Disformal Couplings and Cosmology

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CFP

Based on work with Jack Morrice and Susan Vu; arXiv: 1303. 1773 (v3)

Scalar-Tensor Theories:

- Simple extensions of GR
- Describe corners of other extensions of GR (e.g. extra dimensions (incl. brane worlds), massive gravity, ...,see Danielle Wills talk!)
- Useful for phenomenological approaches to modified gravity

Form considered here is:

$$\mathcal{S} = \int \sqrt{-g} d^4x \left[\frac{M_{\rm Pl}^2}{2} \mathcal{R}(g) - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

$$+ \sum S_i[\tilde{g}^i_{\mu\nu}, \chi_i],$$

with

$$\tilde{g}^{(i)}_{\mu\nu} = C^{(i)}(\phi) g_{\mu\nu} + D^{(i)}(\phi) \phi_{,\mu} \phi_{,\nu}$$

C is called conformal factor, D is the disformal factor. D is not well constrainted by local experiments (Brax (2012), Noller (2012), Brax et al (2012)).

Field equations for only a single coupled field:

Koivisto et al (2012); CvdB & G. Sculthorpe (2012)

$$G_{\mu\nu} = \kappa \left(T^{(\phi)}_{\mu\nu} + T^{\rm matter}_{\mu\nu} \right)$$

$$\Box\phi - \frac{dV}{d\phi} + Q = 0$$

$$\nabla^\mu T_{\mu\nu} = Q \nabla_\nu \phi$$

with

$$Q = \frac{C_{,\phi}}{2C} g^{\mu\nu} T_{\mu\nu} + \frac{D_{,\phi}}{2C} \phi_{,\mu} \phi_{,\nu} T^{\mu\nu} - \nabla_\nu \left[\frac{D}{C} \phi_{,\mu} T^{\mu\nu} \right]$$

FRW , conformal time:

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV}{d\phi} = a^2 \bar{Q}$$

$$a^2\bar{Q} = -\frac{\rho}{2(C + D(\rho - \dot{\phi}/a^2))}\left[a^2\frac{dC}{d\phi}(1-3w) - 2D\left(3\mathcal{H}\dot{\phi}(1+w) + a^2\frac{dV}{d\phi} + \frac{C_{,\phi}}{C}\dot{\phi}^2\right) + D_{,\phi}\dot{\phi}^2\right]$$

Goal: extend this to multiple couplings (radiation + matter)

Extension:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = \sum Q_i \ ,$$

$$\dot{\rho}_i + 3H(\rho_i + p_i) = -Q_i\dot{\phi} \ ,$$

with

$$Q_i = \frac{C_i'}{2C_i}T_i + \frac{D_i'}{2C_i}\phi_{,\mu}\,\phi_{,\nu}\,T_i^{\mu\nu}$$
$$-\nabla_\mu\left(\frac{D_i}{C_i}\phi_\nu T_i^{\mu\nu}\right) \ .$$

FRW spacetime, 2 coupled species:

$$Q_1 = \frac{\mathcal{A}_2}{\mathcal{A}_1\mathcal{A}_2 - D_1 D_2 \rho_1 \rho_2}\left(\mathcal{B}_1 - \frac{\mathcal{B}_2 D_1 \rho_1}{\mathcal{A}_2}\right)$$

$$Q_2 = \frac{\mathcal{A}_1}{\mathcal{A}_1\mathcal{A}_2 - D_1 D_2 \rho_1 \rho_2}\left(\mathcal{B}_2 - \frac{\mathcal{B}_1 D_2 \rho_2}{\mathcal{A}_1}\right) .$$

$$\mathcal{A}_i = C_i + D_i(\rho_i - \dot{\phi}^2) \ ,$$

$$\mathcal{B}_i = \left[\frac{C_i'}{2}\left(-1 + 3\frac{p_i}{\rho_i}\right) - \frac{D_i'}{2}\dot{\phi}^2 \right.$$

$$\left. + D_i\left\{3H\left(1 + \frac{p_i}{\rho_i}\right)\dot{\phi} + V' + \frac{C_i'}{C_i}\dot{\phi}^2\right\}\right]\rho_i \ .$$

Consider radiation and matter coupled.

Goal: study consequences of disformal couplings on radiation ($D_i = M_i^{-4}$)

Relation between energy momentum tensor in different frames:

$$T^{\mu\nu} = \sqrt{1 - \frac{\dot{\phi}^2}{M^4}}\,\tilde{T}^{\mu\nu} \ , \qquad \text{leads to}$$

$$\frac{p}{\rho} = \frac{\tilde{p}}{\tilde{\rho}}\left(1 - \frac{\dot{\phi}^2}{M^4}\right) \ .$$

Equation of state frame dependent!

Another consequence: distribution function is frame-dependent!

$$dN = dx^1 dx^2 dx^3 dP_1 dP_2 dP_3 f \ .$$

Lower case momenta frame independent for conformal transformations, but not for disformal transformations.

The following holds:

$$\frac{d}{dt}\left(\frac{d\tilde{\lambda}}{d\lambda}f\right) = 0 \ . \qquad d\tilde{\lambda}/d\lambda = (1-\dot{\phi}^2/M^4)^{1/2}$$

For photons:

$$\tilde{P}^\mu \tilde{\nabla}_\mu \tilde{P}^\nu = 0^\nu \ , \qquad \tilde{g}_{\mu\nu}\tilde{P}^\mu \tilde{P}^\nu = 0 \ ,$$

Liouville operator:

$$\hat{\mathcal{L}} f = P^0 \frac{\partial f}{\partial t} - H\delta_{ij}P^i P^j \frac{M^4}{M^4-\dot{\phi}^2}\frac{\partial f}{\partial P^0}$$

$$= \frac{\dot{\phi}\ddot{\phi}}{M^4-\dot{\phi}^2} P^0 f \ .$$

We found the same equation for energy density as from the action. Consistent kinetic picture! Can derive equation for photon number density:

$$\dot{n} + 3Hn = -\frac{\dot{F}}{2F}n, \qquad F = (d\tilde{\lambda}/d\lambda)^2$$

Specific entropy not constant (Lima et al (2000)):

$$nTd\sigma = d\rho - \frac{\rho + p}{n}dn \qquad \dot{\sigma} = \frac{\Psi}{nT\left(\frac{\partial\rho}{\partial T}\right)_n}\left[\rho + p - \frac{nC_x}{\Psi}\right]$$

$$C_x = -Q_\gamma\dot{\phi} \qquad \Psi = -\frac{\dot{F}}{2F}n$$

Going back to two species coupled:

Measured speed of light (natural units):

$$c^2_{\rm obs} = 1 - \left(\frac{1}{M^4_\gamma} - \frac{1}{M^4_{\rm m}}\right)\left(\frac{d\phi}{d\tau_{\rm m}}\right)^2$$

written in Einstein frame quantities:

$$c^2_{\rm obs} = \frac{1 - \frac{\dot\phi^2}{M^4_\gamma}}{1 - \frac{\dot\phi^2}{M^4_{\rm m}}}$$

Distribution function obeys:

$$\frac{d}{dt}\left(\frac{d\tilde{\lambda}}{d\lambda}f\right) = 0 \ .$$

Therefore

$$\left[\frac{d\tilde{\lambda}}{d\lambda}f\right]_{\text{initial}} = \left[\frac{d\tilde{\lambda}}{d\lambda}f\right]_{\text{final}}$$

In our work we assume scalar field is dark energy (exponential potential), so field rolls only very late.

$$\Rightarrow f_{\text{final}} = \left[\frac{d\tilde{\lambda}}{d\lambda}\right]^{-1}_{\text{final}} f_{\text{initial}}$$

Write final distribution function in the form

$$f_{\rm final} = \frac{1}{(\exp(\nu/T + \mu) - 1)}$$

For small μ, we find (see also Brax et al 1306.4168) for similar work)

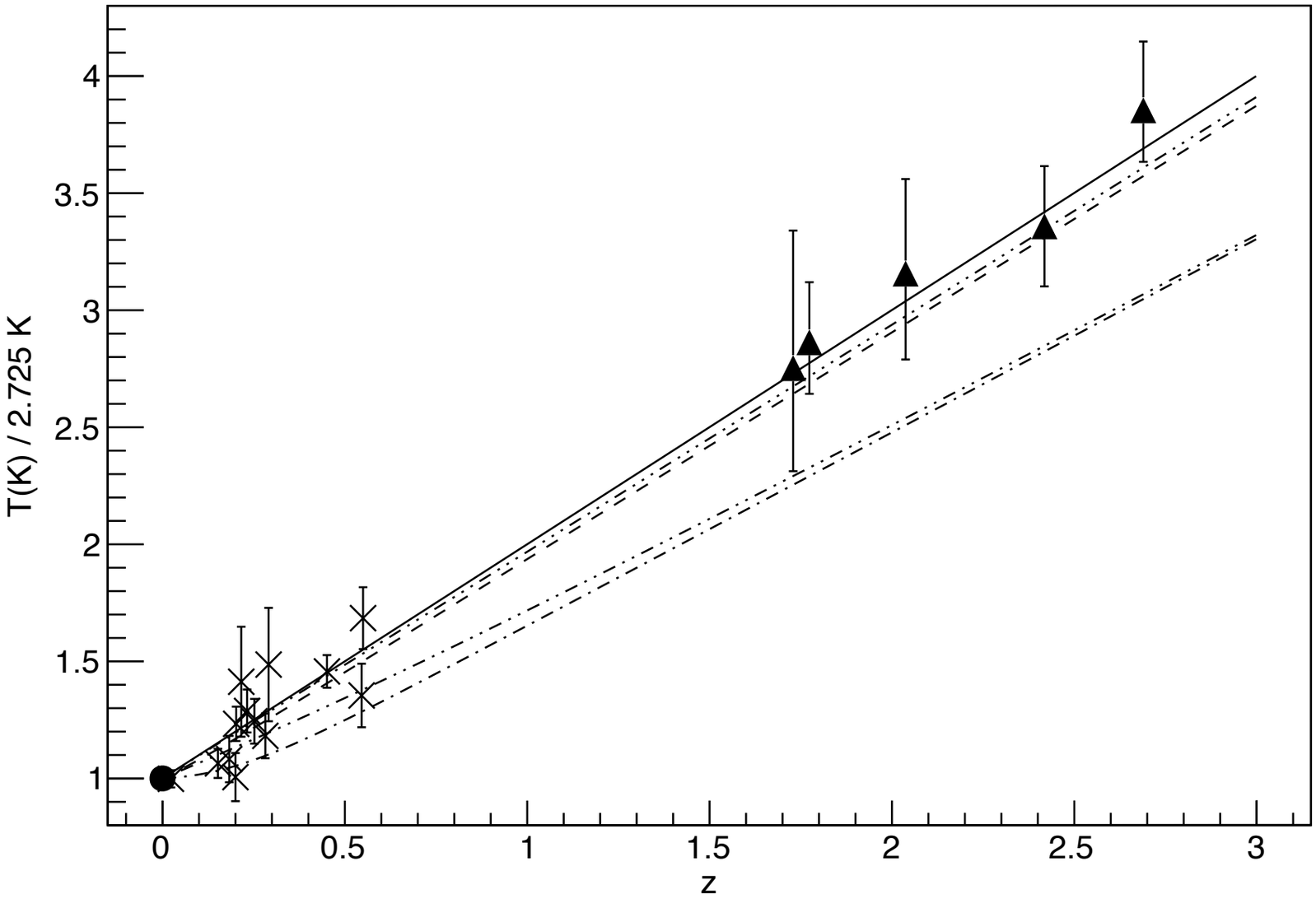
$$\mu = (c_{\rm obs} - 1)\,(1 - e^{-\nu/T})$$

Also, define an effective temperature

$\rho_\gamma^{\rm J} \propto T^4$ (Jordan frame)

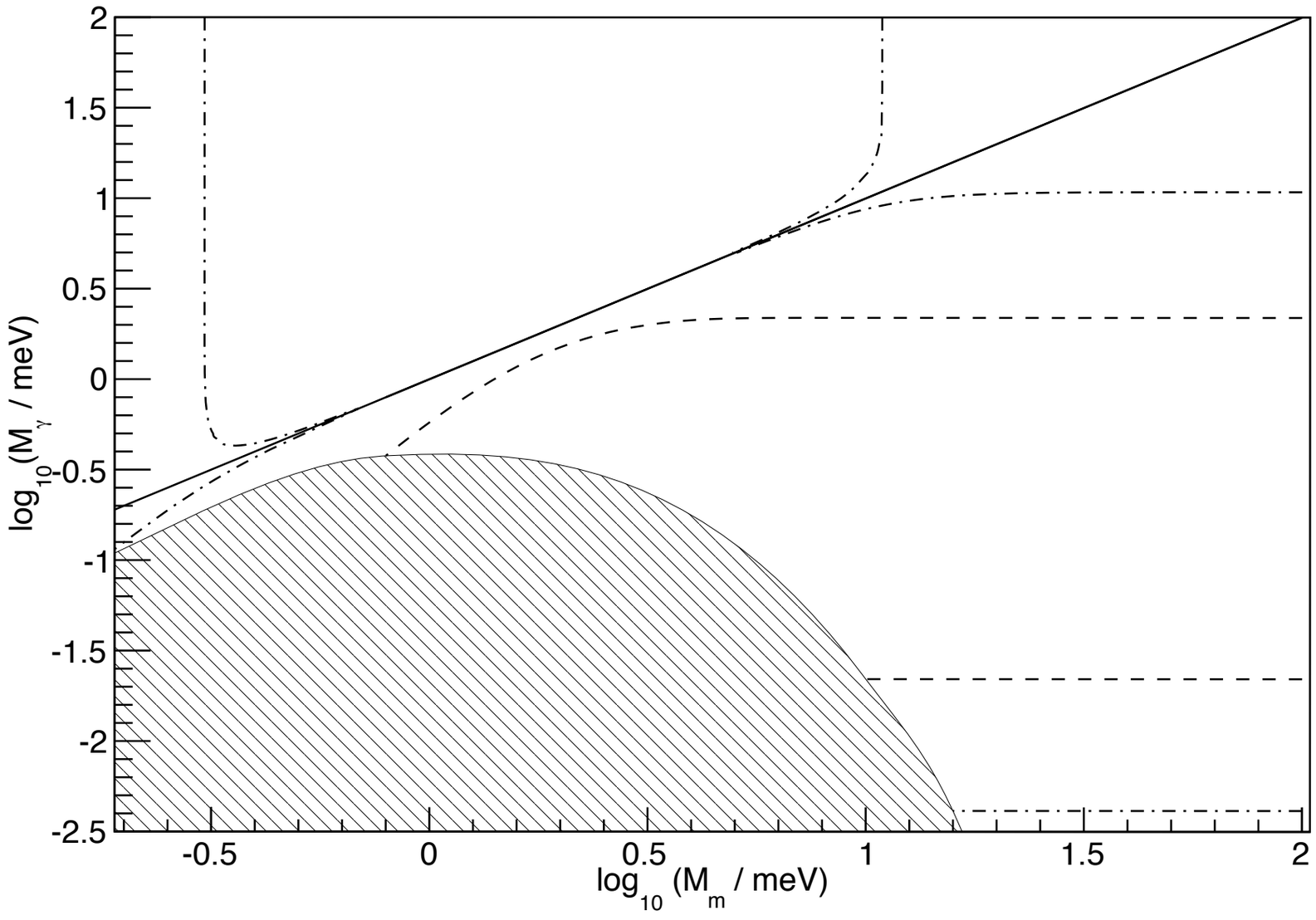
Temperature does not obey $T \propto (1 + z)$

Temperature evolution:



$$V(\phi) = V_0 e^{-\phi}$$

Exclusion region:



$$V(\phi) = V_0 e^{-\phi}$$

Summary: Disformal couplings introduce new phenomenological signatures, in particular on radiation. If $M_\gamma \neq M_{\rm m}$

- Measured equation of state is not 1/3, speed of light not constant and/or 1.
- CMB temperature is not linear in $1+z$ in general.
- Spectrum does not stay Planckian if scalar evolves.

Effects vanish if $M_\gamma = M_{\rm m}$