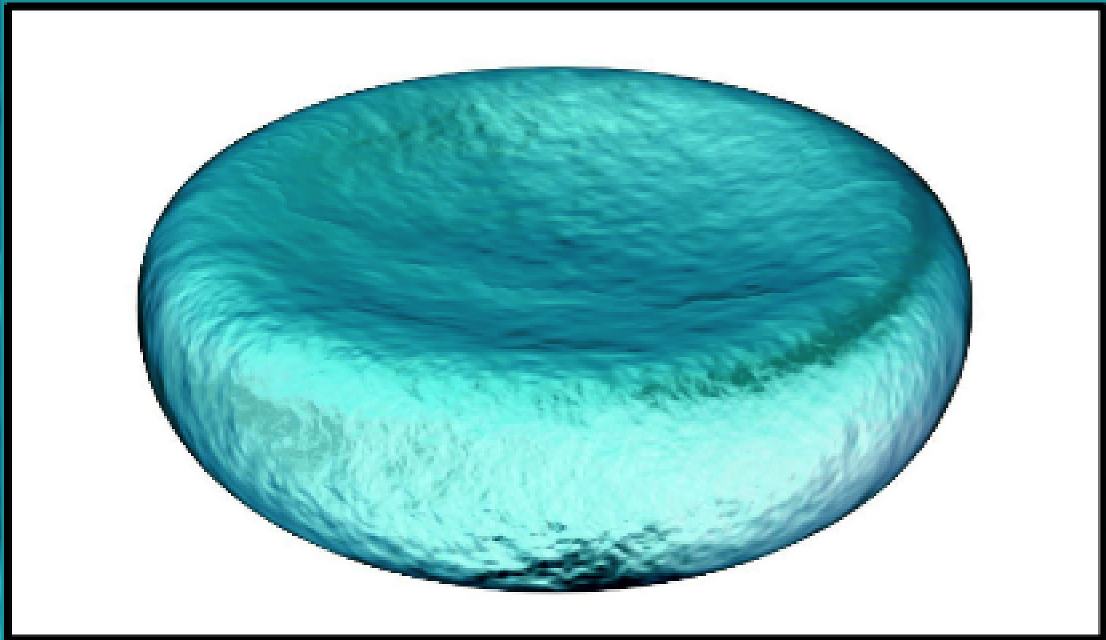
Elastic Expansion, Biophysical (Mem)-branes and Higher-order Blackfold Approach



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based on:

J.Armas, arXiv:1304.7773 (JHEP)
J. Armas, arXiv:1312.0597
JA & T. Harmark, arXiv:1402.6330, arXiv:1404.xxxx

Motivations (I):

. Perturbative construction of higher-dimensional black holes

. Effective action for higher-dimensional black holes (blackfolds)

. Generic perturbations of black branes (viscous + elastic)

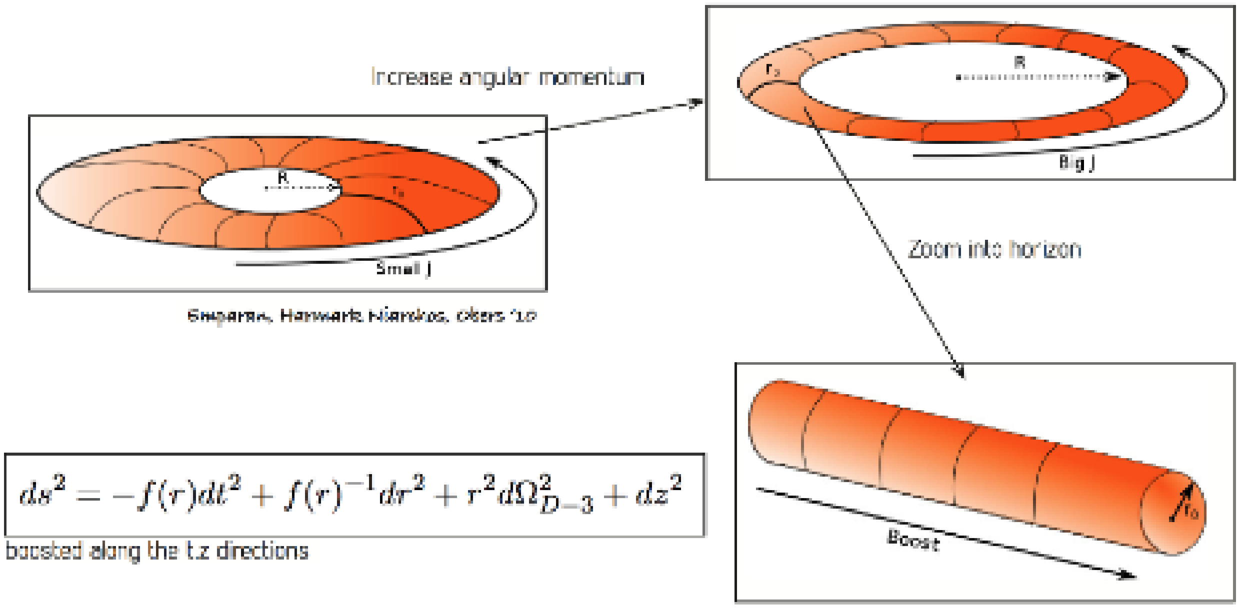
. More general theories of hydrodynamics (confined fluids)

. Fluid membranes | Cellular membranes

. AdS/CFT at finite temperature

Elastic matter (I):

An observation:

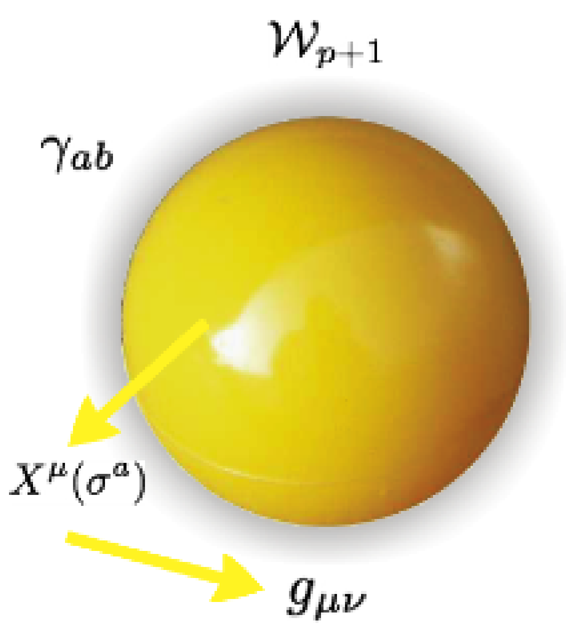


$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-3} + dz^2$$

boosted along the t,z directions

Need effective theory of embedded fluids.

Elastic matter (II):



DBI-type action:

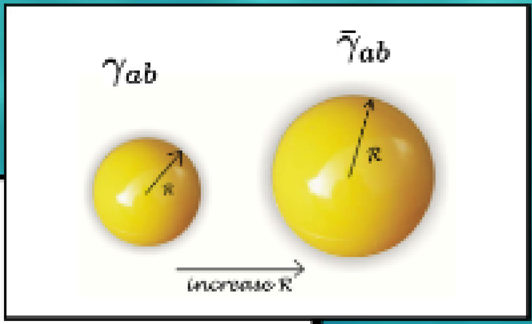
$$I[X^\mu] = \alpha \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}$$

$$T^{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta \mathcal{L}}{\delta \gamma_{ab}} = \alpha \gamma^{ab}$$

$$\nabla_a T^{ab} = 0$$

$$T^{ab} K_{ab}{}^{i} = 0$$

Elastic matter (III):



Lagrangian strain:

$$U_{ab} = -\frac{1}{2}\left(\gamma_{ab} - \bar{\gamma}_{ab}\right)$$

infinitesimal deformation →

$$dU_{ab} = K_{ab}{}^{i}\Phi_i$$

$$T^{ab}K_{ab}{}^{i} = 0$$

contract with orthogonal vector →

minimal surface

$$d\mathcal{V} = 0$$

$$d\mathcal{V} \equiv -\frac{1}{2}\gamma^{ab}dU_{ab}$$

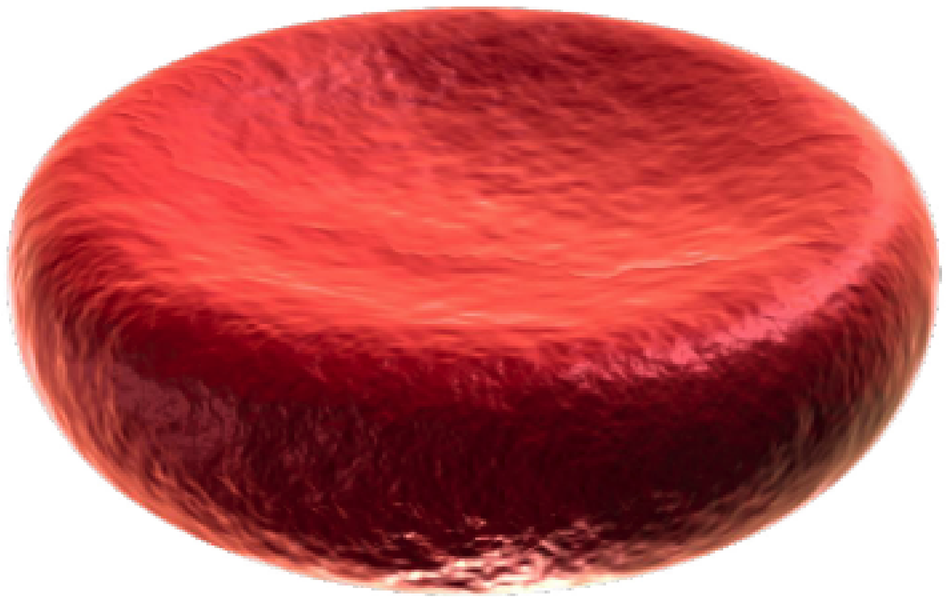
small perturbation ↓

Elasticity Tensor:

$$E^{abcd}K_{ab}{}^{i}K_{cd}{}^{j}\Phi_j + T^{ab}n^{i}{}_{\mu}\nabla_a\nabla_b\left(\Phi^j n^{\mu}{}_{j}\right) = T^{ab}R^{i}{}_{abj}\Phi^j$$

$$E^{abcd} = 2\alpha\gamma^{a(c}\gamma^{d)b}$$

Elastic matter (IV):



red blood cell: erythrocyte

Elastic matter (V):

Helfrich-Canham propose in the 70's an additional piece:

$$I[X^\mu] = \int_{W_{p+1}} \sqrt{-\gamma}\,(\alpha + \lambda_1 K^i K_i)$$

This is enough to explain the biconcave shape of the cell, and many more! (see review by Seifert (1997))

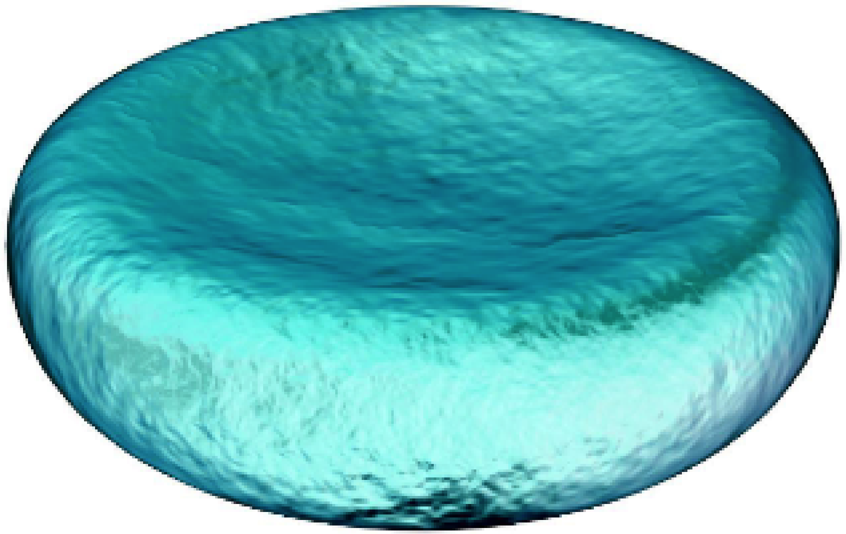
→ In the 80's Polyakov and Kleinert make the same proposal for an improved action of QCD.

→ In the context of cosmic strings the most general elastic action to second order and codimension > 1 is written down:

$$I[X^\mu] = \int_{W_{p+1}} \sqrt{-\gamma}\,(\alpha + \lambda_1 K^i K_i + \lambda_2 K^{ab}{}_i K_{ab}{}^i + v_2 \mathcal{R})$$

Fluid branes (I):

Consider a fluid brane which is in stationary motion:



Fluid branes (II):

Stationarity implies: $\mathbf{k}^a\partial_a = \partial_\tau + \Omega^{(a)}\partial_{\phi_{(a)}}$

Therefore the action for non-extremal branes: $I[X^\mu] = \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\lambda_0(\mathbf{k})$

and hence the stress-energy tensor:

$$T^{ab} = \lambda_0(\mathbf{k})\gamma^{ab} - \lambda_0'(\mathbf{k})\mathbf{k}u^a u^b \qquad u^a = \frac{\mathbf{k}^a}{\mathbf{k}}$$

identify: $P = \lambda_0(\mathbf{k}) \;\; , \;\; \epsilon + P = \mathcal{T}s = -\lambda_0'(\mathbf{k})\mathbf{k}$

Fluid branes (III):

Along worldvolume directions the brane behaves like a fluid:

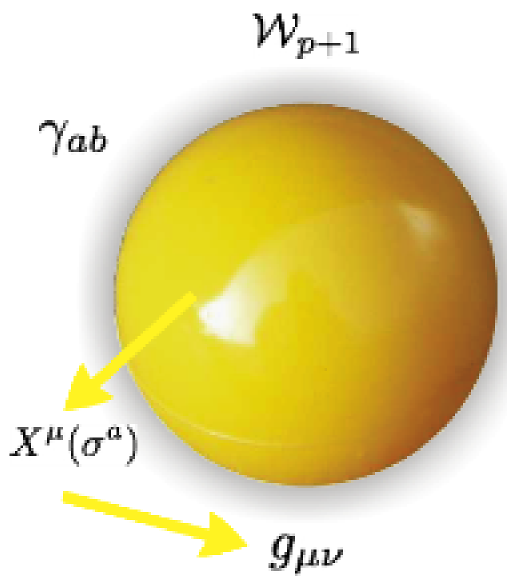
$$d\epsilon = \mathcal{T} ds \quad , \quad dP = s d\mathcal{T}$$

Along orthogonal directions it behaves like an elastic brane:

$$T^{ab} K_{ab}{}^{i} = 0 \quad \longrightarrow \quad dP = -P d\mathcal{V}$$

$$E^{abcd} = 2\left(\lambda_0(\mathbf{k})\gamma^{a(c}\gamma^{d)b} - \left(\frac{\partial \lambda_0(\mathbf{k})}{\partial \gamma^{ab}}\right)\gamma^{cd} - 2\left(\frac{\partial\partial\lambda_0(\mathbf{k})}{\partial\gamma_{ab}\partial\gamma_{cd}}\right)\right)$$

The Elastic Expansion (I):



The generic action should be constructed from:

$$\gamma_{ab}\ ,\ \mathbf{k}^a\ ,\ \nabla_a\ ,\ K_{ab}{}^i\ ,\ \omega_a{}^{ij}\ ,\ \mathcal{R}_{abcd}\ ,\ R_{abcd}$$

hence consider:

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \mathcal{L}\left(\sqrt{-\gamma}, \gamma_{ab}, \mathbf{k}^a, \nabla_a, K_{ab}{}^i, \omega_a{}^{ij}\right)$$

define the bending moment and spin current:

$$\mathcal{D}^{ab}{}_i = \frac{1}{\sqrt{-\gamma}}\frac{\delta \mathcal{L}}{\delta K_{ab}{}^i}\quad,\quad \mathcal{S}^a{}_{ij} = \frac{1}{\sqrt{-\gamma}}\frac{\delta \mathcal{L}}{\delta \omega_a{}^{ij}}$$

$$K_{ab}{}^i = n^i{}_\rho \nabla_a u_b{}^\rho\quad,\quad \omega_a{}^{ij} = -n^j{}_\rho \nabla_a n^{i\rho}$$

The Elastic Expansion (II):

Consider the following hydrodynamic corrections:

$$\upsilon_1(\mathbf{k})\nabla_a\nabla^a\mathbf{k} \;,\; \upsilon_2(\mathbf{k})\mathcal{R} \;,\; \upsilon_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathcal{R}_{ab} \;,$$

$$\upsilon_4(\mathbf{k})\nabla_{[a}\mathbf{k}_{b]}\nabla^{[a}\mathbf{k}^{b]} \;,\; \upsilon_5(\mathbf{k})\nabla_a\mathbf{k}\nabla^a\mathbf{k} \;,\; \upsilon_6(\mathbf{k})R^a{}_{ba}{}^b \;,\; \upsilon_7(\mathbf{k})\mathbf{k}^a\mathbf{k}^b R^c{}_{acb}$$

We get the following action:

$$I[X^\mu] = \int_{W_{p+1}} \sqrt{-\gamma}\left(\lambda_0(\mathbf{k}) + \upsilon_1(\mathbf{k})\nabla_a\nabla^a\mathbf{k} + \upsilon_2(\mathbf{k})\mathcal{R} + \upsilon_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathcal{R}_{ab}\right)$$

Banerjee, Battachary(y)a, Jain, Minwalla, Sharma, arXiv:1203.3544
Bhattacharyya, Hubeny, Minwalla, Rangamani, arXiv:0712.2456

With equations of motion:

$$\nabla_a T^{ab} = 0$$
$$T^{ab}K_{ab}{}^i = 0$$

and stress-energy tensor:

Scalar	Π_n^{ab}
$\upsilon_1(\mathbf{k})\mathcal{V}_1$	$\upsilon_1(\mathbf{k})\mathcal{V}_1\gamma^{ab} - \upsilon_1'(\mathbf{k})\mathbf{k}\mathcal{V}_1 u^a u^b - \gamma^{ab}\nabla_c\left(\upsilon_1(\mathbf{k})\nabla^c\mathbf{k}\right) - \mathbf{k}u^a u^b\nabla_c\nabla^c\upsilon_1(\mathbf{k}) - 2\upsilon_1(\mathbf{k})\nabla^a\nabla^b\mathbf{k} + 2\nabla^{(a}\left(\upsilon_1(\mathbf{k})\nabla^{b)}\mathbf{k}\right)$
$\upsilon_2(\mathbf{k})\mathcal{V}_2$	$\upsilon_2(\mathbf{k})\mathcal{V}_2\gamma^{ab} - \upsilon_2'(\mathbf{k})\mathbf{k}\mathcal{V}_2 u^a u^b - 2\upsilon_2(\mathbf{k})\mathcal{R}^{ab} + 2\nabla^a\nabla^b\upsilon_2(\mathbf{k}) - 2\gamma^{ab}\nabla_c\nabla^c\upsilon_2(\mathbf{k})$
$\upsilon_3(\mathbf{k})\mathcal{V}_3$	$\upsilon_3(\mathbf{k})\mathcal{V}_3\gamma^{ab} - \upsilon_3'(\mathbf{k})\mathbf{k}\mathcal{V}_3 u^a u^b - \nabla_c\nabla^c\left(\upsilon_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\right) + \gamma^{ab}\nabla_c\nabla_d\left(\upsilon_3(\mathbf{k})\mathbf{k}^c\mathbf{k}^d\right) - 2\nabla_c\nabla^{(a}\left(\upsilon_3(\mathbf{k})\mathbf{k}^{b)}\mathbf{k}^c\right)$

The Elastic Expansion (III):

Consider the following elastic corrections:

$$\lambda_1(\mathbf{k})K^iK_i \;\;,\;\; \lambda_2(\mathbf{k})K^{abi}K_{abi} \;\;,\;\; \lambda_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^bK_{ac}{}^iK^c{}_{bi} \;\;,$$

$$\lambda_4(\mathbf{k})\mathbf{k}^a\mathbf{k}^bK_{ab}{}^iK_i \;\;,\;\; \lambda_5(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^dK_{ab}{}^iK_{cdi} \;\;.$$

We get the following action:

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\left(\lambda_0(\mathbf{k}) + \frac{1}{2}\mathcal{D}^{ab}{}_iK_{ab}{}^i\right)$$

With equations of motion:

$$\nabla_aT^{ab} = u_\mu{}^b\nabla_a\nabla_c\mathcal{D}^{ac\mu} + \mathcal{D}^{aci}R^b{}_{aic}$$

$$T^{ab}K_{ab}{}^i = n^i{}_\rho\nabla_a\nabla_b\mathcal{D}^{ab\rho} + \mathcal{D}^{abj}R^i{}_{ajb}$$

Landau-Lifshitz, 1954, Co-dimension-1

and stress-energy tensor:

Scalar	τ_α^{ab}	$\mathcal{D}_\alpha^{abi}$
$\lambda_1(\mathbf{k})\mathcal{L}_1$	$\lambda_1(\mathbf{k})\mathcal{L}_1\gamma^{ab} - \lambda_1'(\mathbf{k})\mathbf{k}\mathcal{L}_1u^au^b - 4\lambda_1(\mathbf{k})K^{ab}{}_iK^i$	$2\lambda_1(\mathbf{k})\gamma^{ab}K^i$
$\lambda_2(\mathbf{k})\mathcal{L}_2$	$\lambda_2(\mathbf{k})\mathcal{L}_2\gamma^{ab} - \lambda_2'(\mathbf{k})\mathbf{k}\mathcal{L}_2u^au^b - 4\lambda_2(\mathbf{k})K^{ac}{}_iK^b{}_c{}^i$	$2\lambda_2(\mathbf{k})K^{abi}$
$\lambda_3(\mathbf{k})\mathcal{L}_3$	$\lambda_3(\mathbf{k})\mathcal{L}_3\gamma^{ab} - \lambda_3'(\mathbf{k})\mathbf{k}\mathcal{L}_3u^au^b - 2\lambda_3(\mathbf{k})\mathbf{k}^c\mathbf{k}^dK^a{}_{ci}K^b{}_d{}^i$	$2\lambda_3(\mathbf{k})\mathbf{k}^d\mathbf{k}^{(a}K^{b)}{}_d{}^i$
$\lambda_4(\mathbf{k})\mathcal{L}_4$	$\lambda_4(\mathbf{k})\mathcal{L}_4\gamma^{ab} - \lambda_4'(\mathbf{k})\mathbf{k}\mathcal{L}_4u^au^b - 2\lambda_4(\mathbf{k})\mathbf{k}^c\mathbf{k}^dK^{ab}{}_iK_{cd}{}^i$	$\lambda_4(\mathbf{k})\mathbf{k}^a\mathbf{k}^bK^i + \lambda_4(\mathbf{k})\gamma^{ab}\mathbf{k}^c\mathbf{k}^dK_{cd}{}^i$
$\lambda_5(\mathbf{k})\mathcal{L}_5$	$\lambda_5(\mathbf{k})\mathcal{L}_5\gamma^{ab} - \lambda_5'(\mathbf{k})\mathbf{k}\mathcal{L}_5u^au^b$	$2\lambda_5(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^dK_{cd}{}^i$

The Elastic Expansion (III):

Consider the following elastic corrections:

$$\lambda_1(\mathbf{k})K^iK_i \;,\; \lambda_2(\mathbf{k})K^{abi}K_{abi} \;,\; \lambda_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^b K_{ac}{}^i K^c{}_{bi} \;,$$

$$\lambda_4(\mathbf{k})\mathbf{k}^a\mathbf{k}^b K_{ab}{}^i K_i \;,\; \lambda_5(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^d K_{ab}{}^i K_{cdi} \;.$$

We get the following action:

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\left(\lambda_0(\mathbf{k}) + \frac{1}{2}\mathcal{D}^{ab}{}_i K_{ab}{}^i\right)$$

With equations of motion:

$$\nabla_a T^{ab} = u_\mu{}^b \nabla_a \nabla_c \mathcal{D}^{ac\mu} + \mathcal{D}^{aci} R^b{}_{aic}$$

$$T^{ab}K_{ab}{}^i = n^i{}_\rho \nabla_a \nabla_b \mathcal{D}^{ab\rho} + \mathcal{D}^{abj} R^i{}_{ajb}$$

Landau-Lifshitz, 1954, Co-dimension-1

and stress-energy tensor:

Scalar	τ_α^{ab}	$\mathcal{D}_\alpha^{abi}$
$\lambda_1(\mathbf{k})\mathcal{L}_1$	$\lambda_1(\mathbf{k})\mathcal{L}_1\gamma^{ab} - \lambda_1'(\mathbf{k})\mathbf{k}\mathcal{L}_1 u^a u^b - 4\lambda_1(\mathbf{k})K^{ab}{}_i K^i$	$2\lambda_1(\mathbf{k})\gamma^{ab}K^i$
$\lambda_2(\mathbf{k})\mathcal{L}_2$	$\lambda_2(\mathbf{k})\mathcal{L}_2\gamma^{ab} - \lambda_2'(\mathbf{k})\mathbf{k}\mathcal{L}_2 u^a u^b - 4\lambda_2(\mathbf{k})K^{ac}{}_i K^b{}_c{}^i$	$2\lambda_2(\mathbf{k})K^{abi}$
$\lambda_3(\mathbf{k})\mathcal{L}_3$	$\lambda_3(\mathbf{k})\mathcal{L}_3\gamma^{ab} - \lambda_3'(\mathbf{k})\mathbf{k}\mathcal{L}_3 u^a u^b - 2\lambda_3(\mathbf{k})\mathbf{k}^c\mathbf{k}^d K^a{}_{ci} K^b{}_d{}^i$	$2\lambda_3(\mathbf{k})\mathbf{k}^d\mathbf{k}^{(a}K^{b)}{}_d{}^i$
$\lambda_4(\mathbf{k})\mathcal{L}_4$	$\lambda_4(\mathbf{k})\mathcal{L}_4\gamma^{ab} - \lambda_4'(\mathbf{k})\mathbf{k}\mathcal{L}_4 u^a u^b - 2\lambda_4(\mathbf{k})\mathbf{k}^c\mathbf{k}^d K^{ab}{}_i K_{cd}{}^i$	$\lambda_4(\mathbf{k})\mathbf{k}^a\mathbf{k}^b K^i + \lambda_4(\mathbf{k})\gamma^{ab}\mathbf{k}^c\mathbf{k}^d K_{cd}{}^i$
$\lambda_5(\mathbf{k})\mathcal{L}_5$	$\lambda_5(\mathbf{k})\mathcal{L}_5\gamma^{ab} - \lambda_5'(\mathbf{k})\mathbf{k}\mathcal{L}_5 u^a u^b$	$2\lambda_5(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\mathbf{k}^d K_{cd}{}^i$

The Elastic Expansion (IV):

The bending moment can be written as:

$$\mathcal{D}^{abi} = \mathcal{Y}^{abcd} K_{cd}{}^{i}$$

where the Young modulus is:

$$\mathcal{Y}^{abcd} = 2\left(\lambda_1 \gamma^{ab}\gamma^{cd} + \lambda_2 \gamma^{a(c}\gamma^{d)b} + \lambda_3 \mathbf{k}^{(a}\gamma^{b)(c}\mathbf{k}^{d)} + \frac{\lambda_4}{2}\left(\gamma^{ab}\mathbf{k}^{c}\mathbf{k}^{d} + \gamma^{cd}\mathbf{k}^{a}\mathbf{k}^{b}\right) + \lambda_5 \mathbf{k}^{a}\mathbf{k}^{b}\mathbf{k}^{c}\mathbf{k}^{d}\right)$$

→ this can be measured from gravity!

$$\mathcal{Y}^{abcd} = \mathcal{Y}^{(ab)(cd)} = \mathcal{Y}^{cdab}$$

The Elastic Expansion (V):

Consider the following spin corrections:

$$\varpi_1(\mathbf{k})\omega^{a}{}_{ij}\omega_a{}^{ij} \quad , \quad \varpi_2(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\omega_{aij}\omega_b{}^{ij}$$

We get the following action:

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\left(\lambda_0(\mathbf{k}) + \frac{1}{2}\mathcal{S}^a{}_{ij}\omega_a{}^{ij}\right)$$

With equations of motion:

$$\nabla_a T^{ab} = \mathcal{S}^a{}_{ji}\Omega_a{}^{bij}$$

$$T^{ab}K_{ab}{}^i = 2n^i{}_\rho\nabla_b\left(\mathcal{S}_a{}^{j\rho}K^{ab}{}_j\right) + \mathcal{S}^{akj}R^i{}_{akj}$$

Papapetrou for Spinning point particles

and stress-energy tensor:

Scalar	Θ^{ab}_{α}	$\mathcal{S}^{aij}_{\alpha}$
$\varpi_1(\mathbf{k})\mathcal{W}_1$	$\varpi_1(\mathbf{k})\mathcal{W}_1\gamma^{ab} - \varpi_1'(\mathbf{k})\mathbf{k}\mathcal{W}_1 u^a u^b - 2\varpi_1(\mathbf{k})\omega^{aij}\omega^b{}_{ij}$	$2\varpi_1(\mathbf{k})\omega^{aij}$
$\varpi_2(\mathbf{k})\mathcal{W}_2$	$\varpi_2(\mathbf{k})\mathcal{W}_2\gamma^{ab} - \varpi_2'(\mathbf{k})\mathbf{k}\mathcal{W}_2 u^a u^b$	$2\varpi_2(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\omega_b{}^{ij}$

The Elastic Expansion (VI):

For codimension-1 surfaces we need to add a piece:

$$I[X^\mu] = \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\left(\vartheta_1(\mathbf{k})K + \vartheta_3(\mathbf{k})\mathbf{k}^a\mathbf{k}^b\mathbf{k}^c\nabla_a K_{bc}\right)$$

The hydrodynamics modes are coupled to the elastic modes through the Gauss-Codazzi equation:

$$R_{abcd} = \mathcal{R}_{abcd} - K_{ac}{}^{i}K_{bdi} + K_{ad}{}^{i}K_{bci}$$

The Elastic Expansion (VII):

Summary of the transport coefficients:

- 3 hydrodynamic, 3 elastic and 1 spin transport coefficient for codimension > 1 surfaces

- 3 hydrodynamic and 5 elastic transport coefficients for codimension-1 surfaces

- 1 hydrodynamic and 4 elastic for fluid membranes in 3-dimensional flat space (hydrodamic transport coefficient and 2 elastic have not been measured yet)

J.Armas, arXiv:1304.7773

Hydrodynamics on embedded surfaces (I):

Take the general equations of motion:

$$\nabla_a T^{ab} = u_\mu{}^b \nabla_a \nabla_c \mathcal{D}^{ac\mu} + \mathcal{D}^{aci} R^b{}_{aic} + \mathcal{S}^a{}_{ji} \Omega_a{}^{bij}$$

$$T^{ab} K_{ab}{}^i = n^i{}_\rho \nabla_a \nabla_c \mathcal{D}^{ac\rho} + \mathcal{D}^{acj} R^i{}_{ajb} + 2n^i{}_\rho \nabla_b \left(\mathcal{S}_a{}^{j\rho} K^{ab}{}_j \right) + \mathcal{S}^{akj} R^i{}_{akj}$$

$$n^i{}_\rho n^j{}_\lambda \nabla_a \mathcal{S}^{a\rho\lambda} = 0$$

$$\mathcal{D}^{ab[i} K_{ab}{}^{j]} = 0$$

Impose positivity of the entropy current:

$$\nabla_a J^a_s \geq 0$$

S. & J Bhattachary(y)a, Minwalla, 2011

S. Bhattacharyya, 2012

Hydrodynamics on embedded surfaces (II):

We make the following assumptions:

- We assume a spinless fluid.
- We assume the existence of a worlvolume entropy current.
- We consider a first order dissipative theory for codimension-1 surfaces and a non-dissipative theory to second order for codimension higher than one.
- We assume the first law of thermodynamics and the Gibbs-Duhem relations.

$$d\epsilon = \mathcal{T} ds \quad , \quad \epsilon + P = \mathcal{T} s \quad , \quad dP = s d\mathcal{T}$$

J. Armas , arXiv:1312.0597

Hydrodynamics on embedded surfaces (III):

Under these assumptions the equations of motion are:

$$\nabla_a T^{ab} = n_\rho{}^i \mathcal{D}^{ac}{}_i \nabla_a K_{ac}{}^\rho - 2\nabla_a \left(\mathcal{D}^{ac}{}_i K_c{}^{bi} \right) \quad ,$$

$$T^{ab} K_{ab}{}^i = n^i{}_\mu \nabla_a \nabla_b \mathcal{D}^{ab\mu} + \mathcal{D}^{abj} R^i{}_{ajb} \quad ,$$

$$\mathcal{D}^{ab[i} K_{ab}{}^{j]} = 0$$

Need to classify the following structures to second order:

$$T^{ab} \quad , \quad \mathcal{D}^{abi} \quad , \quad J_s^a$$

J. Armas , arXiv:1312.0597

Hydrodynamics on embedded surfaces (IV):

We classify all on-shell independent terms to second order in the Landau gauge and in a specific choice of surface:

$$\Pi^{ab}u_b = 0 \ \ , \ \ \mathcal{D}^{abi} \neq \alpha u^a u^b K^i$$

Decompose the derivative of the fluid velocity as:

$$\begin{aligned}
\nabla_a u_b &= -u_a \mathfrak{a}_b + \sigma_{ab} + \omega_{ab} + \frac{\theta}{p}\gamma_{ab} \\
\theta &= \nabla_a u^a \ , \\
\mathfrak{a}^a &= u^b \nabla_b u^a \ , \\
\sigma^{ab} &= P^{ac}P^{bd}\left(\nabla_{(c}u_{d)} - \frac{\theta}{p}\gamma_{cd}\right) \ , \\
\omega^{ab} &= P^{ac}P^{bd}\nabla_{[c}u_{d]} \ .
\end{aligned}$$

Hydrodynamics on embedded surfaces (V):

Classify all terms: first order data

1st order data	Before imposing EOM	EOM	Independent data
Scalars fluid (1)	$u^a\nabla_a\mathcal{T}$, θ	$u_b\nabla_a T^{ab}=0$	θ
Vectors fluid (1)	$P^{ab}\nabla_b\mathcal{T}$, $\mathfrak{a}^a$	$P^c{}_b\nabla_a T^{ab}=0$	$\mathfrak{a}^a$
Tensors fluid (1)	σ^{ab}		σ^{ab}
Scalars elastic (1)	$\mathfrak{a}^i$, K^i , $u^a u^b K_{ab}{}^i$	$T^{ab}K_{ab}{}^i=0$	K^i
Vectors elastic (2)	$u_b K^{abi}$, $u^a K^i$		$u_b K^{abi}$, $u^a K^i$
Tensors elastic (4)	K^{abi} , $u^a u^b K^i$ $\gamma^{ab}K^i$, $u^c u^{(a}K_c{}^{b)i}$		K^{abi} , $u^a u^b K^i$ $\gamma^{ab}K^i$, $u^c u^{(a}K_c{}^{b)i}$

Hydrodynamics on embedded surfaces (VI):

Classify all terms: second order data

2nd order data	Before imposing EOM	EOM	Independent data
Scalars elastic (3)	K^iK_i , $K^{abi}K_{abi}$ $u^au^bK_a{}^{ci}K_{bci}$		K^iK_i , $K^{abi}K_{abi}$ $u^au^bK_a{}^{ci}K_{bci}$
Scalars fluid-elastic (3)	θK^i , $\sigma^{ab}K_{ab}{}^i$ $a^au^bK_{ab}{}^i$, $u^a\nabla_aK^i$	$u^c\nabla_c\left(T^{ab}K_{ab}{}^i\right)=0$	θK^i , $\sigma^{ab}K_{ab}{}^i$, $u^a\nabla_aK^i$
Vectors elastic (4)	$u^aK^iK_i$, $u^aK^{bci}K_{bci}$ $u^au^bu^cK_b{}^{di}K_{cdi}$, u_bK^{abi}		$u^aK^iK_i$, $u^aK^{bci}K_{bci}$ $u^au^bu^cK_b{}^{di}K_{cdi}$, u_bK^{abi}
Vectors fluid-elastic (10)	a^aK^i , a_bK^{abi} $\sigma^{ab}u^cK_{bc}{}^i$, $\omega^{ab}u^cK_{bc}{}^i$ ∇^aK^i , ∇_bK^{abi} $u^a\theta K^i$, $u^a\sigma^{bc}K_{bc}{}^i$ $u^aa^bu^cK_{bc}{}^i$, $u^au^c\nabla_cK^i$ $\theta u_bK_a{}^{bi}$	$P^{dc}\nabla_c\left(T^{ab}K_{ab}{}^i\right)=0$	a^aK^i , a_bK^{abi} $\sigma^{ab}u^cK_{bc}{}^i$, $\theta u_bK_a{}^{bi}$ ∇^aK^i , ∇_bK^{abi} $u^a\theta K^i$, $u^a\sigma^{bc}K_{bc}{}^i$ $u^aa^bu^cK_{bc}{}^i$, $u^au^c\nabla_cK^i$
Tensors elastic (6)	$K^{abi}K_i$, $K^{(a}{}_{ci}K^{b)ci}$ $u^cu^{(a}K^{b)}{}_{ci}K^i$, $P^{ab}K^iK_i$ $P^{ab}K^{cdi}K_{cdi}$ $P^{ab}u^cu^dK_c{}^{ei}K_{dei}$		$K^{abi}K_i$, $K^{(a}{}_{ci}K^{b)ci}$ $u^cu^{(a}K^{b)}{}_{ci}K^i$, $P^{ab}K^iK_i$ $P^{ab}K^{cdi}K_{cdi}$ $P^{ab}u^cu^dK_c{}^{ei}K_{dei}$

Hydrodynamics on embedded surfaces (VII):

Classify all terms: third order data

3rd order data	Before imposing EOM	EOM	Independent data
Scalars fluid-elastic (14)	$\theta K^i K_i \;,\; \theta K^{abi} K_{abi}$ $\theta u^a u^b K_a{}^{ci} K_{bci} \;,\; \sigma^{ab} K_{ab}{}^i K_i$ $\sigma^{ab} K_a{}^{ci} K_{bci}$ $\sigma^{ab} u^c u^d K_{ac}{}^i K_{bdi}$ $\mathfrak{a}^a u^b K_{ab}{}^i K_i \;,\; \mathfrak{a}^a u^b K_a{}^{ci} K_{bc}{}^i$ $u^a K_i \nabla_a K^i \;,\; u^a K_{abi} \nabla^b K^i$ $u_b K_i \nabla_a K^{abi} \;,\; u^a K^{bc}{}_i \nabla_b K_{bc}{}^i$ $u_c K^{abi} \nabla_a K^c{}_{bi}$ $u_c K^c{}_{bi} \nabla_a K^{abi}$ $u^a u^b u^d K_d{}^{ci} \nabla_b K_{ac}{}^i$	$u^c K_i \nabla_c \left(T^{ab} K_{ab}{}^i \right) = 0$	$\theta K^i K_i \;,\; \theta K^{abi} K_{abi}$ $\theta u^a u^b K_a{}^{ci} K_{bci} \;,\; \sigma^{ab} K_{ab}{}^i K_i$ $\sigma^{ab} K_a{}^{ci} K_{bci}$ $\sigma^{ab} u^c u^d K_{ac}{}^i K_{bdi}$ $u_b K_i \nabla_a K^{abi} \;,\; \mathfrak{a}^a u^b K_a{}^{ci} K_{bc}{}^i$ $u^a K_i \nabla_a K^i \;,\; u^a K_{abi} \nabla^b K^i$ $u_c K^{abi} \nabla_a K^c{}_{bi}$ $u_c K^c{}_{bi} \nabla_a K^{abi}$ $u^a u^b u^d K_d{}^{ci} \nabla_b K_{ac}{}^i$

Hydrodynamics on embedded surfaces (IX):

Codimension-1 surfaces to first order:

$$T^{ab} = T^{ab}_{(0)} + \eta\sigma^{ab} + \xi\theta P^{ab} + \alpha_1 K P^{ab} + \alpha_2 P^{ac} P^{bd} K_{cd}$$

$$\mathcal{D}^{ab} = \lambda\gamma^{ab}$$

$$J^a_s = s u^a + \beta_1 \theta u^a + \beta_2 \mathfrak{a}^a + \beta_3 K u^a + \beta_4 u^b K_b{}^a$$

Positivity of the divergence implies:

$$\eta \geq 0 \quad , \quad \xi \geq 0 \quad , \quad \beta_1 = \beta_2 = 0$$

$$\alpha_2 = \beta_4 \mathcal{T} \quad , \quad \beta_3 = \frac{\lambda}{\mathcal{T}} \quad , \quad \beta_4 = -2\frac{\lambda}{\mathcal{T}}$$

$$\alpha_1 = -\frac{2}{\mathcal{T}}\frac{\partial\lambda}{\partial\mathcal{T}} P - \mathcal{T} s \frac{\partial\beta_3}{\partial s} + \mathcal{T}\beta_3 - P\frac{\partial\beta_4}{\partial s}$$

Hydrodynamics on embedded surfaces (X):

For higher codimension and to second order we have:

$$
\begin{aligned}
T^{ab} &= T^{ab}_{(0)} + \eta\sigma^{ab} + \xi\theta P^{ab} \\
&+ \mathcal{T}\left(\kappa_1 \mathcal{R}^{<ab>} - \kappa_2 \mathcal{R}_c{}^{<ab>}{}_d u^c u^d + \kappa_3 \omega^{c<a}\omega^{b>}{}_c + \kappa_4 \mathfrak{a}^{<a}\mathfrak{a}^{b>}\right) \\
&+ \mathcal{T} P^{ab}\left(\eta_2 \mathcal{R} + \eta_3 \mathcal{R}_{cd} u^c u^d - \eta_4 \omega_{cd}\omega^{cd} + \eta_5 \mathfrak{a}^c \mathfrak{a}_c\right) \\
&+ P^{ab}\left(\alpha_1 K^i K_i + \alpha_2 K^{cdi} K_{cdi} + \alpha_3 u^c u^d K_c{}^{fi} K_{dfi}\right) \\
&+ P^a{}_c P^b{}_d\left(\alpha_4 K^{cd}{}_i K^i + \alpha_5 K^{cfi} K^d{}_{fi} + u^f u^h K^c{}_{fi} K^d{}_h{}^i\right)
\end{aligned}
$$

$$
\mathcal{D}^{abi} = \lambda_1 \gamma^{ab} K^i + \lambda_2 K^{abi} + \lambda_3 u^{(a} K^{b)}{}_c{}^i u^c
$$

Hydrodynamics on embedded surfaces (XI):

Summarizing:

→ For codimension-1 surfaces and to 1st order we have 2+1 independent transport coefficients (dissipative)

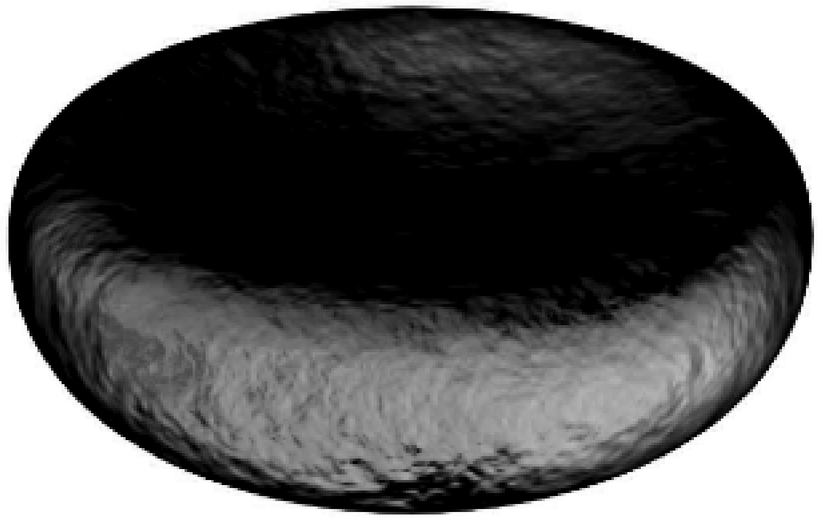
→ For codimension higher we have 10+3 independent transport coefficients (non-dissipative)

→ The constraints match those obtained from equilibrium partition functions

J. Armas , arXiv:1312.0597

Measurement from Gravity (I):

The fluid becomes black:



Measurement from Gravity (II):

To connect with gravity we need an equivalent formulation in terms of space-time tensors:

$$T^{\mu\nu}(x^{\alpha}) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma}\left[T^{\mu\nu}_{(0)}(\sigma^{a})\frac{\delta^{D}(x^{\alpha}-X^{\alpha}(\sigma^{a}))}{\sqrt{-g}} - \nabla_{\rho}\left(T^{\mu\nu\rho}_{(1)}(\sigma^{a})\frac{\delta^{D}(x^{\alpha}-X^{\alpha}(\sigma^{a}))}{\sqrt{-g}}\right) + \ldots\right]$$

Vasilic,Vojinovic, arXiv:0707.3395

where:

$$T^{\mu\nu\rho}_{(1)} = u_{b}{}^{(\mu}j^{(b)\nu)\rho} + u^{\mu}_{a}u^{\nu}_{b}d^{ab\rho} + u^{\rho}_{a}T^{\mu\nu a}_{(1)}$$

JA, Camps, Harmark, Obers, arXiv:1110.4835

equations of motion are obtained by solving:

$$\nabla_{\nu}T^{\nu\mu} = 0$$

Measurement from Gravity (III):

Action formulation and multipole expansion are equivalent provided:

$$T^{ab} = T^{ab}_{(0)} + 2d^{(aci}K^{b)}{}_{ci} \quad , \quad d^{abi} = -\mathcal{D}^{abi} \quad , \quad j^{aij} = 2\mathcal{S}^{aij}$$

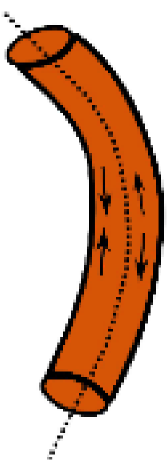
J.Armas, arXiv:1304.7773

the dipole moment is the bending moment:

$$D^{ab\rho} = \int_\Sigma d^{D-1}x\sqrt{-g}\,T^{\mu\nu}u^a_\mu u^b_\nu x^\rho = \int_{B_p} d^p\sigma\sqrt{-\gamma}\,d^{ab\rho}$$

the total spin is the integral over the current:

$$J^{\mu\nu}_\perp = \int_\Sigma d^{D-1}x\sqrt{-g}\,\left(T^{\mu0}x^\nu - T^{\nu0}x^\mu\right) = \int_{B_p} d^p\sigma\sqrt{-\gamma}\,j^{0\mu\nu}$$



Measurement from Gravity (IV):

We take a Schwarzschild black brane and bend it:

$$ds^2_{(1)} = \left(\eta_{ab} - 2K_{ab}{}^{i} r\cos\theta + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1-\frac{r_0^n}{r^n}} + r^2 d\theta^2 + r^2 \sin^2\theta d\Omega^2_{(n)}$$
$$+ h_{\mu\nu}(r,\theta) dx^\mu dx^\nu + \mathcal{O}\left(r^2/R^2\right) \quad .$$

Emparan, Harmark, Niarchos, Obers, Rodriguez, 07
Emparan, Camps, 12

$$h_{\mu\nu}(r,\theta) = \cos\theta \hat{h}_{\mu\nu}(r)$$

split the metric into monopole and dipole contributions:

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{(M)}_{\mu\nu} + h^{(D)}_{\mu\nu} + \mathcal{O}(r^{-n-2})$$

$$\frac{r_0}{R} \ll 1$$

measure by looking at the metric far away:

$$\nabla^2_\perp \bar{h}^{(D)}_{\mu\nu} = 16\pi G d_{\mu\nu}{}^{r_\perp} \partial_{r_\perp} \delta^{(n+2)}(r)$$

Measurement from Gravity (V):

The dipole moment takes the form:

$$\mathcal{D}^{abi} = \mathcal{Y}^{abcd} K_{cd}{}^{i}$$

The Young modulus is:

$$\mathcal{Y}^{abcd} = -P(\mathbf{k})\, r_0^2(\mathbf{k})\, \xi(n) \left(\frac{1}{n+2} \gamma^{a(c}\gamma^{d)b} + 2u^{(a}\gamma^{b)(c}u^{d)} + \frac{3n+4}{n+2} u^a u^b u^c u^d \right) + k\, P(\mathbf{k})\, \xi(n) \left(2\gamma^{ab}\gamma^{cd} - n \left(u^a u^b \gamma^{cd} + u^c u^d \gamma^{ab} \right) \right) ,$$

JA, Camps, Harmark, Obers, arXiv:1110.4835
Camps, Emparan, arXiv:1201.3506

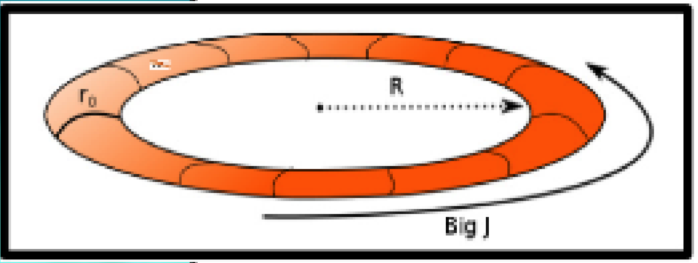
$$P(\mathbf{k}) = -\frac{\Omega_{(n+1)}}{16\pi G} \left(\frac{n}{4\pi T} \right)^n \mathbf{k}^n$$

$$\lambda_1(\mathbf{k}) = k\, P(\mathbf{k})\, \xi(n) \ , \quad \lambda_2(\mathbf{k}) = -\frac{P(\mathbf{k})\, r_0^2(\mathbf{k})\, \xi(n)}{2(n+2)} \ , \quad \lambda_3(\mathbf{k}) = -\frac{P(\mathbf{k})\, r_0^2(\mathbf{k})\, \xi(n)}{\mathbf{k}^2}$$

$$\lambda_4(\mathbf{k}) = \frac{k\, n\, P(\mathbf{k})\, \xi(n)}{\mathbf{k}^2} \ , \quad \lambda_5(\mathbf{k}) = -\frac{3n+4}{2(n+2)} \frac{P(\mathbf{k})\, r_0^2(\mathbf{k})\, \xi(n)}{\mathbf{k}^4} \ .$$

J.Armas, arXiv:1304.7773

Measurement from Gravity (VI):



A ring embedded in flat space:

$$ds^2 = -d\tau^2 + R^2 d\phi^2 \quad , \quad \mathbf{k}^a\partial_a = \partial_\tau + \Omega\partial_\phi$$

The free energy is:

$$\mathcal{F}[R] = -2\pi R\left(P + \tilde{\lambda}_1 K^i K_i\right)$$

J.Armas, arXiv:1304.7773

The solution is:

$$\Omega = \Omega_{(0)} + \Omega_{(2)}$$

$$\Omega_{(0)} = \frac{1}{R}\frac{1}{\sqrt{n+1}}$$

$$\Omega_{(2)} = \frac{(n-4)\sqrt{n+1}}{2n^2(n+2)R}\xi(n)\frac{r_0^2}{R^2}$$

All other charges can be predicted!

Measurement from Gravity (VII):

Corrected phase diagram expressed in physical quantities:

Emparan, Harmark, Niarchos, Obers, Rodrigues, arXiv:0708.2181

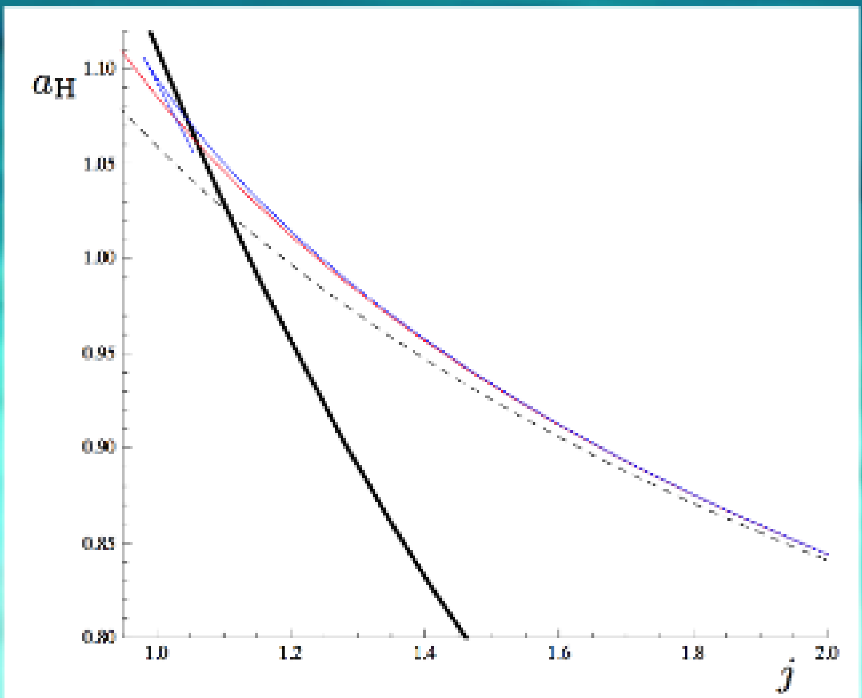
$$a_{\rm H}(j) = \frac{2^{\frac{n-2}{n(n+1)}}}{j^{\frac{1}{n}}}\left(1 + \frac{(n+1)(3n+4)}{2^{\frac{3n+4}{n}} n^3 (n+2)} \frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

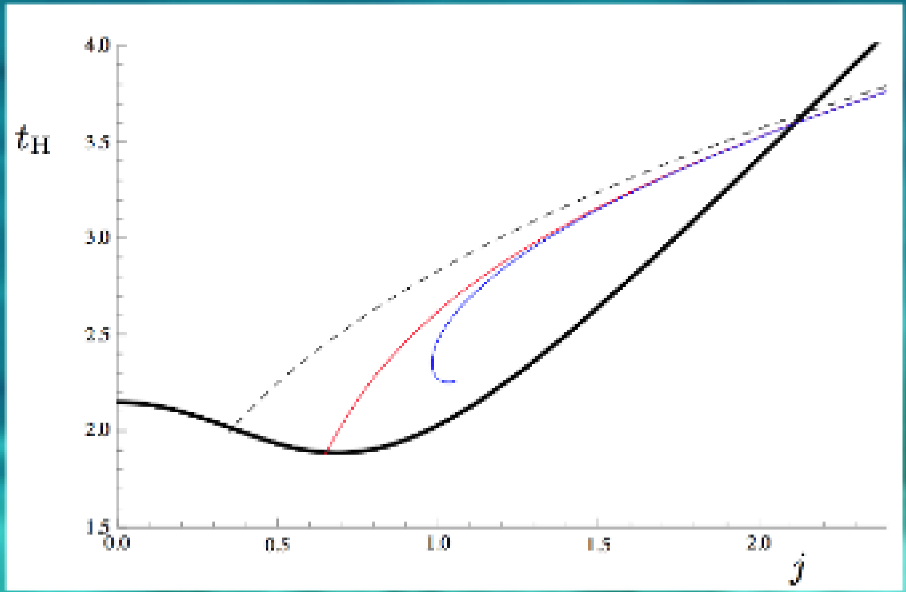
$$\omega_{\rm H}(j) = \frac{1}{2j}\left(1 + \frac{(n+1)(3n+4)}{2^{\frac{2(n+2)}{n}} n^2 (n+2)} \frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right) ,$$

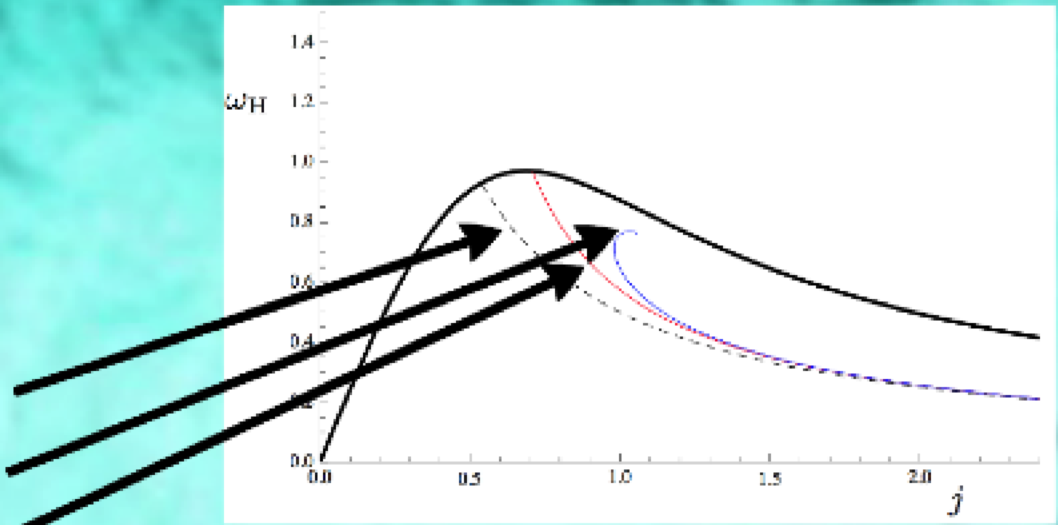
$$t_{\rm H}(j) = \frac{n j^{\frac{1}{n}}}{2^{\frac{n-2}{n(n+1)}}}\left(1 - \frac{3(n+1)(3n+4)}{2^{\frac{3n+4}{n}} n^3 (n+2)} \frac{\xi(n)}{j^{\frac{2(n+1)}{n}}}\right)$$

JA & T. Harmark, arXiv:1402.6330, arXiv:1404.xxxx

Measurement from Gravity (VIII):







Emparan, Harmark, Niarchos, Obers, Rodrigues, arXiv:0708.2181

Dias, Santos, Way, arXiv:1402.6345

JA & T. Harmark, arXiv:1402.6330

Charged black rings (I):

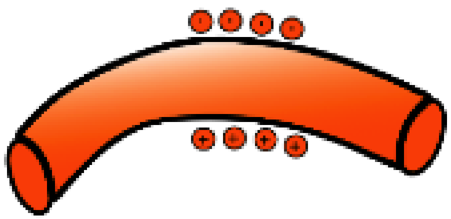
The same can be done for charged branes:

$$J^{\mu_1\cdots\mu_{q+1}}(x^\alpha) = \int_{\mathcal{W}_{p+1}} d^{p+1}\sigma\sqrt{-\gamma}\left[J_{(0)}^{\mu_1\cdots\mu_{q+1}}\frac{\delta^D(x^\alpha - X^\alpha)}{\sqrt{-g}} - \nabla_\rho\left(J_{(1)}^{\mu_1\cdots\mu_{q+1}\rho}\frac{\delta^D(x^\alpha - X^\alpha)}{\sqrt{-g}}\right) + \ldots\right]$$

JA, Gath, Obers, arXiv:1209.5197 (PRL), arXiv:1307.504

Decompose the dipole correction as:

$$J_{(1)}^{\mu\nu} = m^{\mu\nu} + u_a^{\mu}p^{a\nu} + J_{(1)}^{\mu a}u_a^{\nu}$$



Split the gauge field as:

$$A_\mu = A_\mu^{(M)} + A_\mu^{(D)} + \mathcal{O}\left(r^{-n-2}\right)$$

→

$$\nabla_\perp^2 A_\nu^{(D)} = 16\pi G p_\nu{}^{r_\perp}\partial_{r_\perp}\delta^{(n+2)}(r)$$

Charged black rings (II):

The electric dipole moment is of the form:

$$p^{a\rho} = \tilde{\kappa}^{abc} K_{bc}{}^{\rho}$$

for charged dilatonic branes from KK reduction:

$$\tilde{\kappa}_a{}^{bc} = -\xi_2(n) r_0^2 \left(\frac{Q}{n} \delta_a{}^{(b} u^{c)} + \bar{k} J_a^{(0)} \eta^{bc} \right)$$

JA, Gath, Obers, arXiv:1209.5197, arXiv:1307.504

Conclusions (I):

A summary of the results:

- Generic effective action of fluid branes to second order
- First order dissipative theory of (confined) hydrodynamics and second order non-dissipative theory.
- Measurement of transport coefficients from gravity
- Systematic method for finding corrections to black hole charges, good to compare with numerics. Can also study stability.

Future directions:

- Including backreaction corrections in the effective theory
- AdS/CFT interpretation of the Young modulus | bending D3-brane
- Anomalous couplings, Chern-Simons terms
- Universality of transport coefficients
- Full dissipative theory and non-relativistic theory.
- Spinning actions and thermodynamics to all orders J. Armas, Troels Harmark (to appear)

THANK YOU