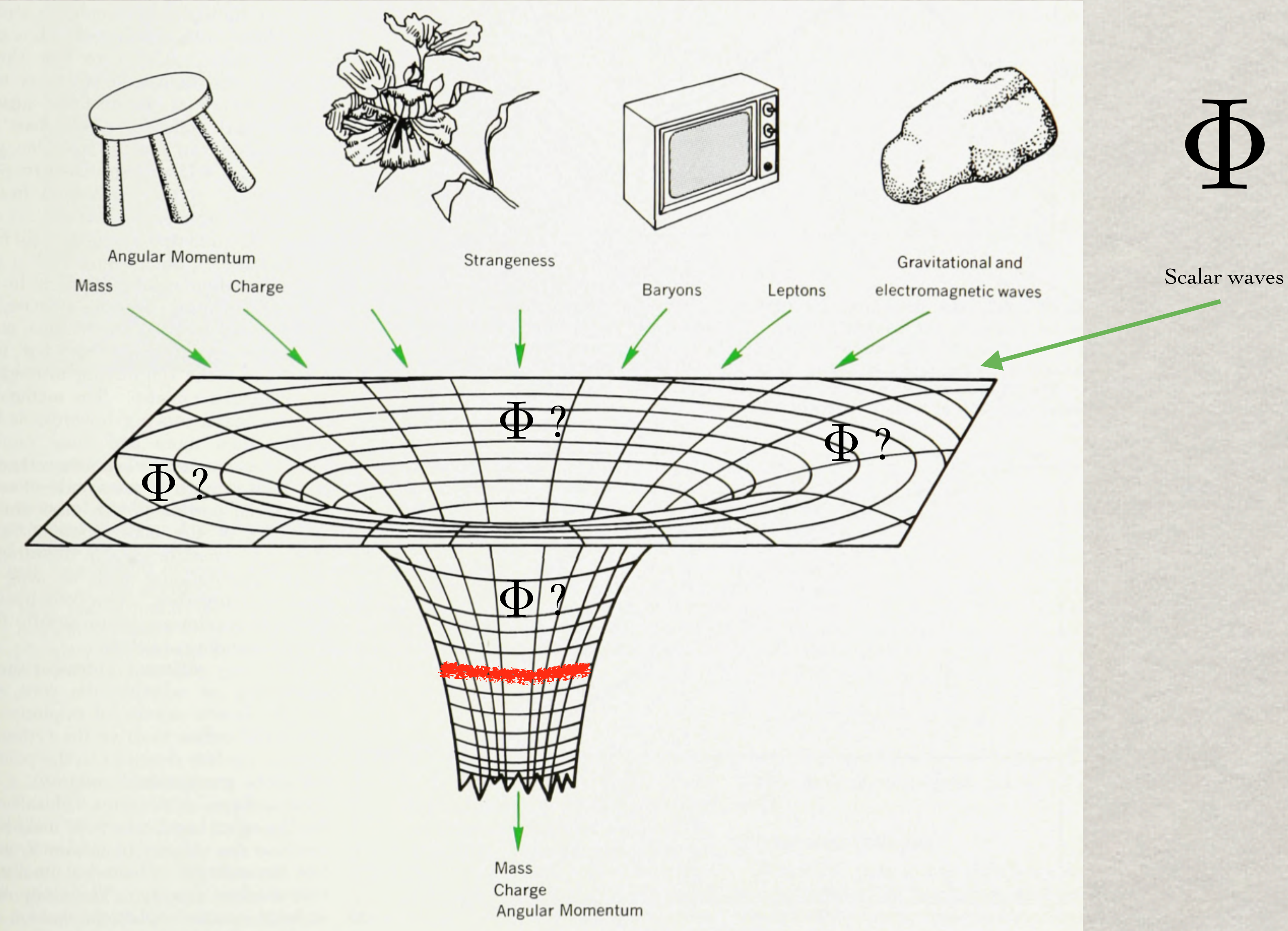
Kerr black holes with scalar hair



C. Herdeiro
Departamento de Física da Universidade de Aveiro, Portugal

New Frontiers in Dynamical Gravity, Cambridge, 24 March 2014

based on arXiv:1403:2757 with E. Radu

The "no-hair" idea

The collapse leads to a black hole endowed with mass and charge and angular momentum but, so far as we can now judge, no other adjustable parameters: "a black hole has no hair." Make one black hole out of matter;

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Ruffini, Wheeler (1971)

Box 33.1 A BLACK HOLE HAS NO "HAIR"

The following theorems come close to proving that *the external gravitational and electromagnetic fields of a stationary black hole* (a black hole that has settled down into its "final" state) *are determined uniquely by the hole's mass M, charge Q, and intrinsic angular momentum S*—i.e., the black hole can have no "hair" (no other independent characteristics). For a detailed review, see Carter (1973).

Misner, Thorne, Wheeler (1973)

Original idea:
collapse leads to equilibrium black holes uniquely determined by M,J,Q -
asymptotically measured quantities subject to a Gauss law
and no other independent characteristics (hair)

Motivated by uniqueness theorems

e.g: Israel 1967, 1968; Carter 1970; Hawking 1972; Robinson 1975, 1977; and many others
Overview: "Four decades of black hole uniqueness theorems" D. Robinson (2004, 2009)

Hairy black hole solutions exist (D=4, asymptotically flat):

Early example: Einstein-Yang-Mills theory

Bizón 1990; Kunzle and Masood-ul-Alam, 1990; Volkov and Galtsov, 1990

Other examples were obtained in: Einstein-Skyrme, Einstein-Yang-Mills-Dilaton, Einstein-Yang-Mills-Higgs, Einstein-non-Abelian-Proca, etc

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Picture of hairy black holes as bound states of BHs with gravitating solitons...
Ashtekar, Corichi and Sudarsky (2001)

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Picture of hairy black holes as bound states of BHs with gravitating solitons...
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...but, apparently, no bound state of *boson stars* with (hairless) BHs.

Boson stars:

Kaup (1968); Ruffini and Bonazzola (1969)
Review: Liebling and Palenzuela (2012)

Einstein-Klein-Gordon theory:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - g^{ab}\partial_a\Phi^*\partial_b\Phi - \mu^2\Phi^*\Phi\right]$$

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Rotating boson stars:
Yoshida and Eriguchi (1997)
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$$\Phi = \phi(r,\theta)e^{i(m\varphi - wt)}$$

Three input parameters: (w,m,n)

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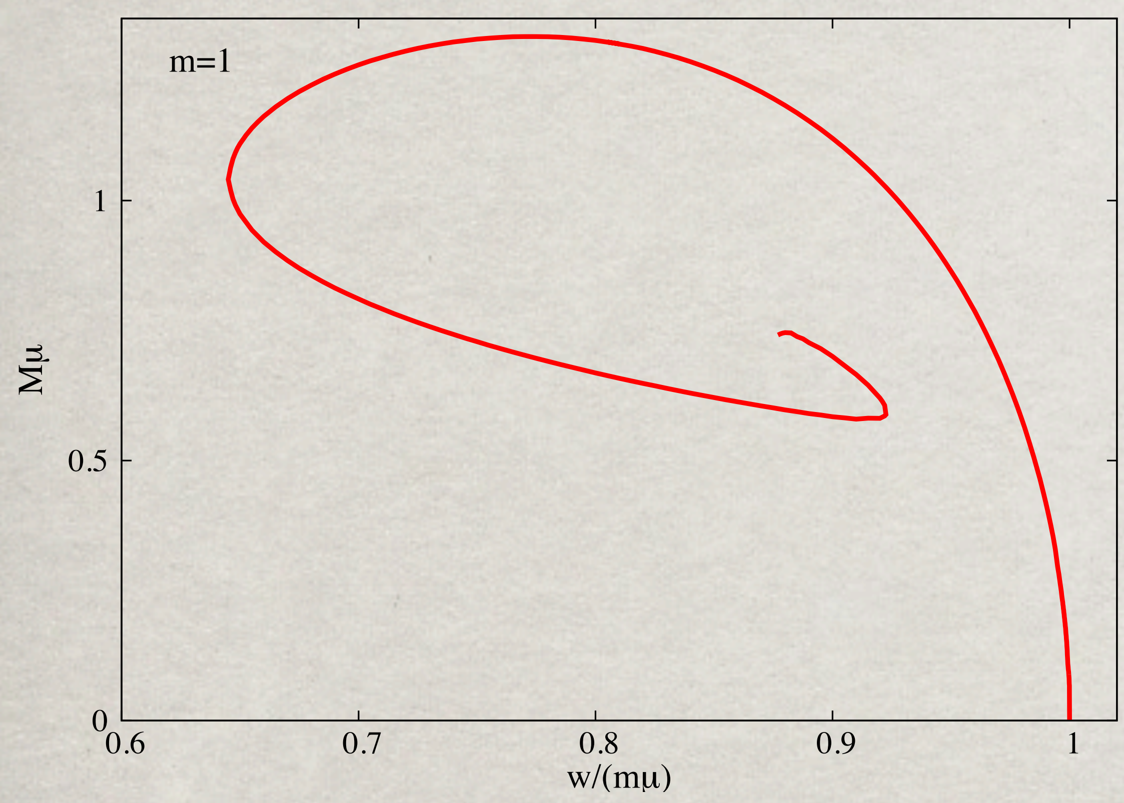
Solutions preserved by a single helicoidal Killing vector field:

$$\frac{\partial}{\partial t} + \frac{w}{m}\frac{\partial}{\partial\varphi}$$

Boson stars phase space (nodeless):



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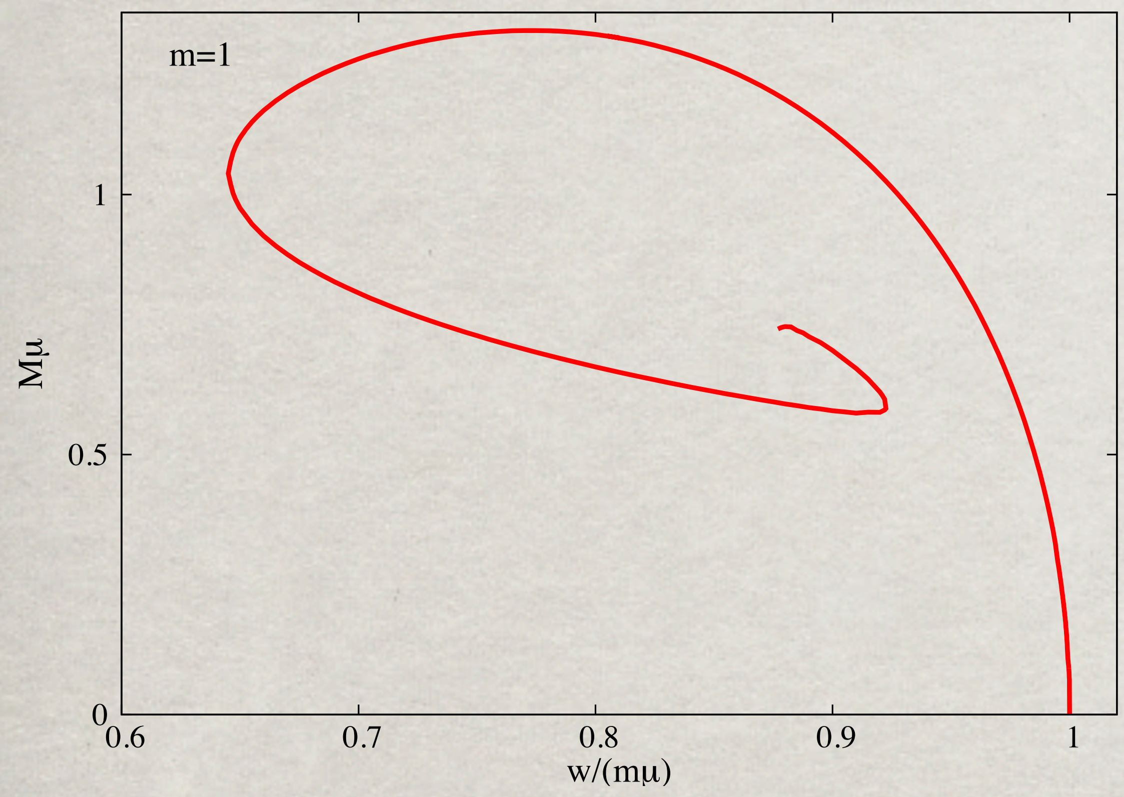

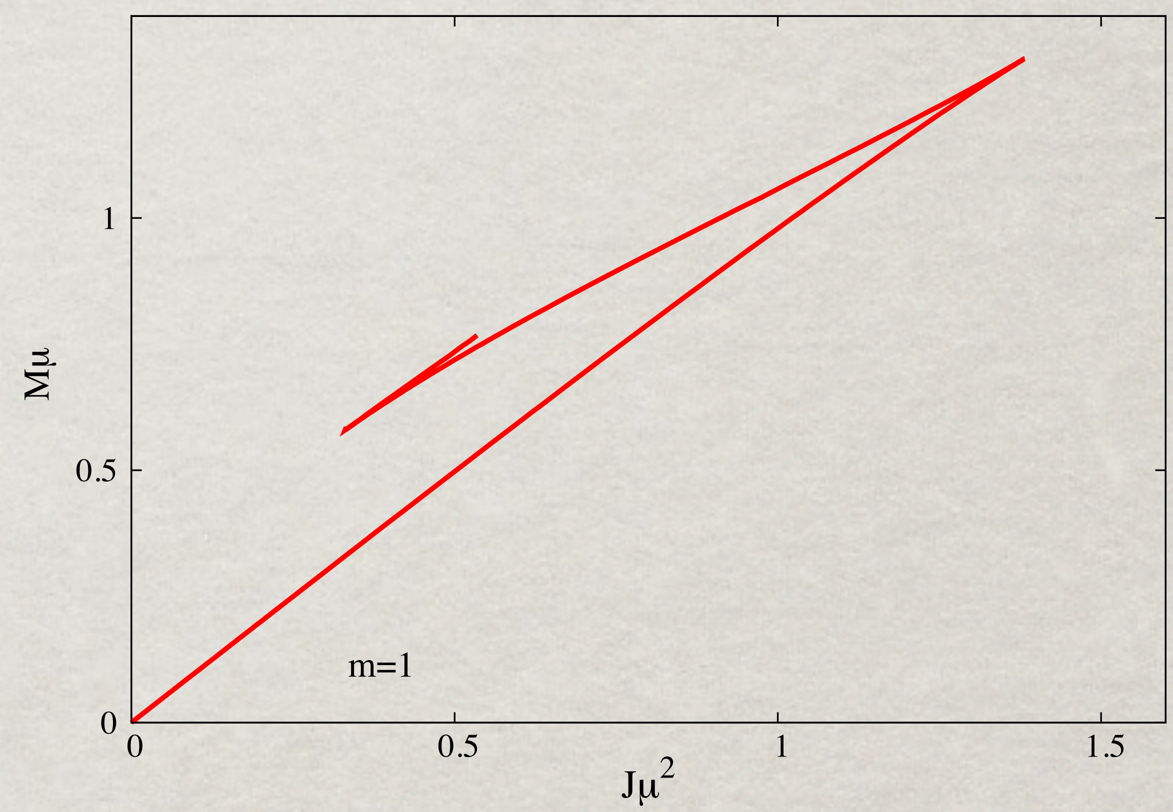

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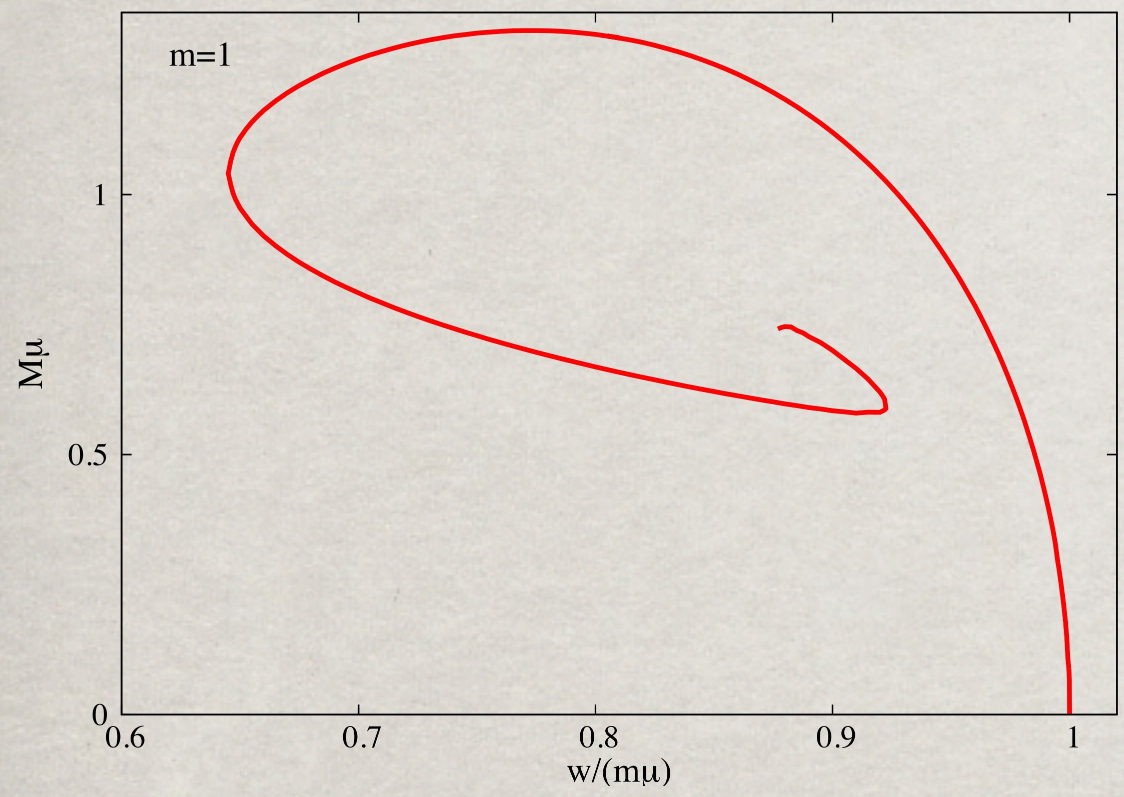
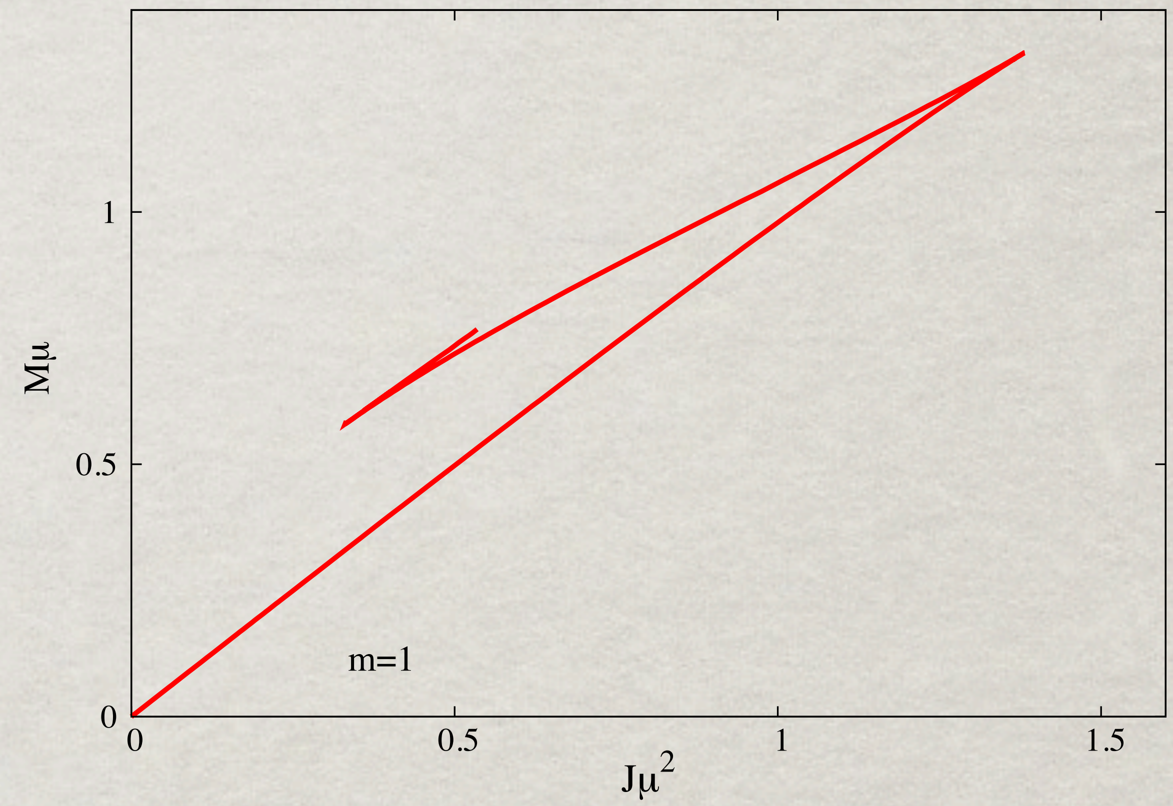




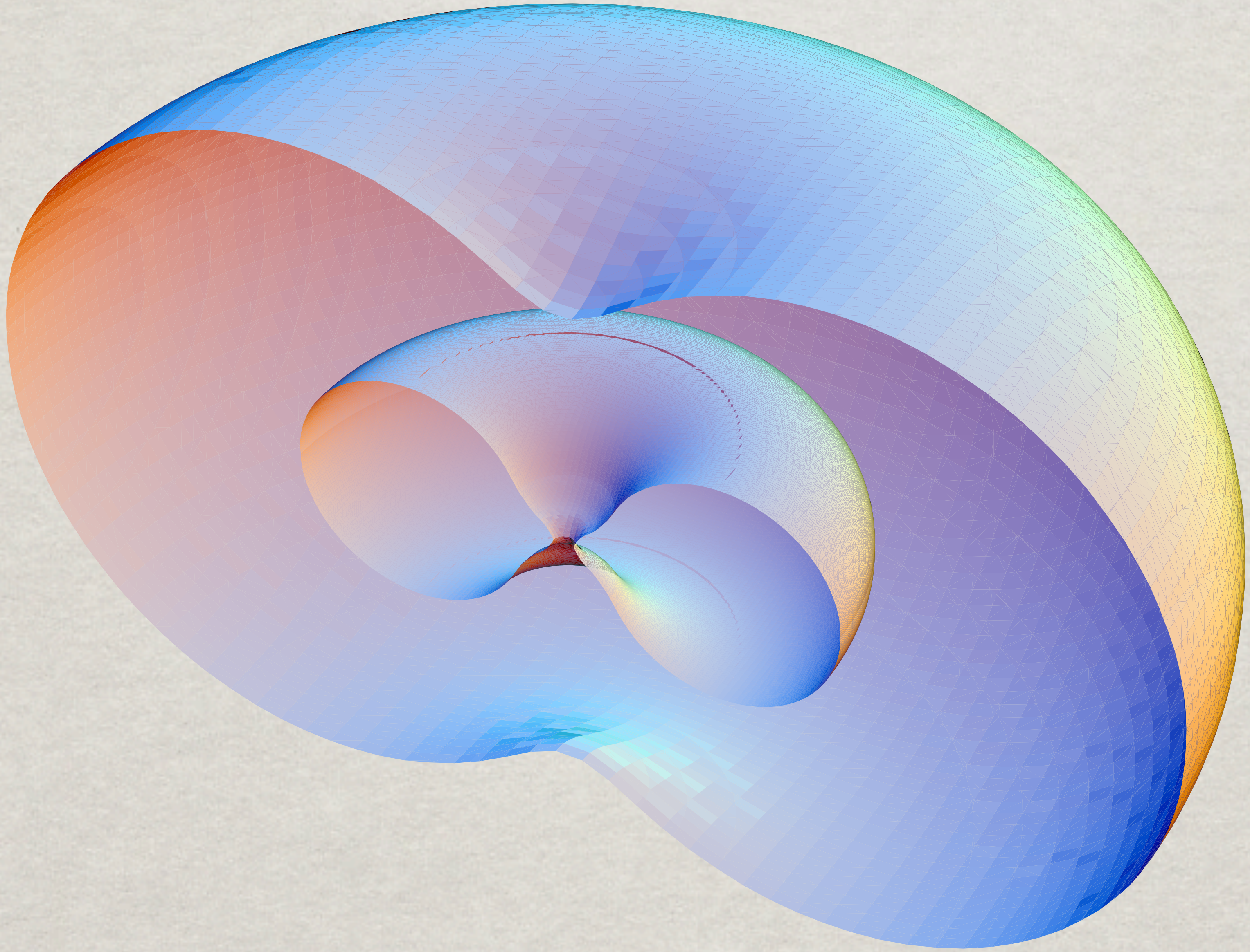
Conserved Noether charge: $$Q = \int_\Sigma dr d\theta d\varphi j^t \sqrt{-g}$$

For rotating boson stars: $$J = mQ$$

Schunck and Mielke (1998)

Convenient parameter: $$q \equiv \frac{mQ}{J}$$

Surfaces of constant scalar energy density



C. H., Radu, 2014 (to appear)

Black holes with scalar hair? (no other fields)

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Chase 1970
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But Kerr has an instability in the presence of a massive scalar field: the **superradiant** instability

Linear analysis: Klein-Gordon equation in Kerr

$$\Box\Phi = \mu^2\Phi \qquad\qquad \Phi = e^{-iwt}e^{im\varphi}S_{\ell m}(\theta)R_{\ell m}(r)$$

Radial Teukolsky equation: Teukolsky (1972)

$$\frac{dR_{\ell m}}{dr} = \left(a^2w^2 - 2maw + \mu^2r^2 + A_{\ell m} + \frac{K^2}{\Delta}\right)R_{\ell m}$$

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$$\omega = \omega_R + i\omega_I$$

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$w_I = 0$ if $w = w_c$ true bound states: *clouds*

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Press and Teukolsky (1972)

Klein-Gordon (linear) clouds around Kerr:

Damour, Deruelle and Ruffini (1976); Zouros and Eardley (1979); Detweiler (1980); Hod 2012; (...); Yakov Shilapentokh-Rothman (2014)

Clouds for extremal Kerr: discrete set labelled by (n,l,m) subject to one quantization condition which yields BH mass. Hod (2012)

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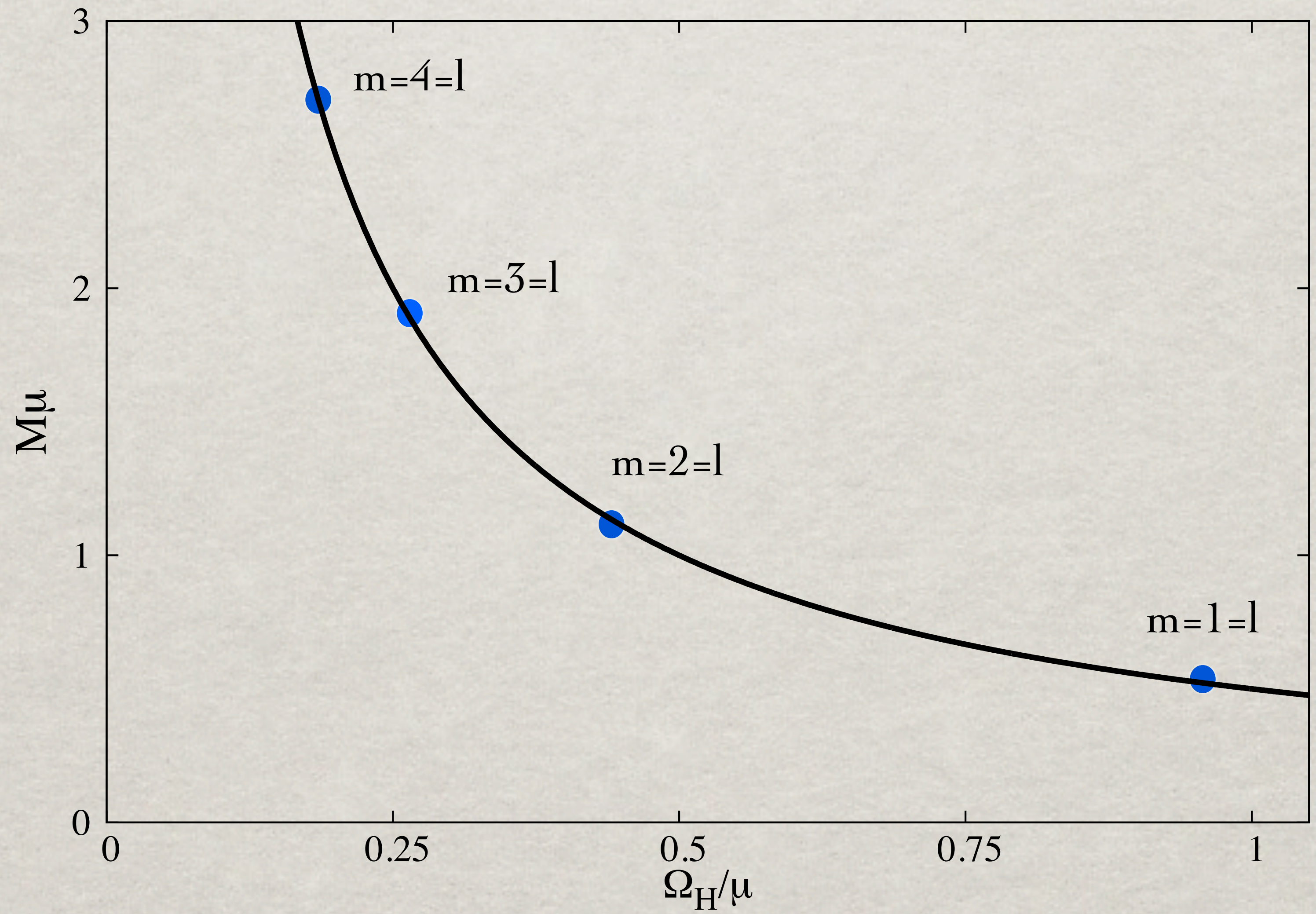

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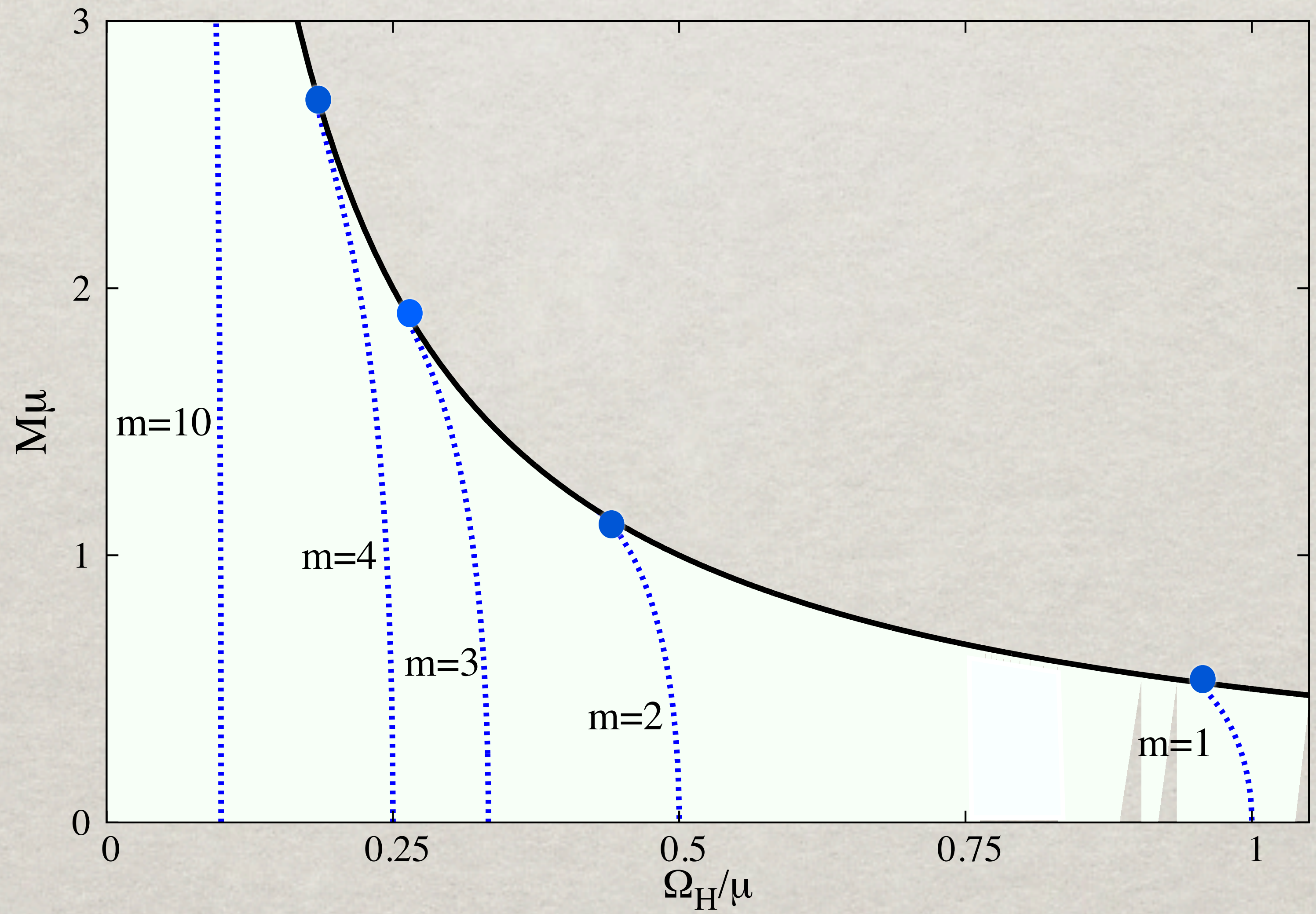

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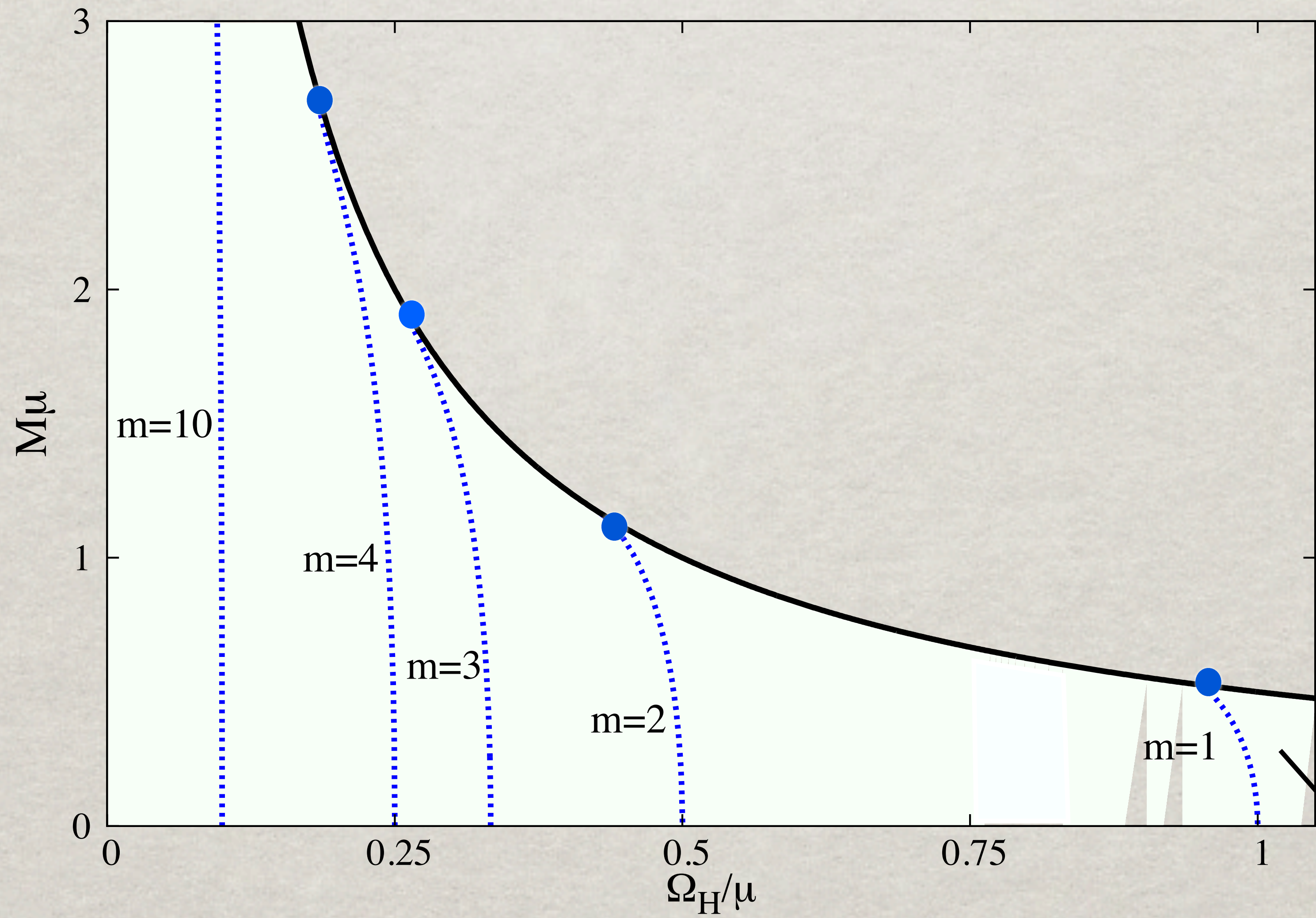

Unstable Kerr solutions against m=1

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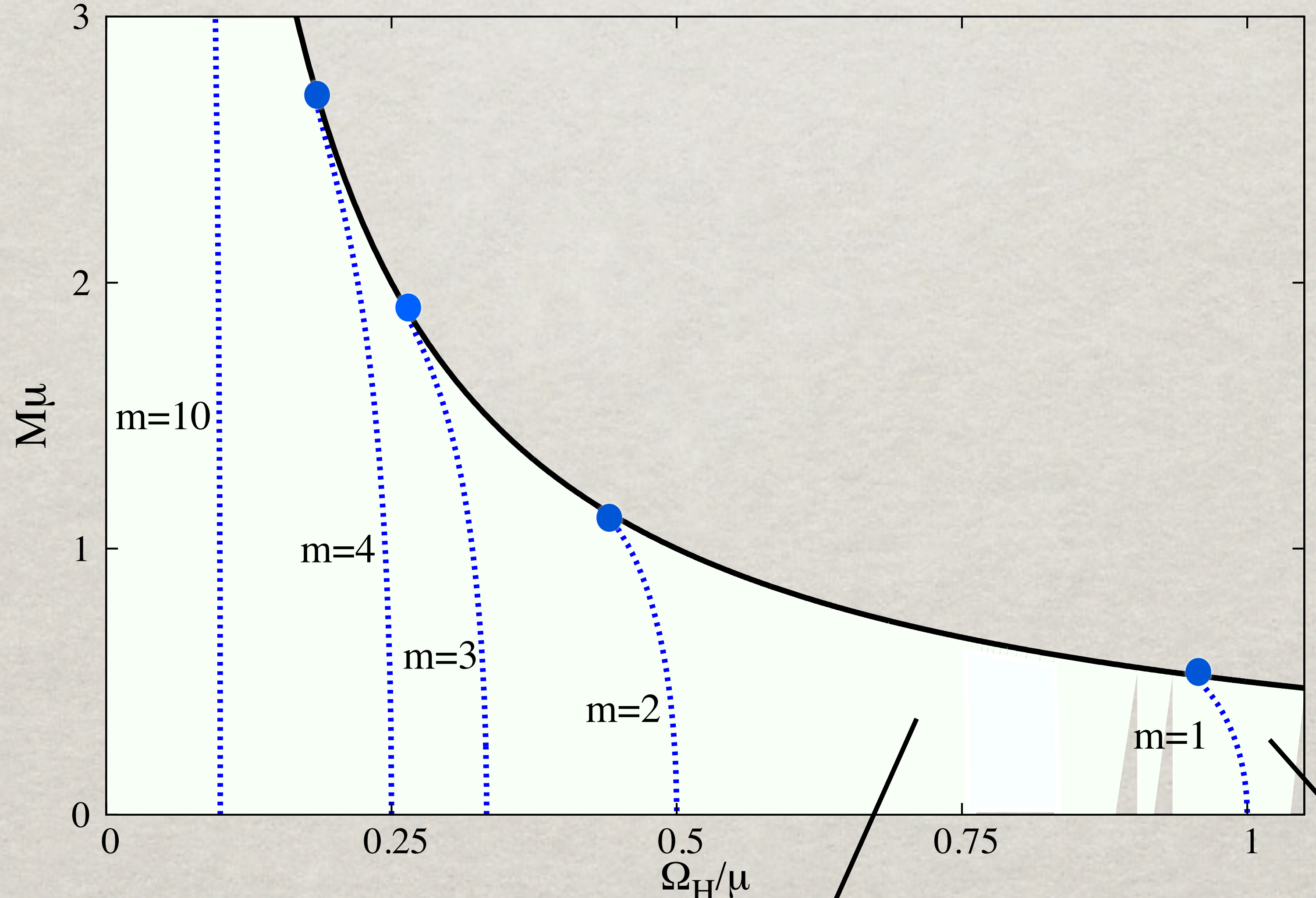

Stable Kerr solutions against m=1

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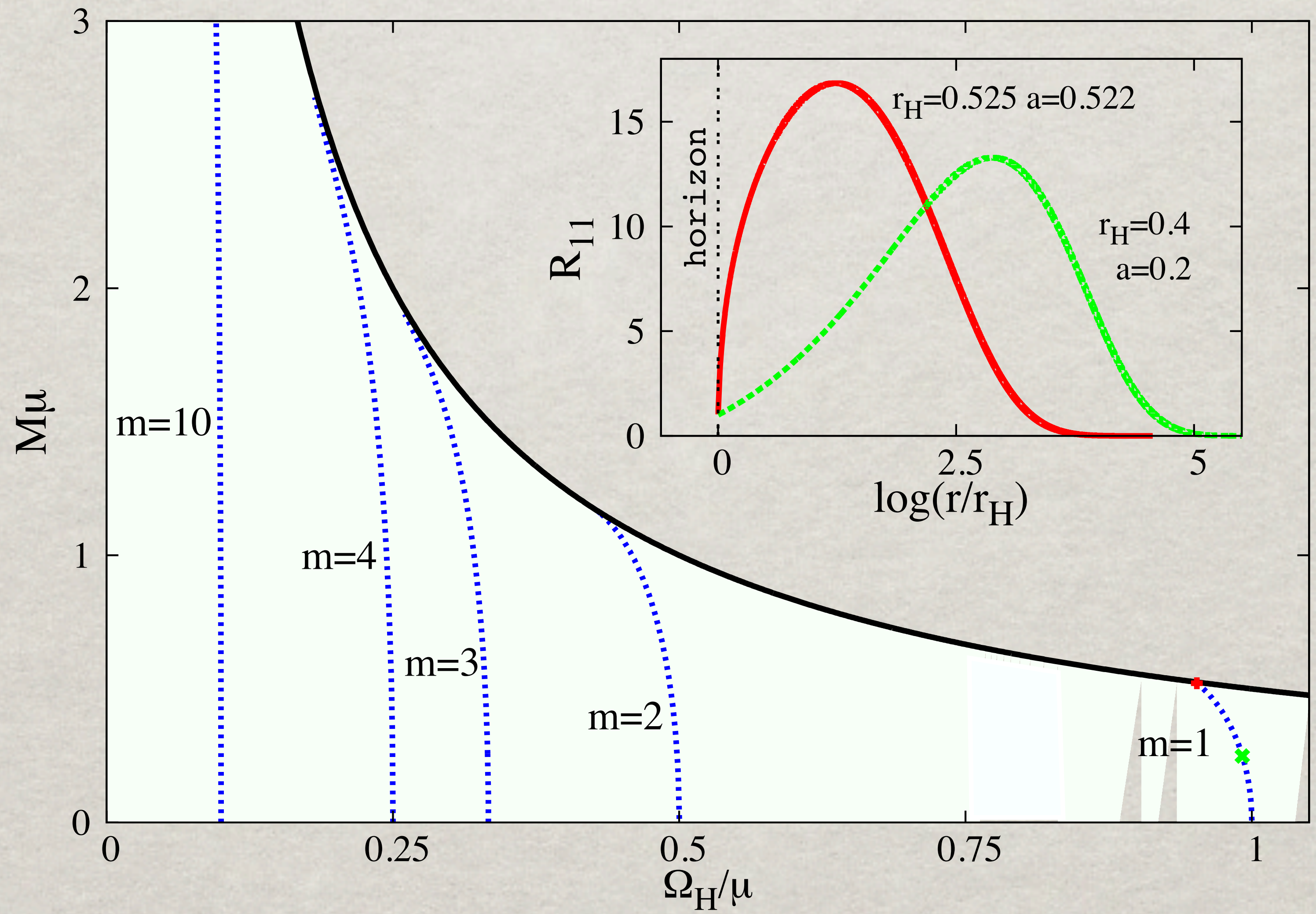
Clouds radial profile



Einstein Klein-Gordon: non-linear setup

Ansatz:

$$ds^2 = -e^{2F_0(r,\theta)} N dt^2 + e^{2F_1(r,\theta)} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \, (d\varphi - W(r,\theta) dt)^2 \qquad N = 1 - \frac{r_H}{r}$$

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take: $w < \mu$

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Near the horizon:

$$x \equiv \sqrt{r^2 - r_H^2}$$

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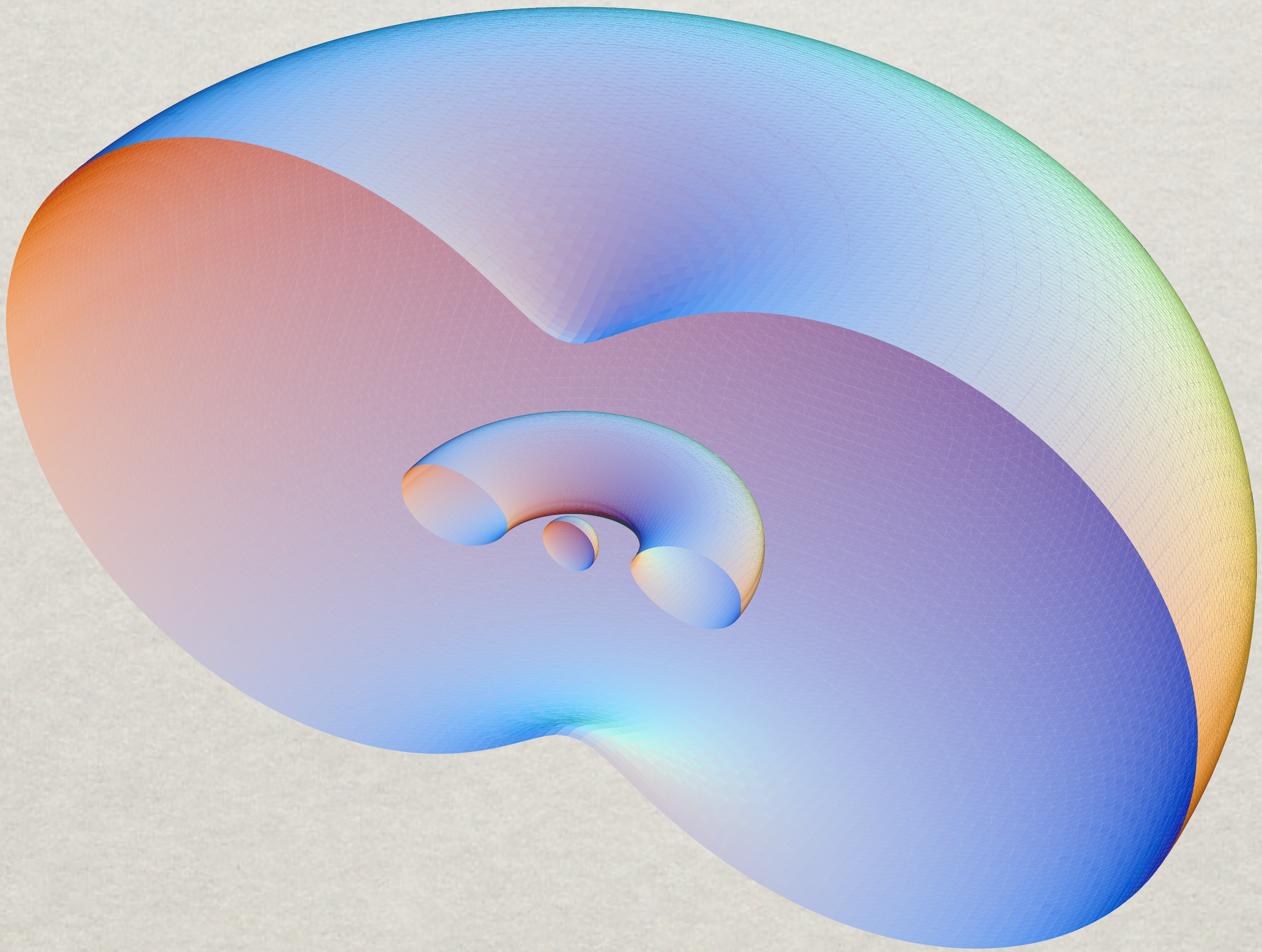
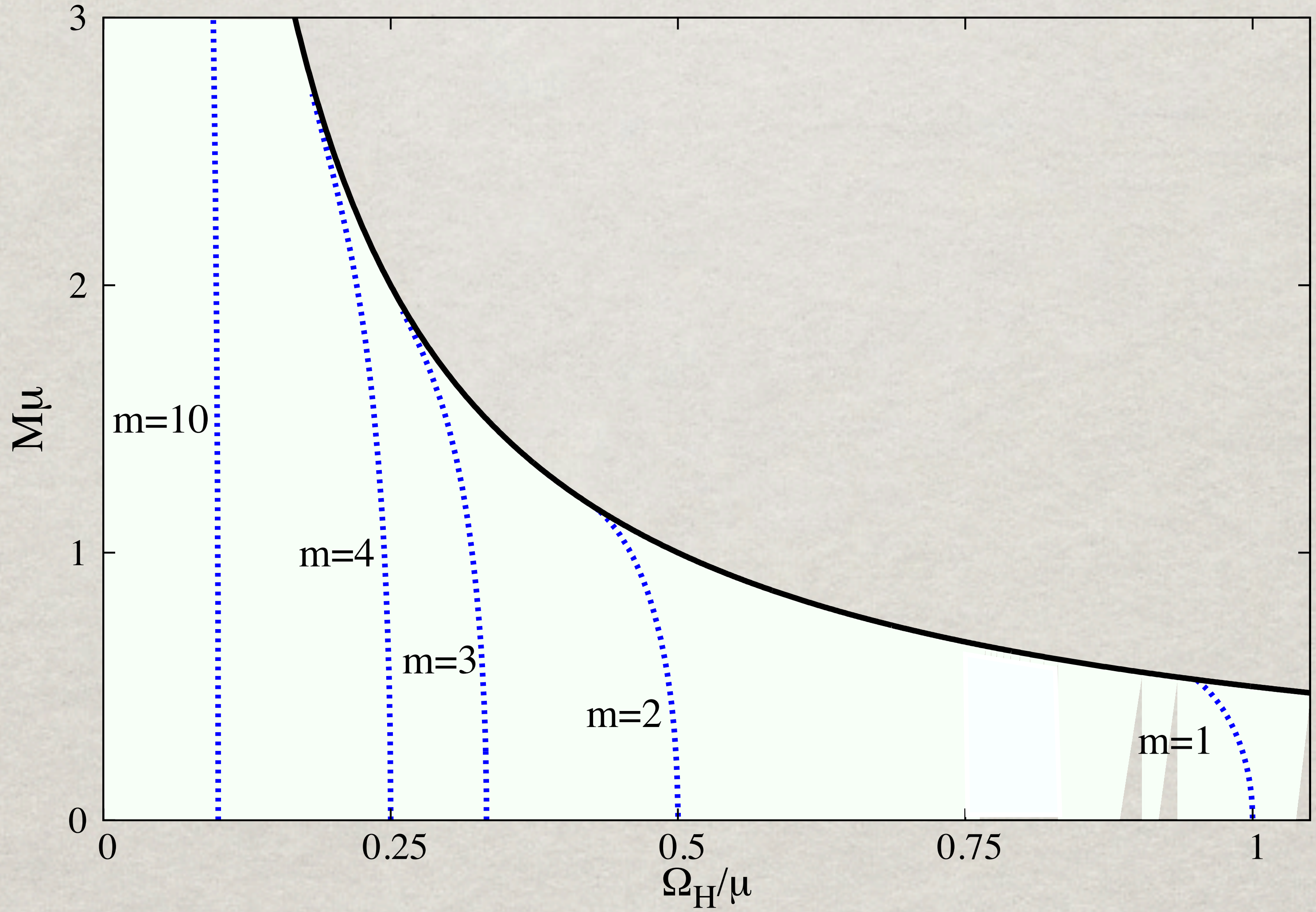
Four input parameters: m, w, r_H, n

Hairy black holes



C. H., Radu, 2014 (to appear)

Hairy black holes phase space



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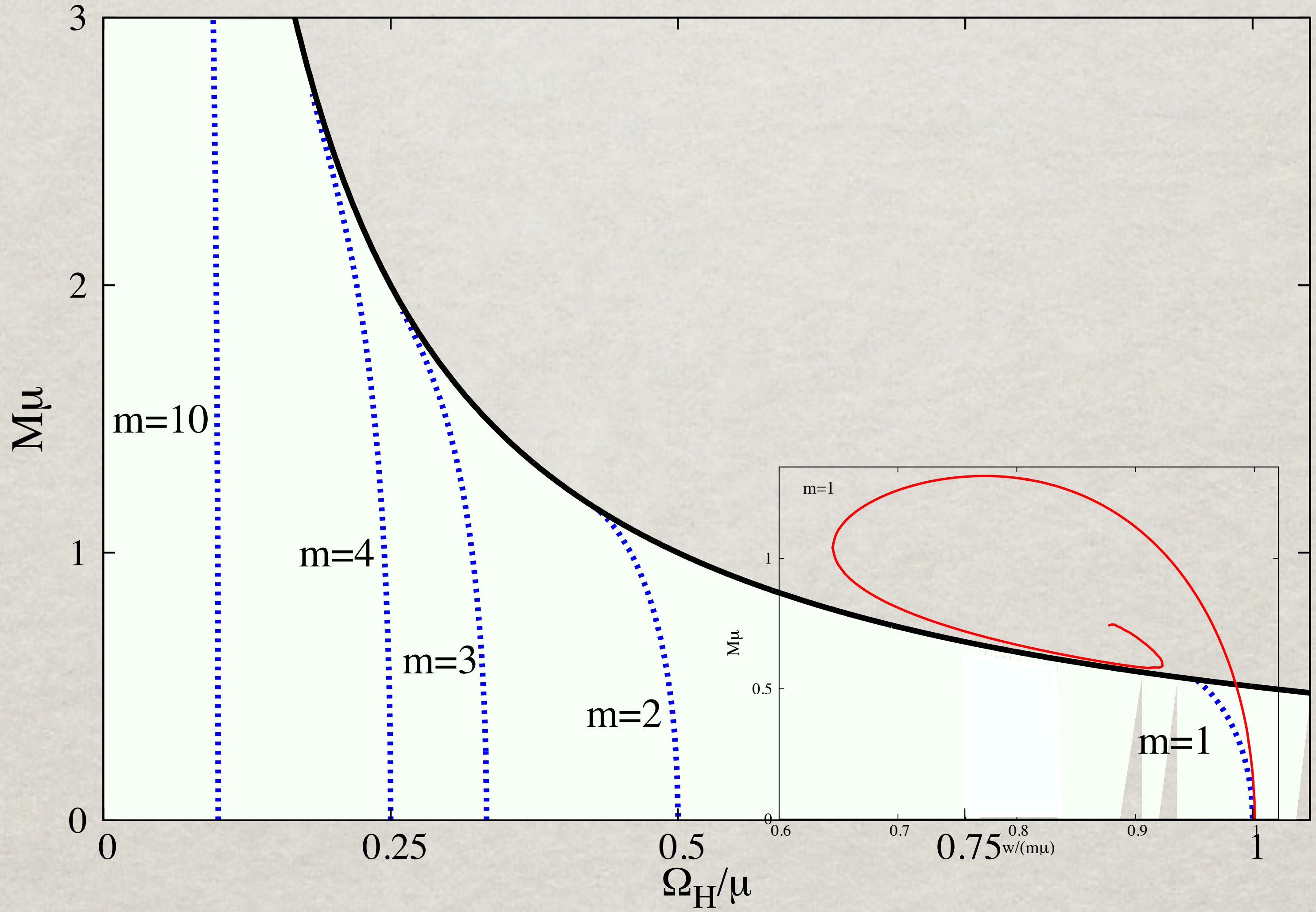

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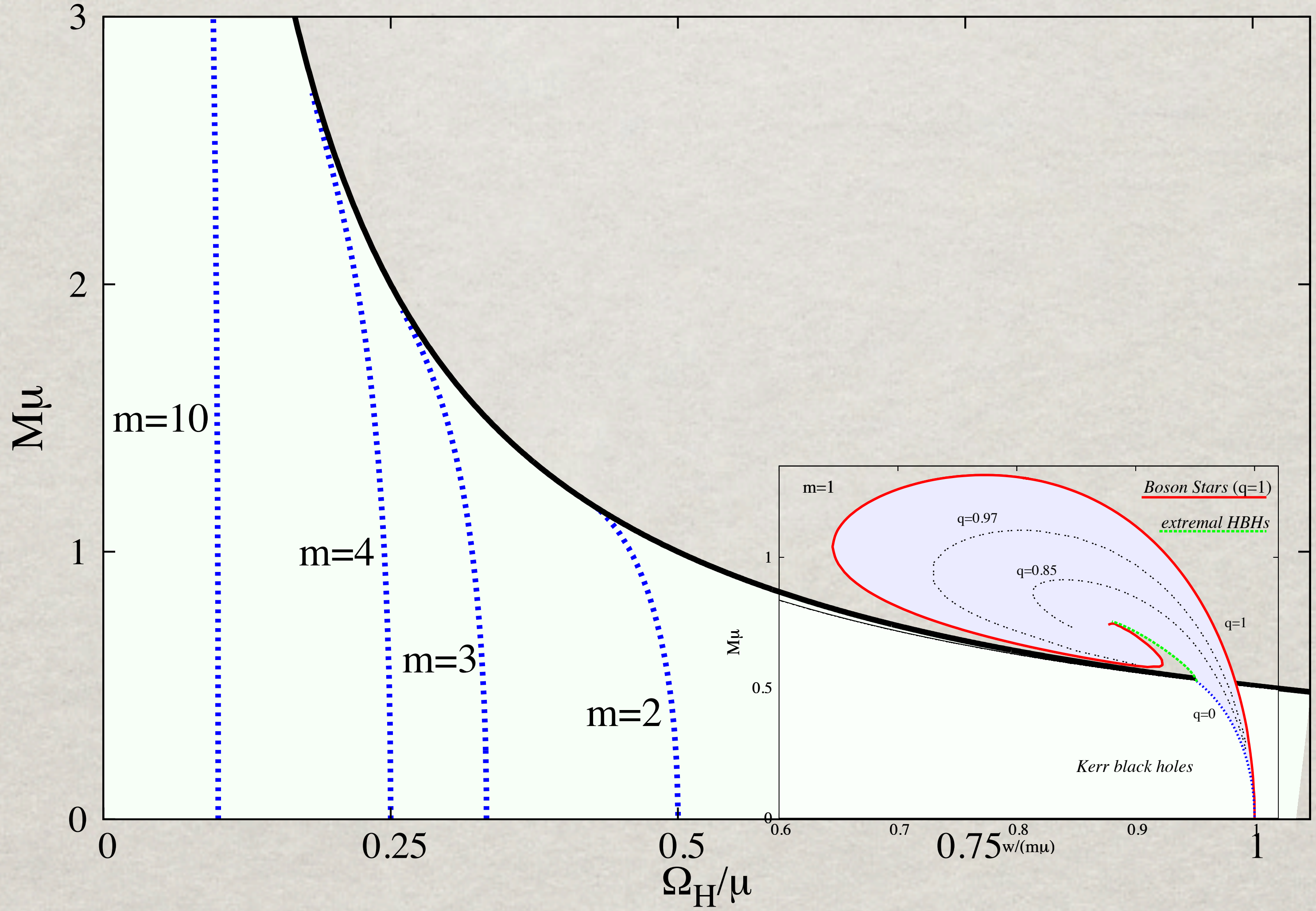

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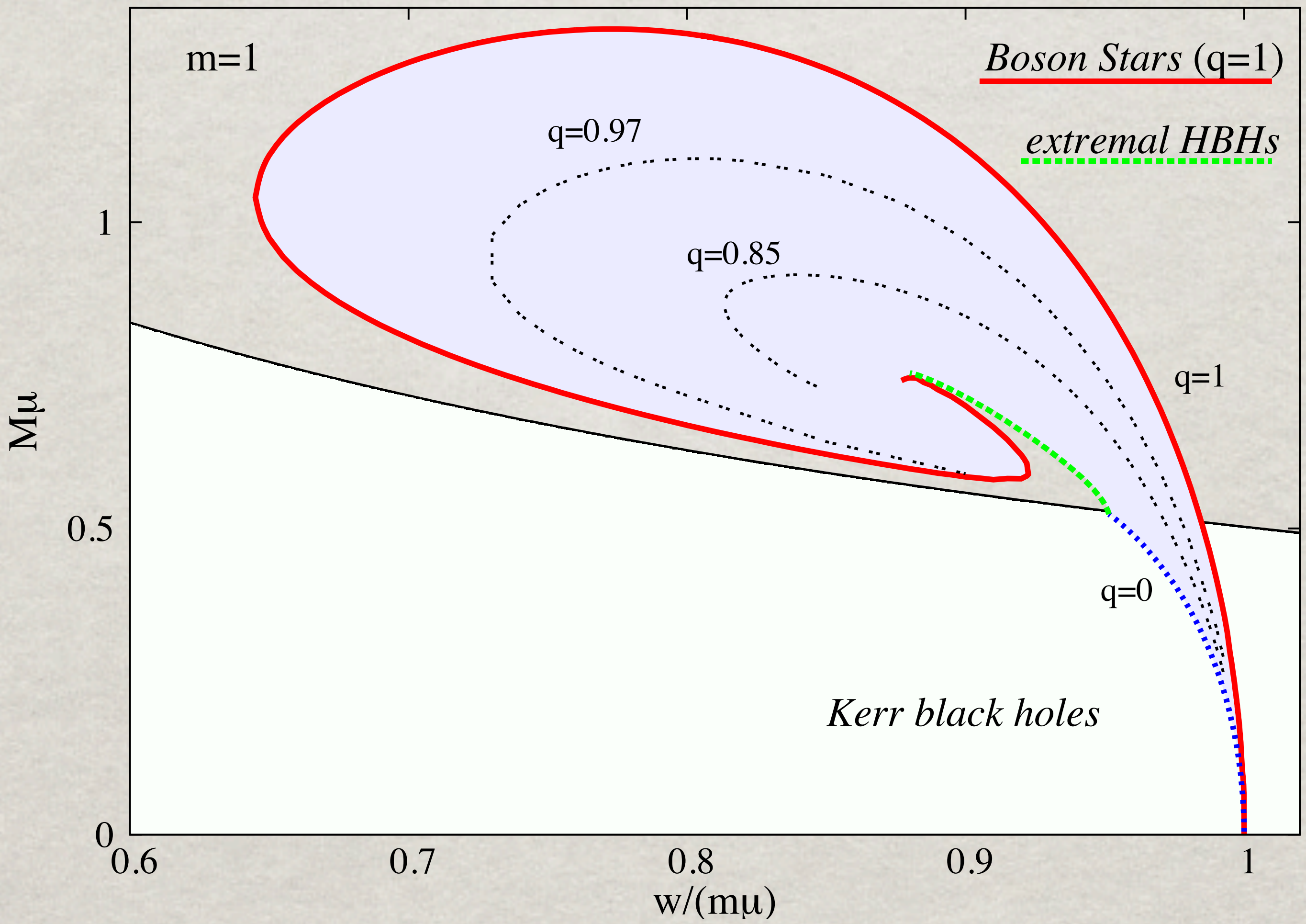

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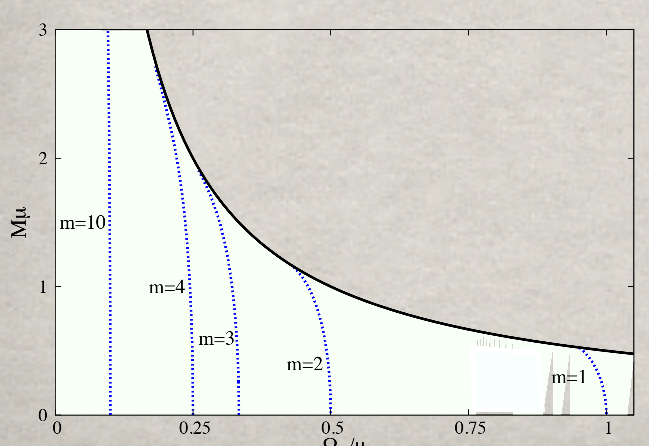

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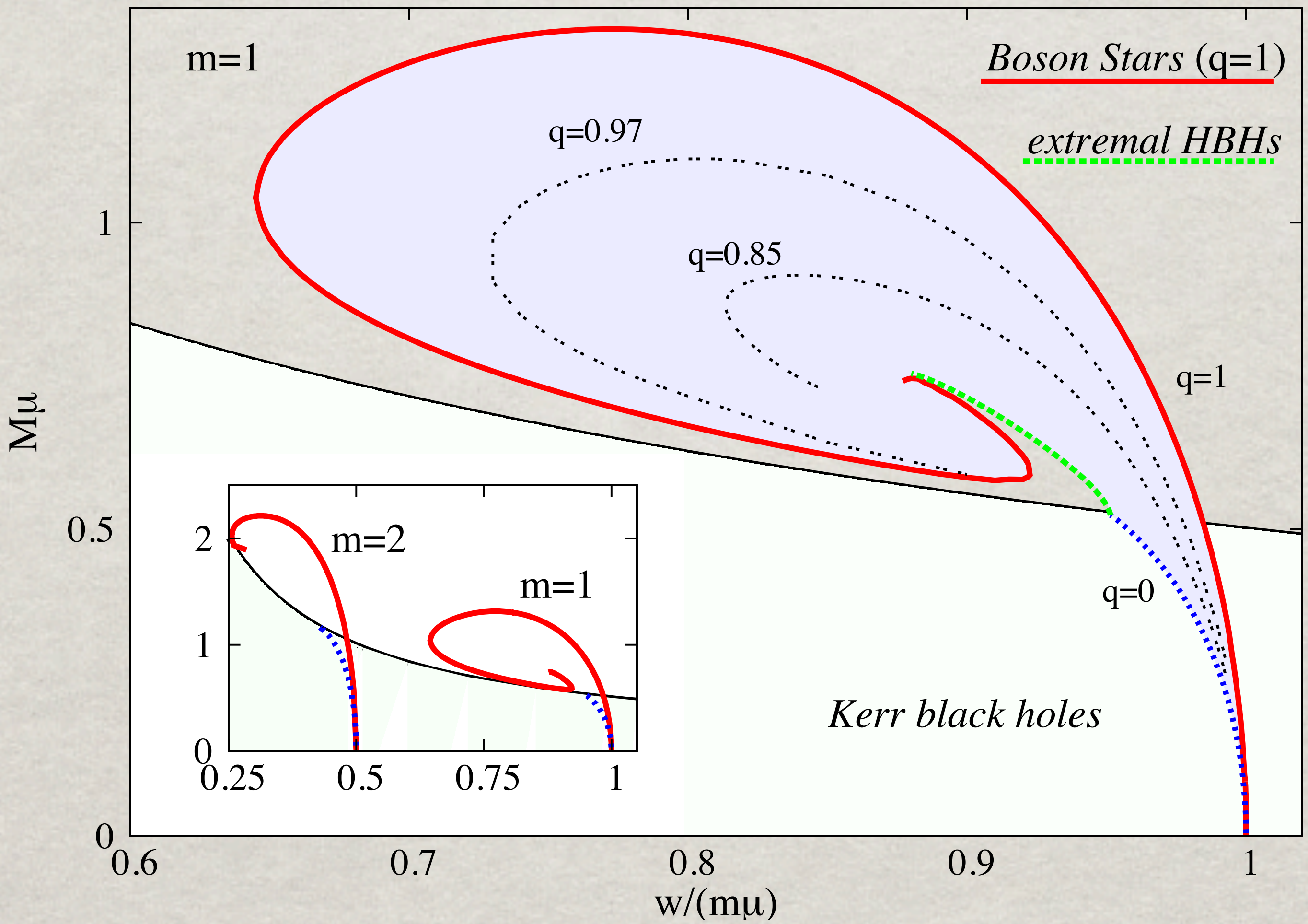

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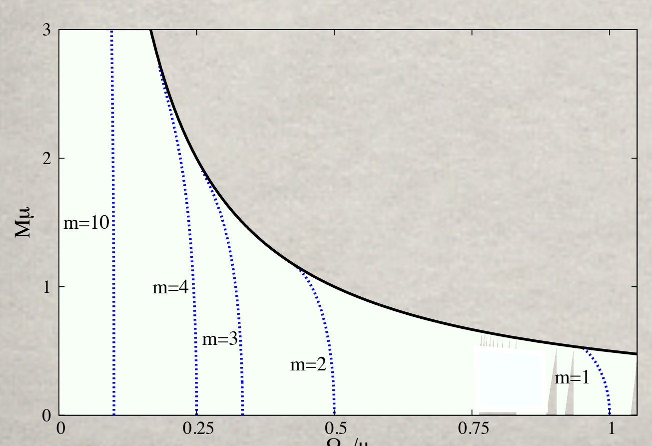

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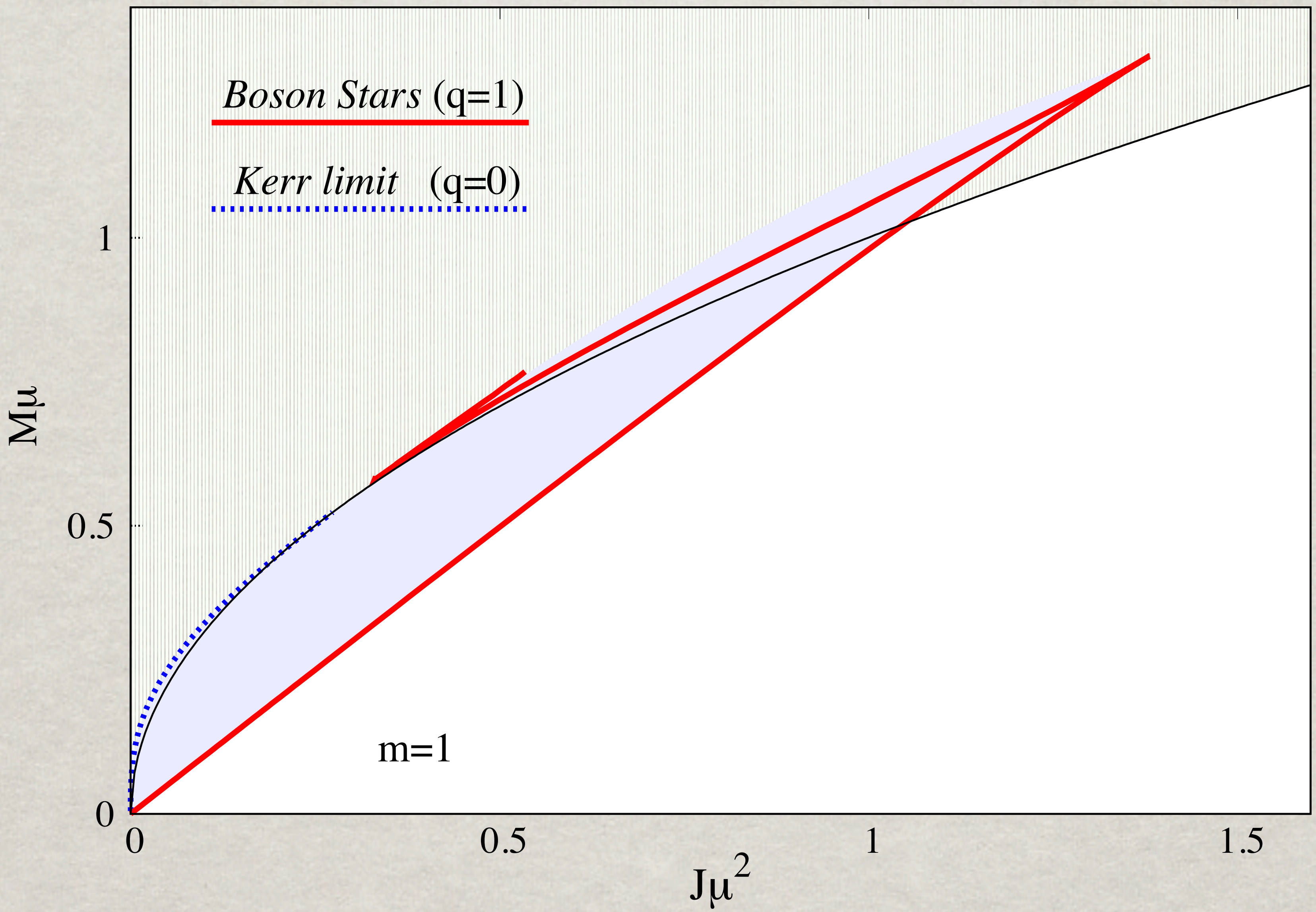

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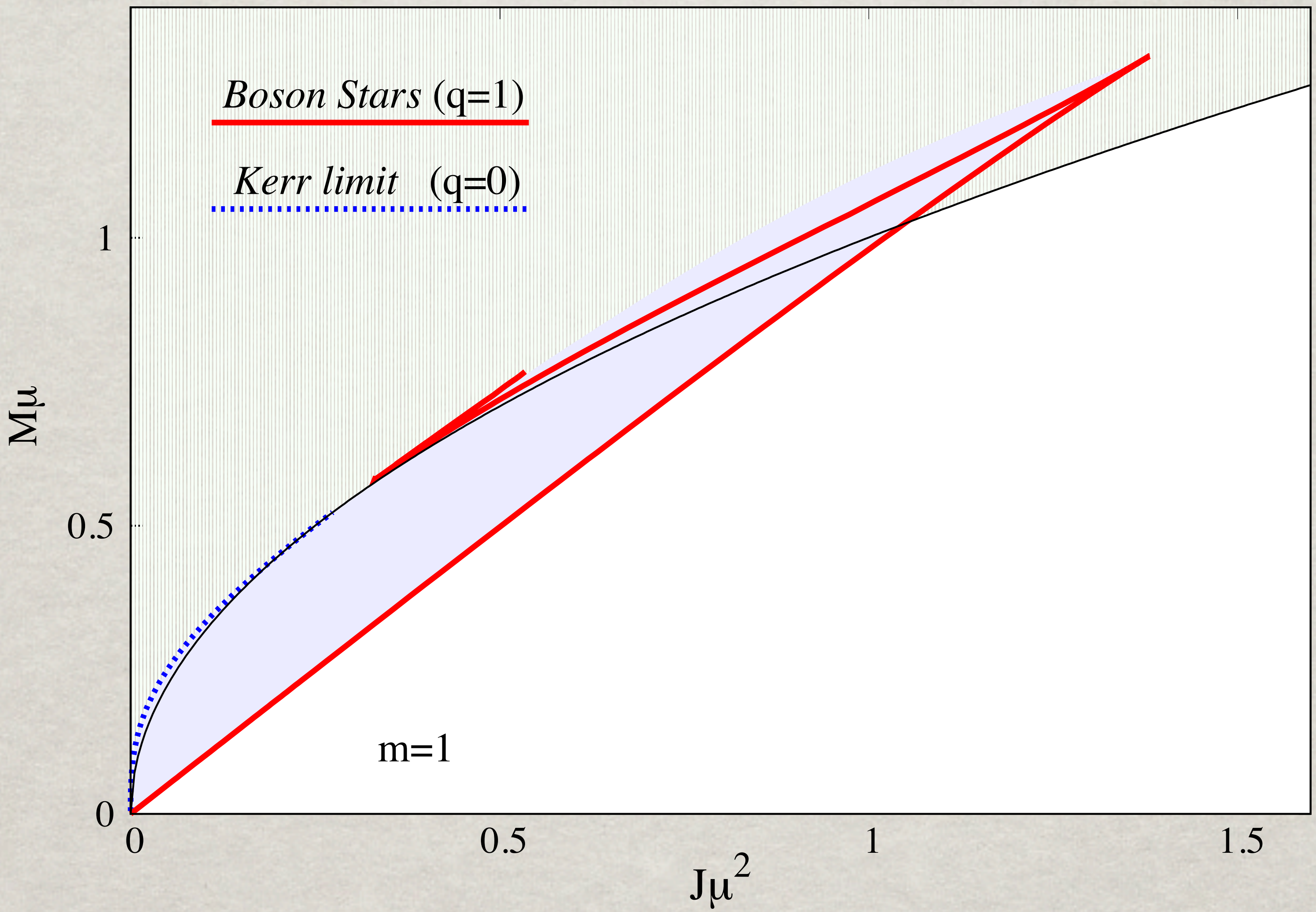
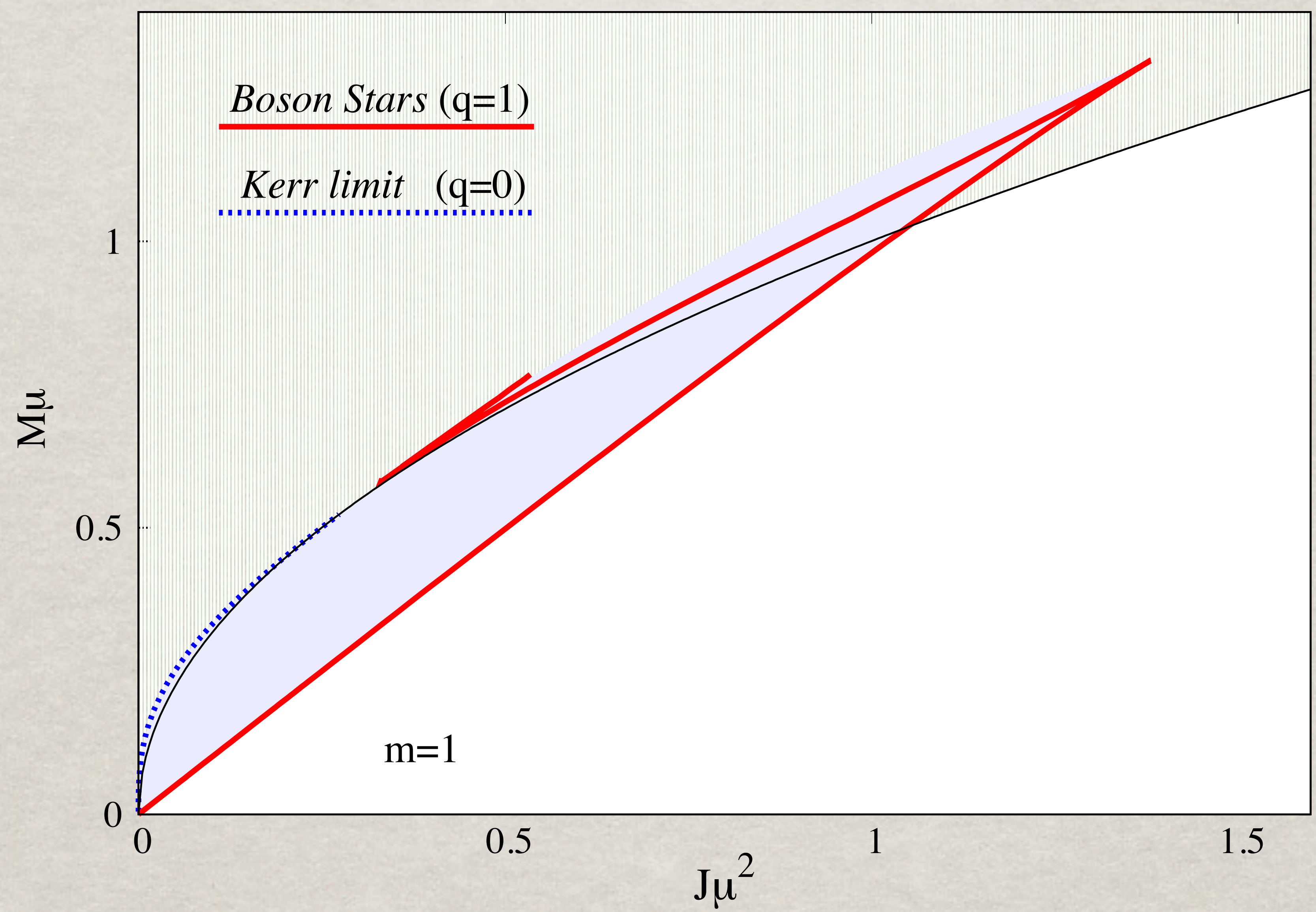


- Can violate Kerr bound

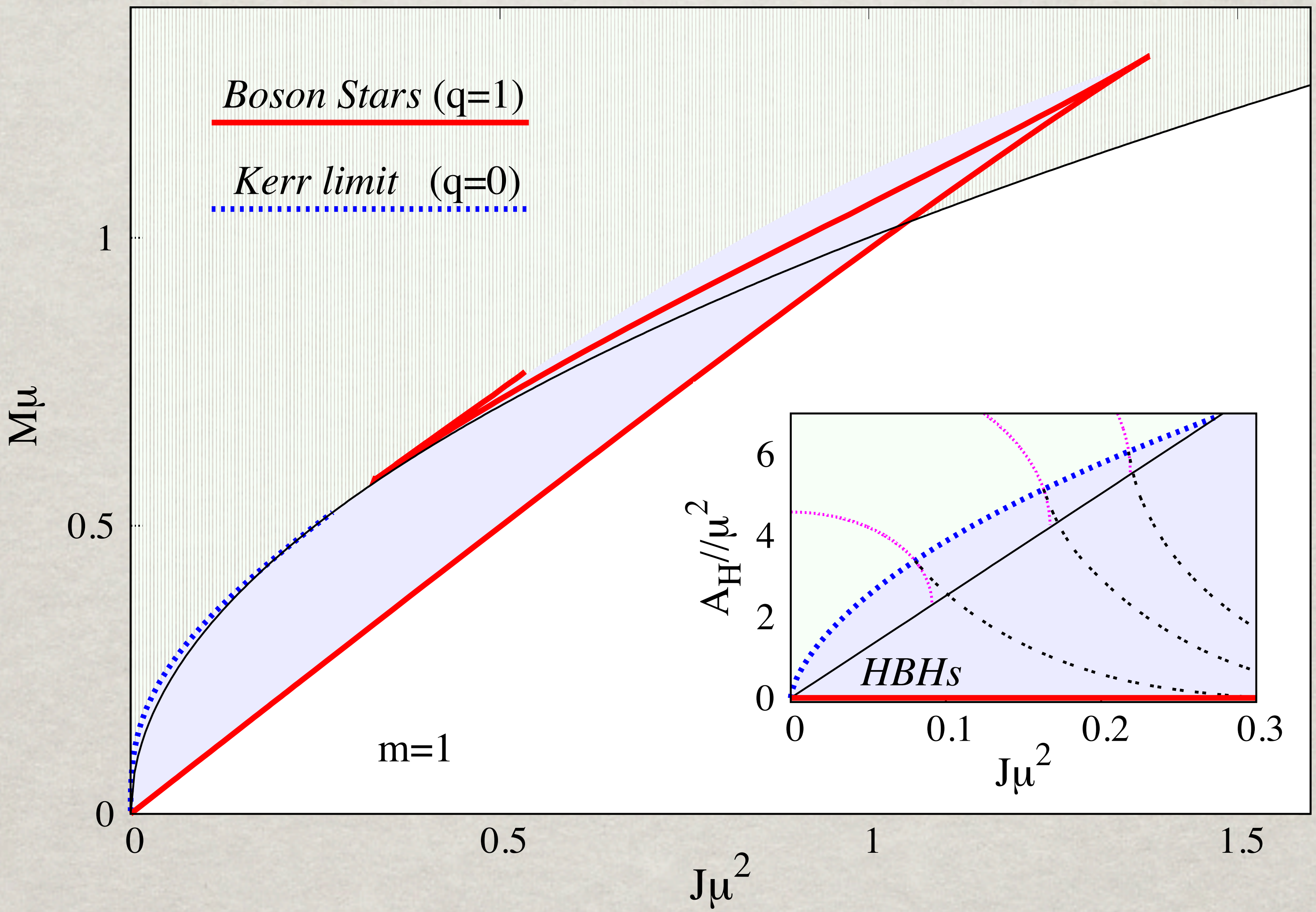
Hairy black holes phase space



- Can violate Kerr bound

- Non-uniqueness (different solutions for same M,J); but degeneracy raised with q

Hairy black holes phase space

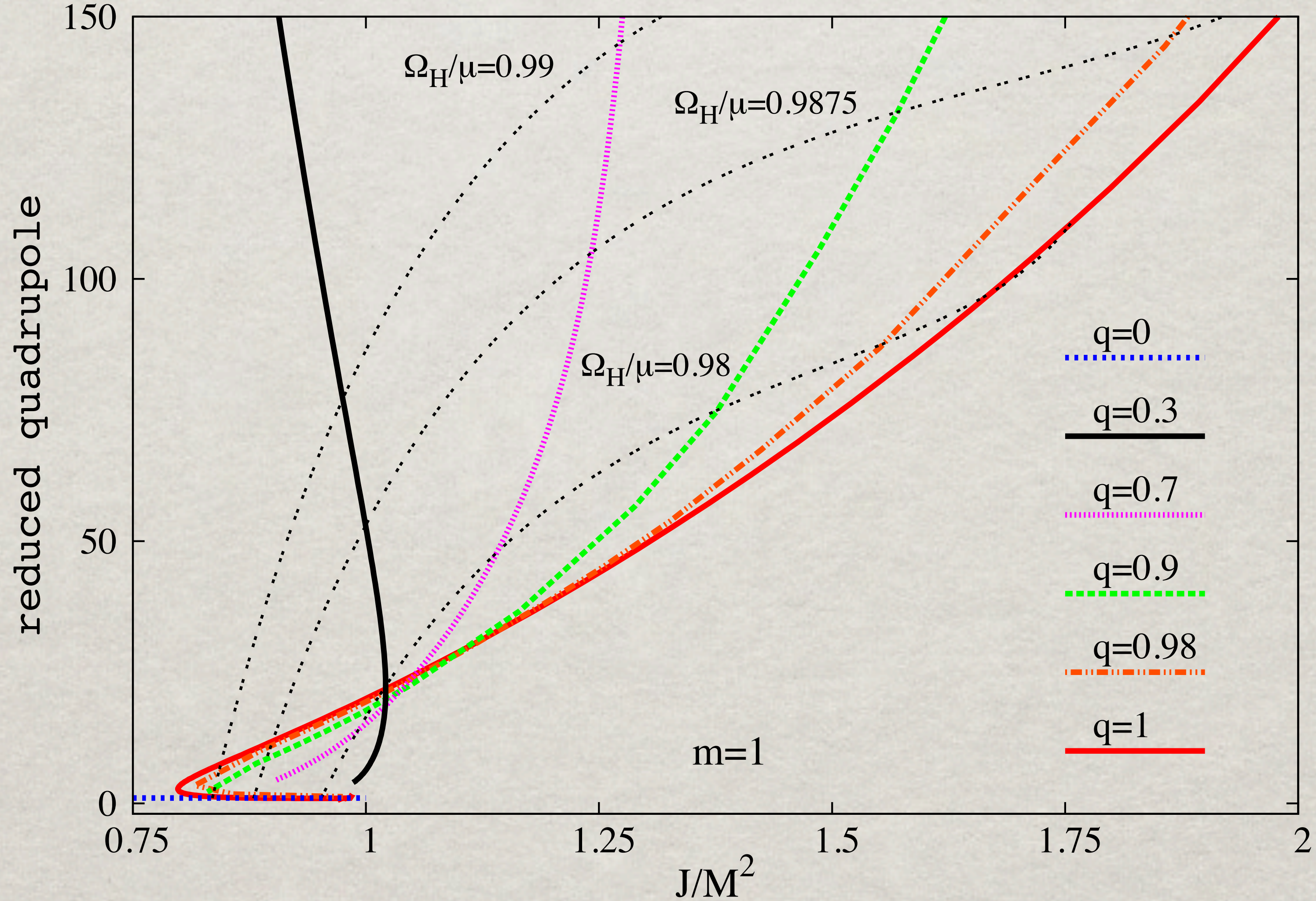


- Entropically favoured;

Hairy black holes are more *star-like*

Geroch-Hansen quadrupole moment:

Geroch (1970); Hansen (1974); Pappas and Apostolatos (2012)



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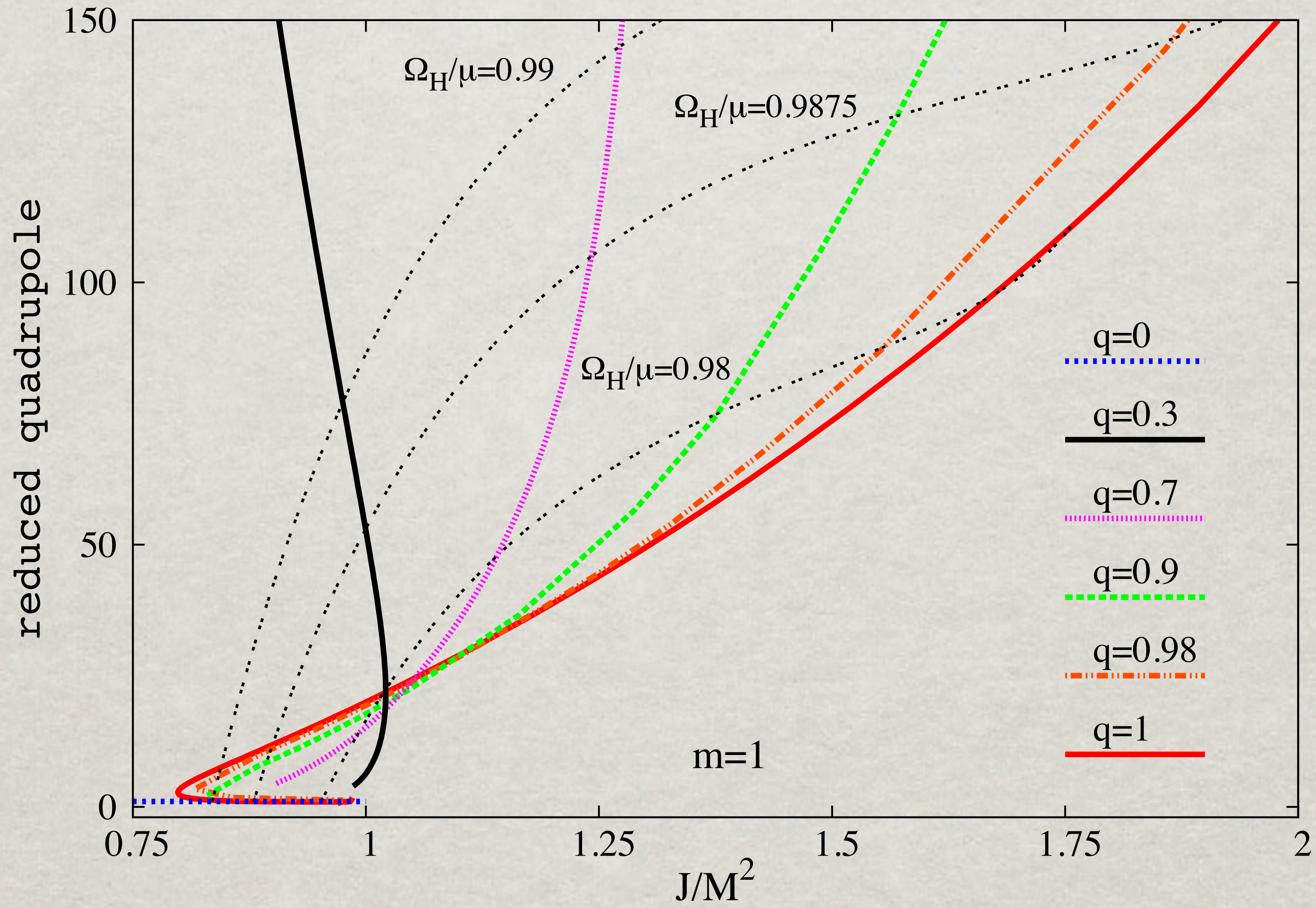

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Similar considerable deviations occur for the orbital frequency at the ISCO.

Final remarks:

Hairy black holes interpolate between Kerr and boson stars.
Two viewpoints:

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General mechanism?
A (hairless) BH which is afflicted by the superradiant instability of a given field must allow a hairy generalization with that field.

E.g. in AdS, the first BH example with a single KVF. Dias, Horowitz and Santos (2011)

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Hairy black holes interpolate between Kerr and boson stars.
Two viewpoints:

Boson stars: one can add a BH for spinning configurations
Kerr black holes: branching towards a new family of solutions due to superradiant instability.

General mechanism?
A (hairless) BH which is afflicted by the superradiant instability of a given field must allow a hairy generalization with that field.

E.g. in AdS, the first BH example with a single KVF. Dias, Horowitz and Santos (2011)

Stability ?

Thank you for your attention!