# Newtonian and Post-Newtonian Gravity in Cosmology : $c \to \infty$ and Beyond

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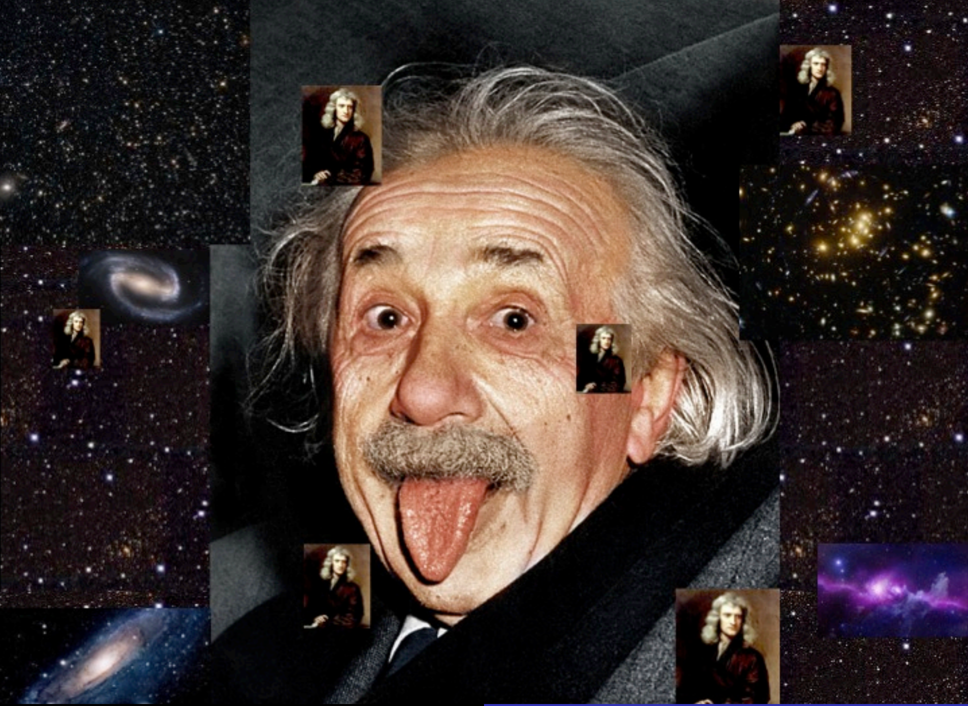

# Gravity in Cosmology

Einstein gravity (GR) used on large scales and Newtonian gravity used on small scales.

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$\Rightarrow$ We need to go beyond Newtonian theory on small scales

# Post-Friedmann Formalism

- Framework for structure formation beyond Newtonian limit
- Uses Post-Newtonian style $\frac{1}{c}$ expansion in Poisson gauge
- PN philosophy in Cosmology different from Solar System:

  Metric evolution $\Rightarrow$ Need consistent solution of Einstein Equations
- Linearisation $\rightarrow$ linear perturbation theory (large scales)
- Leading order $\frac{1}{c}$ expansion $\rightarrow$ Newtonian+ regime (small scales)

# post-Friedmann metric

- $g_{00} = -\left(1 - \frac{2U_N}{c^2} + \frac{1}{c^4}(2U_N^2 - 4U_P)\right)$
- $g_{0i} = -a\left(\frac{B_i^N}{c^3} + \frac{B_i^P}{c^5}\right)$
- $g_{ij} = a^2\left(\left[1 + \frac{2V_N}{c^2} + \frac{2V_N^2+4V_P}{c^4}\right]\delta_{ij} + \frac{h_{ij}}{c^4}\right)$

$B_i^N$ is divergenceless $B_{i,i}^N = 0$

$h_{ij}$ is transverse and traceless $h_{ij,i} = 0$,$h_{ii} = 0$

At this level, $h_{ij}$ is non-dynamical $\rightarrow$ Not gravitational waves

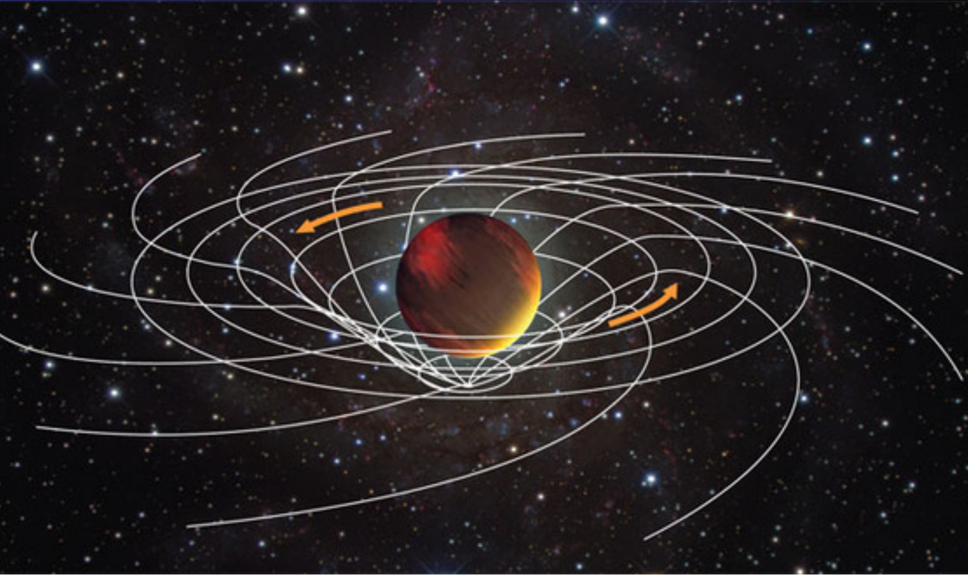
# Leading Order Gravitational Equations

$$\frac{1}{c^2}\nabla^2 V_N = -\frac{4\pi G a^2 \rho_b}{c^2}\delta$$

$$\frac{2}{c^2 a^2}\nabla^2 (V_N - U_N) = 0$$

$$\frac{1}{c^3}\nabla^2 B_i^N = -\frac{16\pi G a^2 \rho_b}{c^3}(1+\delta)v_i + \frac{2}{c^3}\left(\dot{a}U_{N,i} + a\dot{V}_{N,i}\right)$$

# Frame Dragging



# Frame-Dragging Potential

- What is it physically?

  An object's rotation affects spacetime; it "drags" spacetime around

- How can we calculate it[1]?

$$\nabla \times \nabla^2 \vec{B}^N = -\left(16\pi G \rho_b a^2\right) \nabla \times [(1+\delta)\vec{v}]$$

  Sourced by purely Newtonian quantities

  Doesn't affect matter dynamics at this order

  Could affect e.g. photon geodesics

[1] A similar equation appears in Green&Wald '12,Takada&Futamase '99, Hwang&Noh '12

# N-body simulations & tesselation

- Theory

  An N-body simulation should contain $\delta$ and $\vec{v}$

  $\Rightarrow$ We can extract the source term for the Vector potential

- Practice

  Extracting velocities non-trivial

  Delauney Tesselation Field Estimator (DTFE)[2]

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[2] Bernardeau, F. & van de Weygaert, R. 1996, MNRAS, 279, 693
Schaap, W. E. & van de Weygaert, R. 2000, A& A, 363, L29 astro-ph/0011007
Schaap, W. PhD thesis "DTFE: the Delaunay Tessellation Field Estimator"
van de Weygaert, R. & Schaap, W. 2009, 665, 291 arXiv: 0708.1441
Cautun, M. & van de Weygaert, R., "The DTFE public software" arXiv: 1105.0370

# Extracted Power Spectra

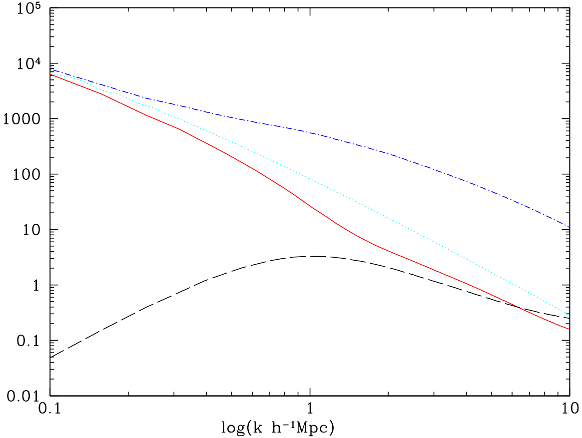


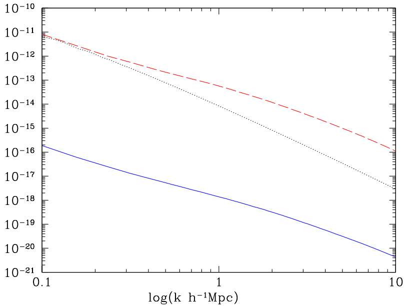

Figure: Power Spectra of density (blue), linear density (cyan), velocity divergence (red) and vorticity (black). Divergence and vorticity divided by $(\mathcal{H}f)^2$

# Quiz Time

How much smaller is the power spectrum of $B_i^N$ compared to the Newtonian potential?

- $\sim 10^3$ times smaller
- $\sim 10^5$ times smaller
- $\sim 10^7$ times smaller
- $\sim 10^9$ times smaller

# Results




Power spectra of the scalar potential (red)

and vector potential (blue)

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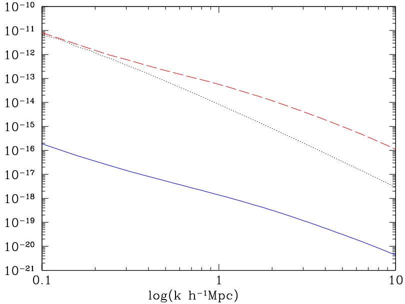


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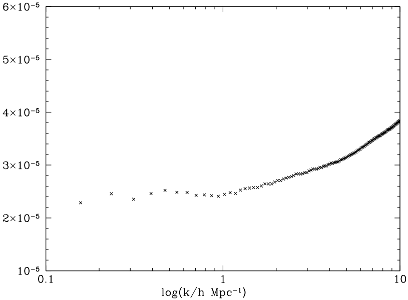


Ratio of vector potential to

scalar potential

# Should we care?

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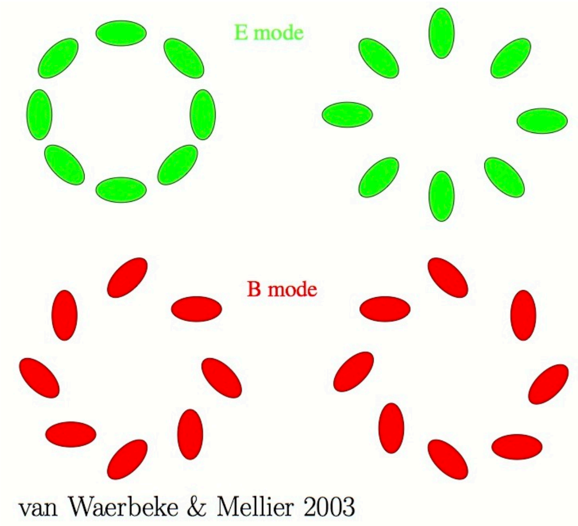

# Conclusions

- We need to go beyond Newtonian gravity on small scales
- Post-Friedmann approach allows calculation of relativistic quantities
- We have computed the vector potential generated in the metric by cosmological large scale structure
  - This is required in the Newtonian regime for consistency of the Einstein equations
  - The first calculation of an intrinsically relativistic quantity in full non-linearity in LSS
- This vector potential could be observed via the weak-lensing B-mode

# Tesselation

- Why?

  "Normal" estimators-CIC etc, give mass-averaged not

  volume-averaged velocities

  Velocity field artificially set to zero in sparse regions

- How?

  Constructs tetrahedra, with nodes located at the particles' positions

  No particle inside the circumsphere of another tetrahedron

  Velocities interpolated across tetrahedra $\Rightarrow \vec{v}$ known everywhere

  Field sampled at random points within grid cell and averaged

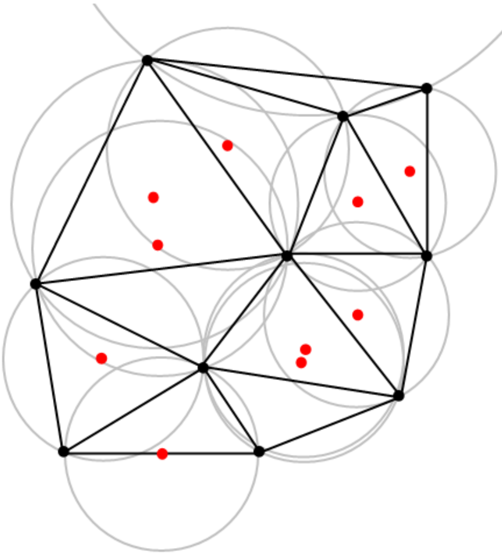

Figure: 2D Delaunay tesselation of black points. Centres of circumcircles in red.