Weak-lensing B-modes as a test of local (an)isotropy

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Cosmological models

Their construction relies on 4 hypothesis

1) Theory of gravity [General relativity]

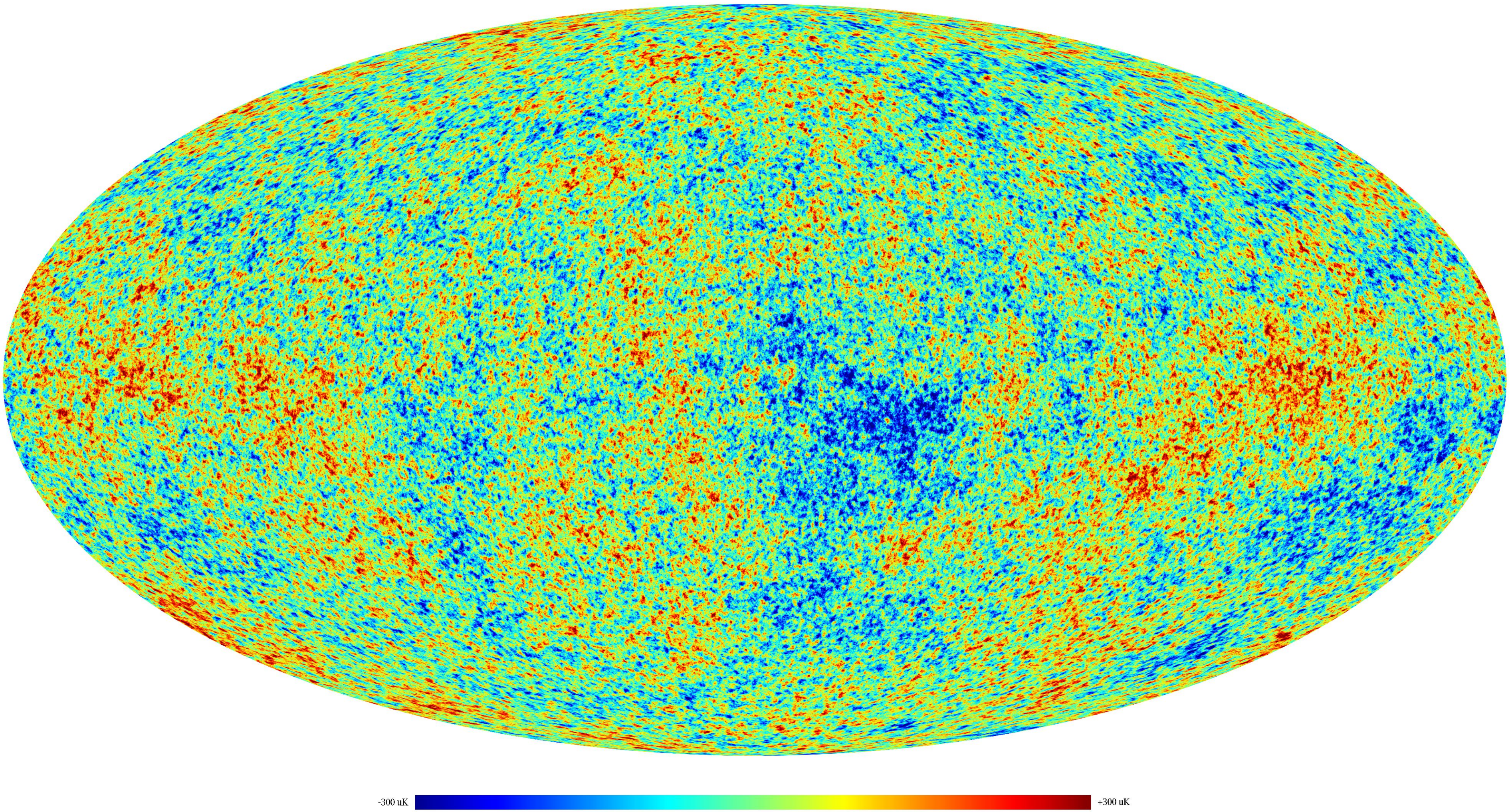
2) Matter [Standard model fields + CDM + Λ]

3) Symmetry hypothesis [Copernican Principle]

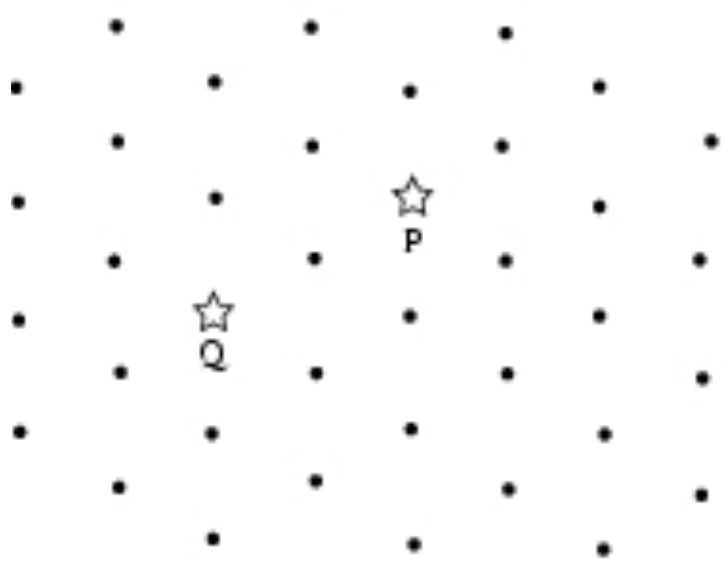
4) Global structure [Topology of space is trivial]

Isotropy: motivation

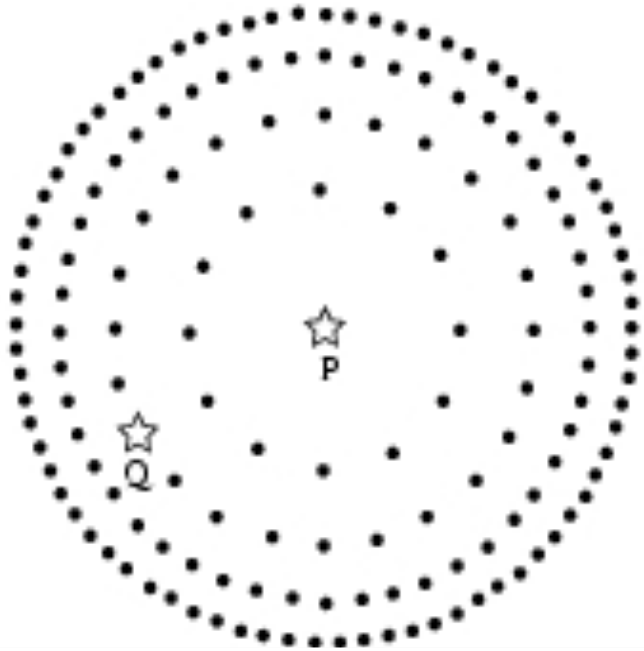
Observationally, the universe seems very isotropic around us.



Two possibilities to achieve this:



Spatially homogeneous & isotropic



Spherically symmetric
Universe has a center

Copernican Principle: we do not occupy a particular spatial location in the universe

Geometrical implication

There exists a priviledged class of observers for which the spatial sections look isotropic and homogeneous.

The spatial sections are constant curvature hypersurfaces.

The spacetime is of the Friedmann-Lemaître type

Consequences:

1- The dynamics of the universe reduces to the one of the scale factor

2- It is dictated by the Friedmann equations

$$3\left(H^2+\frac{K}{a^2}\right)=8\pi G\rho$$

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3P)$$

Several ideas are currently developed to test the Copernican principle.

Testing anisotropy with weak lensing

[CP, Jean-Philippe Uzan, Pereira, arXiv:1203.4069]

Homogeneous (but anisotropic) spacetimes

The solutions of Einstein equations can be classified according to their symmetries. The symmetries are characterized by the set of Killing vectors:

$$\pounds_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0.$$

The Killing vectors satisfy $[\xi_a, \xi_b] = C^c_{ab}\xi_c$:

1) Homogeneity implies three spatial Killing vectors with structure $\boxed{C^i_{\ jk}}$

It can be viewed as an homogeneous but direction dependent curvature

-> *Bianchi spaces classification*

2) We can get different expansion rates for distinct directions.

The difference in expansion is captured by the shear tensor $\boxed{\sigma_{ij} = \gamma'_{ij}}$

Lensing (geodesic deviation)

-We define: k^μ as the tangent vector of the null-geodesic $k^\mu \nabla_\mu k^\nu = 0$

u^μ as the 4-velocity of the observer

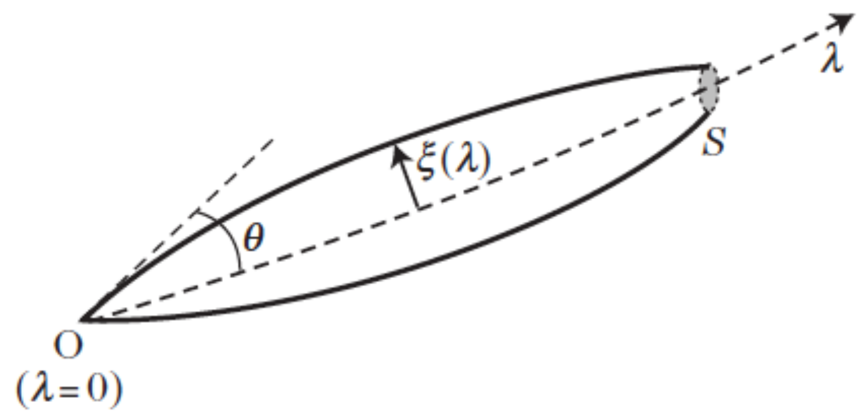
We then introduce n^μ, the spacelike unit vector pointing along the line of sight

$$\hat{k}^\mu \equiv U^{-1} k^\mu = -u^\mu + n^\mu \qquad u^\mu n_\mu = 0\,, \quad n_\mu n^\mu = 1$$

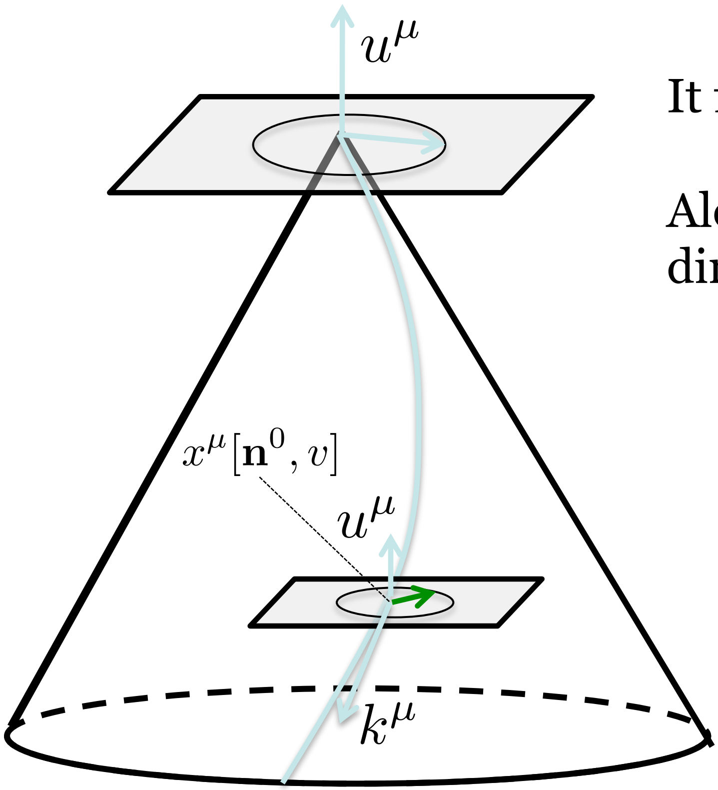
The deviation vector is the taken from a reference geodesic in the bundle

$$x^\mu(\lambda) = \bar{x}^\mu(\lambda) + \xi^\mu(\lambda),$$

$$\frac{D^2}{d\lambda^2}\xi^\mu = R^\mu{}_{\nu\alpha\beta} k^\nu k^\alpha \xi^\beta$$



Screen description



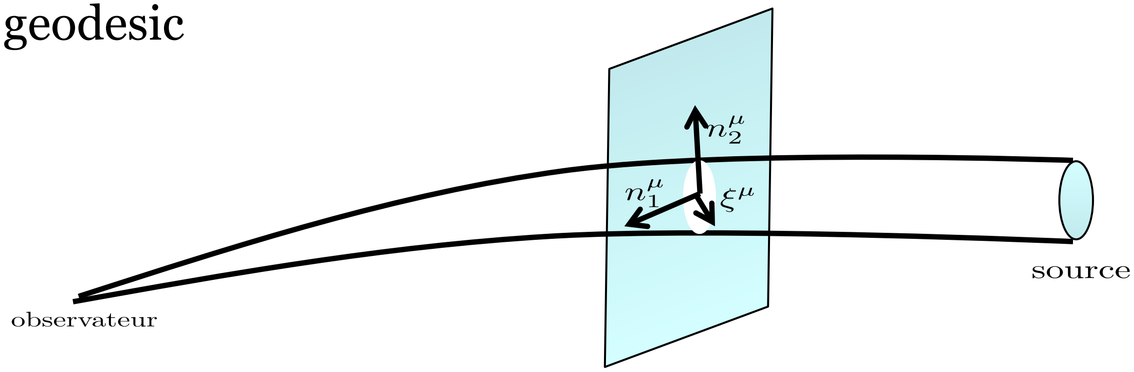
The integration of the geodesic equation gives $x^\mu[\boldsymbol{n}^o, v]$.

It follows that we get $\boldsymbol{n}[\boldsymbol{n}^o, v]$.

Along the geodesic, we can define a basis of the 2-dimensional space orthogonal to $\boldsymbol{n}$.

$$n_a^\mu n_{b\mu} = \delta_{ab}\,, \quad n_a^\mu u_\mu = n_a^\mu n_\mu = 0\,, \quad (a = 1, 2)$$

This basis can be parallely transported along the geodesic



From this basis, we can define the helicity basis by

$$\mathbf{n}_\pm = \frac{1}{\sqrt{2}}(\mathbf{n}_1 \pm i\mathbf{n}_2)$$

Sachs equation

We decompose the connecting vector on the basis of the screen to obtain

$$\frac{\mathrm{d}^2\eta_a}{\mathrm{d}v^2} = \mathcal{R}_{ab}\eta^b \qquad \text{with} \qquad \mathcal{R}_{ab} \equiv R_{\mu\nu\alpha\beta}k^\nu k^\alpha n_a^\mu n_b^\beta$$

The linearity of this equation implies that $\eta^a(v) = \mathcal{D}^a_b(v)\left(\mathrm{d}\eta^b/\mathrm{d}v\right)|_{v=0}$

Sachs equation	Initial condition
$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c\mathcal{D}^c_b$	$\mathcal{D}^a_b(0) = 0\,, \quad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$

Program

Sachs equation

$$\frac{\mathrm{d}^2}{\mathrm{d}v^2}\mathcal{D}^a_b = \mathcal{R}^a_c \mathcal{D}^c_b$$

Initial condition

$$\mathcal{D}^a_b(0) = 0\,, \quad \frac{\mathrm{d}\mathcal{D}^a_b}{\mathrm{d}v}(0) = \delta^a_b$$

1- Express the projected Riemann tensor
in terms of Ricci / E&B of the Weyl

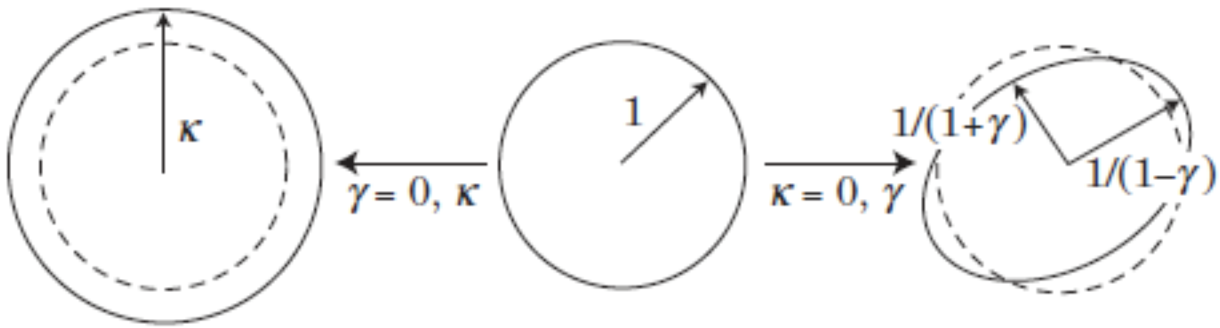
2- Decompose the Jacobi matrix
in terms of a convergence/shear (E&B)/rotation

3- Derive an equation of propagation for these components

4- Perform an harmonic decomposition.

Decomposition of the Jacobi matrix

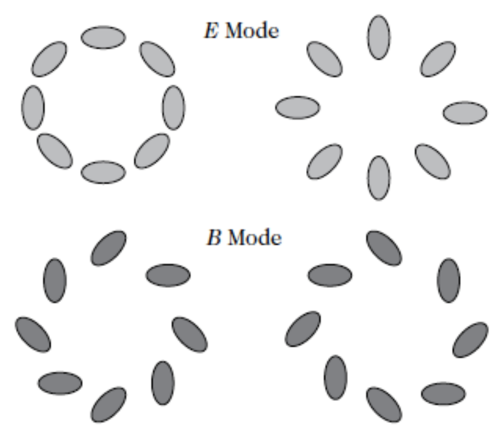
$$\mathcal{D}_{ab} \equiv \kappa I_{ab} + V\epsilon_{ab} + \gamma_{ab} \qquad \epsilon_{ab} = 2\mathrm{i}n^-_{[a}n^+_{b]}$$



The shear can be shown to be a spin-2 and can thus be decomposed as

$$\gamma_{ab}(\boldsymbol{n}^o, v) \equiv \sum_{\lambda=\pm} \gamma^\lambda(\boldsymbol{n}^o, v) n^\lambda_a n^\lambda_b$$

$$\gamma^\pm(\boldsymbol{n}^o, \hat{v}) = \sum_{\ell,m} \left[E_{\ell m}(\hat{v}) \pm \mathrm{i}B_{\ell m}(\hat{v})\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$



Expression of the projected Riemann tensor

The projected Riemann tensor takes the form

$$\mathcal{R}_{ab} = U^2\left(\mathcal{R} I_{ab} + \mathcal{W}_{ab}\right)$$

Ricci

Weyl

$$\mathcal{R} \equiv -\frac{1}{2} R_{\mu\nu}\hat{k}^\mu \hat{k}^\nu\,, \quad \mathcal{W}_{ab} \equiv C_{\mu\rho\sigma\nu}\hat{k}^\rho \hat{k}^\sigma n_a^\mu n_b^\nu$$

The Ricci term is a scalar.

The Weyl term is a spin-2.

$$\mathcal{W}_{ab}(\boldsymbol{n}^o, v) \equiv -2\sum_{\lambda=\pm} \mathcal{W}^\lambda(\boldsymbol{n}^o, v) n_a^\lambda n_b^\lambda .$$

It can be expressed as

$$\mathcal{W}_{ab}(\boldsymbol{n}^o, v) = -2 n^\mu_{\langle a} n^\nu_{b\rangle} \left[\mathcal{E}_{\mu\nu} + \mathcal{B}_\mu{}^\sigma \epsilon_{\sigma\nu}(\boldsymbol{n})\right]\Big|_{\substack{x^\mu(\boldsymbol{n}^o,\, v) \\ \boldsymbol{n}(\boldsymbol{n}^o,\, v)}}$$

with $\quad \mathcal{E}_{\mu\nu} \equiv C_{\mu\rho\nu\sigma} u^\rho u^\sigma\,, \quad \mathcal{B}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\alpha\beta\sigma} u^\sigma C_{\nu\rho}{}^{\alpha\beta} u^\rho$

Propagation of the degrees of freedom

The Sachs equation gives the equation of propagation for the components of the Jacobi matrix

$$\boxed{\left(\frac{\mathrm{d}^2}{\mathrm{d}\hat{v}^2} + H_{\|}\frac{\mathrm{d}}{\mathrm{d}\hat{v}} - \mathcal{R}\right)\begin{pmatrix}\kappa \\ \mathrm{i}V \\ \gamma^{\pm}\end{pmatrix} = -2\begin{pmatrix}\mathcal{W}^{(-}\gamma^{+)} \\ \mathcal{W}^{[-}\gamma^{+]} \\ \mathcal{W}^{\pm}(\kappa \pm \mathrm{i}V)\end{pmatrix}}$$

The integration of this system requires:

- to determine the light-cone structure

  $\boldsymbol{n}^a(\boldsymbol{n}^o,v)$, $H_{//}(\boldsymbol{n}^o,v)$,....

  in this sense, this equation is non-local.

  in general $\boldsymbol{n}(\boldsymbol{n}^o,v) \neq \boldsymbol{n}^o$

- the extraction of the component +/- also depends on $\boldsymbol{n}^a(\boldsymbol{n}^o,v)$

  $\boldsymbol{n}^o$ is the direction of observation.

To go further we insert the multipolar decomposition in this equation.

Multipolar decomposition

	Scalars	Spin-2
Jacobi matrix *[shape of the bundle]*	$\kappa(\boldsymbol{n}^o,\hat v) = \sum_{\ell,m} \kappa_{\ell m}(\hat v) Y_{\ell m}(\boldsymbol{n}^o)$ $V(\boldsymbol{n}^o,\hat v) = \sum_{\ell,m} V_{\ell m}(\hat v) Y_{\ell m}(\boldsymbol{n}^o)$	$\gamma^\pm(\boldsymbol{n}^o,\hat v)$
Source terms *[Properties of the spacetime]*	$\mathcal{R}(\boldsymbol{n}^o,\hat v) = \sum_{\ell,m} \mathcal{R}_{\ell m}(\hat v) Y_{\ell m}(\boldsymbol{n}^o)$ $H_{\|}(\boldsymbol{n}^o,\hat v) = \sum_{\ell,m} h_{\ell m}(\hat v) Y_{\ell m}(\boldsymbol{n}^o)$	$\mathcal{W}^\pm(\boldsymbol{n}^o,\hat v)$

$$\mathcal{W}^\pm(\boldsymbol{n}^o,\hat v) = \sum_{\ell,m} \left[\mathcal{E}_{\ell m}(\hat v) \pm \mathrm{i}\mathcal{B}_{\ell m}(\hat v)\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$

$$\gamma^\pm(\boldsymbol{n}^o,\hat v) = \sum_{\ell,m} \left[\underset{\downarrow}{E_{\ell m}(\hat v)} \pm \mathrm{i}\underset{\downarrow}{B_{\ell m}(\hat v)}\right] Y^{\pm 2}_{\ell m}(\boldsymbol{n}^o)$$

$(-1)^\ell \qquad (-1)^{\ell+1}$

Multipolar hierachy

I skip the details but it involves decomposing products of spherical (spinned) harmonics, hence the C-coefficients.

$$\frac{\mathrm{d}^2 E_{\ell m}}{\mathrm{d}\hat{v}^2} = {}^2C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\Big[\Big(\mathcal{R}_{\ell_1 m_1} - h_{\ell_1 m_1}\frac{\mathrm{d}}{\mathrm{d}\hat{v}}\Big)\left(\delta^+_L E_{\ell_2 m_2} + \mathrm{i}\delta^-_L B_{\ell_2 m_2}\right) - 2\kappa_{\ell_1 m_1}\left(\delta^+_L \mathcal{E}_{\ell_2 m_2} + \mathrm{i}\delta^-_L \mathcal{B}_{\ell_2 m_2}\right)$$
$$+2V_{\ell_1 m_1}\left(-\mathrm{i}\delta^-_L \mathcal{E}_{\ell_2 m_2} + \delta^+_L \mathcal{B}_{\ell_2 m_2}\right)\Big]$$
$$\frac{\mathrm{d}^2 B_{\ell m}}{\mathrm{d}\hat{v}^2} = {}^2C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\Big[\Big(\mathcal{R}_{\ell_1 m_1} - h_{\ell_1 m_1}\frac{\mathrm{d}}{\mathrm{d}\hat{v}}\Big)\left(\delta^+_L B_{\ell_2 m_2} - \mathrm{i}\delta^-_L E_{\ell_2 m_2}\right) - 2\kappa_{\ell_1 m_1}\left(\delta^+_L \mathcal{B}_{\ell_2 m_2} - \mathrm{i}\delta^-_L \mathcal{E}_{\ell_2 m_2}\right)$$
$$-2V_{\ell_1 m_1}\left(\delta^-_L \mathrm{i}\mathcal{B}_{\ell_2 m_2} + \delta^+_L \mathcal{E}_{\ell_2 m_2}\right)\Big]$$
$$\frac{\mathrm{d}^2 \kappa_{\ell m}}{\mathrm{d}\hat{v}^2} = \Big\{{}^0C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\Big(\mathcal{R}_{\ell_1 m_1}\kappa_{\ell_2 m_2} - h_{\ell_1 m_1}\frac{\mathrm{d}\kappa_{\ell_2 m_2}}{\mathrm{d}\hat{v}}\Big)$$
$$-2(-1)^{m_1}\,{}^2C^{-m_2 - m m_1}_{\ell_2\ \ell\ \ell_1}\left[\delta^+_L\left(E_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} + B_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right) + \mathrm{i}\delta^-_L\left(B_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} - E_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right)\right]\Big\}$$
$$\frac{\mathrm{d}^2 V_{\ell m}}{\mathrm{d}\hat{v}^2} = \Big\{{}^0C^{m m_1 m_2}_{\ell \ell_1 \ell_2}\Big(\mathcal{R}_{\ell_1 m_1}V_{\ell_2 m_2} - h_{\ell_1 m_1}\frac{\mathrm{d}V_{\ell_2 m_2}}{\mathrm{d}\hat{v}}\Big)$$
$$+2(-1)^{m_1}\,{}^2C^{-m_2 - m m_1}_{\ell_2\ \ell\ \ell_1}\left[\delta^-_L\mathrm{i}\left(E_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} + B_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right) - \delta^+_L\left(B_{\ell_1 m_1}\mathcal{E}_{\ell_2 m_2} - E_{\ell_1 m_1}\mathcal{B}_{\ell_2 m_2}\right)\right]\Big\}$$

-Hierarchy similar to CMB Boltzmann hierarchy
- does not depend on the choice of any background spacetime [but needs h_{lm} etc..]
- never been derived and generalized the particular FL case
- non-vanishing Weyl implies E/B modes due to coupling to convergence

Linear perturbations in the FL case

We work at linear order in perturbation

3 types of modes: S, V, T. Only S are important at low redshift.

$$\mathcal{R}^{(1)}_{ab} = -D_a D_b(\Phi + \Psi)$$

This implies that the Weyl has no magnetic part: $\mathcal{B}^{(1)}_{ab} = 0$

We can work in the Born approximation: $\boldsymbol{n}(\boldsymbol{n}^o, \hat{v}) = \boldsymbol{n}^o$ so that only $h_{oo} \neq 0$

Shear: E and B modes

$$E^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right) E^{(1)}_{\ell m} - 2\kappa^{(0)}_{00}\,\mathcal{E}^{(1)}_{\ell m}$$

$$B^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right) B^{(1)}_{\ell m}$$

Only E-modes are sourced.

Linear perturbations

Convergence and rotation:

$$\kappa^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right)\kappa^{(1)}_{\ell m} + \mathcal{R}^{(1)}_{\ell m}\kappa^{(0)}_{00}$$

$$V^{(1)}_{\ell m} \to \left(\mathcal{R}^{(0)}_{00} - h^{(0)}_{00}\frac{d}{d\hat{v}}\right)V^{(1)}_{\ell m}$$

Rotation is not sourced.

$$\boxed{\begin{array}{c} V^{(1)}_{\ell m} = B^{(1)}_{\ell m} = 0 \\ \kappa^{(1)}_{\ell m} \neq 0 \ \& \ E^{(1)}_{\ell m} \neq 0 \end{array}}$$

This reproduces the standard lore.

Bianchi I universes (vanishing structure) $C^i_{jk} = 0$

There are major differences with the FL case

1- At background level the Weyl is non vanishing

$$\mathcal{E}^{(0)}_{ij} = H\sigma_{ij} + \frac{1}{3}\sigma^2\gamma_{ij} - \sigma_{ik}\sigma^k_j$$
$$\mathcal{B}^{(0)}_{ij} = 0$$

This implies *l*=2 terms $\mathcal{E}^{(0)}_{2\pm 0} = \sqrt{\frac{\pi}{5}}(\mathcal{E}_{xx} - \mathcal{E}_{yy})$ $\mathcal{E}^{(0)}_{20} = \sqrt{\frac{6\pi}{5}}\mathcal{E}_{zz}$

2- The geodesic structure implies that $\mathbf{n}(\mathbf{n}^0, \hat{v}) \neq \mathbf{n}^0$

One can demonstrate that this sources B-modes for all multipoles for large shear

3- At linear level, everything is possible. Wait and see.

Conclusions

We have demonstrated that a violation of local spatial isotropy implies the existence of non-vanishing B-modes for the cosmic shear.

This is based on a new formalism providing a multipolar hierarchy for weak-lensing independently of the choice of a particular background spacetime.

It recovers the standard lore when the background is FL.

For a pure Bianchi I universe, we have shown that B-modes will be non-vanishing on all multipoles for large shear.

The exact amplitude of the expected B/E-modes requires to work out the complete perturbation theory. This is undergoing (Pitrou/Pereira/Uzan).

Hopefully, it will allow to set strong constraints from the Euclid observations.

This is important for some dark energy model (with anisotropic stress)

Thanks for your attention