Trans-Planckian Considerations in Inflationary Cosmology

Or: *Can unknown high energy physics alter predictions from inflationary cosmology?*

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Quantum Fluctuations in de Sitter Space I

Recall the standard picture

$$\hat{\phi}(\tau, \mathbf{x}) = \bar{\phi}(\tau) + \underbrace{\delta\hat{\phi}(\tau, \mathbf{x})}_{Fluctuation}$$

Introducing a rescaled field $\hat{f} = a(\tau)\hat{\phi}$, it may be shown that

$$f_k'' + \left(k^2 - \frac{z''}{z}\right) f_k = 0,$$

where $z = a\bar{\phi}'/\mathcal{H}$ with $\mathcal{H} = aH$. For slow-roll in de Sitter we have

$$\frac{z''}{z} \approx \frac{a''}{a} = \frac{2}{\tau^2}.$$

Quantum Fluctuations in de Sitter Space II

Hence, recast above as

$$f_k'' + \left(k^2 - \frac{2}{\tau^2}\right) f_k = 0.$$

We need to define a vacuum $a_k|0\rangle = 0$. Idea: Go to the infinite past and to arbitrarily short distances to let space resemble Minkowski space, i.e.

$$f_k'' + k^2 f_k = 0.$$

Thus have the solution

$$f_k(\tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}.$$

Quantum Fluctuations in de Sitter Space III

The general solution is

$$f_k(\tau) = A_k \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + B_k \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right)$$

Combine initial condition and general solution to yield unique mode function

$$f_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right).$$

The Power Spectrum

The vacuum fluctuations of the field $\delta\hat{\phi} = a^{-1}\hat{f}$ may be calculated as

$$\begin{aligned}\langle 0|\hat{\phi}_k^\dagger \hat{\phi}_{k'}|0\rangle &= \delta(k-k')a^{-2}|f_k|^2 \\ &= \frac{H^2}{2k^3}(1+k^2\tau^2) \\ &\to \frac{H^2}{2k^3},\end{aligned}$$

or

$$P_\phi = \frac{k^3}{2\pi^2}\langle 0|\hat{\phi}_k^\dagger \hat{\phi}_{k'}|0\rangle = \left(\frac{H}{2\pi}\right)^2.$$

Approach 1: Modified Dispersion Relations I

Recall the standard setting

$$f_k'' + \underbrace{\left(k^2 - \frac{z''}{z}\right)}_{=\omega^2} f_k = 0.$$

Replace $k \to aF(k,\tau)$ where $aF \to k$ for $k \ll k_c$ for some cutoff scale k_c. We thus have

$$f_k'' + \left(a^2F^2 - \frac{z''}{z}\right) f_k = 0.$$

Approach 1: Modified Dispersion Relations II

Consider WKB solution

$$f_k^{WKB}(\tau) = \frac{1}{\sqrt{2\omega(\tau)}} e^{-i\int_{\tau_i}^{\tau_f} \omega' d\tau'},$$

where the exact solution may be cast as

$$f_k(\tau) = A_k(\tau) f^{WKB}(\tau) + B_k(\tau) \left(f^{WKB}(\tau)\right)^* .$$

For our initial condition, $B_k(\tau) = 0$ unless ω **violates adiabaticity**, i.e. $\partial_\tau \omega/\omega^2 = O(1)$. A violation yields

$$\begin{aligned} P_\phi &\propto k^3 |f_k|^2 \\ &= \underbrace{|f_k^{(0)}|^2}_{B_k \to 0} [|A_k|^2 + |B_k|^2 + A_k^* B_k + B_k^* A_k]. \end{aligned}$$

Approach 2: New Physics Hypersurface I

Idea: Introduce timelike hypersurface, i.e. physical cut-off scale Λ, where mode evolution starts. As comoving and physical momentum are related via $k = ap$, mode evolution starts once $k = a\Lambda$. In de Sitter, $a = -(H\tau)^{-1}$, thus have initial time of mode evolution

$$\tau_i = -\frac{\Lambda}{Hk}.$$

Influence of unknown physics is then encoded in initial conditions. Consider

$$f_k(\tau) = A_k \frac{e^{-ik\tau}}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) + B_k \frac{e^{ik\tau}}{\sqrt{2k}} \left(1 + \frac{i}{k\tau}\right),$$

Approach 2: New Physics Hypersurface II

and find it's conjugate momentum to be

$$g_k(\tau) = A_k \sqrt{\frac{k}{2}} e^{-ik\tau} - B_k \sqrt{\frac{k}{2}} e^{ik\tau}.$$

When fixing the vacuum

$$a_k(\tau_i)|0, \tau_i\rangle = 0$$

and considering $|A_k|^2 - |B_k|^2 = 1$, one then finds

$$B_k = \frac{ie^{-2ik\tau_i}}{2k\tau_i + i} A_k.$$

Approach 2: New Physics Hypersurface III

Computing the power spectrum yields

$$\begin{aligned} P_\phi &= \frac{k^3}{2\pi^2 a^2} \langle 0 | f_k^\dagger f_{k'} | 0 \rangle \\ &\to \underbrace{\left(\frac{H}{2\pi}\right)^2}_{B_k=0} \left(|A_k|^2 + |B_k|^2 + A_k^* B_k + A_k B_k^*\right), \end{aligned}$$

From the above, one can deduce

$$|A_k|^2 = \left(1 - \left|\frac{i}{2k\tau_i + i}\right|^2\right)^{-1}$$

Approach 2: New Physics Hypersurface IV

and

$$|B_k|^2 = \frac{1}{(2k\tau_i)^2} = \frac{H^2}{4\Lambda^2}$$

so that

$$\begin{aligned} P_\phi &= \left(\frac{H}{2\pi}\right)^2 \left[1 + |B_k|^2 \left(2 + \frac{2k\tau_i + i}{i} e^{2ik\tau_i} + \frac{2k\tau_i - i}{-i} e^{-2ik\tau_i}\right)\right] \\ &= \left(\frac{H}{2\pi}\right)^2 \left[1 + \mathcal{O}\left(\frac{H}{\Lambda}\right)\right]. \end{aligned}$$

Approach 2: New Physics Hypersurface V

Finite-duration inflation?

Having considered an initial time

$$\tau_i = -\frac{\Lambda}{Hk}$$

above might suggest to use the formalism and choose τ_i to be the time of the onset of inflation. As spacetime might not be de Sitter before τ_i, access to the infinite past is lost and Bunch-Davies prescription is inconsistent? However above approach **fails** as onset of inflation $\neq$ physical cut-off! So more work required...

Challenges I
Back Reaction

When considering a non-Bunch-Davies state, additional energy density $\langle \rho \rangle = \langle 0 | T^0_0 | 0 \rangle$ is present, which must not be bigger than that of the background. One finds

$$\langle \rho(\tau) \rangle = \frac{1}{4\pi^2} \int_H^C dp |B_k|^2 \left(p^3 + \frac{1}{2} H^2 p \right).$$

For a physical cut-off $C = \Lambda$, one thus requires

$$|B_k|^2 < \frac{M_{pl}^2 H^2}{\Lambda^4}.$$

Challenges II

Cut-off and Mode Creation

However, a physical cut-off requires a **mode creation** mechanism for the finite excitations not to inflate away. Compare with comoving cut-off $C = a^{-1}\Lambda$ so that

$$\langle \rho(\tau) \rangle \propto B_k \frac{\Lambda}{a^4},$$

which inflates away exponentially.

Interlude

Due to a reparametrisation, this initial state is called α-vacuum

$$A_k = \frac{1}{\sqrt{1 - e^{\alpha+\alpha^*}}}$$
$$B_k = \frac{e^{\alpha}}{\sqrt{1 - e^{\alpha+\alpha^*}}},$$

where $\alpha \to -\infty$ corresponds to the Bunch-Davies state.

Challenges III

"Non-Thermality"

Let us couple a detector to the inflaton field

$$ic\langle E_1, \phi_1| \int m(\tau)\phi[x(\tau)]d\tau |0, E_0\rangle$$

For the Bunch-Davies vacuum one finds

$$\mathcal{R}_{ij} \to \frac{\rho(E_j)}{\rho(E_i)} = e^{-2\pi\delta E},$$

yet for an α-vacuum one has

$$\mathcal{R}_{ij} \to e^{2\alpha},$$

i.e. it is non-thermal!

Challenges IV
Regularisation

The below expression is divergent

$$\langle \rho \rangle = \frac{1}{2(2\pi)^3 a^4} \int d^3k \cdot k \left[\left(|A_k|^2 + |B_k|^2 \right) \left(1 + \frac{1}{2(k\tau)^2} \right) \right],$$

regularising the α-contribution does not regularise the Bunch-Davies part, whereas for the usual Bunch-Davies vacuum, any regulator regulates flat space as well.

Challenges V

And more...

Locality is violated (arXiv:hep-th/0209113, arXiv:hep-th/0209159, arXiv:hep-th/0209231)!

Approach 3: EFT

Consider e.g. the following action (arXiv:hep-th/0201158)

$$S_{eff}[\phi] = \int d^4p\phi(p)\phi(-p)[\frac{p^2}{2} + \frac{H^2}{2} + c_0 H^2 \left(\frac{H^2}{\Lambda^2}\right) + c_1 p^2 \left(\frac{H^2}{\Lambda^2}\right) + c_2 \left(\frac{p^4}{\Lambda^2}\right) + c_3 \left(\frac{p^4}{\Lambda^2}\right)\left(\frac{H^2}{\Lambda^2}\right) + c_4 \frac{p^6}{\Lambda^4} + ...].$$

which then leads to

$$\langle\phi(p)\phi(-p)\rangle|_{p=H} = H^2(1 + c_0 \left(\frac{H^2}{\Lambda^2}\right) + c_1 \left(\frac{H^2}{\Lambda^2}\right) + c_2 \left(\frac{H^2}{\Lambda^2}\right) + c_3 \left(\frac{H^2}{\Lambda^2}\right)^2 + c_4 \left(\frac{H^2}{\Lambda^2}\right)^2 + ...).$$

Conclusions

- Modifying Dispersion relation shows that in principle modifications of the power spectrum are possible, yet specific form hard to motivate
- Introducing a physical cut-off scale looks promising at first but brings with it severe challenges
- In the end, EFT based approaches give corrections quadratic in H/Λ
- Bunch-Davies prescription valid even if spacetime might not be de Sitter before some finite τ_i?

Thank you very much for your attention!