

Outline

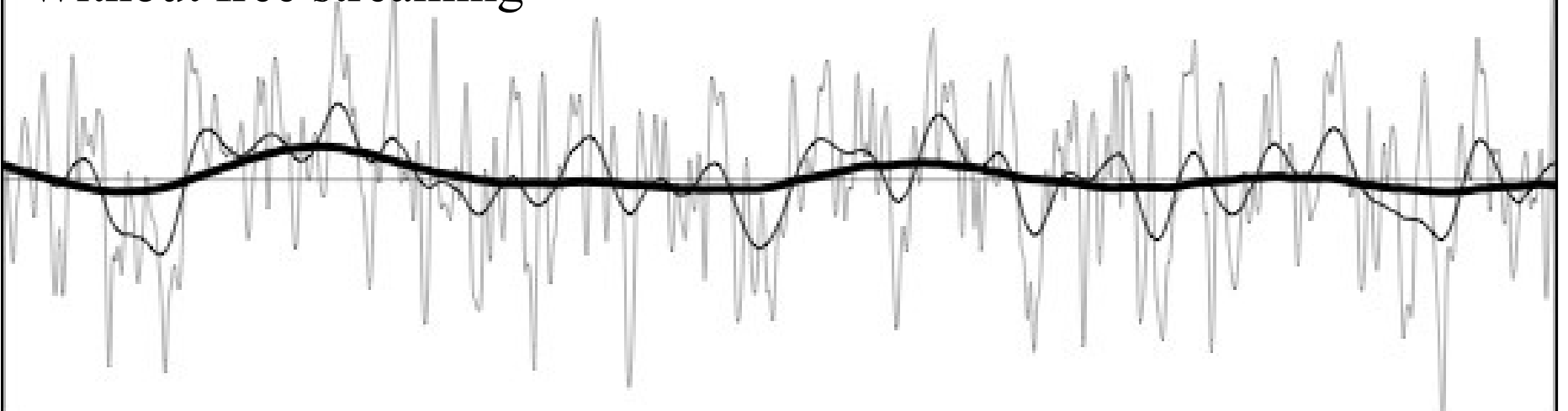
Structure Formation and Free Streaming

**Modeling the Halo Mass Function
with WDM**

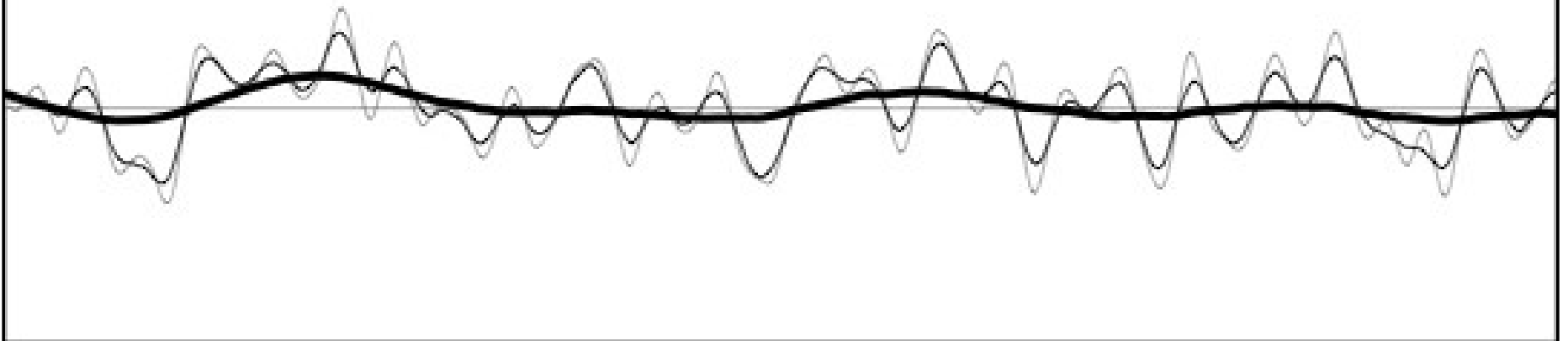
Predictions for CDM

Structure formation: linear density field

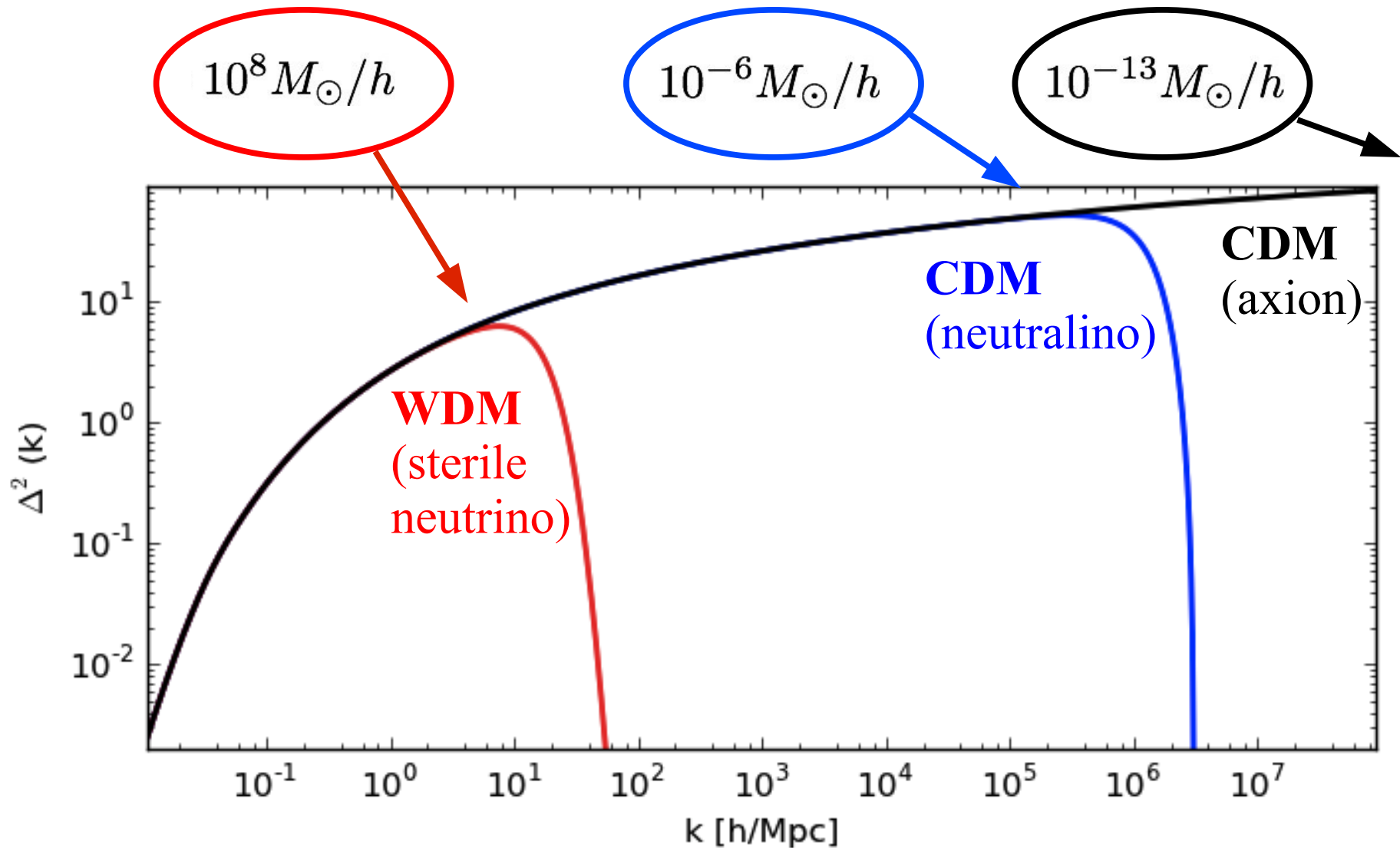
Without free streaming



With free streaming



Structure formation: linear density field



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Structure Formation and Free Streaming

**Modeling the Halo Mass Function
with WDM**

Predictions for CDM

Mass Function: Sheth-Tormen model

$$\frac{dn}{d \log M} = -\frac{1}{2} \frac{\bar{\rho}}{M} f(\nu) \frac{d \log \sigma^2}{d \log M} \quad \nu = \frac{\delta_c^2}{\sigma^2(M)}$$

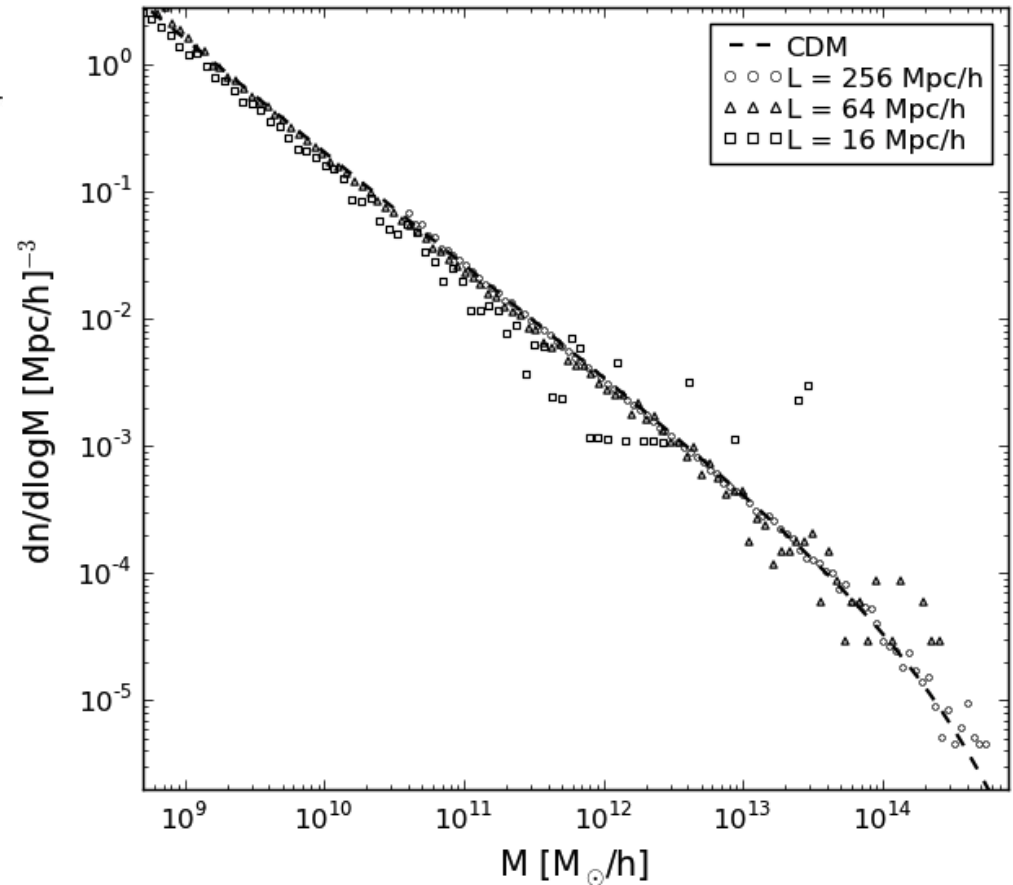
$$\sigma^2(R) = \int \frac{d\mathbf{k}^3}{(2\pi)^3} P_{\text{Lin}}(k) W_{\text{TH}}^2(kR)$$

$$W_{\text{TH}} = 3 [\sin y - y \cos y] / y$$

$$f(\nu) = A \sqrt{\frac{2q\nu}{\pi}} [1 + (q\nu)^{-p}] e^{-q\nu/2},$$

$$A = 0.322, \quad p = 0.3, \quad q = 0.707.$$

$$M = \frac{4\pi}{3} \bar{\rho} R^3$$



Mass Function: Sheth-Tormen model

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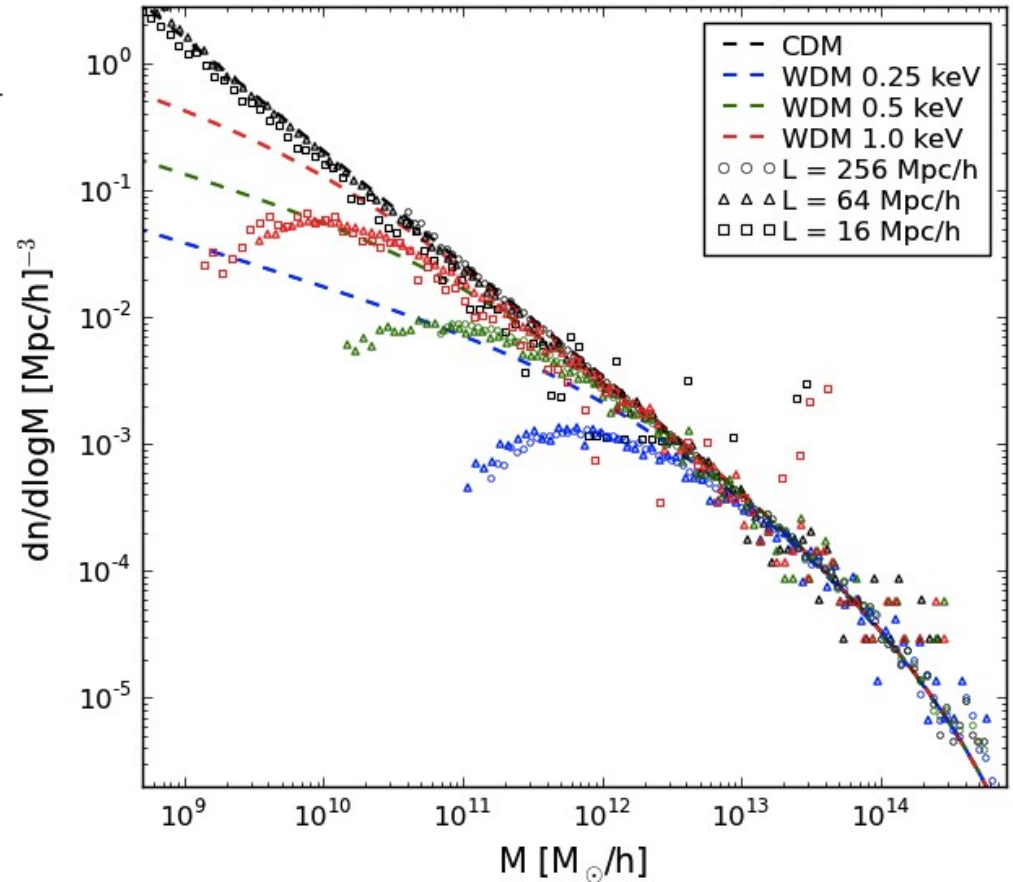
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WDM limit: $\lim_{M \rightarrow 0} \frac{dn}{d \log M} \propto M^{-1/3} = \infty$

Mass Function: Sharp-k model

$$\frac{dn}{d \log M} = -\frac{1}{2} \frac{\bar{\rho}}{M} f(\nu) \frac{d \log \sigma^2}{d \log M} \quad \nu = \frac{\delta_c^2}{\sigma^2(M)}$$

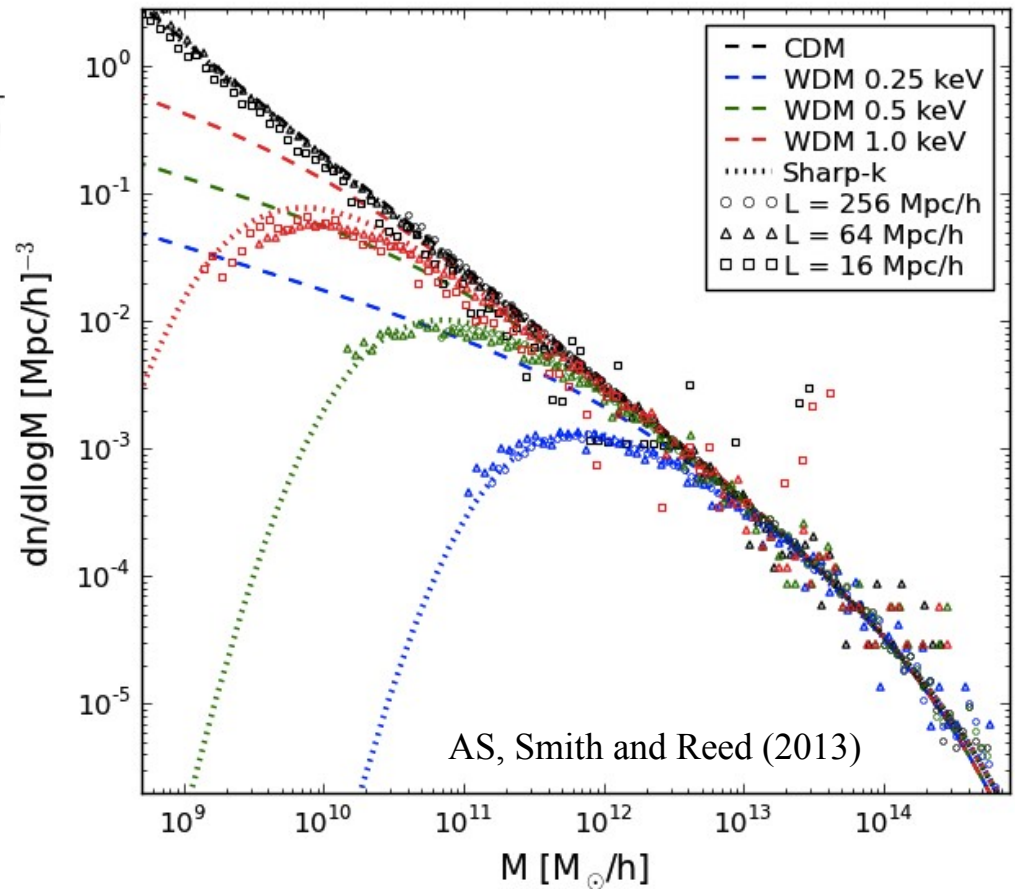
$$\sigma^2(R) = \int \frac{d\mathbf{k}^3}{(2\pi)^3} P_{\text{Lin}}(k) W_{\text{SK}}^2(kR)$$

$$W_{\text{SK}} = \Theta(1 - kR)$$

$$f(\nu) = A \sqrt{\frac{2q\nu}{\pi}} [1 + (q\nu)^{-p}] e^{-q\nu/2},$$

$$A = 0.322, \quad p = 0.3, \quad q = 1.0.$$

$$M = \frac{4\pi}{3} \bar{\rho} [cR]^3, \quad c = 2.7$$



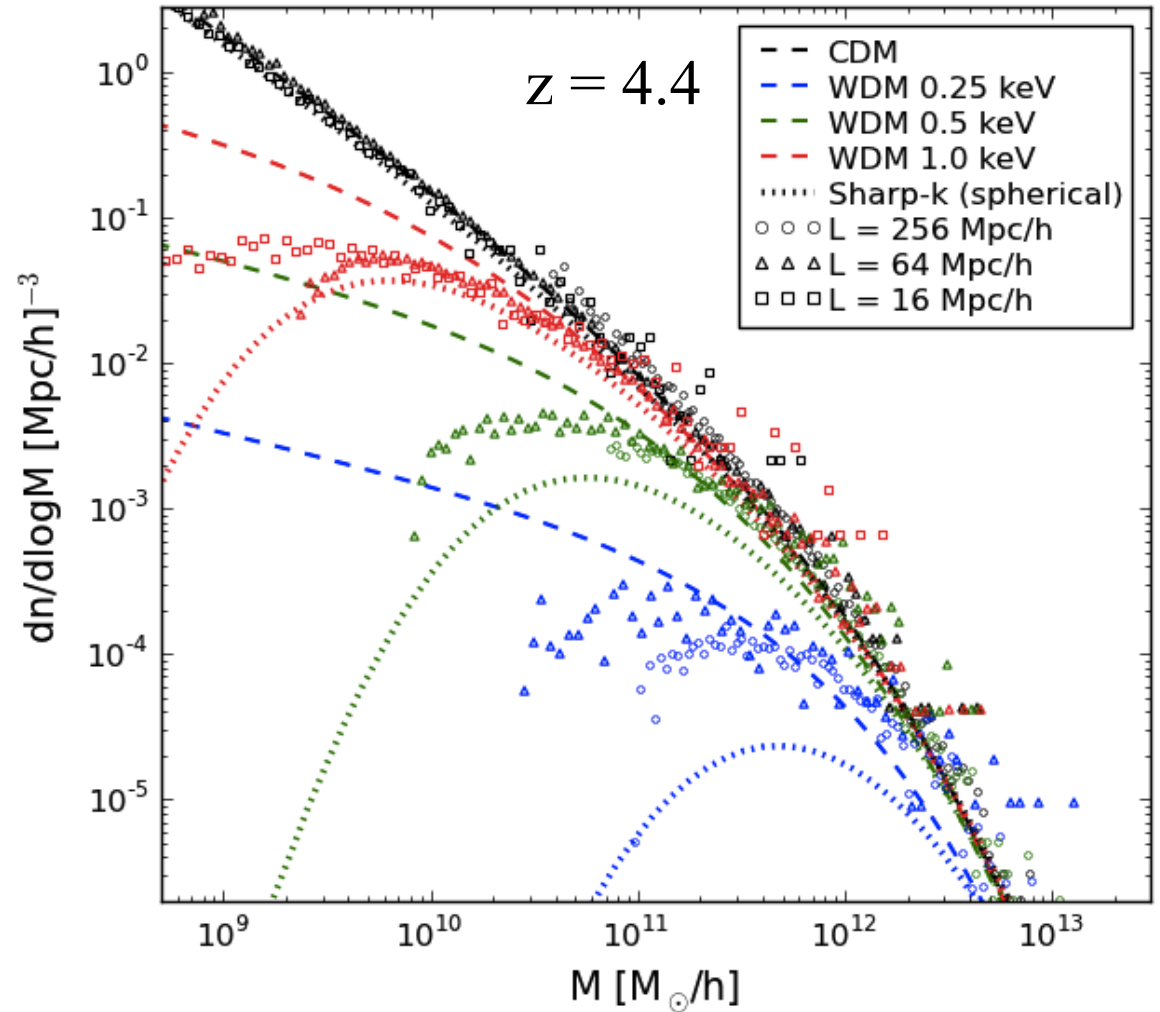
WDM limit: $\lim_{M \rightarrow 0} \frac{dn}{d \log M} \propto M^{6-n/3} = 0$

Mass Function: Sharp-k model

Problem solved ?

Mass Function: Sharp-k model

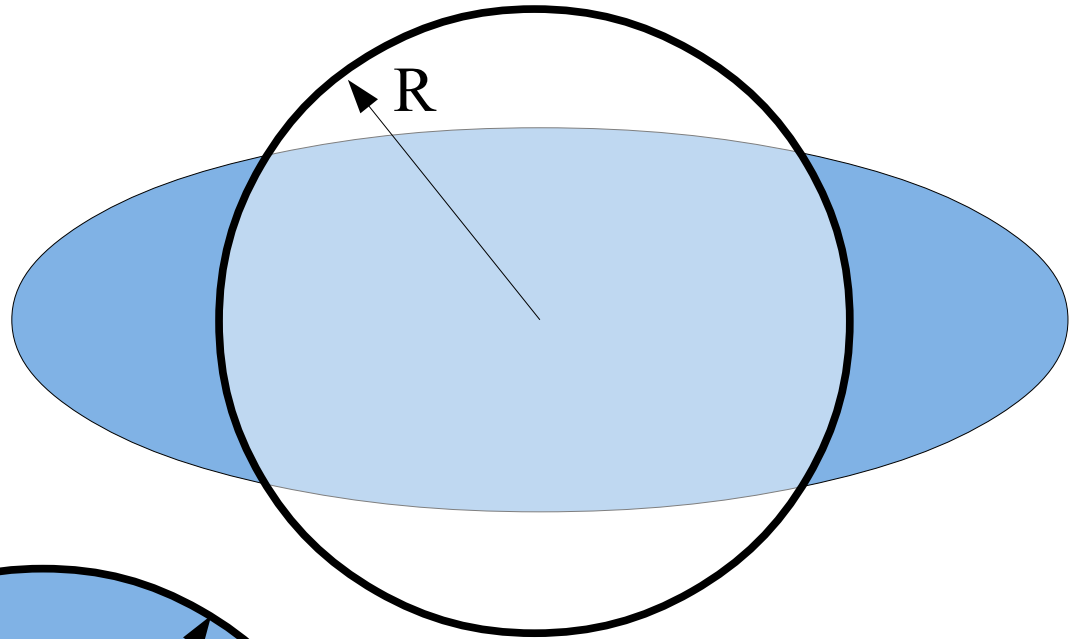
Problem solved ?



...not at high redshift!

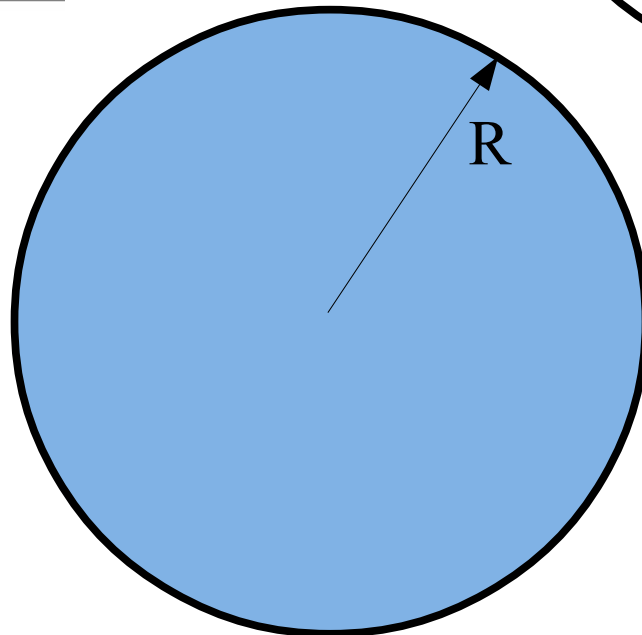
Mass Function: ellipsoidal correction

$$\frac{dn}{d \log M} = -\frac{1}{2} \frac{\bar{\rho}}{M} f(\nu) \frac{d \log \sigma^2}{d \log M}$$



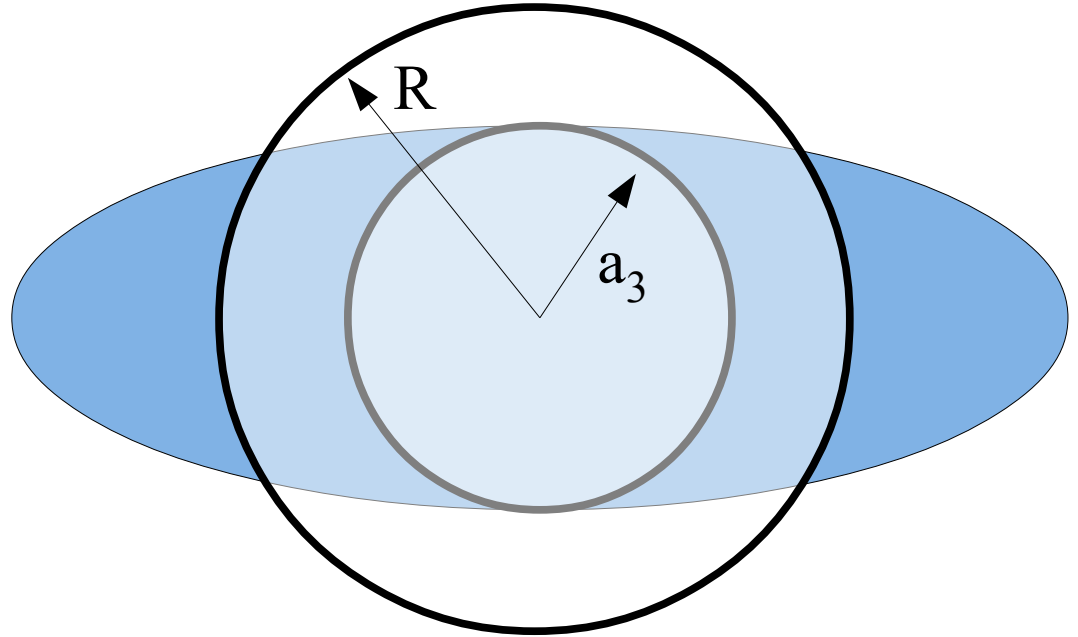
$$\frac{d \log \sigma^2}{d \log M} = \frac{1}{6\pi^2 \sigma^2} \frac{P_{\text{Lin}}(1/R)}{R^3}$$

$$M = \frac{4\pi}{3} \bar{\rho} [cR]^3$$



Mass Function: ellipsoidal correction

$$\frac{dn}{d \log M} = -\frac{1}{2} \frac{\bar{\rho}}{M} f(\nu) \frac{d \log \sigma^2}{d \log M}$$



$$\frac{d \log \sigma^2}{d \log M} = \frac{1}{6\pi^2 \sigma^2} \frac{P_{\text{Lin}}(1/a_3)}{a_3^3} \frac{da_3}{dR} \xi(R)$$

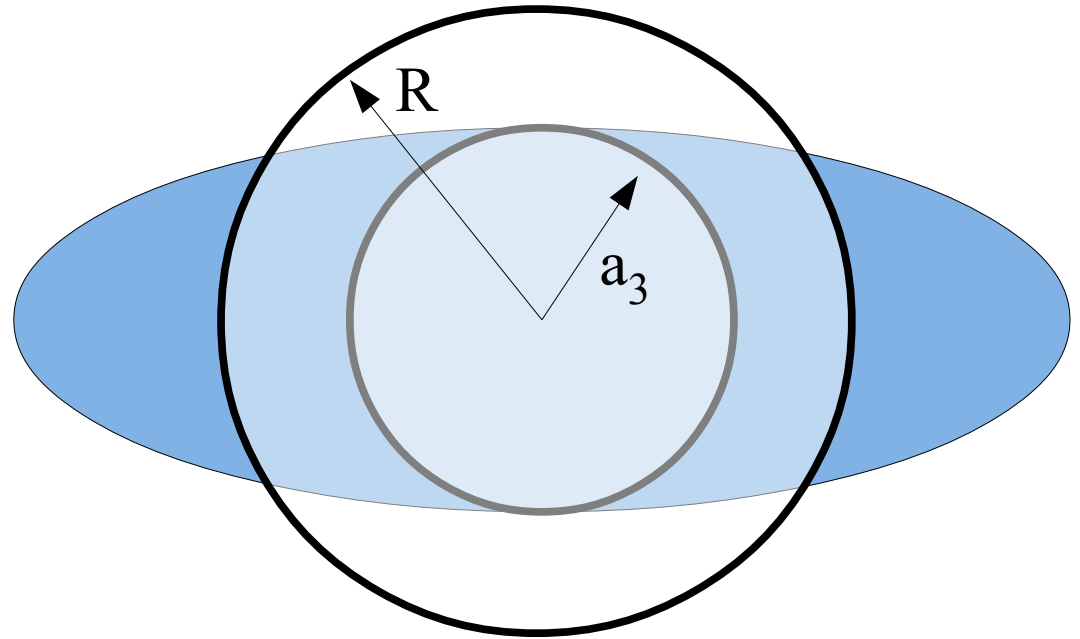
$$R^3 = a_1 a_2 a_3 = \left(\frac{a_1 a_2}{a_3 a_3} \right) a_3 = (\xi a_3)^3,$$

$$a_3(R) = \frac{R}{\xi(R)}$$

$$M = \frac{4\pi}{3} \bar{\rho} [cR(a_3)]^3$$

Mass Function: ellipsoidal correction

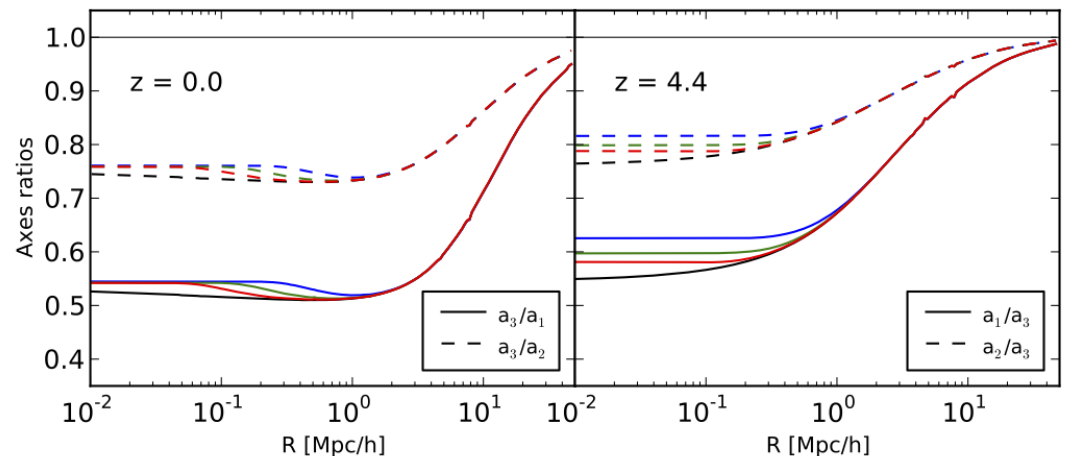
$$\frac{dn}{d \log M} = -\frac{1}{2} \frac{\bar{\rho}}{M} f(\nu) \frac{d \log \sigma^2}{d \log M}$$



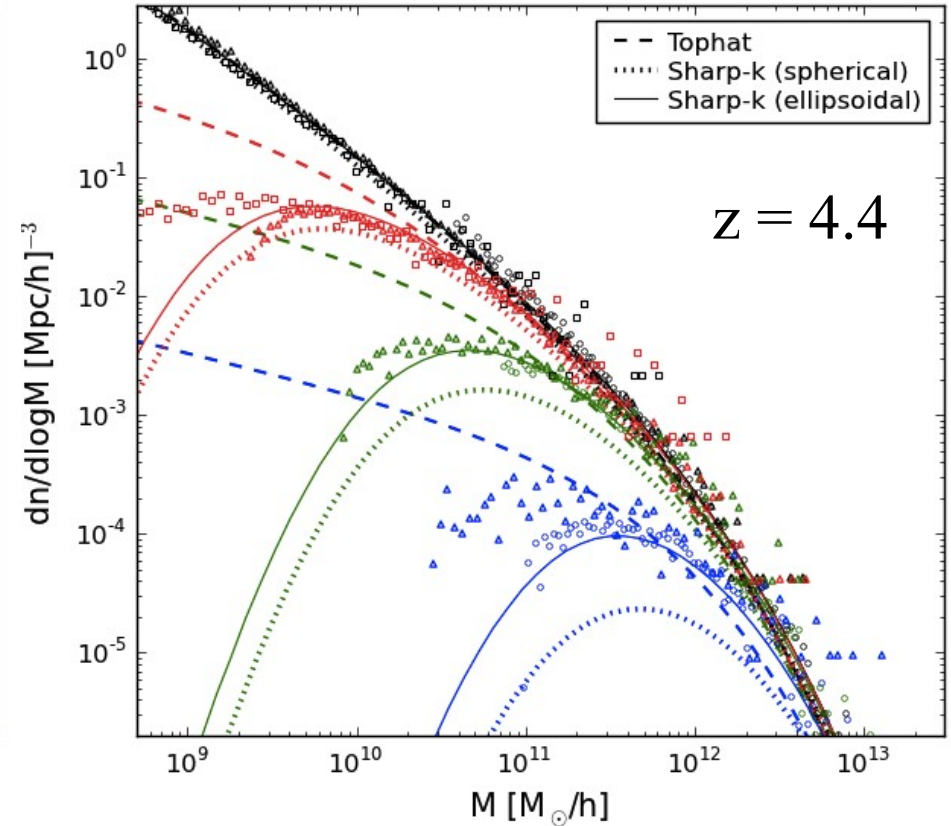
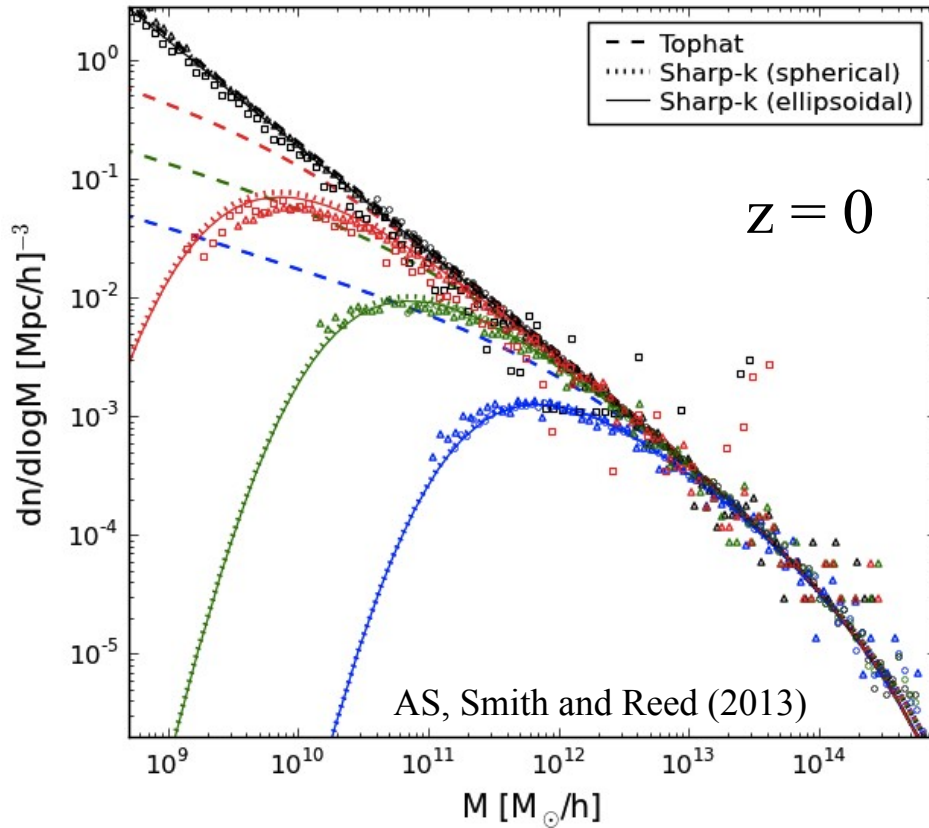
$$\frac{d \log \sigma^2}{d \log M} = \frac{1}{6\pi^2 \sigma^2} \frac{P_{\text{Lin}}(1/a_3)}{a_3^3} \frac{da_3}{dR} \xi(R)$$

$$R^3 = a_1 a_2 a_3 = \underbrace{\left(\frac{a_1}{a_3} \frac{a_2}{a_3} \right)}_{\xi} a_3 = (\xi a_3)^3$$

Shape of initial patches
depend on scale and redshift
(Bardeen et al 1986)



Mass Function: ellipsoidal correction



- Spherical sharp-k: $A = 0.322$, $p = 0.3$, $q = 1.0$. $M = \frac{4\pi}{3} \bar{\rho} [cR]^3$, $c = 2.7$
- Ellipsoidal sharp-k: $A = 0.322$, $p = 0.3$, $q = 0.75$. $M = \frac{4\pi}{3} \bar{\rho} [cR(a_3)]^3$, $c = 2.0$

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Cold Dark Matter: comparison

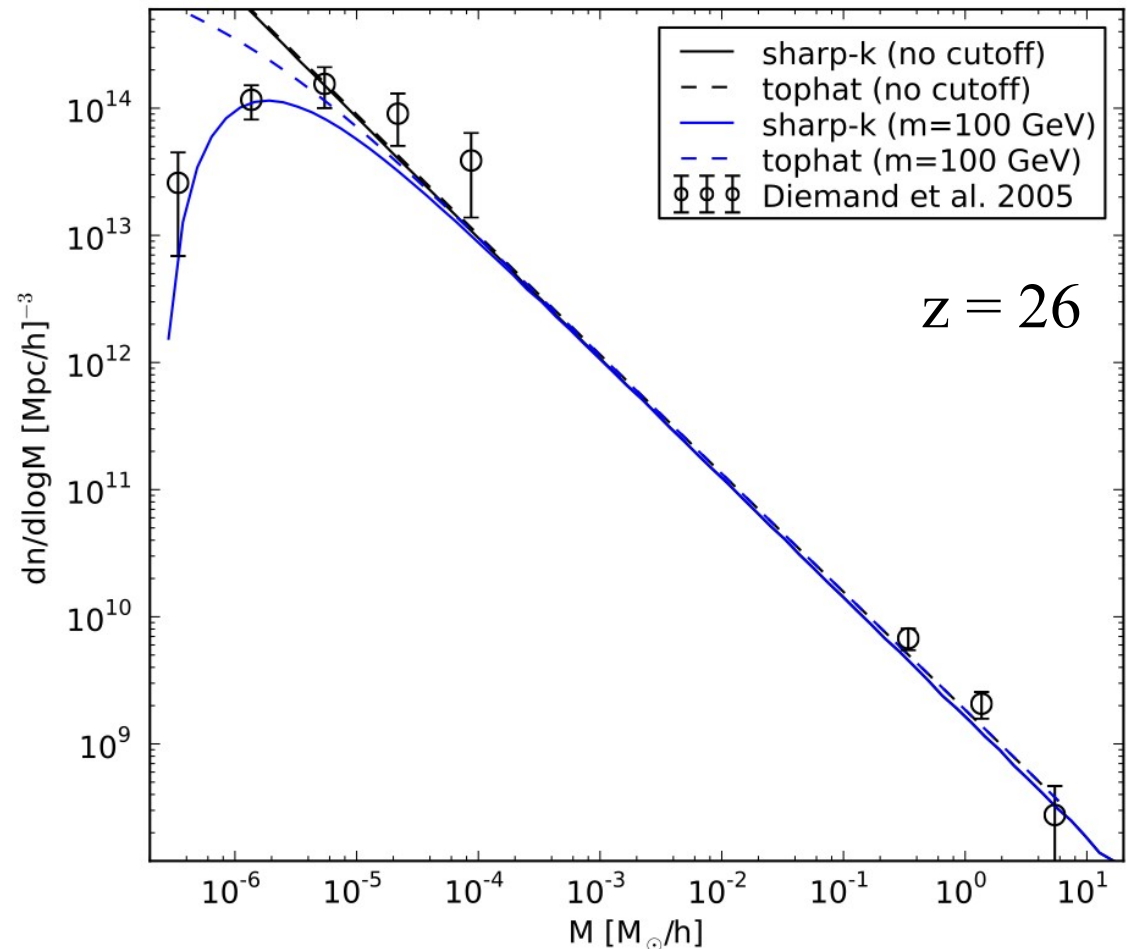
Does model work for
neutralino-CDM?

- Free streaming scale:
 10^{15} times smaller!
- Different cutoff

Cold Dark Matter: comparison

Does model work for neutralino-CDM?

- Free streaming scale:
 10^{15} times smaller!
- Different cutoff



Diemand et al. (2006):
 $m = 100 \text{ GeV}$, $T_{\text{dk}} = 28 \text{ MeV}$

