Stellar Oscillations in Modified Gravity

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JS arXiv: 1309.0495
see also Jain, Vikram, JS arXiv: 1204.6044
Davis, Lim, JS, Shaw arXiv: 1102.5278
Chang & Hui 1011.4107

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Cosmological Tests of Modified Gravity

Upcoming surveys such as EUCLID will probe the nature of gravity on cosmological scales to unprecedented levels!

- Signals are very small but lots of data and different probes.
- Cosmological scales are highly degenerate between different theories.
- Some theories e.g. chameleons don't show effects on linear scales.

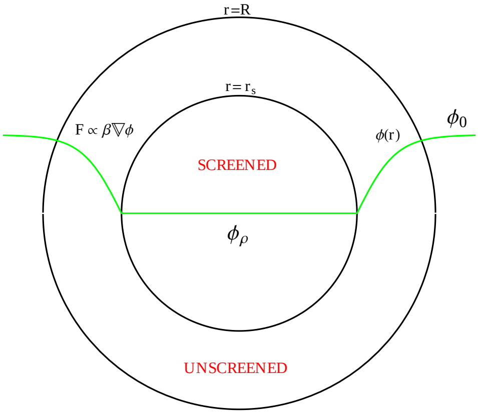
Astrophysical Tests of Modified Gravity?

We expect novel features of astrophysical scales:

- Get very large, $\mathcal{O}(1)$ signals.
- These are messy - need to disentangle them from other astrophysical effects.
- This is possible using the environment-dependence of the screening - we can look for systematic offsets.

Chameleon Screening

Specialise to Chameleon-like theories:



Screening Parametrisation

There is a model-independent parametrisation perfect for small-scale tests:

$$\alpha: \; G(r) = G(1+\alpha) \quad \text{when fully unscreened.}$$

$f(R)$ gravity has $\alpha = 1/3$.

$$\chi_0 \text{ - the self-screening parameter.}$$

Rule of thumb: an object is partially unscreened if

$$\chi_0 > \Phi_{\mathrm{N}}.$$

Φ_{N} is the Newtonian Potential.

Self-Screening Parameter

These theories screen according to the Newtonian potential.

The Newtonian potential of the Sun and Milky Way is $\mathcal{O}(10^{-6})$.

Need to test these theories in less-dense environments:

- Post-main-sequence stars $\Phi_{\rm N} \sim 10^{-7} - 10^{-8}$.
- Isolated dwarf galaxies in voids $\Phi_{\rm N} \sim 10^{-8}$.

This is possible thanks to a recent screening map of the nearby universe Cabre et al. 12.

Stellar Structure Tests

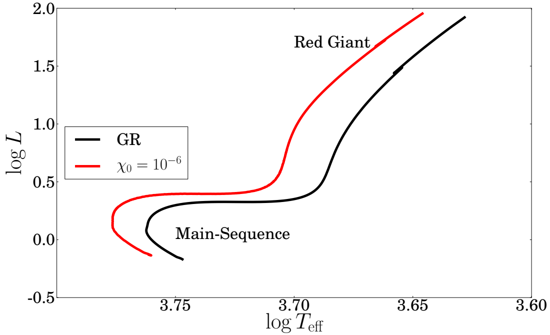
Stars in modified gravity:

- Need to burn more fuel per unit time to stave off gravitational collapse.
- More Compact
- Hotter
- Brighter

We have a modified stellar structure code, MESA, which can simulate real stars including modified gravity.

Modified Gravity with MESA

$$M = 1M_\odot \quad \alpha = 1/3$$



Davis, Lim, JS & Shaw 2011

Cepheid Distance Indicator Tests

We exploit these effects by comparing distances from screened and unscreened indicators.

The period of Cepheid pulsations changes in modified gravity:

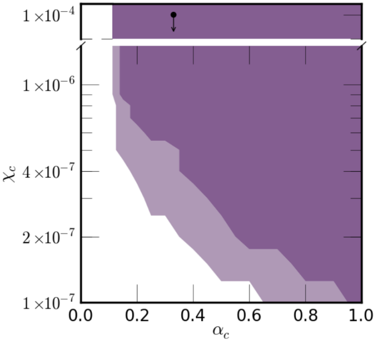
$$\log d \propto \log \tau$$

$$\tau \propto G^{-\frac{1}{2}}$$

$$\frac{d_{\rm MG} - d_{\rm GR}}{d_{\rm GR}} \approx -0.3 \frac{\Delta G}{G}$$

We compute $\frac{\Delta G}{G}$ using MESA profiles and compare with tip of the red giant branch distances (insensitive to modified gravity).

Constraints



Jain, Vikram & JS 2012

$$\chi_0 \lesssim \ 4\times10^{-7} \quad (f(R)\ \text{Gravity})$$

These are currently the strongest constraints in the literature.

Motivation

$$\frac{d_{\rm MG}-d_{\rm GR}}{d_{\rm GR}}\approx -0.3\frac{\Delta G}{G}$$

is an approximation - need full hydrodynamic perturbation theory.

There are three new features in modified gravity hydrodynamics

1. The modified periods are even smaller than GR.
2. The stars are more stable
3. Oscillations source scalar radiation and vica versa (I will not discuss this in this talk).

Modified Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r) e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

General Relativity:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 \Gamma_{1,0} P_0 \frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3 \frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0} - 4\right) P_0\right] \xi + r^4 \rho_0 \omega^2 \xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the periods of oscillation.

Modified Linear Adiabatic Wave Equation

$$\frac{\delta r}{r} = \xi(r) e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

Modified Gravity:

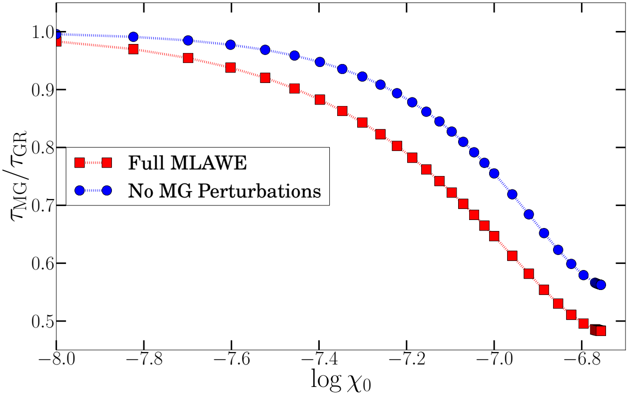
$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^4 \Gamma_{1,0} P_0 \frac{\mathrm{d}\xi}{\mathrm{d}r}\right) + r^3 \frac{\mathrm{d}}{\mathrm{d}r}\left[\left(3\Gamma_{1,0} - 4\right) P_0\right] \xi - 4\pi\alpha G r^4 \rho_0^2 \xi + r^4 \rho_0 \omega^2 \xi = 0$$

This is a Sturm-Liouville problem.

The eigenfrequencies give the modified periods of oscillation.

Example: Lane-Emden Models

Semi-analytic model (Davis, Lim, JS & Shaw 2011) with $P = K\rho^{\frac{5}{3}}$:



$$\frac{GM}{R} = 10^{-7}$$

Cepheid Distances

α	χ_0	$\Delta d/d$ (approx)	$\Delta d/d$ (LAWE)	$\Delta d/d$ (MLAWE)
$1/3$	4×10^{-7}	-0.03	-0.04	-0.12
$1/2$	4×10^{-7}	-0.05	-0.06	-0.16
1	2×10^{-7}	-0.06	-0.07	-0.19

Full hydrodynamic prediction is 3 times larger than the equilibrium prediction!

Can improve the constraints using the same data sets.

Stellar Stability

$$\frac{\delta r}{r} = \xi(r) e^{i\omega t} \quad \Gamma_{1,0} = \frac{\partial \ln P_0}{\partial \ln \rho_0}$$

We can use the variational method to find an upper bound on the fundamental frequency.

$$\omega_0^2 \leq F[P_0, \rho_0]$$

The star is unstable if this is imaginary.

General Relativity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3}$$

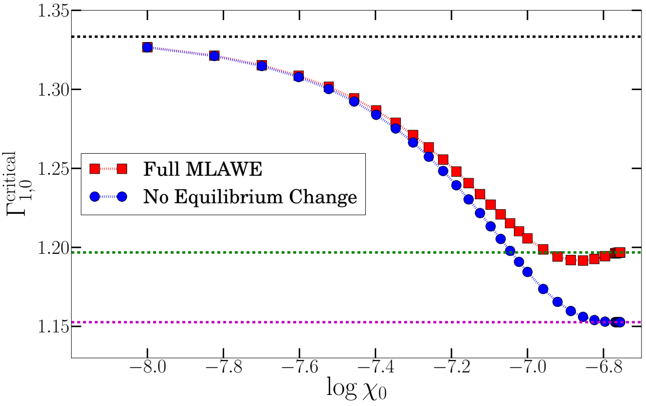
Stellar Stability

Modified Gravity:

$$\Gamma_{1,0}^{\text{critical}} = \frac{4}{3} - \alpha g(\chi_0)$$

$g(\chi_0)$ is an $\mathcal{O}(1)$ composition dependent factor which depends on how unscreened the star is.

Stability of Lane-Emden Models



The dip is present because the modified equilibrium structure competes with the perturbations.

Summary

- Astrophysical tests can constrain chameleon gravity to levels unreachable by other means.
- Using an approximation we have placed the tightest constraints to date.
- We have investigated this by deriving the full equations governing perturbations in modified gravity hydrodynamics.
- We find that the period of Cepheid oscillations could be three times shorter than we previously predicted.
- This means we can place even tighter constraints with the same data sets.
- Stars in modified gravity are more stable and there is no universal bound on $\Gamma_{1,0}$.

Future Directions

- Need to investigate the effects of scalar radiation $\Rightarrow$ MLAWE becomes two coupled equations.
- Applications to other modified gravity theories such as Galileons.

Thank You!

Code Comparisons

We compare our GR dimensionless frequencies

$$\tilde{\omega}^2 = \frac{(n+1)\omega^2}{4\pi G\rho_c}$$

to those found by Hurley, Roberts & Wright 1966:

n	Hurley, Roberts & Wright	Me
0.5	0.37071	0.370714029
1.0	0.38331	0.38331184243
1.5	0.37640	0.376399032288
2.0	0.35087	0.350866992807
2.5	0.30389	0.303893585012
3.0	0.22774	0.227742000109
3.25	0.17731	0.177307835186
3.5	0.12404	0.124042556661
4.0	0.04056	0.0405613874985