Instabilities during Einstein-Aether Inflation

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Based on work to appear soon:
AS & John D Barrow arXiv:1309.xxxx

September 3$^{\mathrm{rd}}$, 2013

Why consider Lorentz violation?

Lorentz invariance is a key ingredient of the most successful modern theories.

What do we gain from theories in which it breaks down?

Why consider Lorentz violation?
Completeness

Anything as crucial to physics as Lorentz symmetry deserves testing.

More to the point: LI is tested extremely well in the standard model Mattingly 2005 gr-qc/0502097, but significant room for improvement remains in gravity, dark matter, and inflation.

Lorentz-violating theories (Examples)

- *Gravity*: Einstein-aether theory, Hořava-Lifschitz gravity
- *Dark Matter*: Blas, Ivanov, & Sibiryakov 2012 arXiv:1209.0464
- *Inflation*: This talk

Why consider Lorentz violation?

Beyond the Standard Model/Quantum Gravity

Beyond the Standard Model/Quantum Gravity:

Lorentz symmetry is not guaranteed to survive all the way to the Planck scale.

Example: Hořava-Lifschitz gravity Hořava 2009 arXiv:0901.3775

- Lorentz-violating gravity theory — power-counting renormalizable, UV completion of GR?

Note that the low-energy limit of HL is closely related to our model.

Why consider Lorentz violation?
Modified Gravity

Ties to Modified Gravity:

We will consider LV in the gravitational sector. This is part of the ongoing investigation into modified gravity, with motivations including completeness and dark energy. LV gravity may have something to say about dark energy *e.g.*, Blas & Sibiryakov 2011 arXiv:1104.3579 — closely related to this work.

LV in the gravitational sector: *Einstein-aether theory*

Einstein-aether theory is a modification of GR which spontaneously violates Lorentz invariance.

Ingredients:

- *Metric* $g_{\mu\nu}$ with usual Einstein-Hilbert term
- *Aether* u^μ: Fixed norm and timelike — preferred frame at every point, violating LI.
- *Lagrange multiplier* λ: Enforces fixed norm condition.

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The action - most general up to second derivatives

$$S = \int d^4x\sqrt{-g}\left[\underbrace{\frac{1}{16\pi G}R}_{\text{E-H}} + \mathcal{L}_{\text{kin}} + \underbrace{\lambda\left(u^{\mu}u_{\mu} + m^2\right)}_{\text{Imposes } u^2 = -m^2}\right]$$

$$\mathcal{L}_{\text{kin}} = -c_1\nabla_{\mu}u_{\nu}\nabla^{\mu}u^{\nu} - c_2\left(\nabla_{\mu}u^{\mu}\right)^2 - c_3\nabla_{\mu}u^{\nu}\nabla_{\nu}u^{\mu}$$

LV in the gravitational sector: *Einstein-aether theory*

Major restriction: Couplings between u^μ and SM matter fields are strongly constrained. But what about a scalar field?

Coupling the aether to a scalar field

Donnelly & Jacobson 2010 arXiv:1007.2594

Add canonical scalar field ϕ — allow it to couple to the *divergence* of the aether:

$$\theta \equiv \nabla_\mu u^\mu$$

The coupled action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R + \mathcal{L}_{\rm kin} + \lambda \left(u^\mu u_\mu + m^2 \right) - \frac{1}{2} (\partial\phi)^2 - \underbrace{V(\phi, \theta)}_{\text{coupling}} \right]$$

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NB: In FRW background, $\theta = 3mH$ measures the Hubble rate. Hence this is sometimes considered *expansion-coupled* inflation.

Aether-scalar coupling introduces tachyonic instability

AS & John D Barrow 2013 arXiv:0913.xxxx

Now consider cosmological perturbations around slow-roll background for general $V(\phi, \theta)$.

In background, aether aligns with cosmic rest frame

$$\bar{u}^\mu = \frac{m}{a(\tau)} \delta^\mu{}_0.$$

Define aether perturbations V^i by[1]

$$u^i = \frac{m}{a} V^i.$$

[1] u^0 fixed by $u_\mu u^\mu = -m^2$

Aether-scalar coupling introduces tachyonic instability

Spin-1 Perturbations

During slow-roll inflation, the spin-1 aether perturbations $V^{(\pm 1)}$ obey the wave equation

$$\boxed{\frac{1}{a}(aV)'' + c_s^2 k^2 V + \frac{M_{\rm eff}^2}{H^2\tau^2} V = 0.}$$

Note the effective mass is

$$M_{\rm eff}^2 = \frac{\bar{V}_\phi \bar{V}_{\theta\phi}}{6mc_1 H}\left(1 + \mathcal{O}(\varepsilon)\right).$$

$\longrightarrow$ $V^{(\pm 1)}$ becomes unstable for $M_{\rm eff}^2 < -2H^2$.

Aether-scalar coupling introduces tachyonic instability

Spin-1 Perturbations

The metric shift perturbation $B^{(1)}$ (spin-1 piece of g_{0i}) is related to the aether perturbation by

$$B^{(\pm 1)} = \gamma V^{(\pm 1)}$$

where

$$\gamma \equiv 16\pi G m^2 c_{13}.$$

This means that the off-diagonal metric component blows up exponentially, destroying FRW background.

(Slightly more complicated to show: this instability persists to the spin-0 modes as well.)

Aether-scalar coupling introduces tachyonic instability

Parameter constraints

For the quadratic Donnelly-Jacobson potential DJ 2010 arXiv:1007.2594

$$V(\phi,\theta) = \frac{1}{2}M^2\phi^2 + \mu\theta\phi$$

the previous constraint (flat space stability) was DJ 2010

$$\frac{|\mu|}{M} < \sqrt{2(c_1+c_2+c_3)} \sim \mathcal{O}(1).$$

The absence of the inflationary instability

$$\frac{|\mu|}{M} \lesssim \underbrace{2\sqrt{6}c_1}_{\sim\mathcal{O}(1)} \underbrace{\frac{m}{M_{\rm Pl}}}_{\lesssim\mathcal{O}(10^{-7})} \underbrace{\left(\frac{M}{H}\right)^{-2}}_{\sim\mathcal{O}(10^2)}.$$

This is many orders of magnitude stronger.

Aether-scalar coupling effects are negligible in the CMB

Aether-scalar coupling effects are negligible in the CMB

The spin-0 perturbations (which affect CMB) are difficult to solve:

$$
\begin{aligned}
4\pi\tilde{G}_c\left(-\bar{\phi}'\delta\phi' - a^2\bar{V}_\phi\delta\phi\right) &= \left(3\mathcal{H}^2 - A\right)\Phi - 3\mathcal{H}\Psi' - \frac{\tilde{G}_c}{G}k^2\Psi - 8\pi\tilde{G}_c c_1 m^2 k^2\Phi \\
&+ 8\pi\tilde{G}_c c_1 m^2 k(V' + \mathcal{H}V) - 8\pi\tilde{G}_c\tilde{\alpha}\mathcal{H}kV \\
&+ 4\pi\tilde{G}_c ma\bar{V}_{\theta\phi}(\bar{\phi}'\Phi - 3\mathcal{H}\delta\phi) \\
\frac{1}{8\pi G}(k\mathcal{H}\Phi - k\Psi') &= \frac{k}{2}\bar{\phi}'\delta\phi - \tilde{\alpha}AV + c_1 m^2 a^{-1}(ak\Phi)' \\
&- c_1 m^2\frac{\xi''}{a} + \frac{1}{2}ma\bar{V}_{\theta\phi}\bar{\phi}'V \\
4\pi\tilde{G}_c\left(\bar{\phi}'\delta\phi' - a^2\bar{V}_\phi\delta\phi\right) &= \left(3\mathcal{H}^2 - A\right)\Phi + \mathcal{H}\Phi' - 2\mathcal{H}\Psi' - \Psi'' - \frac{8\pi\tilde{G}_c m^2}{\gamma}\tilde{c}_{123}k^2(\Phi + \Psi) \\
&+ 4\pi\tilde{G}_c\frac{3m^3}{a}A\bar{V}_{\theta\theta\theta}\left(3\Psi' - 3\mathcal{H}\Phi + kV\right). \\
&- 4\pi\tilde{G}_c ma\left[\bar{V}_{\theta\phi}(3\mathcal{H}\delta\phi + \delta\phi') + \bar{V}_{\theta\phi\phi}\bar{\phi}'\delta\phi\right] \\
&+ 4\pi\tilde{G}_c m^2\bar{V}_{\theta\theta\phi}\left[3A\delta\phi - \bar{\phi}'(3\Psi' - 3\mathcal{H}\Phi + kV)\right], \\
k^2(\Phi + \Psi) &= \gamma a^{-2}(a^2kV)',
\end{aligned}
$$

$$
\frac{1}{a}(aV)'' + \frac{\tilde{c}_{123}m^2}{c_1 m^2 + \tilde{\alpha}\gamma}k^2V + \left(\frac{\tilde{\alpha}(1-\gamma)A - \frac{1}{2}ma\bar{V}_{\theta\phi}\bar{\phi}'}{c_1 m^2 + \tilde{\alpha}\gamma}\right)V = \frac{c_1 m^2 + \tilde{\alpha}}{c_1 m^2 + \tilde{\alpha}\gamma}\frac{k(a\Phi)'}{a} - \frac{1}{2}\frac{ma\bar{V}_{\theta\phi}}{c_1 m^2 + \tilde{\alpha}\gamma}k\delta\phi.
$$

Aether-scalar coupling effects are negligible in the CMB

Can make progress by using the smallness of $m/M_{\rm Pl} \lesssim 10^{-7}$.

Strategy: Expand equations of motion in $m/M_{\rm Pl}$ and solve order-by-order.

Aether-scalar coupling effects are negligible in the CMB

Φ evolution

At $O(m/M_{\rm Pl})^0$, metric perturbation Φ on superhorizon scales obeys the usual equation Mukhanov, Feldman, and Brandenberger 1992

$$\Phi'' + \left(2\mathcal{H} - \frac{A'}{A}\right)\Phi' + \left(2\mathcal{H}^2 - 2A - \frac{A'}{A}\mathcal{H}\right)\Phi + \mathcal{O}\left(\frac{m}{M_{\rm Pl}}\right) = 0$$

where $A \equiv \mathcal{H}^2 - \mathcal{H}'$ (vanishes in the de Sitter limit).

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the exact same equation.

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$$\Phi'' + \left(2\mathcal{H} - \frac{A'}{A}\right)\Phi' + \left(2\mathcal{H}^2 - 2A - \frac{A'}{A}\mathcal{H}\right)\Phi + \mathcal{O}\left(\frac{m}{M_{\rm Pl}}\right)^2 = 0$$

the exact same equation.

This is unusual! This tells us:

- No superhorizon isocurvature modes (to $\mathcal{O}(m/M_{\rm Pl})$)
- Coupling effects on CMB are $\mathcal{O}(m/M_{\rm Pl})^2 \lesssim 10^{-15}$ corrections.
- Unexpected cancellations — deeper physical mechanism?

Summary

- The model: Einstein-aether theory coupled to scalar inflaton (fairly general)
- Coupling strongly constrained by instabilities about inflationary background — much stronger (5–6 OOM?) than previous constraints
- No isocurvature modes (to $\mathcal{O}(m/M_{\mathrm{Pl}})$) — surprising for a coupled theory!
- Coupling effects on CMB are $\lesssim \mathcal{O}(10^{-15})$ corrections.
- Unexpected cancellations — deeper physical mechanism? — work ongoing