COSMO 2013

2-6 Sept 2013, Cambridge, UK

Isotropy theorem for cosmological vectors and higher-spin fields

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Phys.Rev**. D87** (2013) 043523

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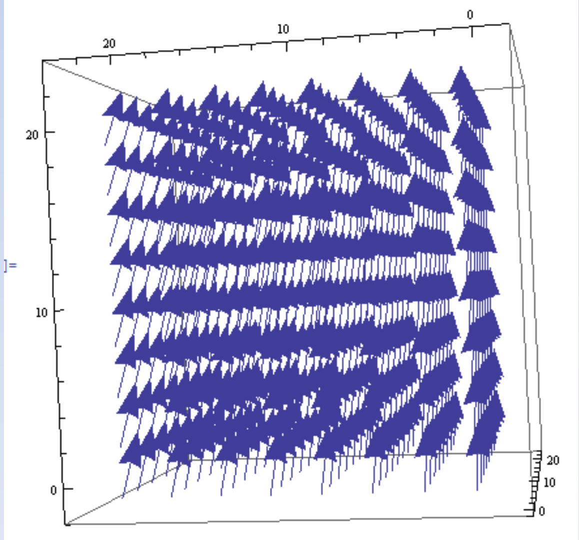
Phys.Rev. **D86** (2012) 021301

Introduction

- Homogeneous scalar fields are ubiquitous in cosmology: inflaton field, dark matter (axions or ALP's) or dark energy (quintessence, k-essence …)

- One of the main limitations for **homogeneous vector** (or higher-spin) fields is the large degree of isotropy of CMB

- Solutions proposed based on the use of only temporal components, triads or large N number of vectors fields.



- What about oscillating fields?

Vectors in cosmology

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(A_\mu A^\mu) \right)$$

$\mu = 0$

$$V'(A^2)A_0 = 0$$

$\mu = i$

$$\ddot{A}_i + H\dot{A}_i - 2V'(A^2)A_i = 0$$

Spatial vectors

$A_\mu = (0, A_i(t))$

$$T^i{}_j = \frac{\dot{A}_i \dot{A}_j}{a^2} + 2V'(A^2)\frac{A_i A_j}{a^2}, \quad i \neq j$$

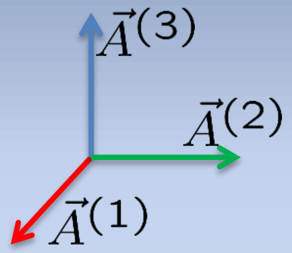
The anisotropy problem

There are different solutions in the literature:

- Using the scalar degree of freedom **A_0** .

Beltran Jimenez, A.L.M., Phys. Rev. D78, 063005 (2008) and JCAP 0903, 016 (2009)

- **Particular solutions**: Triads of orthogonal vectors.



$$A_i^{(a)} \propto \delta_i^a, \text{ a=1,2,3}$$

$$T^i_{\ j} \propto \delta^i_{\ j}$$

Cervero, Jacobs,
Phys. Lett. B78, 427 (1978)

- Large number, N, of **randomly oriented fields**.

$$T^i_j / p_k \sim 1/\sqrt{N}$$

Golovnev, Mukhanov, Vanchurin, JCAP 0806, 009 (2008)

- **Average isotropy** for a linearly polarized Abelian vector coherent oscillations with potential $m^2 A_\mu A^\mu$.

Dimopoulos, Phys. Rev. D 74, 083502 (2006)

Oscillating vectors (abelian case)

$$\ddot{A}_i + H\dot{A}_i - 2V'(A^2)A_i = 0$$

$$\omega_i \sim |\dot{A}_i/A_i|$$

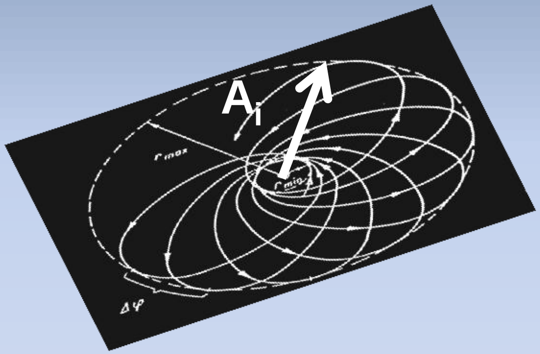
$$\ddot{A}_i = 2V'(A^2)A_i$$

rapid oscillations

$$\omega_i \gg H$$

$$G_{ij} = \frac{\dot{A}_i A_j}{a^2}$$

bounded



Virial theorem

$$0 = \frac{G_{ij}(T) - G_{ij}(0)}{T} = \left\langle 2V'(A^2)\frac{A_i A_j}{a^2}\right\rangle + \left\langle \frac{\dot{A}_i \dot{A}_j}{a^2}\right\rangle$$

Oscillating vectors (abelian case)

1.- Average energy-momentum tensor is diagonal and isotropic

$$\langle T^i{}_0 \rangle = 0 \qquad \langle T^i{}_j \rangle = -\langle p \rangle \delta^i{}_j$$

2.- Average equation of state for $V = \lambda(A_\mu A^\mu)^n$:

$$\omega = \frac{\langle p \rangle}{\langle \rho \rangle} = \frac{n-1}{n+1}$$

Same as for scalar fields
Turner, PRD28 (1983) 1243

Isotropy theorem for Yang-Mills theories

Yang-Mills theories for semi-simple Lie groups:

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{4} F^a{}_{\mu\nu} F^{a\,\mu\nu} - V\left(A^a{}_\mu A^{a\,\mu}\right)\right)$$

$$\begin{aligned} \ddot{A}_i^a &- g c_{abc} \left(2\dot{A}_i^b A_0^c + A_i^b \dot{A}_0^c \right) + g^2 c_{abc} c_{bde} \left(A_i^d A_0^e A_0^c \right. \\ &- \left. A_i^d A_j^e A_j^c \right) - 2V' M_{ab} a^2(\eta) A_i^b = 0 \,, \end{aligned}$$

If the **field evolves rapidly** and $A^a{}_i$ **,** $\dot{A}^a{}_i$ **are bounded** during its evolution,

1.- **The energy momentum tensor is diagonal and isotropic in average.**

2.- **Without potential, the equation of state parameter is w = 1/3 i.e. it behaves as radiation.**

JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Example: SU(2) theory

The self-interaction for non-Abelian theories changes the average equation of state. For high energy densities or large coupling constants it will behave as radiation, in the opposite limit, the Abelian behavior is recovered.

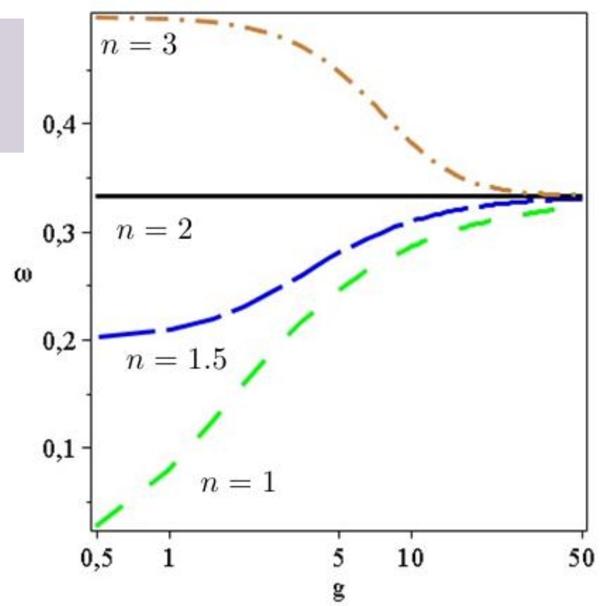
$$V = \frac{1}{2}(-M^2 A^a_\rho A^{a\,\rho})^n$$

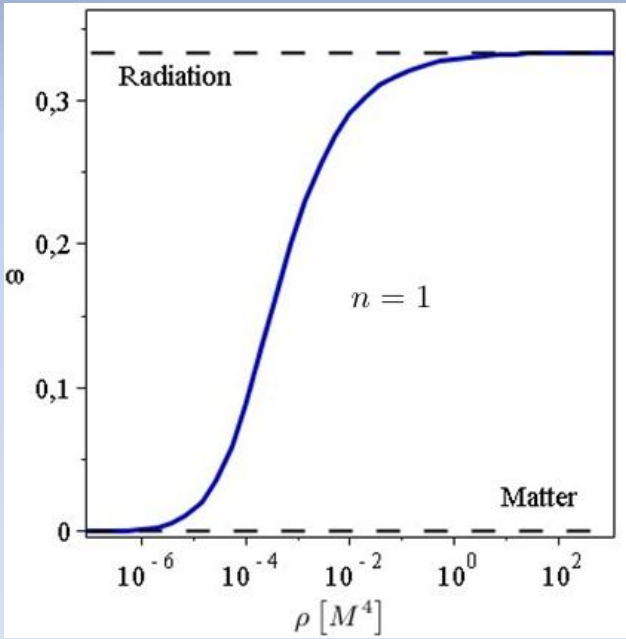
$g \downarrow,\ \rho \downarrow$

$$\omega = \frac{n-1}{n+1}$$

$g \uparrow,\ \rho \uparrow$

$$\omega = \frac{1}{3}$$





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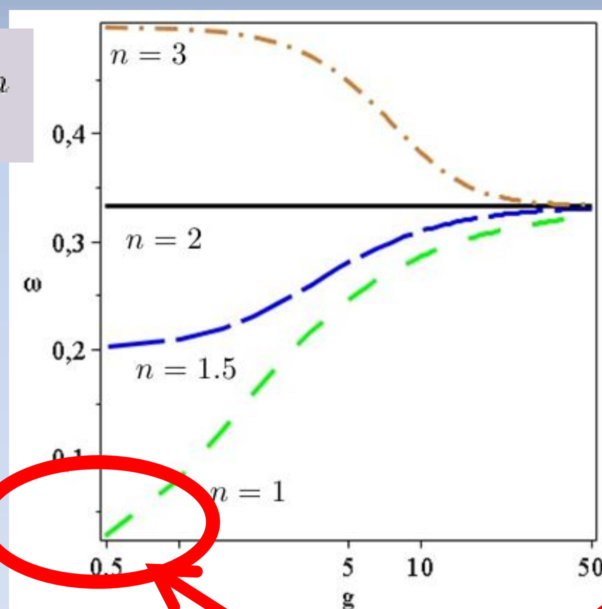

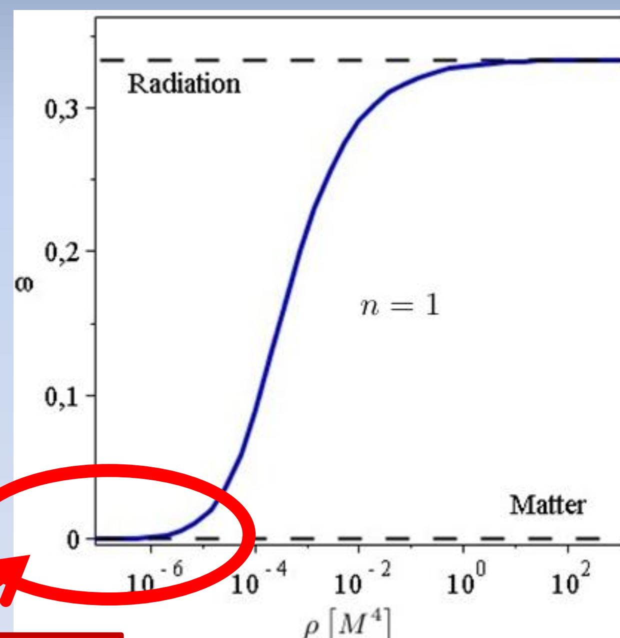

DARK MATTER BEHAVIOR

JARC, Maroto, Nunez Jareno, Phys. Rev. D87 (2013) 043523

Isotropy theorem for higher-spin theories

Consider a generic field ϕ_i with general Lagrangian of the form:

$$\mathcal{L} \equiv \mathcal{L}\left[\phi_i, \nabla_\mu \phi_i\right]$$

Canonical energy-momentum tensor

$$\Theta^{\mu\nu} = g^{\mu\nu}\mathcal{L} - \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi_i\right)}\partial^\nu \phi_i$$

Belinfante-Rosenfeld energy momentum tensor

$$\nabla_\rho \tilde{\Theta}^{\nu\rho;\mu}$$

$$T^{\mu\nu} = \Theta^{\mu\nu} - \frac{1}{2}\nabla_\rho\left(S^{\rho\mu\nu} + S^{\mu\nu\rho} - S^{\nu\rho\mu}\right)$$

$$S^{\mu\nu\rho} = \Pi_i^\mu \Sigma^{\nu\rho}\phi^i$$

Isotropy theorem for higher-spin theories

$$\left\langle g^{\rho\gamma}\nabla_\rho \tilde{\Theta}_{\nu\gamma;\mu}\right\rangle \approx \frac{1}{T}\int_t^{t+T} dt' \partial_0 \tilde{\Theta}_{\nu 0;\mu}(t') = \frac{\tilde{\Theta}_{\nu 0;\mu}(t+T) - \tilde{\Theta}_{\nu 0;\mu}(t)}{T}$$

Virial theorem:

$$H^{-1} \gg T \gg \omega^{-1}$$

Diagonal and isotropic energy-momentum tensor

$$\begin{aligned}
\langle T^{00}\rangle &= \langle \Pi_i^0 \nabla^0 \phi^i - g^{00}\mathcal{L}\rangle \,; \\
\langle T^{0j}\rangle &= T^{0j} = 0 \,; \\
\langle T^{jj}\rangle &= \langle -g^{jj}\mathcal{L}\rangle \,; \\
\langle T^{jk}\rangle &= 0 \,; k \neq j \,.
\end{aligned}$$

$$\mathcal{H} = \left(\lambda^{ij} g^{00} \Pi_{i\,0} \Pi_{j\,0}\right)^{n_T} + \left(M_{ij}\phi^i\phi^j\right)^{n_V}$$

Average equation of state:

$$\omega = \frac{2\, n_V}{1 + \frac{n_V}{n_T}} - 1$$

Beyond Robertson-Walker background

For a general background metric. Riemann normal coordinates:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{1}{3} R_{\mu\alpha\nu\beta} x^\alpha x^\beta + \ldots$$

1.- If $|R^{\mu}{}_{\nu\rho\sigma}| \ll \omega_i^2$ and $|\partial_j \phi_i| \ll |\dot{\phi}_i|$ second term negligible in the averaging time T. $|R^{\mu}{}_{\nu\rho\sigma}| \ll T^{-2} \ll \omega_i^2$

2.- Lagrangian of the form: $\mathcal{L} = \mathcal{L}[\phi_i, \nabla_\mu \phi_i]$

3.- ϕ_i and $\dot{\phi}_i$ bounded in time evolution

The energy-momentum tensor takes the perfect fluid form for any locally inertial observer.

Conclusions

- We have considered homogeneous vectors and higher-spin fields with generic Lagrangian densities.

- For bounded and rapid evolution compared to the rate of expansion, the average energy-momentum tensor is always isotropic for any field configuration.

- For theories with power law kinetic and potential terms, the average equation of state is: $\omega = \frac{2n_V n_T}{n_T + n_V} - 1$

- This result can be extended to arbitrary geometries for any locally inertial observer