLSUSY representation.

(a) The kinetic (free) action with the Fayet-Iliopoulos (FI) D term for the $N = 2$ vector supermultiplet $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$
\begin{aligned}
S_{\mathcal{V}\mathrm{free}} &= \int d^2x \left\{ \int d^2\theta^i \, \frac{1}{32} (\overline{D^i \mathcal{W}^{jk}} D^i \mathcal{W}^{jk} + \overline{D^i \mathcal{W}_5^{jk}} D^i \mathcal{W}_5^{jk}) + \int d^4\theta^i \, \frac{\xi}{2\kappa} \mathcal{V} \right\}_{\theta^i=0} \\
&= \xi^2 S_{N=2\mathrm{NLSUSY}},
\end{aligned}
\qquad (40)
$$

where

$$
\mathcal{W}^{ij} = \bar{D}^i D^j \mathcal{V}, \quad \mathcal{W}_5^{ij} = \bar{D}^i \gamma_5 D^j \mathcal{V}. \qquad (41)
$$

(Note) The FI D term gives the correct sign of the NLSUSY action.

(b) Yukawa interaction terms for $\mathcal{V}$ vanish,

i.e.

$$
\begin{aligned}
S_{\mathcal{V}f} &= \frac{1}{8}\int d^2x\ f\left[\int d^2\theta^i\ \mathcal{W}^{jk}(\mathcal{W}^{jl}\mathcal{W}^{kl} + \mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\right. \\
&\quad \left. + \int d\bar\theta^i d\theta^j\ 2\{\mathcal{W}^{ij}(\mathcal{W}^{kl}\mathcal{W}^{kl} + \mathcal{W}_5^{kl}\mathcal{W}_5^{kl}) + \mathcal{W}^{ik}(\mathcal{W}^{jl}\mathcal{W}^{kl} + \mathcal{W}_5^{jl}\mathcal{W}_5^{kl})\}\right]_{\theta^i=0} \\
&= 0,
\end{aligned} \tag{42}
$$

by means of cancellations among four NG-fermion self-interaction terms.

[Note]

- General mass terms for $\tilde{\mathcal{V}}$ and $\tilde{\Phi}$ vanish as well. $\rightarrow$ Chirality is encoded in the false vacuum.

(c) The *most general* gauge invariant action for Φ^i coupled with $\mathcal{V}$ reduces to $S_{N=2\mathrm{NLSUSY}}$;

$$S_{\mathrm{gauge}} = -\frac{1}{16}\int d^2x \int d^4\theta^i e^{-4e\mathcal{V}}(\Phi^j)^2$$
$$= -(\xi^i)^2 S_{N=2\mathrm{NLSUSY}}. \tag{43}$$

• Here $U(1)$ gauge interaction terms with the gauge coupling constant e produce

four NG-fermion self-interaction terms as

$$S_e(\text{for the minimal off shell multiplet}) = \int d^2x \left\{ \frac{1}{4} e\kappa\xi(\xi^i)^2 \bar{\psi}^j\psi^j\bar{\psi}^k\psi^k \right\}, \tag{44}$$

which are absorbed in the SUSY invariant relation of the auxiliary field $F^i = F^i(\psi)$ by adding four NG-fermion self-interaction terms as (24):

$$F^i(\psi) \longrightarrow F^i(\psi) - \frac{1}{4} e\kappa^2\xi\xi^i\bar{\psi}^j\psi^j\bar{\psi}^k\psi^k|w_{VA}|. \tag{45}$$

Therefore,

• under SUSY invariant relations,
the $N = 2$ NLSUSY action $S_{N=2\mathrm{NLSUSY}}$ is related to $N = 2$ SUSY QED action:

$$f(\xi, \xi^i) S_{N=2\mathrm{NLSUSY}} = S_{N=2\mathrm{SUSYQED}} \equiv S_{\mathcal{V}\mathrm{free}} + S_{\mathcal{V}f} + S_{\mathrm{gauge}} \tag{46}$$

when $f(\xi, \xi^i) = \xi^2 - (\xi^i)^2 = 1$.

$\Longrightarrow$ NL/L SUSY relation gives the relation between
the cosmology and the low energy particle physics in NLSUSY GR (in Sec. 4).

• **SGM scenario predicts the magnitude of the bare gauge coupling constant.**

More general SUSY invariant constraints, i.e. NLSUSY vevs of 0^+ auxiliary fields:

$$\tilde{C} = \xi_c, \quad \tilde{\Lambda}^i = \tilde{M}^{ij} = \tilde{\phi} = \tilde{v}^a = \tilde{\lambda}^i = 0, \quad \tilde{D} = \frac{\xi}{\kappa}, \quad \tilde{B}^i = \tilde{\chi} = \tilde{\nu} = 0, \quad \tilde{F}^i = \frac{\xi^i}{\kappa}. \tag{47}$$

produce

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \ e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \tag{48}$$

where e is the bare gauge coupling constant.

• This mechanism is natural and favorable for SGM scenario as a theory for everything.

Broken LSUSY(QED) gauge theory is encoded
in the vacuum of NLSUSY theory
as composites of NG fermion.

4. Cosmology and Low Energy Physics in NLSUSY GR

The variation of SGM action $L_{N=2SGM}(e,\psi)$ with respect to $e^a{}_\mu$ yields the equation of motion for $e^a{}_\mu$ in Riemann space-time:

$$R_{\mu\nu}(e) - \frac{1}{2} g_{\mu\nu} R(e) = \frac{8\pi G}{c^4} \{\tilde{T}_{\mu\nu}(e,\psi) - g_{\mu\nu} \frac{c^4 \Lambda}{16\pi G}\}, \tag{49}$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction.

• Note that $-\frac{c^4\Lambda}{16\pi G}$ can be interpreted as the negative energy density of space-time, i.e. the dark energy density ρ_D.
(The negative sign in r.h.s is unique.)

4.1. Low Energy Particle Physics of NLSUSY GR :

We have seen in the preceding section that

$N=2$ SGM is essentially N=2 NLSUSY action in Riemann-flat (tangent) space-time.

- The low energy theorem for NLSUSY gives the following superon(massless NG fermion matter)-vacuum coupling

$$< \psi^j{}_\alpha(x)|J^{k\mu}{}_\beta|0 >= i\sqrt{\frac{c^4\Lambda}{16\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{jk} + \cdots, \tag{50}$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{16\pi G}}\gamma^\mu\psi^k + \cdots$ is the conserved supercurrent.

$\sqrt{\frac{c^4\Lambda}{16\pi G}}$ is the coupling constant (g_{sv}) of superon with the vacuum.

For extracting the low energy particle physics of $N = 2$ SGM (NLSUSY GR) we consider in Riemann-flat space-time, where NL/L SUSY relation gives:

$$L_{N=2\mathrm{SGM}} \longrightarrow L_{N=2\mathrm{NLSUSY}} + [\text{suface terms}] = L_{N=2\mathrm{SUSYQED}}. \quad (51)$$

- We study vacuum structures of $N = 2$ LSUSY QED action in stead of $N = 2$ SGM.

The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\mathrm{LSUSYQED}}$,

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e(B^i)^2 \right\} D + \frac{e^2}{2}(A^2 + \phi^2)(B^i)^2. \quad (52)$$

Substituting the solution of the equation of motion for the auxiliary field D we obtain

$$V(A,\phi,B^i) = \frac{1}{2}f^2\left\{A^2-\phi^2-\frac{e}{2f}(B^i)^2-\frac{\xi}{f\kappa}\right\}^2+\frac{1}{2}e^2(A^2+\phi^2)(B^i)^2 \geq 0. \tag{53}$$

The field configurations of the vacua $V_{P.E.}=0$ in (A,ϕ,B^i)-space should firstly satisfy followings with $SO(1,3)$ or $SO(3,1)$ isometry:

(I) For $ef>0$, $\frac{\xi}{f}>0$ case,

$$A^2-\phi^2-(\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i=\sqrt{\frac{e}{2f}}B^i, \quad k^2=\frac{\xi}{f\kappa}\right) \tag{54}$$

(II) For $ef<0$, $\frac{\xi}{f}>0$ case,

$$A^2-\phi^2+(\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i=\sqrt{-\frac{e}{2f}}B^i, \quad k^2=\frac{\xi}{f\kappa}\right) \tag{55}$$

(III) For $ef > 0, \quad \frac{\xi}{f} < 0$ case,

$$-A^2 + \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \tag{56}$$

(IV) For $ef < 0, \quad \frac{\xi}{f} < 0$ case,

$$-A^2 + \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \tag{57}$$

• The low energy particle spectrum is obtained by expanding the fields (A, ϕ, B^i) around the vacuum field configurations.

• We find that

the vacua (I) and (IV) with $SO(1,3)$ isometry are unphysical

and as shown below

the vacua (II) and (III) with $SO(3,1)$ isometry possess two different physical vacua.

• Adopt following expressions for two cases of vacuum (II): with $SO(3,1)$
Case (IIa) with $O(2)$ for $(\tilde{B}^1, \tilde{B}^2)$

$$\begin{aligned}
A &= (k+\rho)\sin\theta\cosh\omega, \\
\phi &= (k+\rho)\sinh\omega, \\
\tilde{B}^1 &= (k+\rho)\cos\theta\cos\varphi\cosh\omega, \\
\tilde{B}^2 &= (k+\rho)\cos\theta\sin\varphi\cosh\omega
\end{aligned}$$

Case (IIb) with $O(2)$ for $(A, \tilde{B}^1)$

$$\begin{aligned}
A &= -(k+\rho)\cos\theta\cos\varphi\cosh\omega, \\
\phi &= (k+\rho)\sinh\omega, \\
\tilde{B}^1 &= (k+\rho)\sin\theta\cosh\omega, \\
\tilde{B}^2 &= (k+\rho)\cos\theta\sin\varphi\cosh\omega.
\end{aligned}$$

• Substituting these expressions into $V(A, \phi, \tilde{B}^i)$
and expanding them around the vacuum configuration:

$\rho \ll 1$ and angles for $\tilde{B}^i = 0$ or $A = \phi = 0$

we obtain the physical particle contents.(Arguments hold for case (III) as well.)

- For (IIa) and (IIIa) we obtain

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \rho)^2 - 2(-ef)k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a \theta)^2 + (\partial_a \omega)^2 - 2(-ef)k^2(\theta^2 + \omega^2)\} \\
& + \frac{1}{2}(\partial_a \varphi)^2 \\
& - \frac{1}{4}(F_{ab})^2 + (-ef)k^2 v_a^2 \\
& + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \sqrt{-2ef}(\bar{\lambda}^1\chi - \bar{\lambda}^2\nu) + \cdots,
\end{aligned}
\tag{58}
$$

and following mass spectra

$$
\begin{aligned}
&m_\rho^2 = m_\theta^2 = m_\omega^2 = m_{v_a}^2 = 2(-ef)k^2 = -\frac{2\xi e}{\kappa}, \\
&m_{\lambda^i} = m_\chi = m_\nu = m_\varphi = 0.
\end{aligned}
\tag{59}
$$

- The vacuum breaks both SUSY and the local $U(1)(O(2))$ spontaneously.

(φ is the NG boson for the spontaneous breaking of $U(1)$ symmetry, i.e. the $U(1)$ phase of $\tilde{B}$, and totally gauged away by the Higgs-Kibble mechanism with $\Omega(x) = \sqrt{e\kappa/2}\varphi(x)$ for the $U(1)$ gauge (26).)

- All bosons have the same mass, and remarkably all fermions remain massless.

- λ^i are not NG fermions of LSUSY. $\leftarrow \; < \delta\lambda > \; \sim \; < D >= 0$

- Off-diagonal mass terms $\sqrt{-2ef}(\bar\lambda^1\chi - \bar\lambda^2\nu) = \sqrt{-2ef}(\bar\chi_{\rm D}\lambda + \bar\lambda\chi_{\rm D})$ would induce mixings of fermions. $\Rightarrow$ pathological?

- For (IIb) and (IIIb) we obtain

$$
\begin{aligned}
L_{N=2\mathrm{SUSYQED}} = & \frac{1}{2}\{(\partial_a \rho)^2 - 4f^2k^2\rho^2\} \\
& + \frac{1}{2}\{(\partial_a \theta)^2 + (\partial_a \varphi)^2 - e^2k^2(\theta^2 + \varphi^2)\} \\
& + \frac{1}{2}(\partial_a \omega)^2 \\
& - \frac{1}{4}(F_{ab})^2 \\
& + \frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i \lambda^i) \\
& + \frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/ \chi + \bar{\nu}\partial\!\!\!/ \nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots .
\end{aligned} \tag{60}
$$

and following mass spectra:

$$
\begin{aligned}
& m_\rho^2 = m_{\lambda^i}^2 = 4f^2k^2 = \frac{4\xi f}{\kappa}, \\
& m_\theta^2 = m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2 = \frac{\xi e^2}{\kappa f}, \\
& m_{v_a} = m_\omega = 0,
\end{aligned}
\tag{61}
$$

which produces mass hierarchy by the factor $\frac{e}{f}$.

• The vacuum breaks both SUSY and $O(2)(U(1))$ for $(A, \tilde{B}^2)$ and restores(maintains) $O(2)(U(1))$ for $(\tilde{B}^1, \tilde{B}^2)$, spontaneously,

which gives soft masses $< A >$ to λ^i and produces NG-Boson ω and massless photon v_a, respectively.

• We have shown explicitly that N=2 LSUSY QED, i.e. the matter sector (in flat-space) of $N=2$ SGM, possesses a unique true vacuum type (b) with $V_{P.E}=0$.

The resulting model describes:

one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$),
one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$),
one massless vector (a photon) (v_a),
one charged scalar ($\phi^c \sim \theta + i\varphi$),
one neutral complex scalar ($\phi^0 \sim \rho(+i\omega)$),

which are composites of superons.

• Remakably the lepton-Higgs sector of SM analogue $SU(2)_{gl} \times U(1)$ appears from $N=2$ LSUSY QED without superpartners.

- **Cosmological meanings of $N = 2$ LSUSY QED in the SGM scenario**:

The unique vacuum (b) explains naturally observed mysterious (numerical) relations:

(dark) energy density of the universe $\sim {m_\nu}^4 \sim (10^{-12} GeV)^4 \sim {g_{sv}}^2$,

provided ${\lambda_D}^0$ is identified with neutrino [in $d = 4$ as well],
which gives a new insight into the origin of mass.

- The vacuum (a) inducing the fermion mixing may be generic for $N > 2$
and deserve further investigations.

6. Summary

NLSUSY GR(SGM) scenario:

• Ultimate entity; **New unstable $d = 4$ space-time U:**$[x^a, {\psi_\alpha}^N; x^\mu]$ described by $[L_{NLSUSYGR}(w)]$ **:** NLSUSY GR on New space-time with $\Lambda > 0$
• Mach principle is encoded geometrically

$\Longrightarrow$ **Big Decay** (due to false vacuum $V_{P.E.} = \Lambda > 0$) **to** $[L_{SGM}(e.\psi)]$;

• The creation of Riemann space-time $[x^a; x^\mu]$ and massless fermionic matter $[{\psi_\alpha}^{\mathbf{N}}]$
$[L_{SGM} = L_{EH}(e) - \Lambda + T(\psi.e)]$ **:** Einstein GR with $V_{P.E.} = \Lambda > 0$ and N superon

$\Longrightarrow$ Formation of gravitational masless composite states:L_{LSUSY}
$\Longrightarrow$ **Ignition of Big Bang Universe**

• Phase transition towards the true vacuum $V_{P.E} = 0$,
achieved by forming composite massless LSUSY and
subsequent oscilations around the true vacuum.

• In flat space-time, broken N-LSUSY theory emerges from the N-NLSUSY cosmological term of $L_{SGM}(e, \psi)$ [NL/L SUSY relation]. $\longleftrightarrow$ BCS vs GL

The cosmological constant is the origin of everything!

Predictions and Conjectures:

@ Group theory of SO(10) sP with $\underline{10} = \underline{5} + \underline{5}^*$.
$\underline{5} = \underline{5}_{SU(5)GUT}$ interpreted as superon-quintet(SQ):

- Spin-$\frac{3}{2}$ lepton-type doublet (Γ^-, ν_Γ); Doubly charged spin 1/2 particles $E^{2\pm}$
- Proton decay diagrams in GUTs are forbidden by selection rules. $\Rightarrow$ stable proton
- neutral $J^P = 1^-$ boson S.
- Neutrino problems(mass and oscillation) are gravitational origin.

@Field theory via Linearization:

- Chiral eigenstates in SM may be a NLSUSY effect.
- NLSUSY GR(SGM) scenario predicts **4 dimensional space-time**.
- The bare gauge coupling constant is determined.
- N-LSUSY from N-NLSUSY $\Longleftrightarrow$ SQ hypothesis for all particles (except gravity)
- Superfluidity of space-time $\Longleftrightarrow$ κ^{-2}: chemical potential for SGM

cosmological constant $\leftrightarrow$ dark energy density $\leftrightarrow$ SUSY Br. $\rightarrow m_\nu$

Many Open Questions ! e.g.,

- Large N, $D = 4$ case (especially N=5 and N=10), Is realistic and minimal?
- SGM scenario suggests $N \geq 2$ low energy MSSM, SUSY GUT

- Meanings of Chiral symmetry, Yukawa and gauge coulings
in SGM composite scenario

- Direct linearization of SGM action in curved space-time.
- Superfield systematics of NL/L SUSY relation for SGM action.

- Superfluidity of sapce-time and matter?

- Equivalence principle and NLSUSYGR.