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Full-sky formulae for weak lensing power spectra from total angular momentum method

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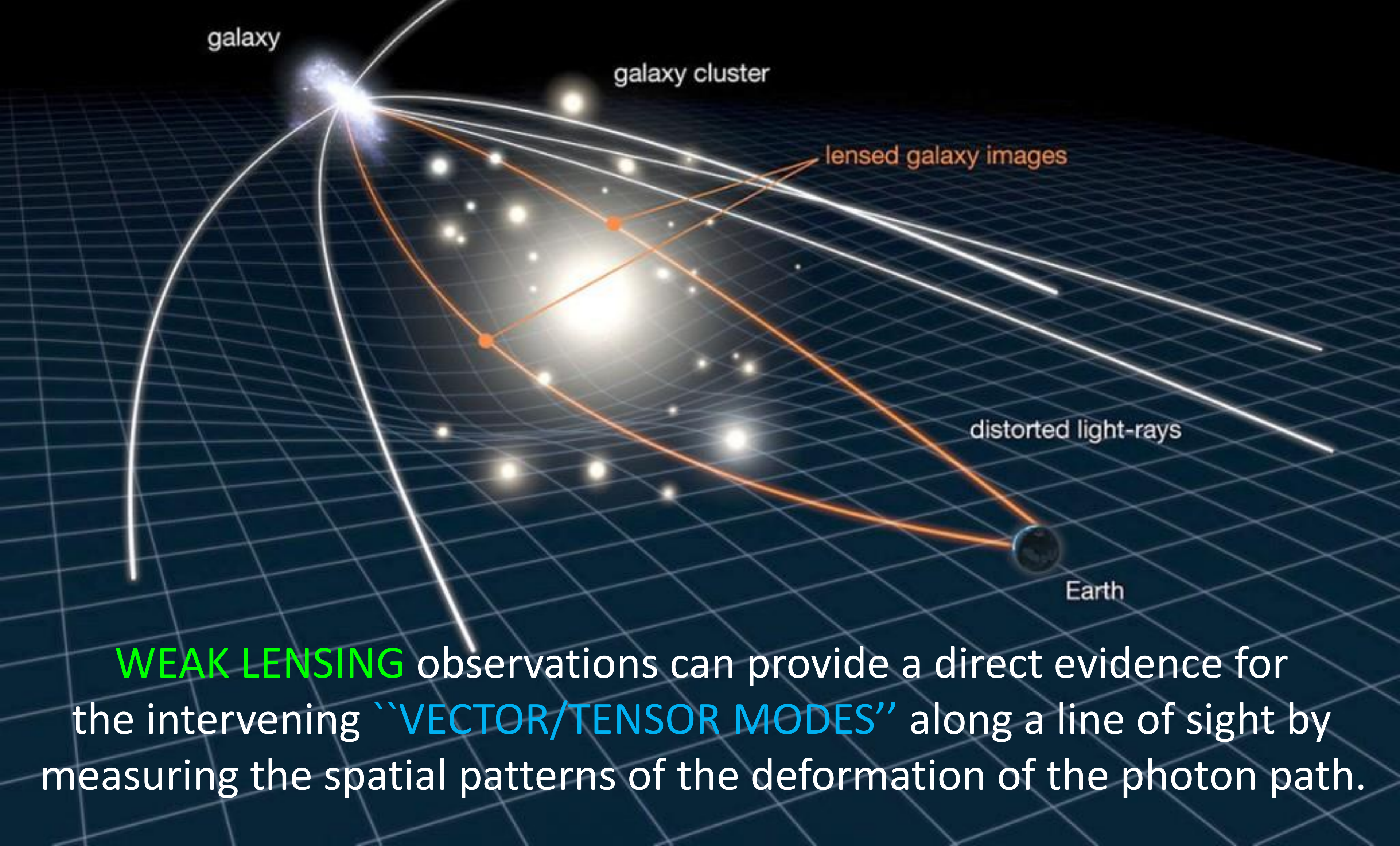
DY, A. Taruya, T. Namikawa, JCAP08(2013)051, arXiv:1305.3348.
T. Namikawa, **DY**, A. Taruya, arXiv:1308.6068.

Gravitational Lensing
= method to ``see'' invisibles

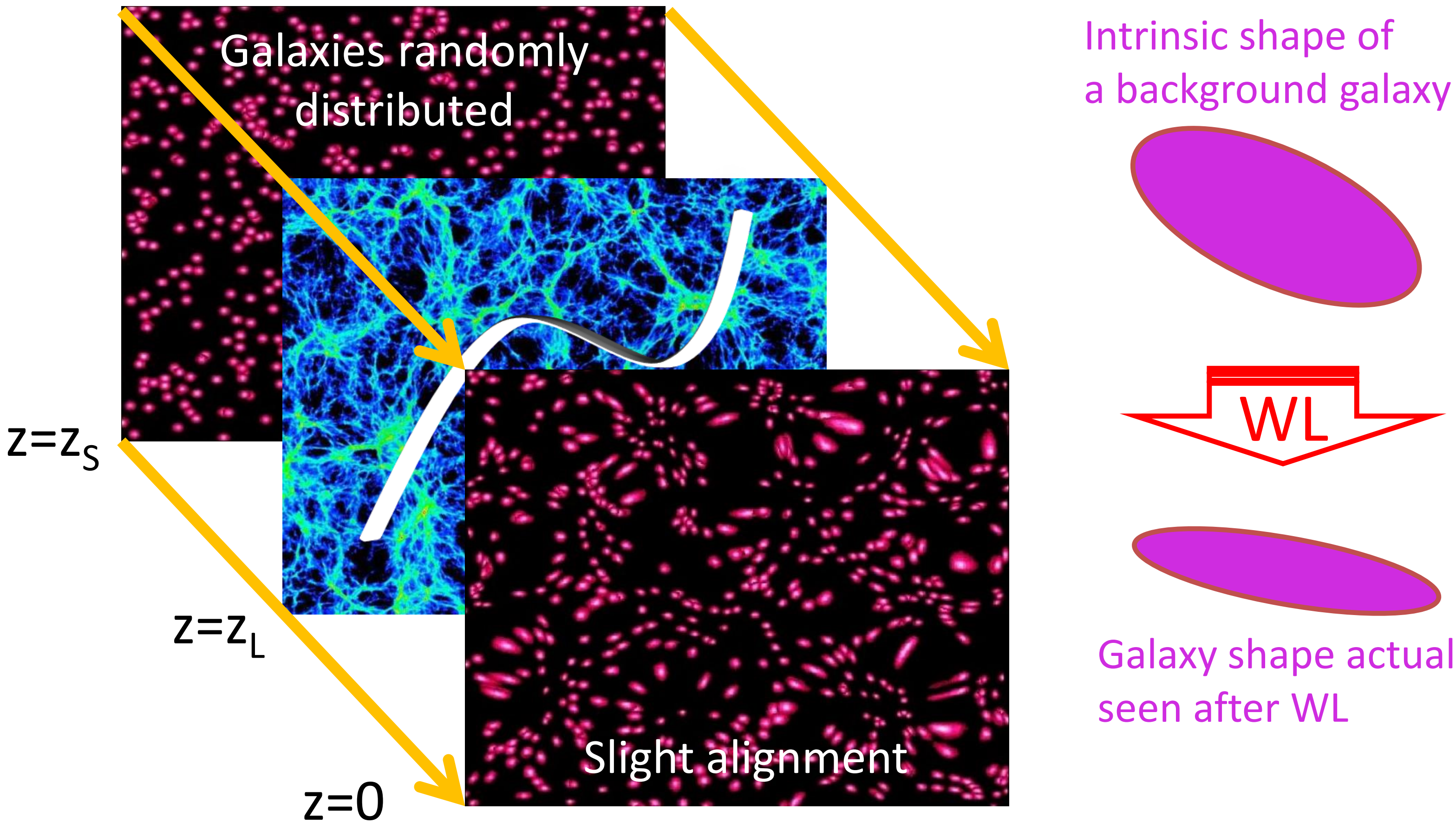
NASA/ESA



WEAK LENSING observations can provide a direct evidence for the intervening ``VECTOR/TENSOR MODES'' along a line of sight by measuring the spatial patterns of the deformation of the photon path.

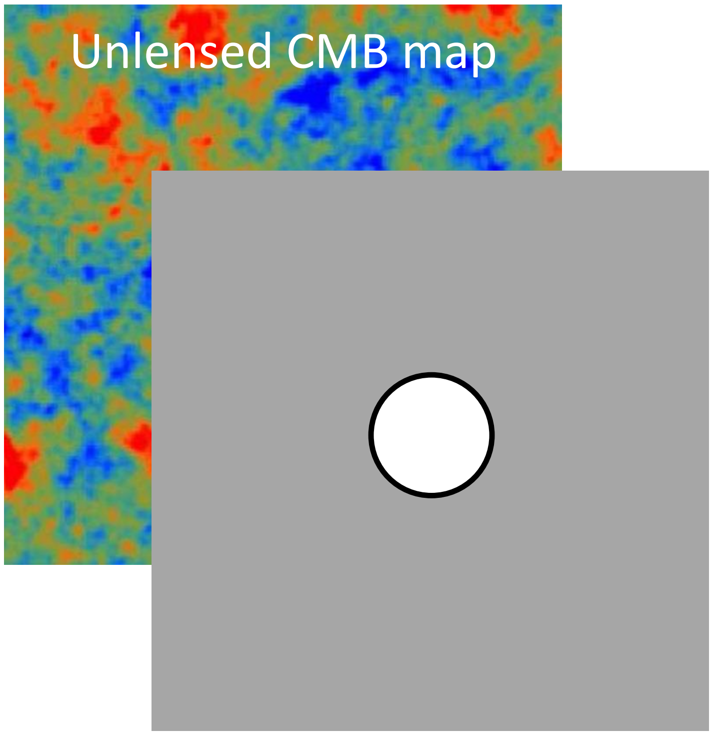
Cosmic shear

What we can measure is the shape of galaxies modified by large scale structure (and exotic objects), which is characterized by the deformation of the two-dim spatial pattern, γ_{ab} (the solution of the geodesic deviation eq.).



Light deflection : CMB lensing

The distortion effect of lensing on the primary CMB anisotropies is expressed by a remapping with two dimensional vector, which can decomposed into gradient- and curl-modes.



$$\boxed{\Delta_a = \phi_{:a}}$$

[Hu+Okamoto(2002)]

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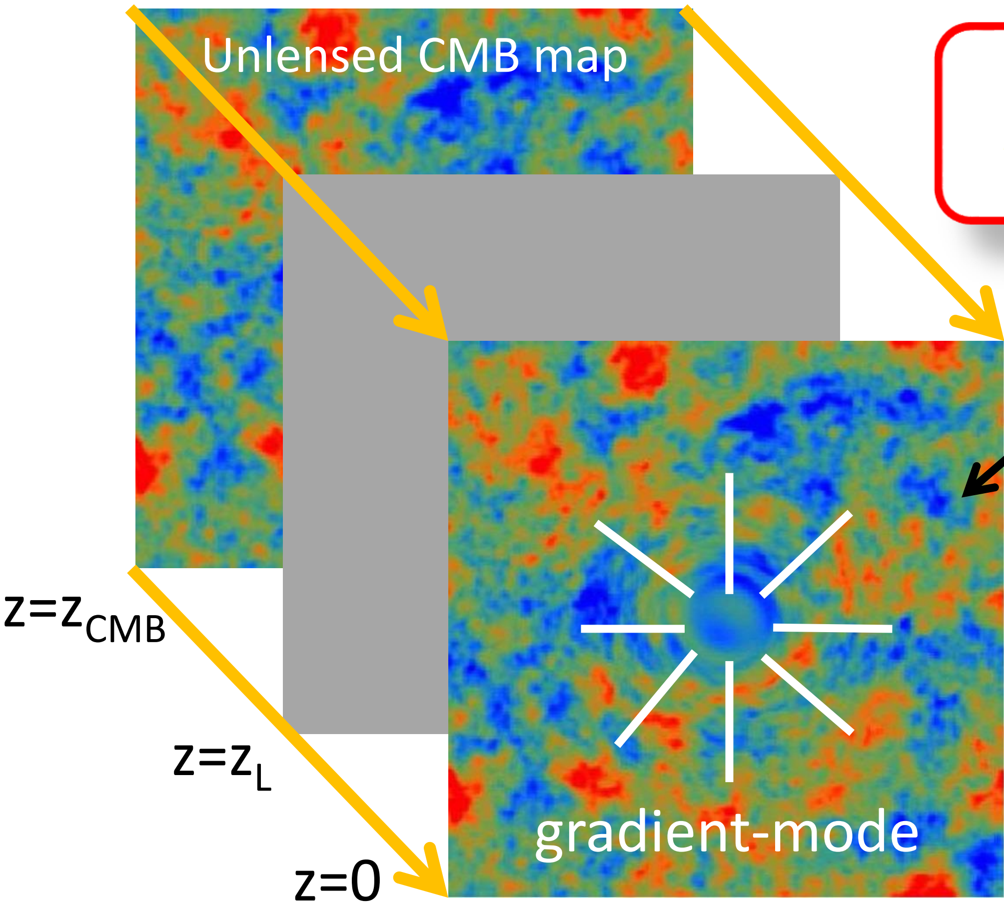


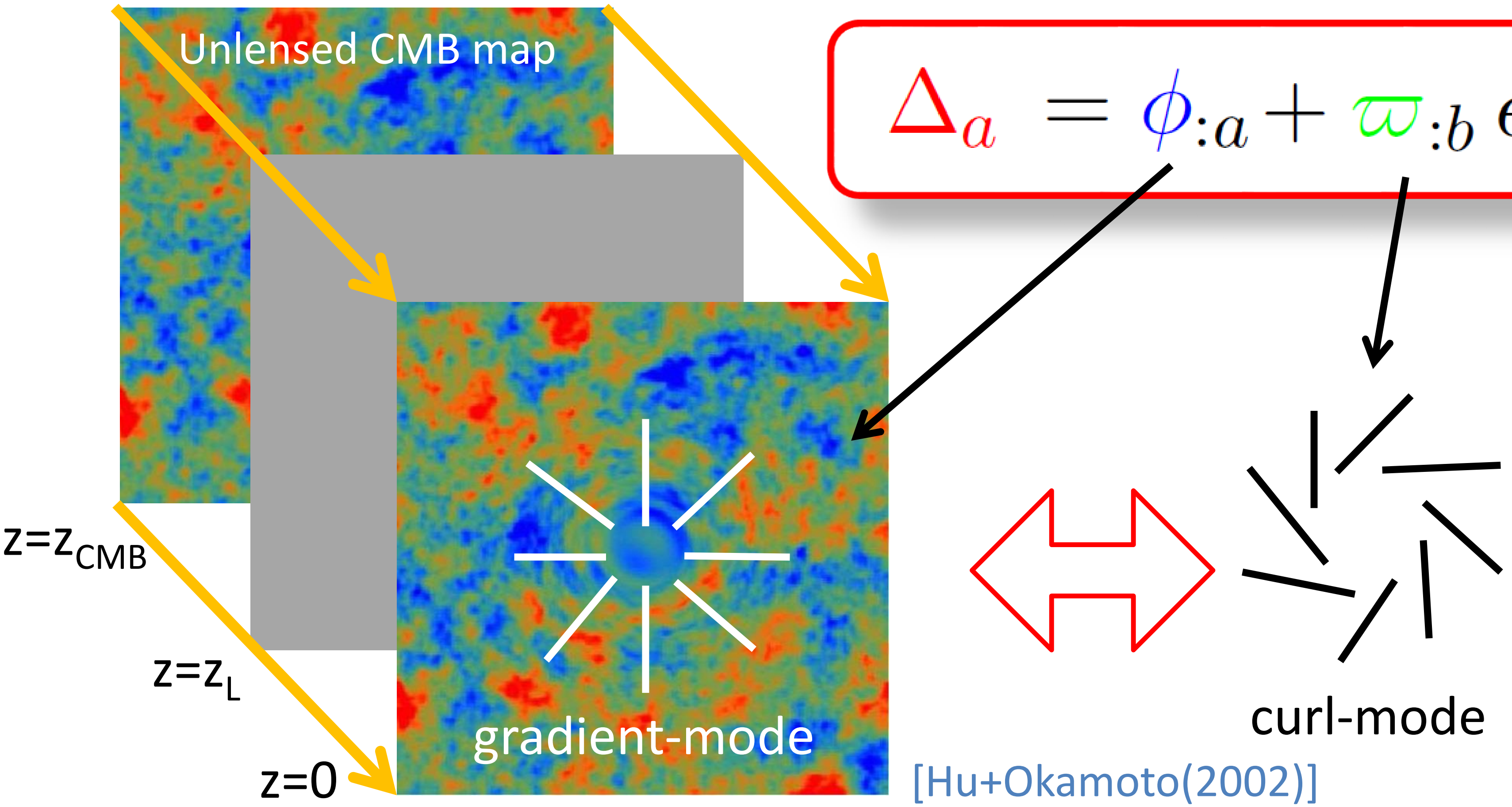
$$\Delta_a = \phi_{:a}$$

[Hu+Okamoto(2002)]

Light deflection : CMB lensing

The distortion effect of lensing on the primary CMB anisotropies is expressed by a remapping with two dimensional vector, which can decomposed into gradient- and curl-modes.

$$\Delta_a = \phi_{:a} + \varpi_{:b}\,\epsilon^{b}{}_{a}$$



[Hu+Okamoto(2002)]

Question

In usual treatment of lensing, the symmetric trace-free part of the gradient of the deflection Δ_a (from geodesic eq.) can be used as a proxy for the shear γ_{ab} (from geodesic deviation eq.).

Is it always true?

$$\gamma_{ab} \stackrel{?}{=} \Delta_{\langle a:b\rangle}$$

NO!

New !

Gauge-inv. shear-deflection relation

Spin-2 contributions

$$\gamma_{ab} = \Delta_{\langle a:b\rangle} + \boxed{\frac{1}{2}\left(h_{\langle ab\rangle}\Big|_{\chi_S} - h_{\langle ab\rangle}\Big|_0\right)}$$

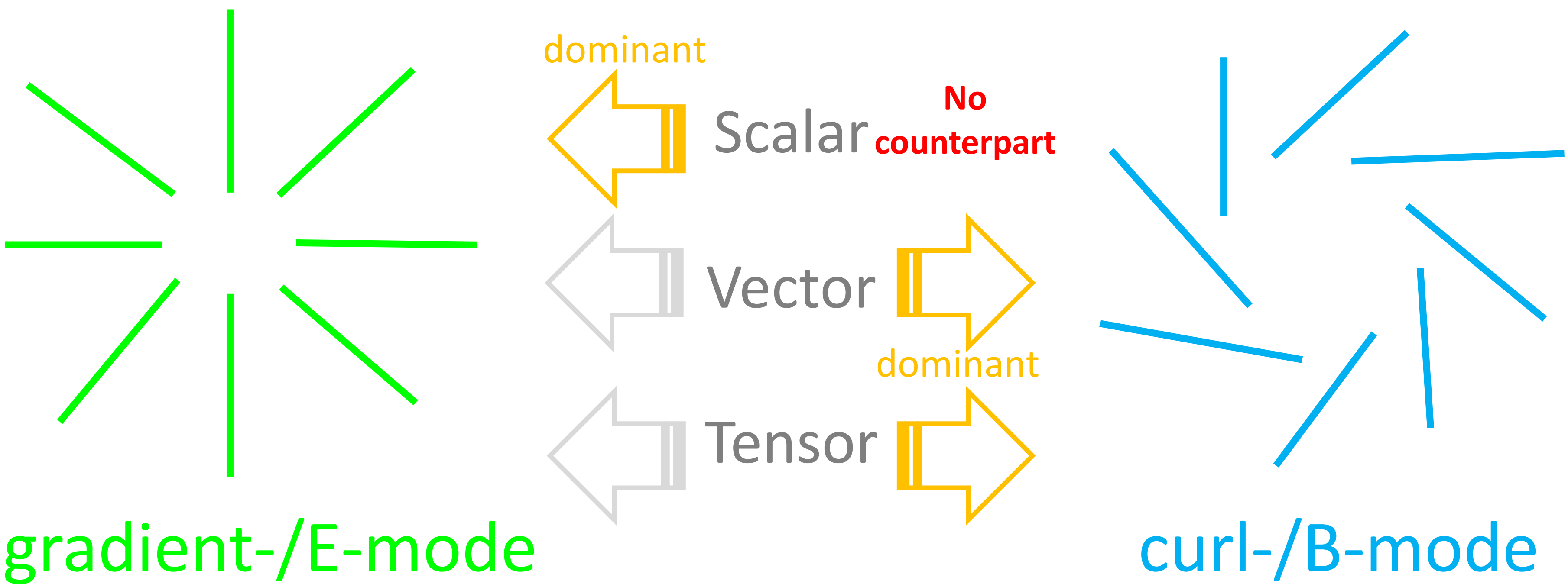
✓ Metric shear [Dodelson+(2003)] / FNC term [Jeong+Schmidt(2012)] (based on the geodesic eq)

- Cosmic shear measurement via galaxy survey are usually referenced to the coordinate in which galaxies are statistically isotropic.
- This is in general different from our reference coordinate (FLRW). The correction from the gravitational potential should appear.

In contrast to the previous studies, the metric shear/FNC term naturally arises in our case from the geodesic deviation eq.

E-/B-mode decomposition in WL

The lensing fields can be decomposed into even-parity part (gradient-/E-mode) and odd-parity part (curl-/B-mode).



The non-vanishing curl- and B-mode shear signal would be a direct evidence for non-scalar metric perturbations.

(See [Dodelson+('03), Schmidt+('12),…] for primordial GW)

Deriving lensing power spectrum,…

One complication is that while the weak lensing observables are defined on a spherical sky, the metric perturbations usually appear in the three-dim space.
Even decomposing the perturbations into the plane waves, they contribute to many multipole due to their angular structure…

E.g.) Bardeen potential

$$\Phi(\boldsymbol{x},\eta)=\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Phi_{\boldsymbol{k}}(\eta)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\sum_{L=0}^{\infty}4\pi(-i)^{L}j_{L}(k\chi){Y_{L}}^{0}(\hat{\boldsymbol{n}})$$

The situation is more complicated for vector/tensor perturbations…

Total angular momentum (TAM)

$$\boxed{{}_sG_\ell{}^m(\boldsymbol{x},\hat{\boldsymbol{n}}) \equiv (-i)^\ell \sqrt{\frac{4\pi}{2\ell+1}}\, {}_sY_\ell{}^m(\hat{\boldsymbol{n}})\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}}$$

[see also Hu+White (2001), Dai+Kamionkowski+Jeong(2012)]

Total angular momentum (TAM)

$$\boxed{{}_sG_\ell{}^m(\boldsymbol{x},\hat{\boldsymbol{n}}) = \sum_{L=0}^{\infty}(-i)^L\sqrt{4\pi(2L+1)}\left({}_s\epsilon_L^{(\ell,m)}(k\chi) + i\mathrm{sgn}(s)\,{}_s\beta_L^{(\ell,m)}(k\chi)\right){}_sY_L{}^m(\hat{\boldsymbol{n}})}$$

[see also Hu+White (2001), Dai+Kamionkowski+Jeong(2012)]

TAM substantially simplifies the derivation of the full-sky formula, and it enables us to simultaneously treat the lensing by vector and tensor modes on an equal footing with those generated by scalar modes.

New !

Lensing power spectrum

The m=0, +-1, +-2 modes corresponds to the scalar, vector, tensor metric perturbations.

$$C_\ell^{\mathrm{XX}'} = \frac{2}{\pi} \sum_{m=-2}^{2} \int_0^\infty \mathrm{d}k\, k^2 \int_0^\infty \mathrm{d}\chi \int_0^\infty \mathrm{d}\chi' \mathcal{S}_{\mathrm{X},\ell}^{(m)}(k,\chi) \mathcal{S}_{\mathrm{X}',\ell}^{(m)}(k,\chi') \mathcal{P}_{|m|}(k,\chi,\chi')$$

Auto-power spectrum for m-mode

- Transfer functions for X and X’

New !

gradient mode (X=ϕ)

$$\mathcal{S}^{(0)}_{\phi,\ell} = 2\frac{\chi_{\rm S}-\chi}{\chi_{\rm S}}\frac{1}{k\chi}\,{}_0\epsilon^{(0,0)}_\ell(k\chi)$$

$$\mathcal{S}^{(\pm1)}_{\phi,\ell} = \frac{1}{k\chi}\left[\frac{\chi_{\rm S}-\chi}{\chi_{\rm S}}\,{}_0\epsilon^{(1,\pm1)}_\ell(k\chi) - \sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(1,\pm1)}_\ell(k\chi)\right]$$

$$\mathcal{S}^{(\pm2)}_{\phi,\ell} = \frac{1}{2k\chi}\left[\frac{\chi_{\rm S}-\chi}{\sqrt{3}\chi_{\rm S}}\,{}_0\epsilon^{(2,\pm2)}_\ell(k\chi) - \sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(2,\pm2)}_\ell(k\chi)\right] + \frac{1}{10\sqrt{3}}\delta_{\ell,2}\delta_{\rm D}(k\chi)$$

Curl mode (X=ω)

$$\mathcal{S}^{(0)}_{\varpi,\ell} = 0$$

Curl-mode is not generated by the m=0 mode (scalar metric perturbations), as is expected.

$$\mathcal{S}^{(\pm1)}_{\varpi,\ell} = -\sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\frac{1}{k\chi}\,{}_1\beta^{(1,\pm1)}_\ell(k\chi)$$

$$\mathcal{S}^{(\pm2)}_{\varpi,\ell} = -\sqrt{\frac{1}{2}\frac{(\ell-1)!}{(\ell+1)!}}\frac{1}{k\chi}\,{}_1\beta^{(2,\pm2)}_\ell(k\chi)$$

New !

E-mode shear

$$\mathcal{S}^{(0)}_{\mathrm{E},\ell}=\sqrt{\frac{(\ell-2)!}{(\ell+2)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\,{}_0\epsilon^{(0,0)}_\ell(k\chi)$$

$$\mathcal{S}^{(\pm1)}_{\mathrm{E},\ell}=\frac{1}{2}\sqrt{\frac{(\ell-2)!}{(\ell+2)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\left[\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}}{}_0\epsilon^{(1,\pm1)}_\ell(k\chi)-\sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(1,\pm1)}_\ell(k\chi)\right]$$

$$\mathcal{S}^{(\pm2)}_{\mathrm{E},\ell}=\frac{1}{4}\sqrt{\frac{(\ell-2)!}{(\ell+2)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\left[\frac{1}{\sqrt{3}}\frac{\chi_\mathrm{S}-\chi}{\chi_\mathrm{S}}{}_0\epsilon^{(2,\pm2)}_\ell(k\chi)-\sqrt{2\frac{(\ell-1)!}{(\ell+1)!}}\,{}_1\epsilon^{(2,\pm2)}_\ell(k\chi)\right]$$
$$+\frac{1}{10\sqrt{2}}\delta_{\ell,2}\delta_\mathrm{D}(k\chi)+\boxed{\frac{1}{2\sqrt{2}}\frac{N(\chi)}{kN_\mathrm{g}}\,{}_2\epsilon^{(2,\pm2)}_\ell(k\chi)}$$

Metric shear/FNC term

B-mode shear

$$\mathcal{S}^{(0)}_{\mathrm{B},\ell}=0$$

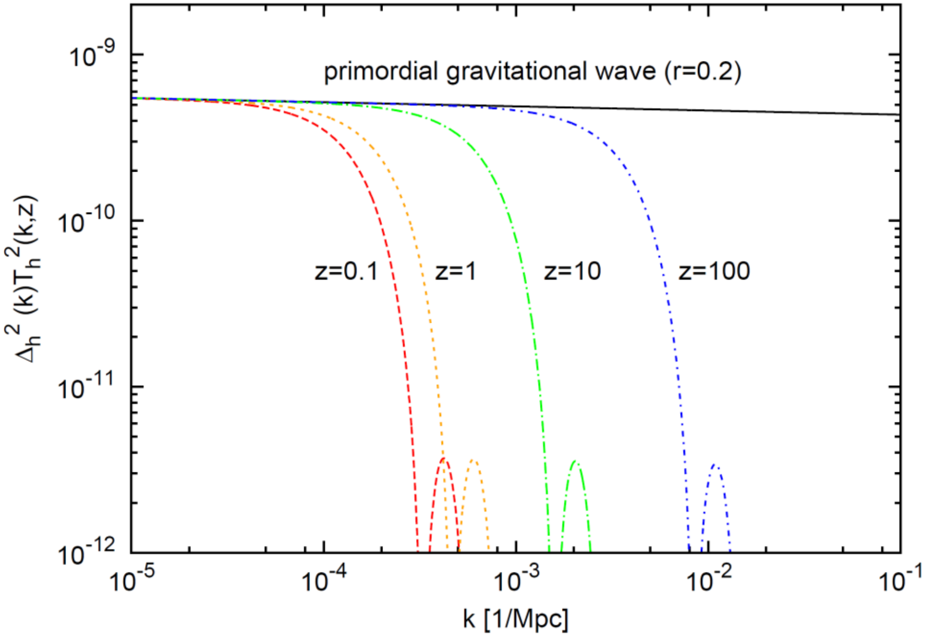
$$\mathcal{S}^{(\pm1)}_{\mathrm{B},\ell}=-\sqrt{\frac{1}{2}\frac{(\ell+2)!(\ell-1)!}{(\ell-2)!(\ell+1)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\,{}_1\beta^{(1,\pm1)}_\ell(k\chi)$$

$$\mathcal{S}^{(\pm2)}_{\mathrm{B},\ell}=-\frac{1}{2}\sqrt{\frac{1}{2}\frac{(\ell+2)!(\ell-1)!}{(\ell-2)!(\ell+1)!}}\frac{1}{k\chi}\int_\chi^\infty \mathrm{d}\chi_\mathrm{S}\frac{N(\chi_\mathrm{S})}{N_\mathrm{g}}\,{}_1\beta^{(2,\pm2)}_\ell(k\chi)+\frac{1}{2\sqrt{2}}\frac{N(\chi)}{kN_\mathrm{g}}\,{}_2\beta^{(2,\pm2)}_\ell(k\chi)$$

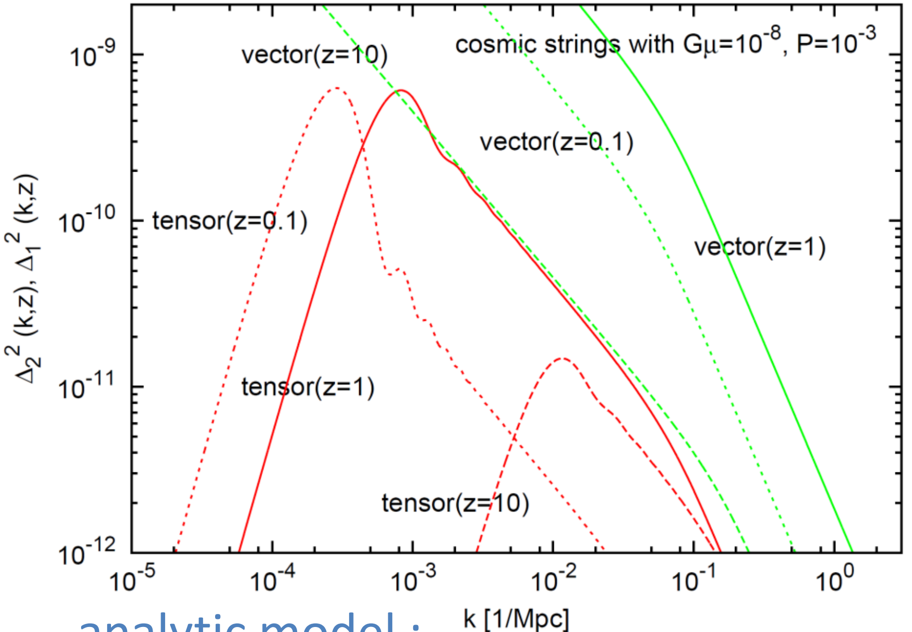
Curl-mode/B-mode

We consider the primordial gravitational waves and the cosmic strings as intriguing examples for vector and tensor perturbations.

passive perturbations
: primordial gravitational wave
(r=0.2)



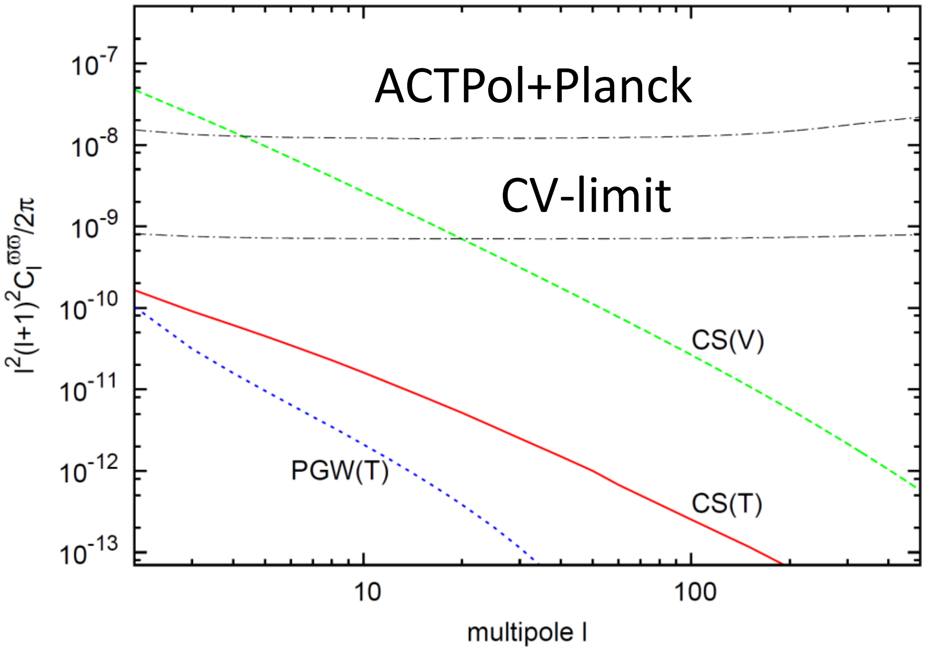
active perturbations
: a cosmic string network
($G\mu=10^{-8}$, $P=10^{-3}$)



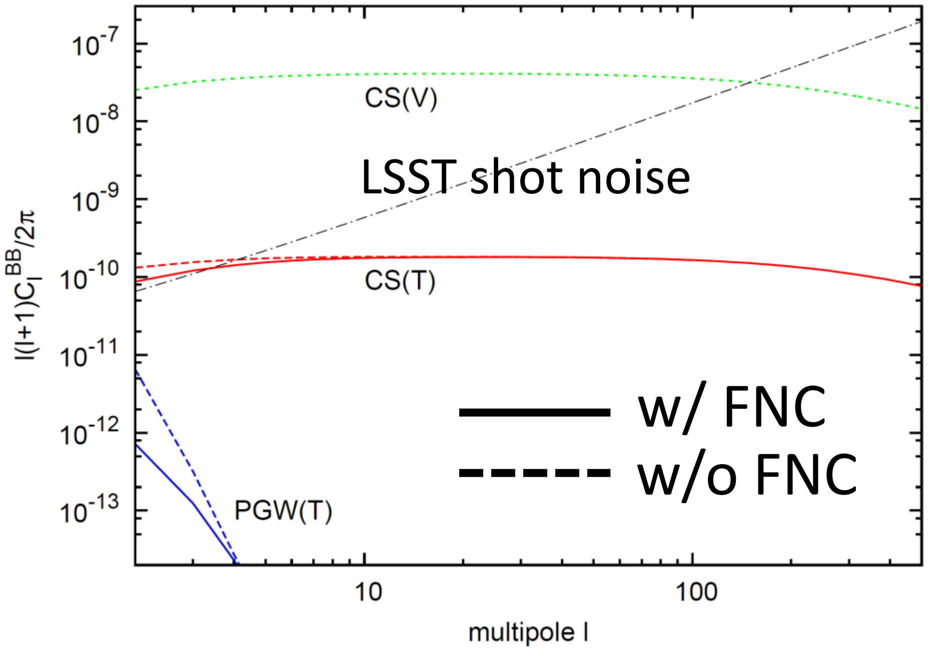
analytic model :
[**DY**+Namikawa+Taruya, 1205.2139]

Curl-mode

Curl-mode estimator and noise
[Namikawa+**DY**+Taruya (2011)]



B-mode

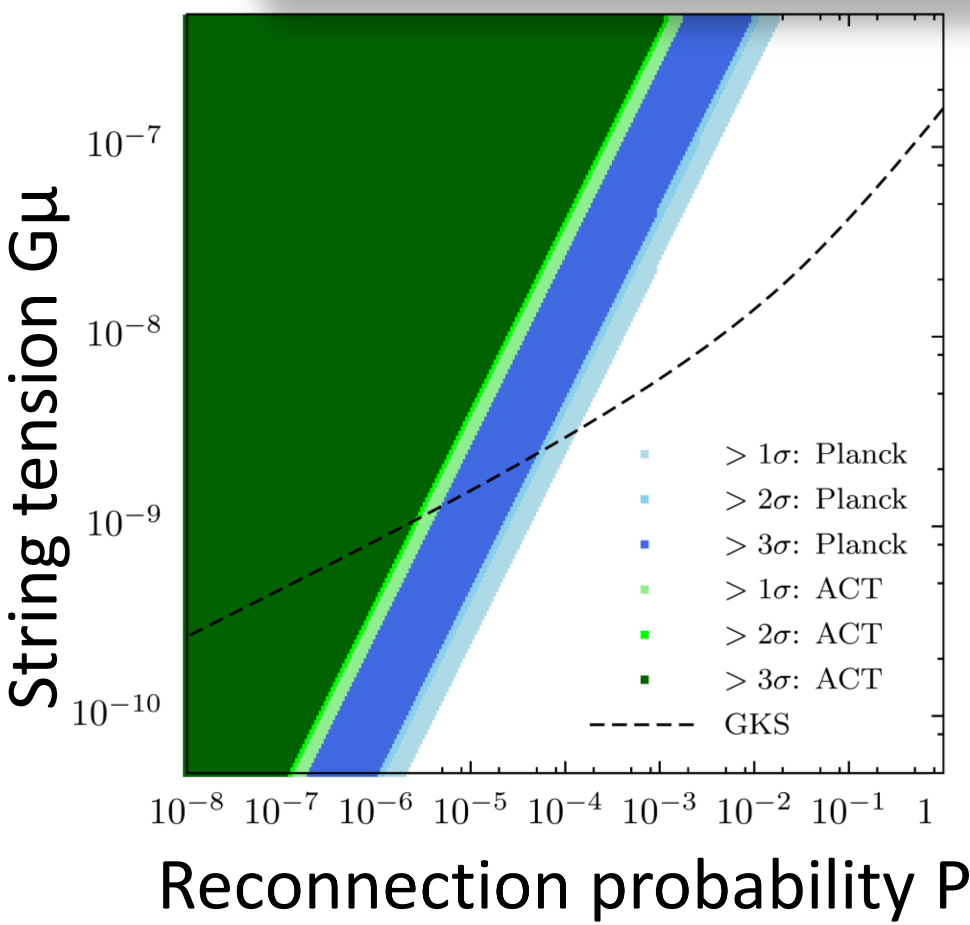


Comparison between the statistical errors and the predictions immediately follows that it is difficult and challenging to detect primordial gravitational waves via the weak lensing measurement, while a cosmic string network is potentially detectable.

Constraint on string parameters from curl mode for ACT and Planck

New !

$$G\mu P^{-1} \leq 3.4 \times 10^{-5}$$ (95%CL, Planck)



Lensing curl mode is more sensitive to small values of reconnection probability P compared to the small scale temperature power spectrum.

[Namikawa+**DY**+Taruya (2013)]

Summary

1. Gauge-invariant deflection-shear relation

$$\boxed{\gamma_{ab} = \Delta_{\langle a:b\rangle} + \frac{1}{2}\left(h_{\langle ab\rangle}\Big|_{\chi_{\rm S}} - h_{\langle ab\rangle}\Big|_{0}\right)}$$

In contrast previous studies, the metric shear/FNC term naturally arises in our case from the geodesic deviation eq.

2. Lensing power spectrum with TAM

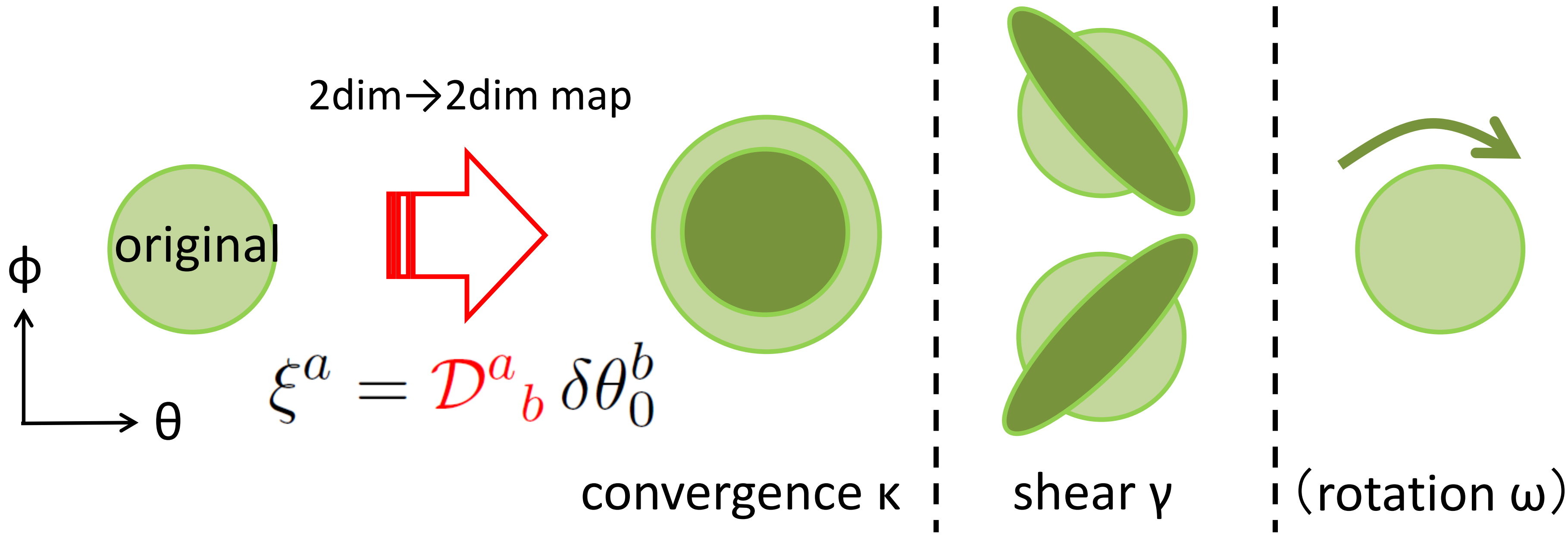
$$\boxed{{}_sG_\ell{}^m(\boldsymbol{x},\hat{\boldsymbol{n}}) = \sum_{L=0}^{\infty}(-i)^L\sqrt{4\pi(2L+1)}\,{}_sj_L^{(\ell,m)}(k\chi)\,{}_sY_L{}^m(\hat{\boldsymbol{n}})}$$

Total angular momentum method substantially simplifies the derivation of the full-sky formula.

Thank you !

APPENDIX

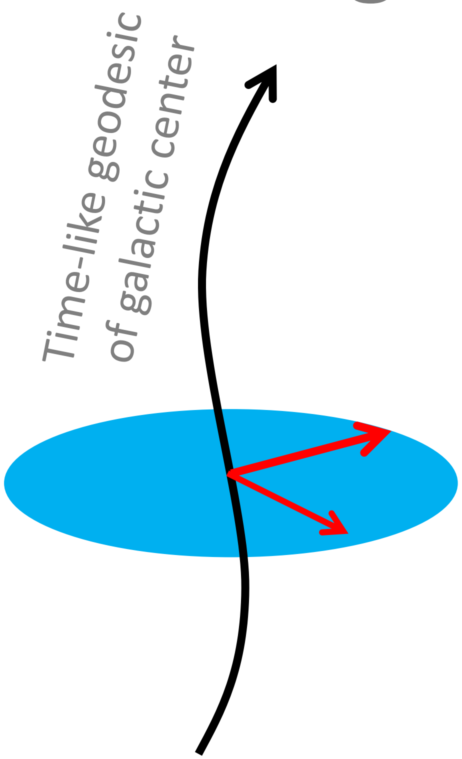
Jacobi map



$$\frac{\mathrm{d}^2}{\mathrm{d}\chi^2}\mathcal{D}^a{}_b = \mathcal{T}^a{}_c\,\mathcal{D}^c{}_b$$

What we can measure is the shape of galaxies modified by lensing, which is characterized by the deformation of the two-dim spatial pattern. Hence we solve the geodesic deviation eq for the shear.

Why does FNC correction appear in geodesic deviation eq?



$$\delta g_{00}^{\mathrm{FNC}} = -R_{0\ell 0m} x^\ell x^m$$

$$\delta g_{0i}^{\mathrm{FNC}} = -\frac{2}{3} R_{0\ell im} x^\ell x^m$$

$$\delta g_{ij}^{\mathrm{FNC}} = -\frac{1}{3} R_{i\ell jm} x^\ell x^m$$

$$\mathcal{T}_{ab}$$

Since the leading correction of the metric in the FNC is known to be described by the Riemann curvature, the FNC contribution is automatically included in the symmetric tidal matrix perturbed around FLRW.