Transporting non-Gaussianity from sub to super-horizon scales

David Mulryne

based on arXiv:1302.4636

(also see earlier work with Daniel Wesley, David Seery, Gemma Anderson, arXiv:1008.3159, arXiv:1203.2635, and forthcoming work with David Seery, Mafalda Dias, Jonny Frazer, Joe Elliston)
Transport introduction

First developed as alternative to $\delta N$ (e.g. Lyth and Rodriguez 2005), based on flow of probability to provide insight to evolution in multi-field models.

Provides set of coupled ODEs for the correlations of inflationary perturbations - easy numerical algorithm in contrast to $\delta N$ or In-In.

Extended to sub-horizon scales and quantum correlation functions in recent paper. Mulryne 2013

Now lots of data (Planck, Ade et al. 2013) - but nearly all (multi-field) models must be tested against it numerically.

Currently we are implementing equations in a code to be publicly released - will calculate full power, bi- (and tri-) spectrum.

This talk will introduce the (sub-horizon) transport equations and some preliminary results.
Transport basics

We care about the Fourier space correlation functions:

\[
\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 P(k_1) \delta^3(k_1 + k_2)
\]

\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3)
\]

Where for inflation

\[ P(k) \approx A k^{-3} \]

And for vanilla inflation

\[ f_{\text{nl}}(k_1, k_2, k_3) \sim \text{slow roll parameters} \]

Cosmological perturbation theory (e.g. review of Malik and Wands 2008), provides evolution equations for perturbations (curvature/isocurved or fields).

Would prefer evolution equations for the correlations of these perturbations.
Transport derivation

Consider M scalar fields with perturbations:

$$x_{\alpha'} = \{\delta \phi_{\alpha'}, \delta \dot{\phi}_{b'}\}$$

Consider their evolution equations in compact notation (after promoting to operators)

$$\frac{dx_{\alpha'}}{dt} = u_{\alpha' \beta'} x_{\beta'} + \frac{1}{2!} u_{\alpha' \beta' \gamma'} \left( x_{\beta'} x_{\gamma'} - \langle x_{\beta'} x_{\gamma'} \rangle \right) + \ldots$$

Where, e.g.

$$\delta \varphi_{1'} = \delta \varphi_1 (k_1)$$

And

$$u_{\alpha' \beta'} = (2\pi)^3 u_{\alpha \beta} (k_\alpha) \delta (k_\alpha - k_\beta), \quad u_{\alpha' \beta' \gamma'} = (2\pi)^3 u_{\alpha \beta \gamma} (k_\alpha, k_\beta, k_\gamma) \delta (k_\alpha - k_\beta - k_\gamma)$$
Transport derivation

Now consider correlation functions

\[ \Sigma_{\alpha' \beta'} = \langle x_{\alpha'} x_{\beta'} \rangle, \quad \alpha_{\alpha' \beta' \gamma'} = \langle x_{\alpha'} x_{\beta'} x_{\gamma'} \rangle \]

Where

\[ \Sigma_{\alpha' \beta'} = (2\pi)^3 \delta(k_\alpha + k_\beta) \Sigma_{\alpha \beta}(k_\alpha) \]
\[ \alpha_{\alpha' \beta' \gamma'} = (2\pi)^3 \delta(k_\alpha + k_\beta + k_\gamma) \alpha_{\alpha \beta \gamma}(k_\alpha, k_\beta, k_\gamma) \]

And employ Ehrenfest's theorem

\[ \frac{d\langle \mathcal{O} \rangle}{dt} = \left\langle \frac{d\mathcal{O}}{dt} \right\rangle \]

We arrive at

\[ \frac{\Sigma^r_{1 \beta}(k_\alpha)}{dt} = u_{\alpha \gamma}(k_\alpha) \Sigma^r_{\gamma \beta}(k_\alpha) + u_{\beta \gamma}(k_\alpha) \Sigma^r_{1 \alpha}(k_\alpha) \]
\[ \frac{d\alpha_{\alpha \beta \gamma}(k_\alpha, k_\beta, k_\gamma)}{dt} = u_{\alpha \lambda}(k_\alpha) \alpha_{\lambda \beta \gamma}(k_\alpha, k_\beta, k_\gamma) + u_{\alpha \lambda \mu}(k_\alpha, k_\beta, k_\gamma) \Sigma^r_{\lambda \beta}(k_\beta) \Sigma^r_{\mu \gamma}(k_\gamma) \]
\[ -\frac{1}{3} u_{\alpha \lambda \mu}(k_\alpha, k_\beta, k_\gamma) \Sigma^i_{\lambda \beta}(k_\beta) \Sigma^i_{\mu \gamma}(k_\gamma) + \text{cyclic} \]
Many further attractive properties, for example:

- Easy to convert to the statistics of $\zeta$. (Used for example in Dias, Frazer and Liddle 2013)

- Evolution can be decomposed into equations for ‘shapes’ such as local $f_{NL}$, $T_{NL}$ and $g_{NL}$. Anderson, Mulryne and Seery (2012)


- Moreover, integral solutions to transport hierarchy in terms of ‘$\Gamma$’ matrices are possible – connect to the integral solutions of the In-In formalism. Seery, Mulryne, Frazer and Ribeiro (2012), Mulryne (2013)

- They turn out to be Taylor coefficients of $\delta N$ style expansion, and satisfy ODEs – providing a differential formulation of $\delta N$. (see recent transport papers and earlier work of Yokoyama et al. 2007)
Transport numerical algorithm

Step 1. Derive the $u$ coefficients for the model at hand (multi-field canonical/non-canonical, curved field space etc).

Step 2. Calculate the initial conditions (perhaps Bunch-Davis) – integral solutions can be used to fix these at arbitrary times (at or long before horizon crossing).

Step 3. Solve the ODEs for the correlations of the field perturbations. If want the bi-spectra for example, one evolution for each triangle of $k$ scales.

Step 4. Convert to any other quantity of interest (zeta correlations – power/bi-spectra – fnl.....)
Transport examples (super-horizon)

Consider potential

\[ V = M^4 \left[ \frac{1}{2} m^2 \phi_1^2 + \frac{1}{2} g^2 \phi_1^2 \phi_2^2 + \frac{\lambda}{4} (\phi_2^2 - v^2)^2 \right] \]
Transport examples (sub-horizon)

Consider potential

\[ V = \frac{1}{2} m_1 \phi_1^2 + \frac{1}{2} m_2 \phi_2^2 \]

(preliminary results, future publication with Dias, Frazer, Mulryne, Seery)
Transport conclusions

- Transport techniques provide a suite of methods for the calculation of the correlations (power, bi-, trispectrum) of inflationary perturbations.

- In particular evolve correlations from sub- to super-horizon scales in a numerically convenient way.

- For the bi-spectrum one triangle (point on bispectrum), takes a couple of seconds on a laptop for a simple two field model - so can be used to build up a picture of bispectrum in reasonable time scale.

- In due course we will release code, initially for canonical multi-field models up to bispectrum, hopefully followed by non-canonical and other generalizations.