Revisiting Scalar Collapse in AdS

New Frontiers in Dynamical Gravity
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Instability of Scalar Field in spher. symm. aAdS

Evolution of a Scalar field in sph. symm., asympt. AdS (aAdS)
[Bizoń -Rostworowski,2011]

- **Fully nonlinear:**
  - Consider Gaussian-type initial data w/ amplitude $\epsilon$ and width $\sigma$
  - Choptuik-type critical behavior
  - However, sub-critical eventually collapses as well

- **Perturbative about pure AdS:**
  - At linear order, uncoupled modes: oscillon
  - Resonance at $O(\epsilon^3)$: $j_r = j_1 + j_2 - j_3$
  - Single mode stable, multiple modes unstable

- **Conjecture:**
  - AdS generically unstable to collapse via weakly nonlinear turbulent cascade
Instability in light of AdS/CFT Correspondence

- Holographic duality between 
  \[(d + 1)\text{-dimensional global AdS (the bulk)}\]
  and
  \[\text{conformal field theory (CFT) on } d - 1\text{-dim. boundary } (S^{d-1} \times R)\]

- Dictionary translates between bulk quantities of aAdS spacetime 
  and quantum operators of CFT

- Interpretation of instability: 
  \textbf{initial data generically thermalizes by BH formation}

...but are there non-thermalizing initial configurations in the CFT?
Paths to Stability

- **Perturbative analysis** showing stable solutions
  - [Dias, Horowitz, Marolf, Santos, 1208.5772]
  - Argue perturbatively for nonlinear stability
  - Geons and boson stars, not necessarily spher. symm.

- **Excite all modes**
  - [Buchel, Lehner, SLL, 1210.0890]
  - Perturbative argument for stability at $\mathcal{O}(\epsilon^3)$
  - $\omega_{jr} \rightarrow \omega_{jr} + \epsilon^2 \sum_{\{j_1,j_2,j_3\}} \frac{A_{j_1}A_{j_2}A_{j_3}}{A_{jr}} \ C_{j_1j_2j_3jr}$, where the triple sum is over all the resonance channels $\omega_{j_1} + \omega_{j_2} = \omega_{jr} + \omega_{j_3}$

- **Time-periodic** solutions
  - [Maliborski, Rostworowski, 1303.3186]
  - Construct time-periodic solutions
  - Argue for nonlinear stability

- **Frustrated resonance**
  - [Buchel, SLL, Lehner, 1304.4166]
Frustrated Resonance

[Buchel,SLL,Lehner,1304.4166]

- Broadly distrib. energy perturbs AdS & introduces dispersion
- Dispersion competes with nonlinear sharpening
- BR data: increasing $\sigma$ increases distribution of energy
- Issues with $\sigma$-parameterization:
  [Maliborski,Rostworowski,1307.2875]
  - Large-$\sigma$ ceases to be broadly distributed (us and
    [Abajo-Arrastia,Silva,Lopez,Mas,Serantes,1403.2632])
  - “window” in $\sigma$ shrinks for higher dims but other ID stable
Perturbed Boson Star

1/791 (1265)  (1.6e+00, 7.6e-04)
0.0000000000e+00  4 dmdr 1
(0.0e+00, 0.0e+00)

4dmdr1.mpg  Perturbed BS
Effect of Mass [Balasubramanian, Buchel, Green, Lehner, SLL, in prep]

- **Motivation**: explore CFT operators of different weight
  ...mass changes decay rate of SF at boundary
- Introduce mass term $-\mu^2|\phi|^2$
- No dispersion at linear order
- Mass changes location of transition $\sigma_{\text{crit}}$
Open Questions

Among others, just two here:

- What’s stable and what’s unstable?
  \[\text{...in other words, can we identify whether initial data will collapse for any amplitude a priori?}\]

- For ID that appears unstable, can we be sure whether it extends to \(\epsilon \to 0\)?

...using “unstable” as ID that collapses for \(\epsilon \to 0\) but \(\epsilon \neq 0\)
Two-Time Formalism (TTF)

- Dynamics characterized by two time scales:
  - fast time $t$—generally $t < \pi$ where $\pi$ is time for a bounce off boundary
  - slow time $\tau$—scale over which energy transfers among oscillons, $\tau \equiv \epsilon^2 t$
- Allow mode amplitudes $A_j(t)$ to be functions of both times $A_j(t, \tau)$
- Enforce at $O(\epsilon^3)$ the absence of secular terms in the scalar field
- **Advantages:**
  - Goes beyond initial transfer of energy (...to time $t > 1/\epsilon^2$)
  - Conserves energy
  - Both direct and inverse cascades
- Solve coupled, cubic, ODEs in complex mode amplitudes $A_j$
- Resembles FPUT paradox
TTF and Fermi-Pasta-Ulam-Tsingou (FPUT, 1953)

- Model 1D atoms in a crystal by masses linked by springs with nonlinear term
- At linear order, Fourier modes decouple
- Nonlinear system may not not approach equipartition, as predicted by classical stat. mech.

...apparently still debated more than 50 years later!

- For $N \to \infty$ (nonlinear string) and small energies, system is similarly resonant

$$V(x) = \frac{1}{2} k x^2 + \frac{\alpha}{3} x^3 + \frac{\beta}{4} x^4$$

[D. Campbell's APS 2010 talk]
TTF and Fully Nonlinear...two-mode ID

- Similar “evolutions”...**both direct and inverse cascades**
- TTF convergent with increasing $j_{\text{max}}$
- In $\epsilon \to 0$ and $j_{\text{max}} \to \infty$ limits, TTF & NL should converge
TTF and Quasi-Periodic (QP) Solutions

- Specify $A_{jq}^{qp}(\tau) = \alpha_j e^{-i\beta_j \tau}$
- Solutions branching from dominant mode $j_r$...two branches for $j_r > 0$
- One-parameter generalizations of MR time-periodic solutions
- Such solutions balance direct and inverse transfers...no energy transfer among modes $\dot{E}_j = 0$
- Approximated by exponential spectrum $E_j = e^{-\mu_j}$
(Approximate) QP Solution Evolved Fully Nonlinearly

\[ \ln |a^2_j| \]

\[ \text{Log}_{10} |\Pi_2(x, 0)/\epsilon^2| \]

\[ \text{lnasqj_qp.mpg} \]
Engineered Initial Data (ID)

- Form of ID for fully nonlinear evolutions
- Specify amplitudes $c_j$ of oscillons present in ID:

$$\Pi(x, 0) = 0$$
$$\phi(x, 0) = \Sigma_j (c_j e^j(x))$$

- Examples:
  - Equal energy 2-mode ID: $c_j = \delta^i_j/(3 + 2i) + \delta^k_j/(3 + 2k)$
  - Exponential amplitude ID: $c_j = e^{-\alpha j}$
  - Exponential energy ID: $c_j = e^{-\alpha j}/(3 + 2j)$
Two-mode Stable Solutions

Equal-Energy Two-Mode Initial Data: Modes 0 and 1: \( c_j = e_0/3 + e_1/5 \)
Two-mode Stable Solutions

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Two-mode Stable Solutions
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4.3. The FPU phenomenon: exhibition of the apparent stabilization. Time-averaged harmonic energies $\bar{E}_k$ versus time in log–log scale, for a time interval much larger than the period of the modes. [Left from: Benettin, Carati, Galgani, Giorgilli, 2008]
Other stable solutions
Three-Mode Initial Data: Modes 1, 3, and 8: \( c_j = e_1/10 + e_3 + e_8/10 \)
Other stable solutions

Three-Mode Initial Data: Modes 1, 3, and 8: $c_j = \frac{c_1}{10} + c_3 + \frac{c_8}{10}$
Other (possibly) stable solutions

Three-Mode Initial Data: Modes 1, 3, and 8: \( c_j = e_1 + e_3 + e_8 \)

not one-mode dominant
Other (possibly) stable solutions

Exponential Amplitude: \( c_j = e^{-\alpha j} \) for \( \alpha = 0.575 \)
Take-home Points

- **TTF formalism** most useful, perturbative approach
- System demonstrates both direct- and indirect cascades
- Similarity to FPUT system...identical structure of equations as TTF; resonant and nonresonant regimes
- Stability regions
  - Existence of stable, quasi-periodic solutions
  - ID w/ broadly distributed energy immune frustrated resonance...(regardless of deficiencies in large-$\sigma$ parameterization)
  - Frustrated resonance continues to higher dimensions
  - Frustrated resonance extends to massive case (qualitatively similar)
  - Other, not just one-mode dominant solutions...equal energy, two-mode ID
  - Poses questions about thermalization/equilibration in CFT
Uncanny resemblance!
FPUT and 2-Mode 0-1 Equal Energy FNL

Plot of energy in each mode $E_j(t)$

Figure 2: FPU recurrence for a FPU-$\alpha$ model with $N = 32$ masses and fixed ends. The plot shows the time evolution of the energy (kinetic + potential) $E_k = (\frac{1}{2} A_k^2 + \omega_k^2 A_k^2)/2$ of each of the three lowest normal modes,