Phenomenology of Unified Dark Matter

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Introduction

- Unified Dark Matter (UDM): a single matter component explains both structure formation and cosmic acceleration
- UDM are appealing because evade the coincidence problem and predict $w_{DE} \approx -1$
- Worrying issues: $c_s^2 \neq 0$ (non-negligible Jeans scale/strong late ISW effect)
- Possible solution: UDM models with fast transition between an early CDM-like phase and a late ΛCDM-like epoch
- UDM density perturbations evolve in a scale-dependent fashion (unlike CDM)
- Our aim: prescribe phenomenological adiabatic UDM models with fast transition that can be easily implemented into numerical codes

UDM

The background

- Flat FRW with $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
- $T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p g_{\mu\nu}$ (perfect fluid)
- Einstein equations imply: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3}$ and $\frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho + 3p\right)$
- Projecting the conservation equations $T^{\mu\nu}{}_{;\nu}$ along u^μ:
  $\dot{\rho} = -3H\left(\rho + p\right) = -3H\rho\left(1 + w\right)$
- The equation of state $w = p/\rho$ characterises the background of our UDM model

- Perturbations of the FRW metric in the longitudinal gauge: $ds^2 = -a^2(\eta)\left[(1+2\Phi)\,d\eta^2 - (1-2\Phi)\,\delta_{ij}dx^i dx^j\right]$
- Defining $u = \frac{2\Phi}{\sqrt{\rho+p}}$ and linearising the 0-0 and 0-i components of Einstein equations: $\frac{d^2u}{d\;\eta^2} + k^2c_\mathrm{s}^2u - \frac{1}{\theta}\frac{d^2\theta}{d\;\theta^2}u = 0$ where $\theta = \sqrt{\frac{\rho}{3(\rho+p)}}(1+z)$ and z is the redshift, $1+z = a^{-1}$.
- c_s^2 characterises the perturbative dynamics of our UDM model, being also crucially involved in the growth of overdensities $\delta\rho$
- Assuming adiabatic perturbations: $c_s^2 = c_\mathrm{ad}^2 = \frac{dp}{d\rho} = \frac{\frac{dp}{d\eta}}{\frac{d\rho}{d\eta}}$
- Define the squared Jeans wave number: $k_\mathrm{J}^2 = \left|\frac{1}{c_\mathrm{s}^2\theta}\frac{d^2\theta}{d\;\theta^2}\right|$ (the squared Jeans length $\lambda_\mathrm{J}^2 = a^2/k_\mathrm{J}^2$).

- $k_{\rm J}^2$ is crucial in determining the viability of a UDM model, because of its effect on perturbations, which is revealed in observables such as the CMB and matter power spectrum
- Any UDM model should satisfy the condition $k_{\rm J}^2 \gg k^2$ for all the scales of cosmological interest, in turn giving an evolution for the gravitational potential $\Phi(\eta, k) \simeq A_{\rm k}\left[1 - \frac{H(\eta)}{a(\eta)}\int a(\hat{\eta})^2 d\hat{\eta}\right]$
- The explicit form of the Jeans wave number is $k_{\rm J}^2 = \frac{3}{2}\rho a^2 \frac{(1+w)}{c_{\rm s}^2}\left|\frac{1}{2}(c_{\rm s}^2 - w) - \rho\frac{dc_{\rm s}^2}{d\rho} + \frac{3(c_{\rm s}^2-w)^2-2(c_{\rm s}^2-w)}{6(1+w)} + \frac{1}{3}\right|$
- If we want an analytic expression for $k_{\rm J}^2$ in order to obtain some insight on the behaviour of perturbations in a given UDM model, we need to be able to obtain analytic expressions for ρ, p, w and c_s^2

Three possible prescriptions for the dynamics

- Unfortunately, it is not possible to find analytic expressions for ρ, p, w and c_s^2 as functions of η (or t), because this requires the knowledge of an analytic expression for the scale factor as function of time, i.e. to solve the Friedmann equation, which in general is only possible for very special cases
- We can disentangle the evolution of (ρ, p, w and c_s^2) from Einstein equations, noticing that we can obtain these quantities as functions of a if we use only the conservation equation.
- The conservation equation becomes: $\rho' = -\frac{3}{a}(\rho + p)$

Three possible prescriptions for the dynamics

Starting from $w(a)$

- Suppose that $p/\rho = w$ is pre-assigned as a function of the scale factor: $w = w(a)$
- The adiabatic speed of sound $c_s^2 = \frac{dp}{d\rho} = \frac{\frac{dp}{da}}{\frac{d\rho}{da}} = \frac{p'}{\rho'} = w - \frac{a\,w'}{3(1+w)}$
- This prescription however doesn't lead to analytic expressions for $\rho(a)$ and $p(a)$ in general, unless $\int \frac{1+w(a)}{a} da$ is integrable

Three possible prescriptions for the dynamics

Starting from $p(a)$

- Prescribing $p = p(a)$ can be useful, e.g. if one is dealing with a scalar field, in which case this is equivalent to prescribe a Lagrangian
- Rewrite the energy conservation $\rho' + \frac{3}{a}\rho = -\frac{3}{a}p(a)$
- The homogeneous solution is $\rho_M \propto a^{-3}$ and for a given $p(a)$ an analytic expression for $\rho(a)$ can be found if $E = 3\int a^2 p(a) da = \int p dV$ is integrable, giving $\rho = E/V$
- c_s^2 is immediately found, given $p(a)$ and the energy conservation, but an analytic expression for $w(a)$ can only be found if that for $\rho(a)$ is found

Three possible prescriptions for the dynamics

Starting from $\rho(a)$

- It is perhaps less obvious what the functional form for $\rho(a)$ should be
- In view of constructing UDM models with fast transition we want to recover $\rho \simeq \rho_M = \rho_{M,0} a^{-3}$ at early times (before the transition) and $\rho \simeq \rho_\Lambda + \rho_{M,0} a^{-3}$ after the transition, in the simplest case that we want to recover a ΛCDM at late times
- Given a (at least C^3) $\rho = \rho(a)$, we have the following expressions for the quantities that enter into the Jeans wave number:

  $w = -\frac{a}{3}\frac{\rho'}{\rho} - 1$

  $c_s^2 = -\frac{a}{3}\frac{\rho''}{\rho'} - \frac{4}{3}$

  $\frac{dc_s^2}{d\rho} = -\frac{1}{3\rho'^2}\left[a\rho''' + \rho'' - a\frac{\rho''^2}{\rho'}\right]$

A phenomenological UDM model with fast transition

- We assume that the universe is well described by an EdS model before the transition, while for generality we choose to describe the post-transition era with an affine model:

$$\rho = \begin{cases} \rho_t \left(\frac{a_t}{a}\right)^3 & a < a_t \\ \rho_\Lambda + (\rho_t - \rho_\Lambda)\left(\frac{a_t}{a}\right)^{3(1+\alpha)} & a > a_t \end{cases} \quad (1)$$

- It is useful to explicitly incorporate a Heaviside function $H(a - a_t)$:
$\rho = \rho_t \left(\frac{a_t}{a}\right)^3 + \left[\rho_\Lambda + (\rho_t - \rho_\Lambda)\left(\frac{a_t}{a}\right)^{3(1+\alpha)} - \rho_t \left(\frac{a_t}{a}\right)^3\right] H(a - a_t)$.
For $\alpha = 0$ this reduces to
$\rho = \rho_t \left(\frac{a_t}{a}\right)^3 + \rho_\Lambda \left[1 - \left(\frac{a_t}{a}\right)^3\right] H(a - a_t)$,
representing a sudden transition to ΛCDM.

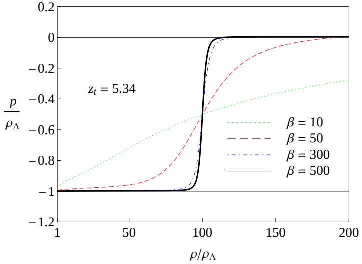
A phenomenological UDM model with fast transition

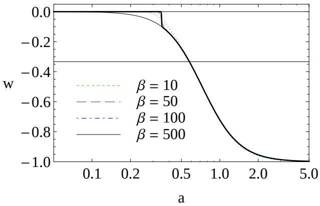
- Replacing $H(a - a_t)$ with a smoother function, we can obtain simple models for a UDM with a fast transition
- We shall consider a continuous approximation to the Heaviside function given by

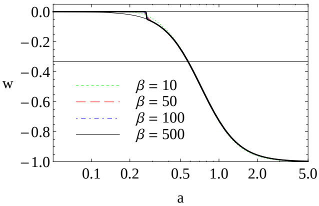
$$H_w(a - a_t) = \frac{1}{2} + \frac{1}{\pi}\arctan(\beta(a - a_t)), \tag{2}$$

- The transition must occur at relatively high redshifts, during the dark matter epoch, such that $\rho_\mathrm{t} \gg \rho_\Lambda$, which corresponds to a minimum value of the redshift z_t. Otherwise, it could be difficult to have a good fit of supernovae and ISW effect data

A phenomenological UDM model with fast transition





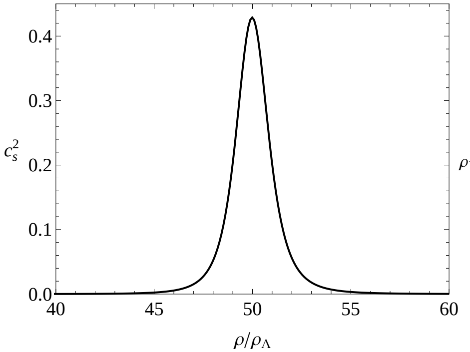


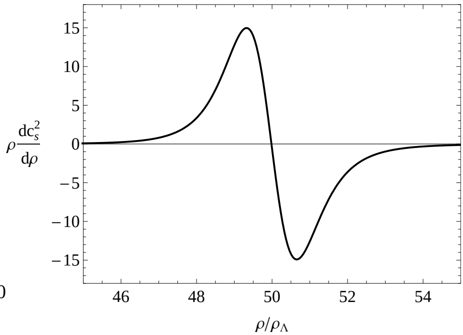
$$k_{\rm J}^2 = \frac{3}{2}\rho a^2 \frac{(1+w)}{c_{\rm s}^2}\left|\frac{1}{2}(c_{\rm s}^2 - w) - \rho\frac{dc_{\rm s}^2}{d\rho} + \frac{3(c_{\rm s}^2-w)^2-2(c_{\rm s}^2-w)}{6(1+w)} + \frac{1}{3}\right|$$

- We focus on k_J for our toy UDM model and investigate its behaviour as a function of c_s^2 around $\rho = \rho_t$, which corresponds to the middle of the transition where c_s^2 is at its peak
- A large $k_{\rm J}^2$ can be obtained not only when $c_{\rm s}^2 \to 0$, but when $c_{\rm s}^2$ changes rapidly as well, i.e, when $k_{\rm J}^2$ is dominated by the $\rho\, dc_{\rm s}^2/d\rho$ term
- Viable adiabatic UDM models can be constructed which do not require $c_{\rm s}^2 \ll 1$ at all times if c_s^2 goes through a rapid change, a fast transition period during which $k_{\rm J}^2$ can remain large, in the sense that $k^2 \ll k_{\rm J}^2$ for all scales of cosmological interest

Analysis of the properties of the perturbations

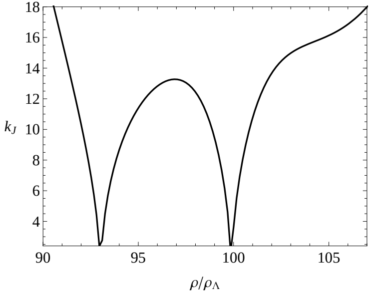
The Jeans scale





Analysis of the properties of the perturbations

The Jeans scale



- k_{J} becomes vanishingly small for extremely short times, so that the effects caused by its vanishing are sufficiently negligible, as we are going to show when we analyse the gravitational potential Φ.
- Before and after the fast transition we need c_s^2 to be very small so that we have a vanishing Jeans length, or a very large k_J

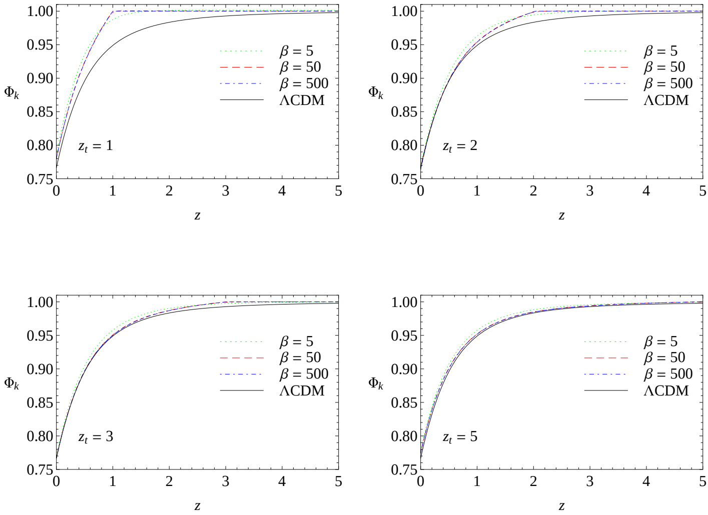
Analysis of the properties of the perturbations

The gravitational potential

- $\frac{d^2\Phi(\mathbf{k},a)}{da^2} + \left(\frac{1}{\mathcal{H}}\frac{d\mathcal{H}}{da} + \frac{4}{a} + 3\frac{c_s^2}{a}\right)\frac{d\Phi(\mathbf{k},a)}{da} + \left[\frac{2}{a\mathcal{H}}\frac{d\mathcal{H}}{da} + \frac{1}{a^2}(1+3c_s^2) + \frac{c_s^2k^2}{a^2\mathcal{H}^2}\right]\Phi(\mathbf{k},a) = 0$ where $\mathcal{H} = a'/a$ is the conformal time Hubble parameter
- The normalised initial conditions are $\Phi_\Lambda(\mathbf{k};a_{\mathrm{rec}}) = 1$ and $d\Phi_\Lambda/da|_{a_{\mathrm{rec}}} = 0$
- We can now analyse the gravitational potential and explore its dependence on the background parameters β and z_{t}

Analysis of the properties of the perturbations

The gravitational potential



Conclusions

- UDM models have the advantage that they can describe the dynamics of the Universe with a single dark fluid which triggers both the accelerated expansion at late times and the LSS formation at earlier times
- UDM models have no coincidence problem by definition and predict an effective cosmological constant at late times
- We have build a phenomenological adiabatic UDM model with a fast transition between an Einstein de Sitter model, and a more recent epoch whose dynamics, background and perturbative, are very close to that of a standard ΛCDM model. This model can be easily and efficiently implemented into numerical codes
- We have shown that for an early enough fast transition with $\beta > 500$ and $z_{\mathrm{t}} > 2$ our UDM model should be compatible with observations. On the other hand, a study of the matter and CMB power spectra is needed to study the viability of models with $10 \lesssim \beta < 500$, and those with $\beta > 500$ and $z_{\mathrm{t}} < 2$. We shall undertake this work in the future.